Towards nuclear matter from dense QCD

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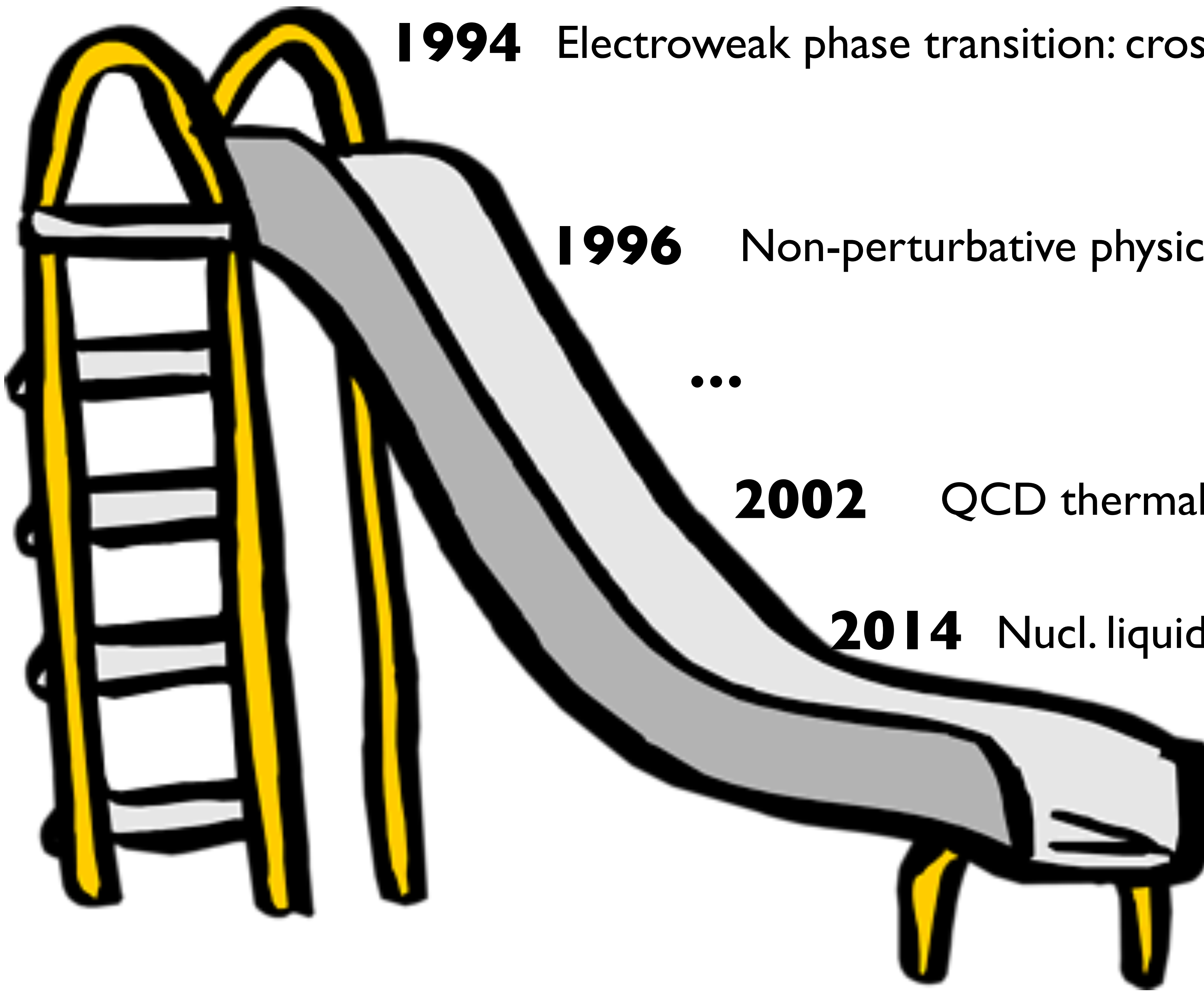
HIC for FAIR Helmholtz International Center

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

- Motivation
- Lattice QCD and the sign problem
- Heavy dense QCD
- Chiral coarse QCD

The slippery slope of (my) research….

1990 DESY: grad student of Wilfried $WW\gamma$ vertex in 500 GeV e^+e^- coll.



1994 Electroweak phase transition: crossover for m_H>70 GeV

1996 Non-perturbative physics, lattice QCD

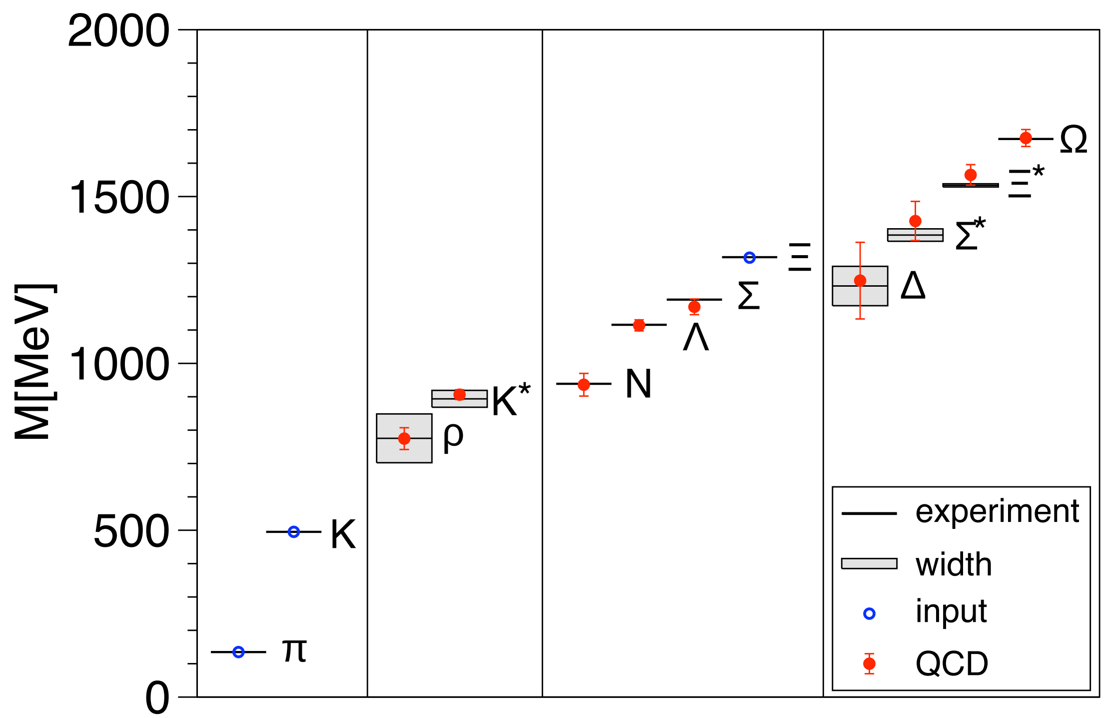
…

2002 QCD thermal transition ~ 200 MeV

2014 Nucl. liquid gas transition ~ 10 MeV

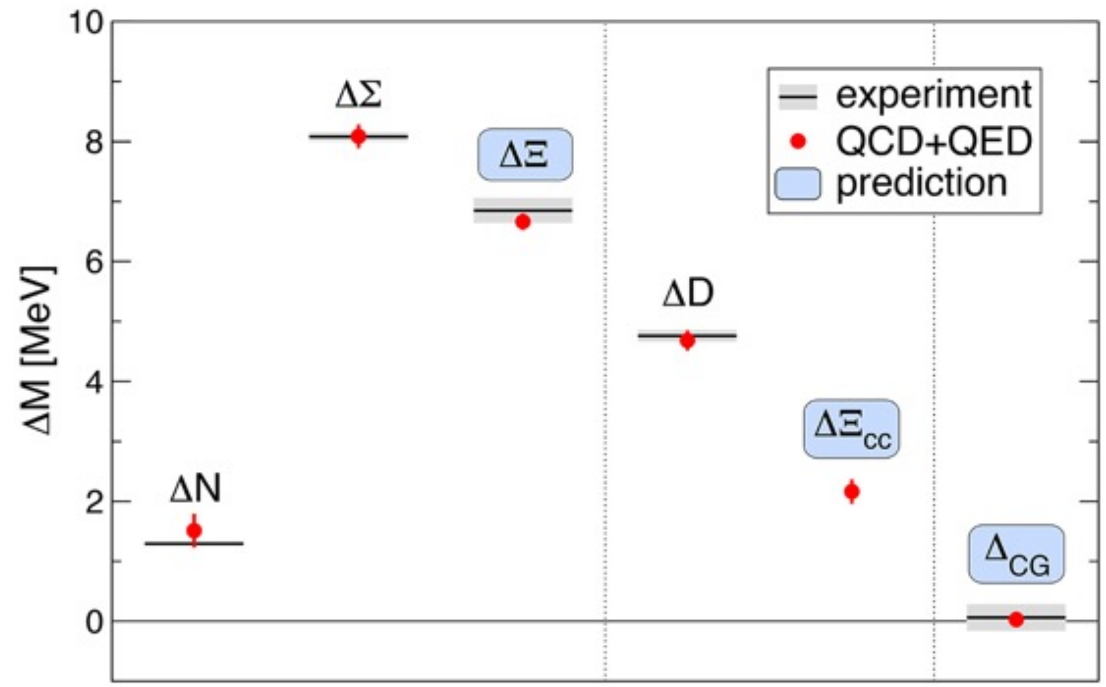
Study of physics: Stuttgart, Hamburg, Cornell

Successes of Lattice QCD



(Budapest, Marseille, Wuppertal) 2010

Group leader: Z. Fodor, former DESY postdoc



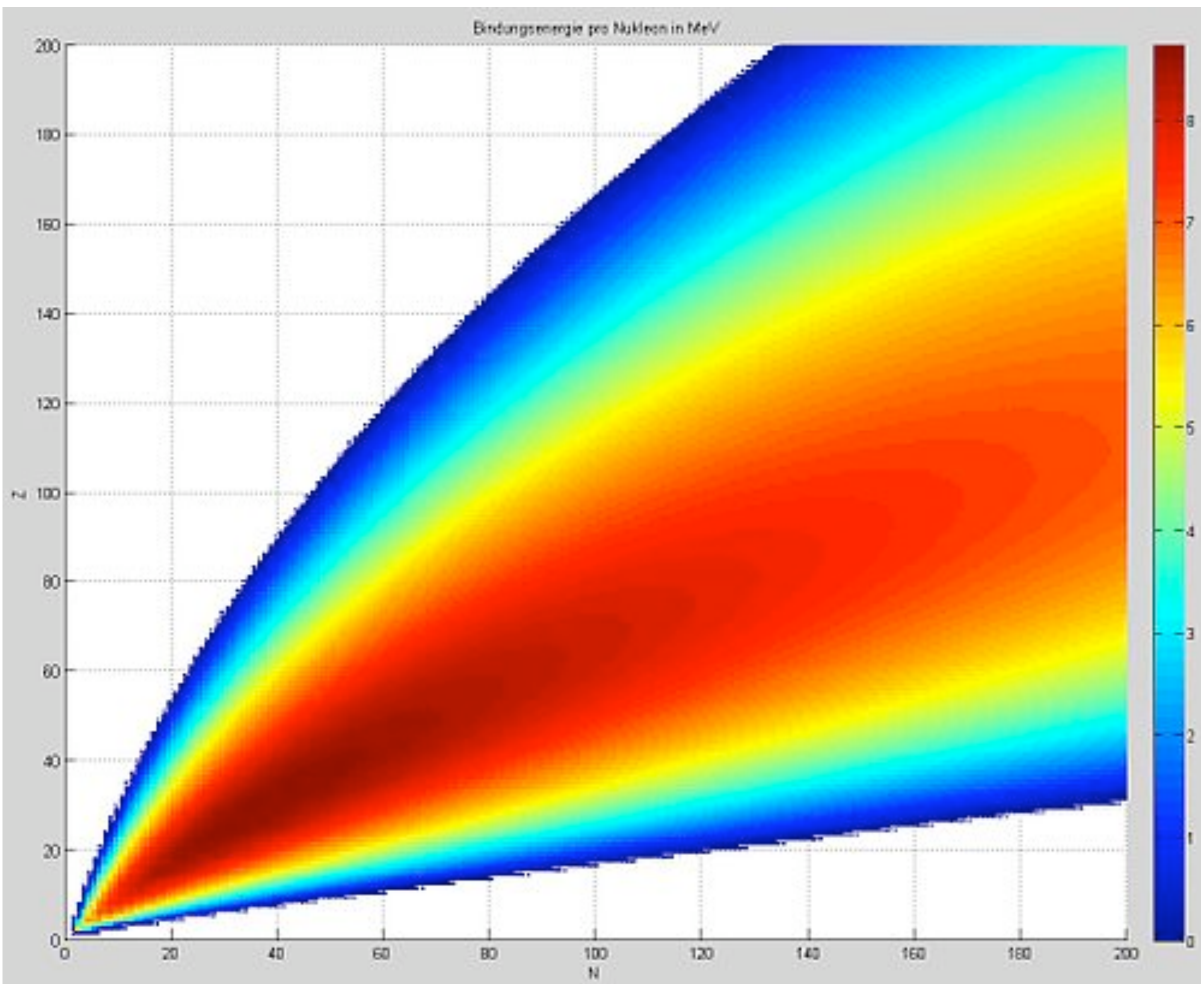
Budapest - Wuppertal 2014

- High precision hadron spectrum
- QED effects included
- Used as discovery tool:

  exotic hadron states, quark gluon plasma, axions,
  beyond SM models, hadronic matrix elements for g-2, relic WIMP abundance….

Completely unsolved: bulk nuclear matter

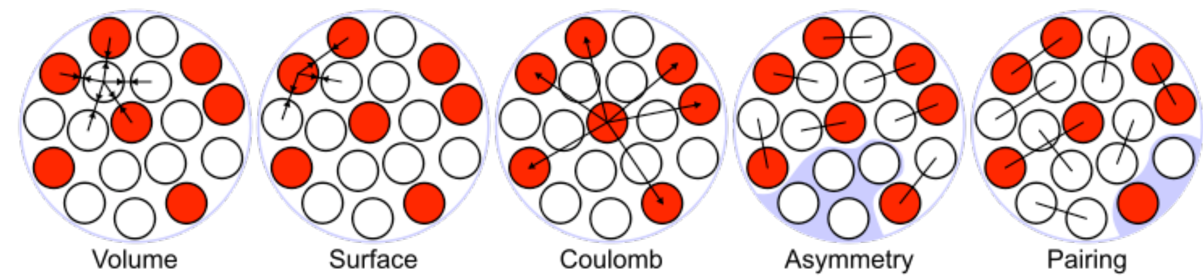
~100 years old, still no fundamental description, Bethe-Weizsäcker droplet model:



Z

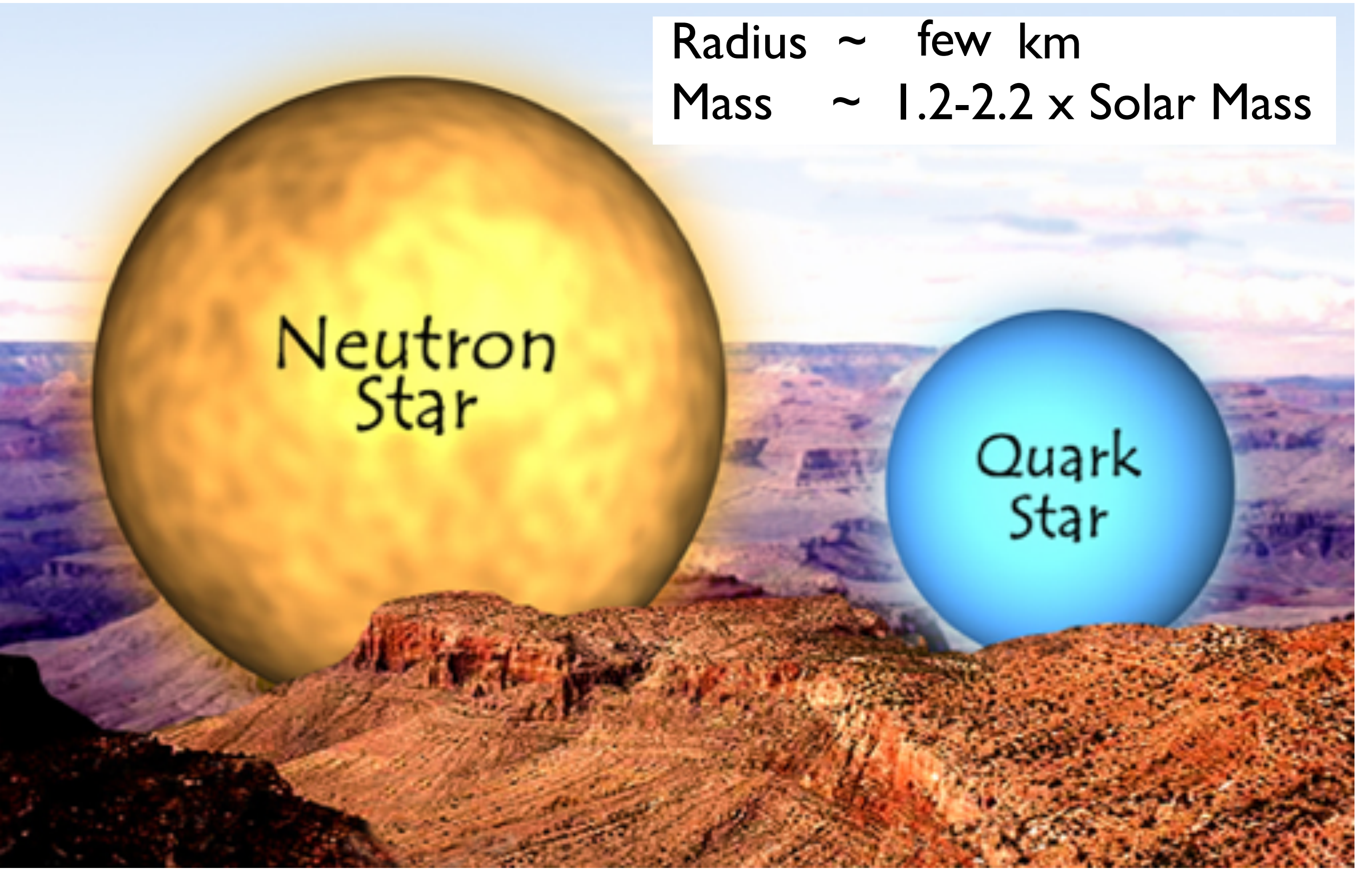
N

Binding energy per nucleon



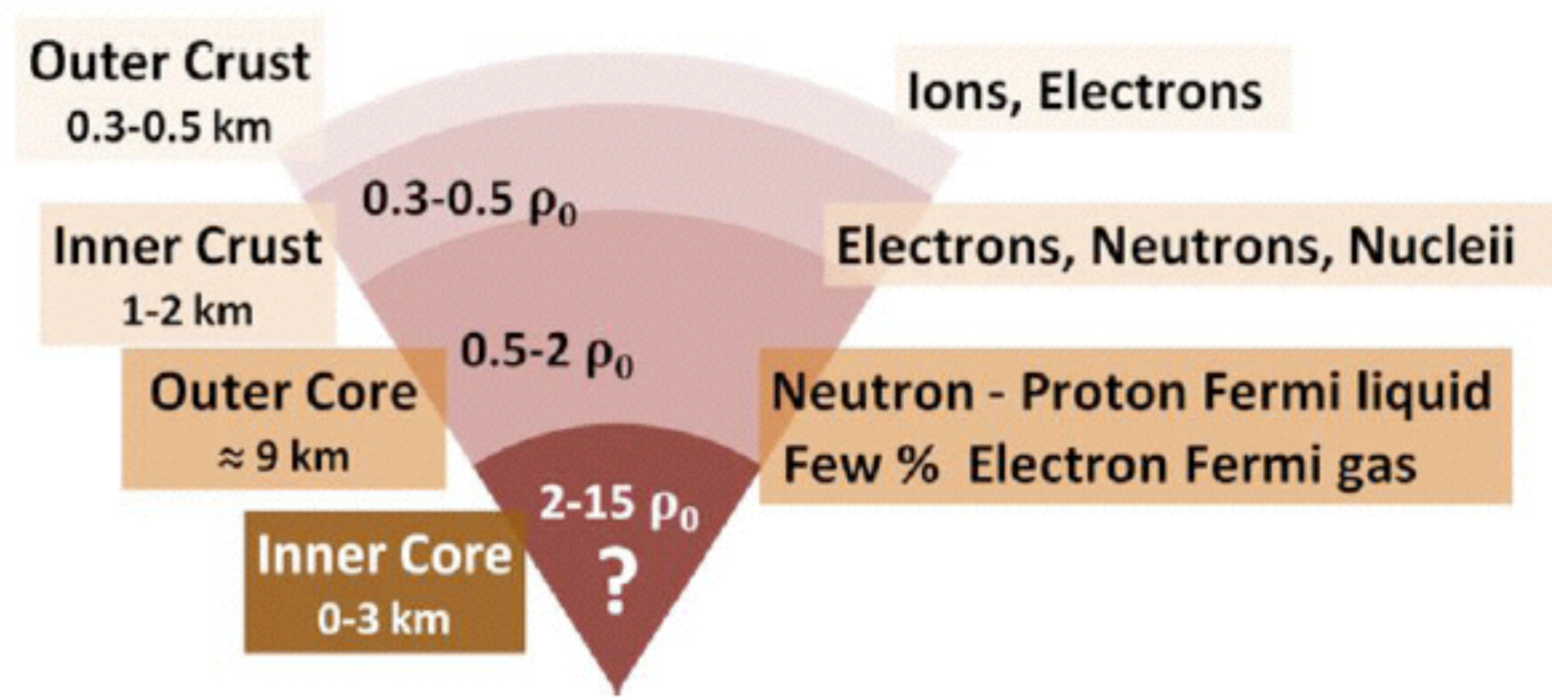
QFT descriptions: Fetter-Walecka model, Skyrme model, ...

What are compact stars made of?



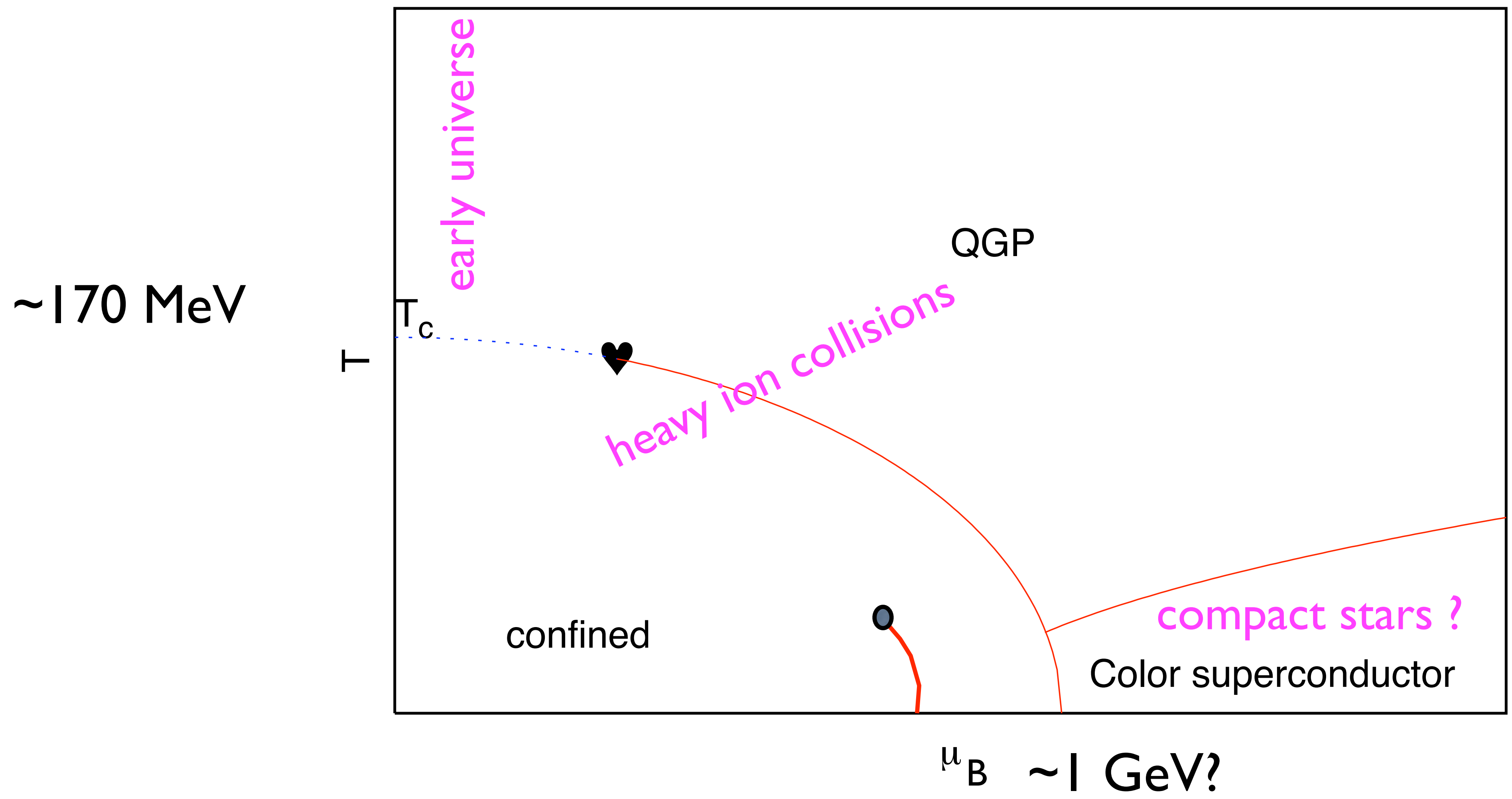
EoS affects:

-mass-radius relationship
-frequency of pulsars
-gravitational wave emission of binaries



ρ_0 : nuclear density

QCD phase diagram: theorist's view (science fiction)

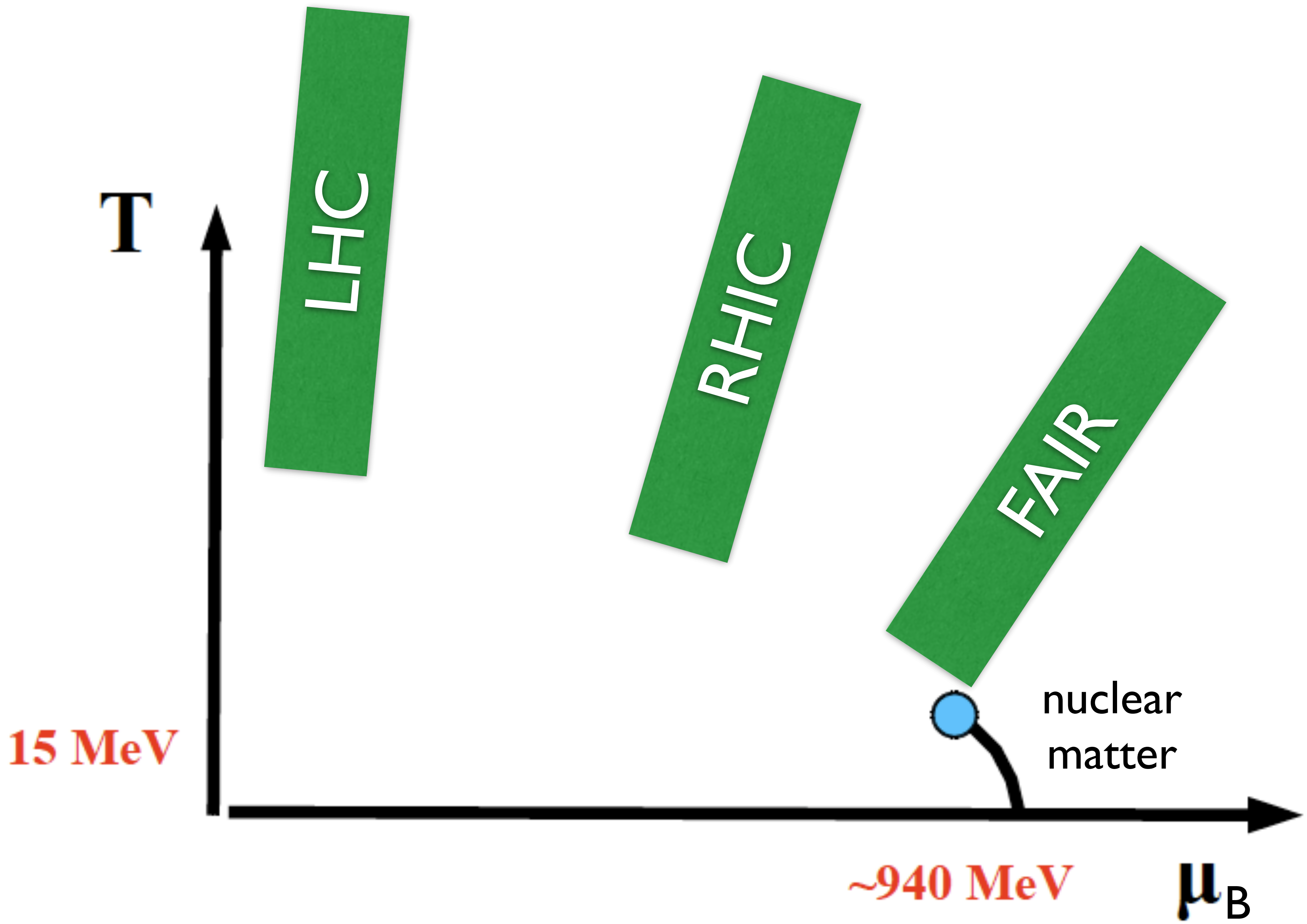


Until 2001: no finite density lattice calculations, sign problem!

Expectation based on simplifying models (NJL, linear sigma model, random matrix models, ...)

Check this from first principles QCD!

The QCD phase diagram established by experiment:



Nuclear liquid gas transition with critical end point

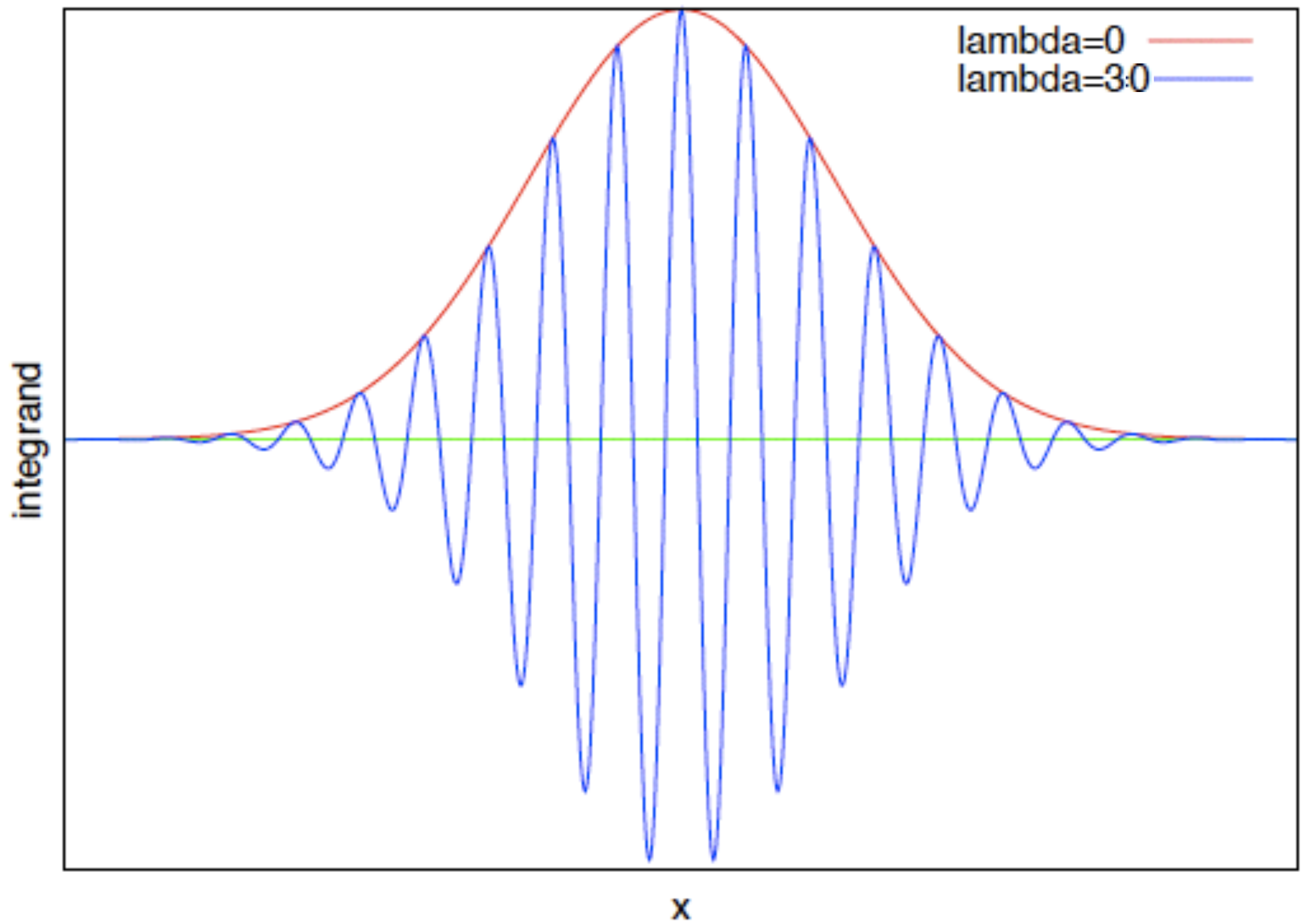
The sign problem for finite density QCD

$$Z = \int DU\, [\det M(\mu)]^f \mathrm{e}^{-S_g[U]}$$

importance sampling requires
positive weights

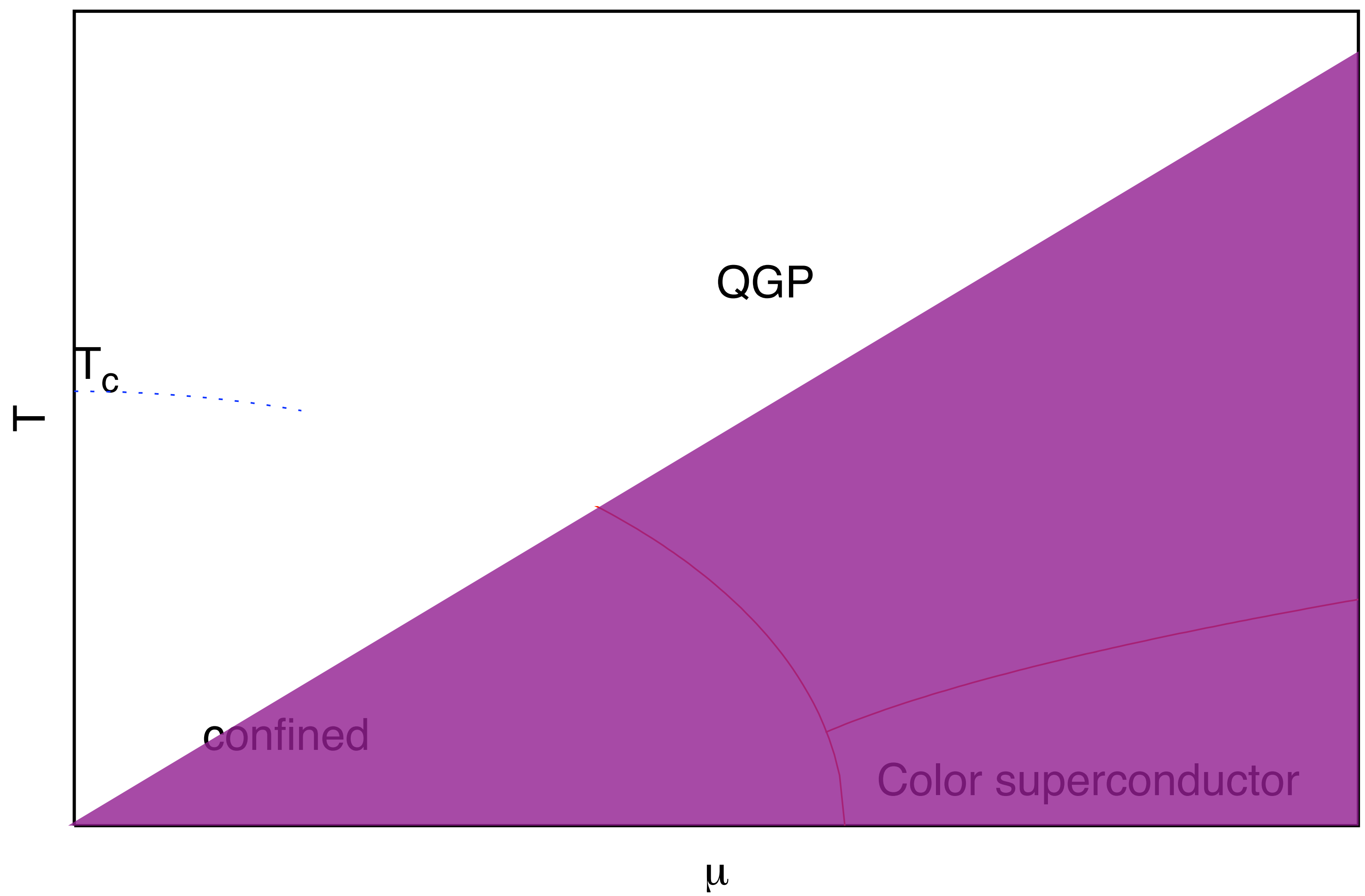
$\Rightarrow \det(M)$ complex for SU(3), $\mu \neq 0$ real positive for $\mu = i\mu_i$

Example: $Z(\lambda) = \int dx \exp(-x^2 + \mathbf{i}\lambda \mathbf{x})$



$Z(\lambda)/Z(0) = \exp(-\lambda^2/4)$: exponential cancellations

The lattice-calculable region of the phase diagram



- Sign problem prohibits direct simulation, circumvented by approximate methods: reweigthing, Taylor expansion, imaginary chem. pot., need $\mu/T \lesssim 1 \quad (\mu = \mu_B/3)$
- No critical point in the controllable region, some signals beyond

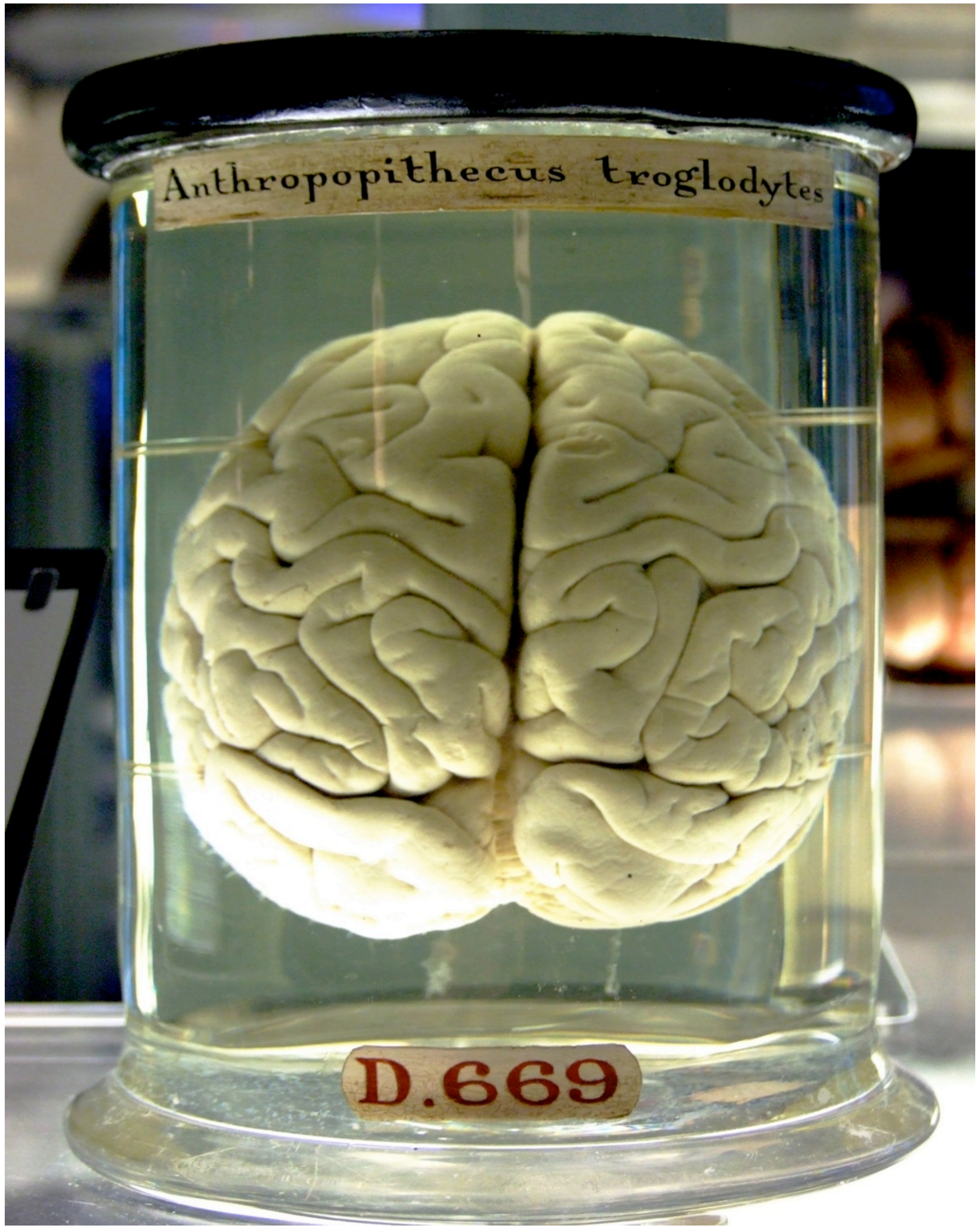
New computational avenues in LQCD:

"(Wall)Time is Money (CPU hrs)"

CPU → GPU

Here, very old-fashioned approach: BPU!

Biological Processing Unit!



Large densities? Effective theories!

Effective lattice theory for heavy dense QCD

O.P. with M.Fromm, J.Langelage, S.Lottini, M.Neuman

- Two-step treatment:

  I. Calculate effective theory analytically
  II. Simulate effective theory

- Step I.: split temporal and spatial link integrations:

$$Z = \int DU_0 DU_i \;\det Q \; e^{S_g[U]} \equiv \int DU_0 e^{-S_{eff}[U_0]} = \int DL \; e^{-S_{eff}[L]}$$

  Spatial integration after analytic strong coupling and hopping expansion

- Truncation: valid for heavy quarks, sufficiently close to the continuum
- Step II: sign problem milder: Monte Carlo, complex Langevin

Starting point: Wilson's lattice Yang-Mills action

Partition function; link variables as degrees of freedom

$$Z = \int \prod_{x,\mu} dU(x;\mu) \exp(-S_{YM}) \equiv \int DU \exp(-S_{YM})$$

Wilson's gauge action

$$S_W = -\frac{\beta}{N} \sum_p \mathrm{Re}\mathrm{Tr}(U_p) = \sum_p S_p \qquad \beta = \frac{2N}{g^2}$$

Plaquette:

$$\square \to 1 + ia^2 g F_{\mu\nu} - \frac{a^4 g^2}{2} F_{\mu\nu} F^{\mu\nu} + O(a^6) + \ldots$$

$U_\mu(x) = \mathrm{e}^{-\mathrm{i}agA_\mu(x)}$

$$T = \frac{1}{aN_t}$$ continuum limit $a \to 0, N_t \to \infty$

Small $\beta(a) \Rightarrow$ small T

- Leading order graph in case of $N_\tau = 4$:

Figure: 4 plaquettes in fundamental representation lead to a 2 Polyakov loop interaction term

- Integration of spatial link variables leads to

$$-S_1 = u^{N_\tau} \sum_{\langle ij \rangle} \mathrm{tr}\, W_i \, \mathrm{tr}\, W_j$$

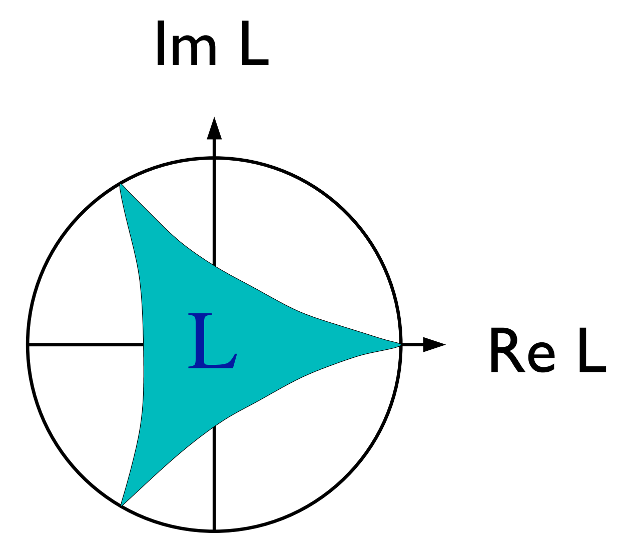
Expansion parameter: $u = a_f(\beta) = \beta/18 + \cdots$

- Possible generalizations: larger distance, higher dimensional representations, larger number of loops involved, ...
- *Here*: Decorate LO graph with additional spatial and temporal plaquettes

Effective one-coupling theory for SU(3) YM

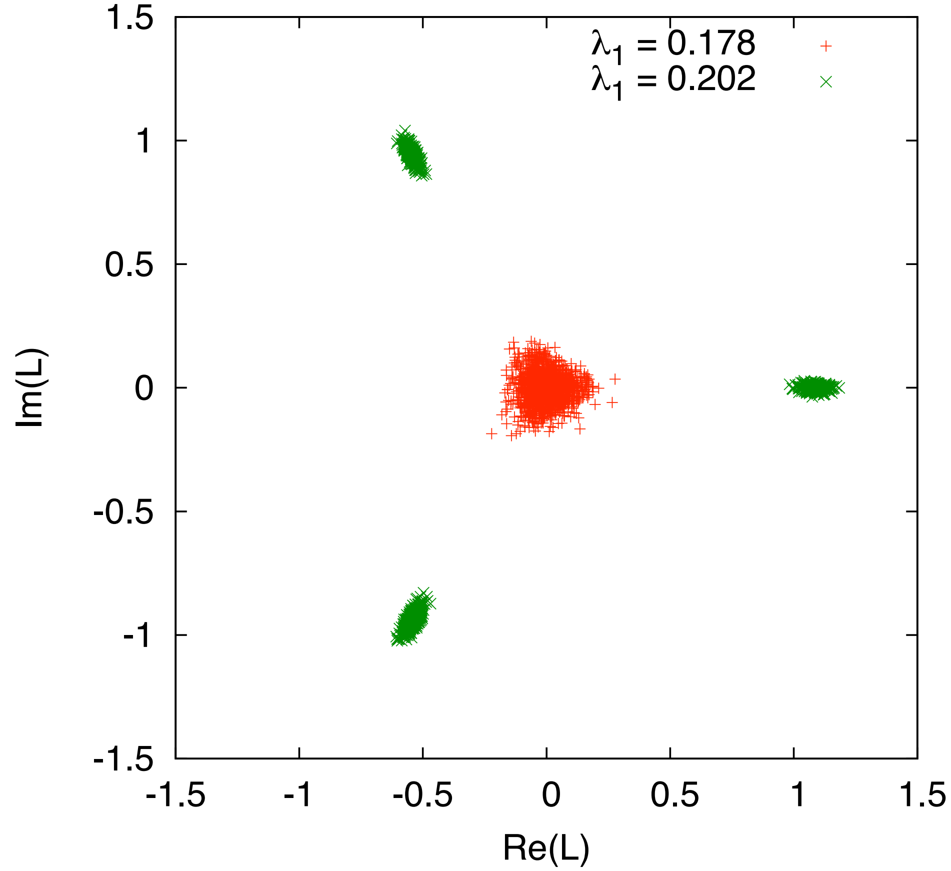
(L= Tr W)

$$
\begin{aligned}
Z &= \int [dL] \exp\left[-S_1 + V_{SU(3)}\right] \\
&= \int [dL] \prod_{\langle ij \rangle} \left[1 + 2\lambda_1 \mathrm{Re}\left(L_i L_j^*\right)\right] * \\
&\quad * \prod_i \sqrt{27 - 18|L_i|^2 + 8\mathrm{Re} L_i^3 - |L_i|^4}
\end{aligned}
$$

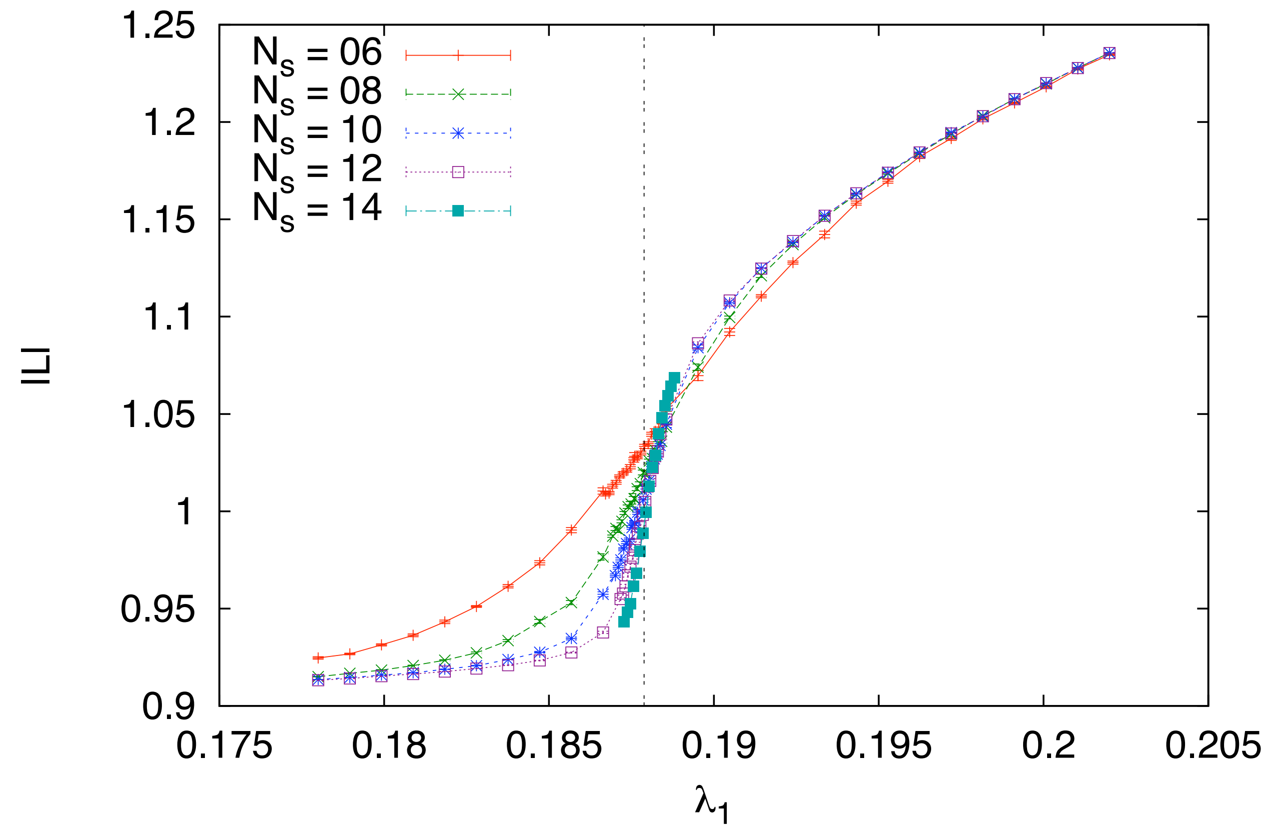


$$
\lambda(u, N_\tau \geq 5) = u^{N_\tau} \exp\left[N_\tau\left(4u^4 + 12u^5 - 14u^6 - 36u^7 + \frac{295}{2}u^8 + \frac{1851}{10}u^9 + \frac{1055797}{5120}u^{10}\right)\right]
$$

Numerical results for SU(3), one coupling



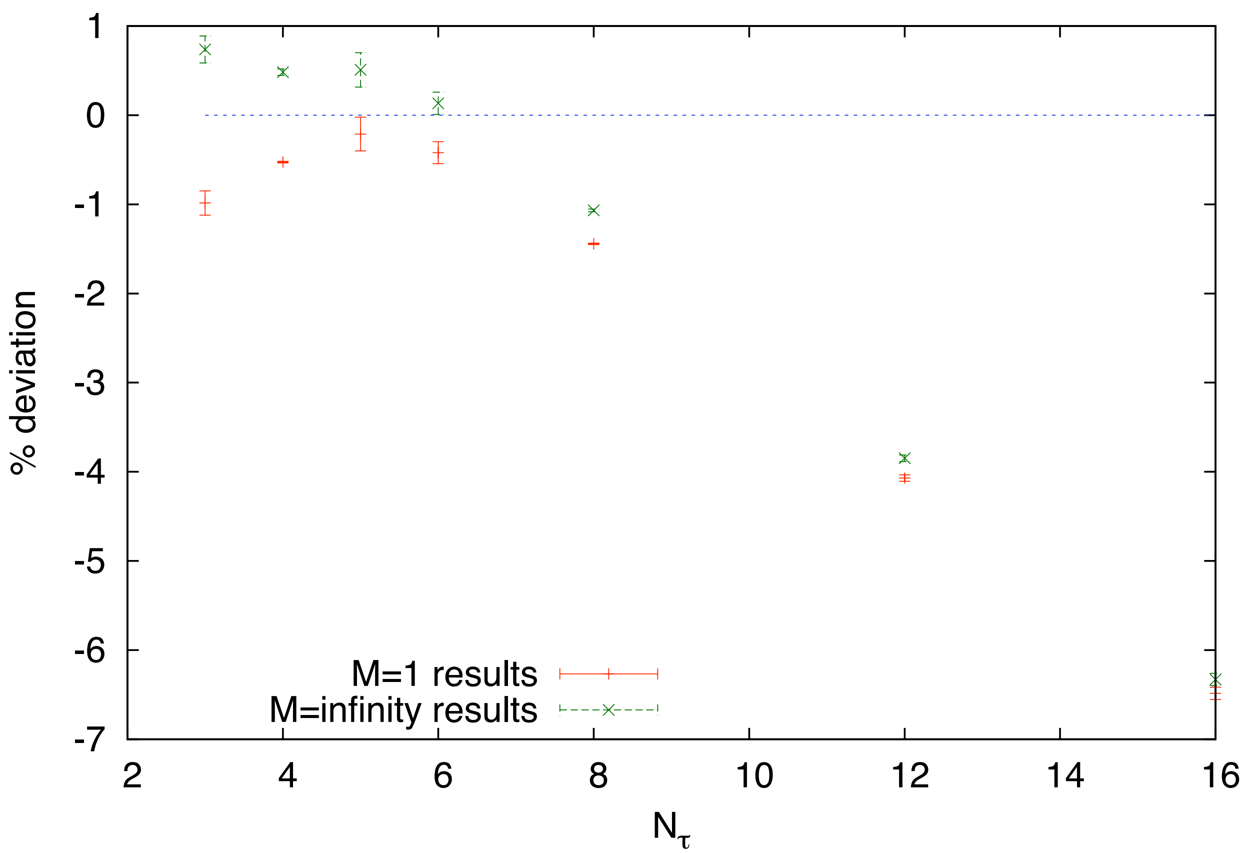
Order-disorder transition
=Z(3) breaking



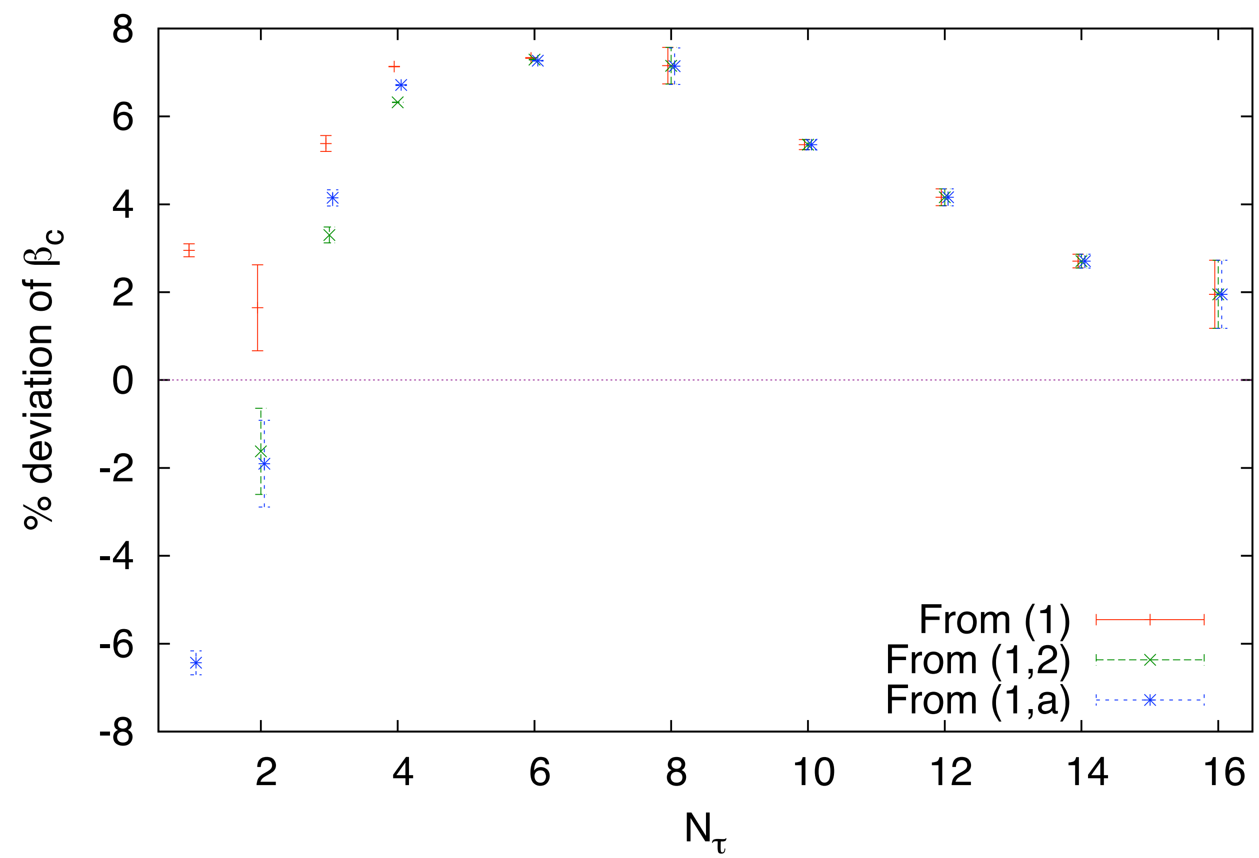
Comparison with 4d Monte Carlo

Relative accuracy for β_c compared to the full theory

SU(2)

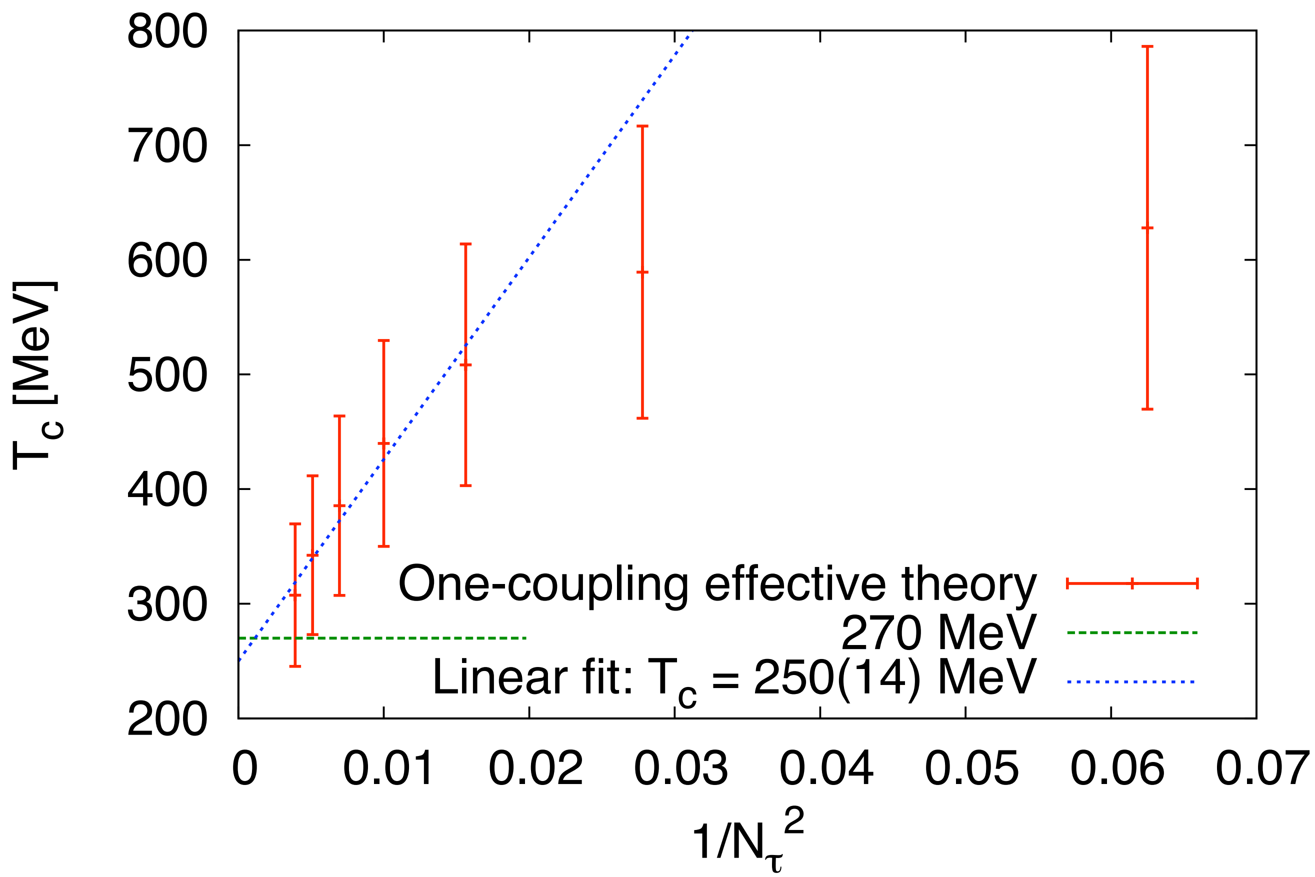


SU(3)



Note: influence of additional couplings checked explicitly!

Continuum limit feasible!



-error bars: difference between last two orders in strong coupling exp.

-using non-perturbative beta-function (4d T=0 lattice)

-all data points from one single 3d MC simulation!

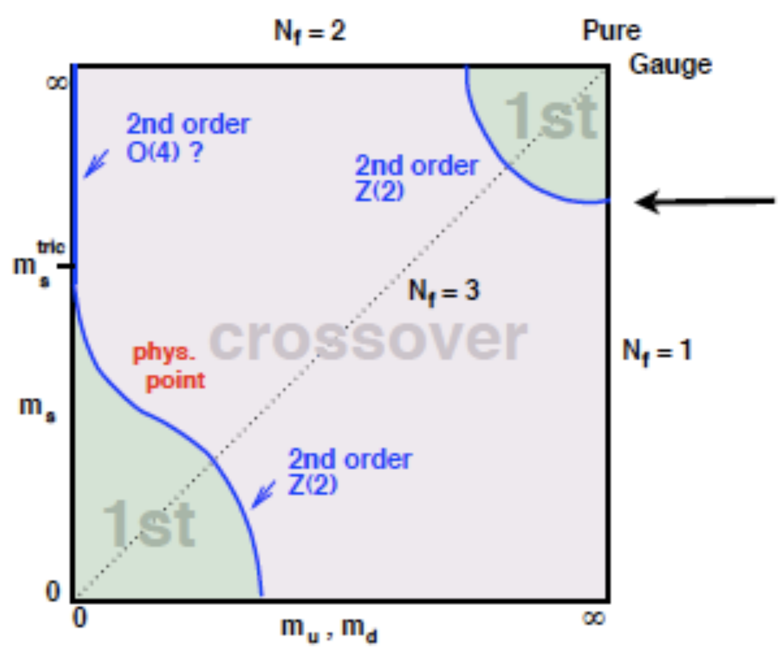
Including heavy, dynamical Wilson fermions

Expand in the *hopping parameter* $\kappa = 1/(2aM+8)$:

$$-S_{\text{eff}} = \sum_i \lambda_i(u,\kappa,N_\tau)S_i^{\mathrm{S}} - 2N_f \sum_i \left[h_i(u,\kappa,\mu,N_\tau)S_i^{\mathrm{A}} + \bar{h}_i(u,\kappa,\mu,N_\tau)S_i^{\dagger\mathrm{A}}\right]$$

Deconfinement transition for heavy quarks

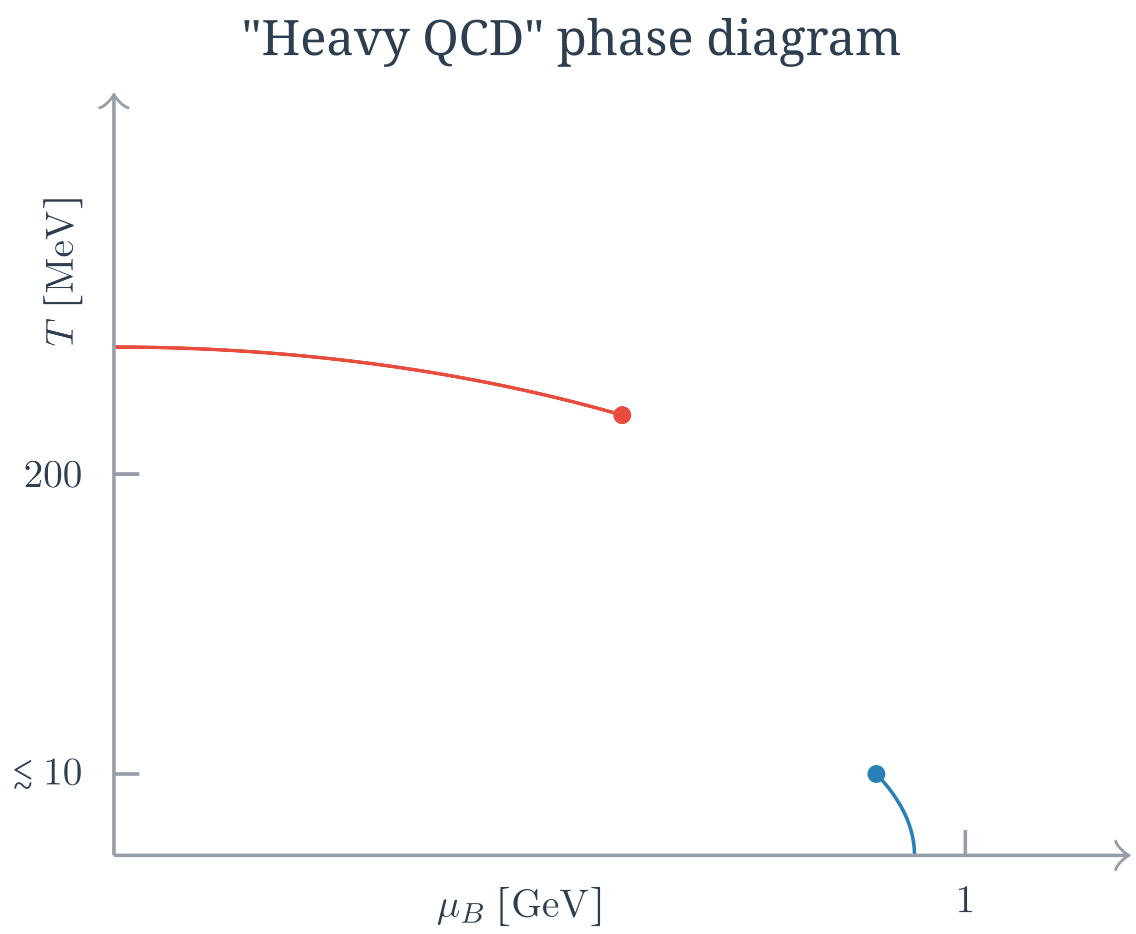
NLO: $\sim \kappa^2$

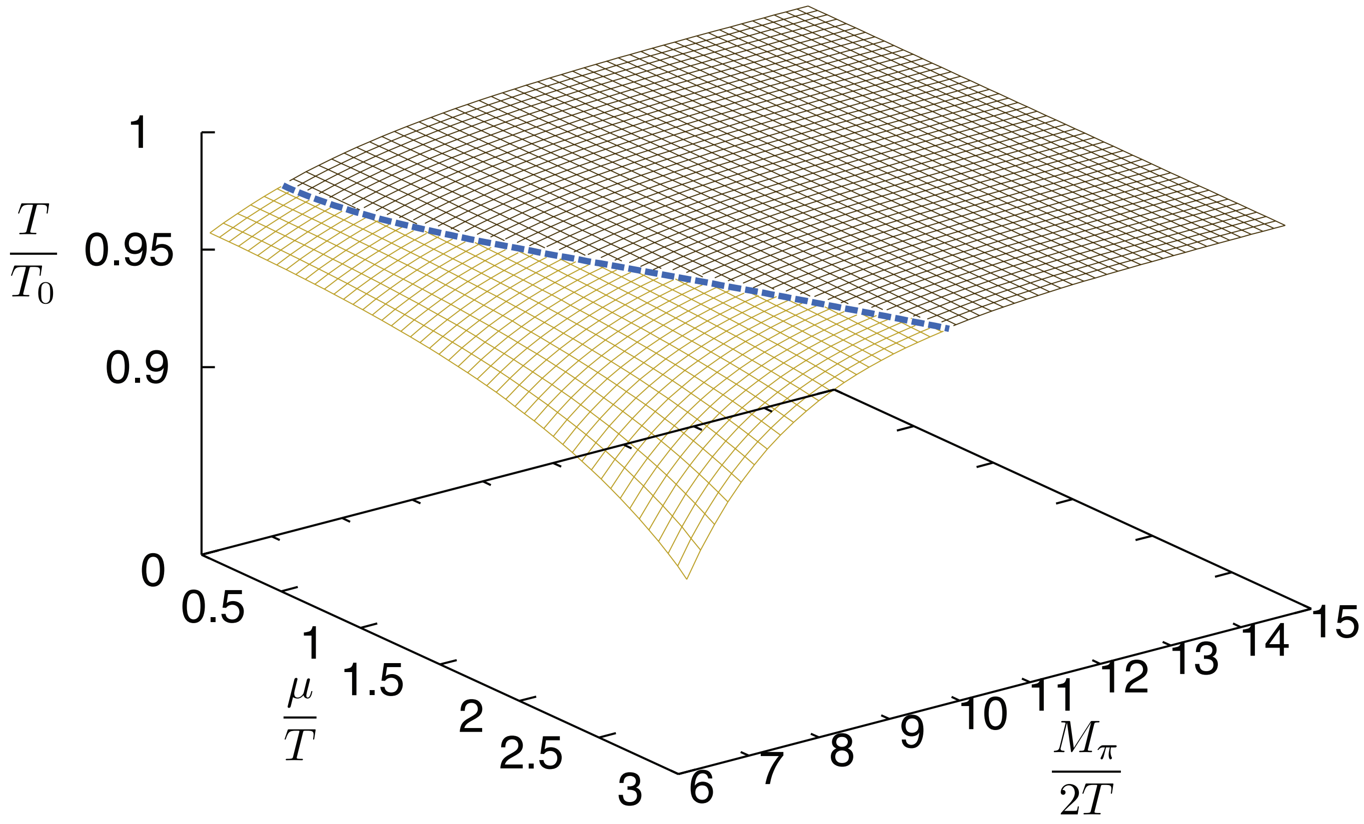


		eff. theory	4d MC,WHOT	4d MC,de Forcrand et al
N_f	M_c/T	$\kappa_c(N_\tau = 4)$	$\kappa_c(4)$, Ref. [23]	$\kappa_c(4)$, Ref. [22]
1	7.22(5)	0.0822(11)	0.0783(4)	$\sim$ 0.08
2	7.91(5)	0.0691(9)	0.0658(3)	–
3	8.32(5)	0.0625(9)	0.0595(3)	–

Accuracy ~5%, predictions for Nt=6,8,... available!

The fully calculated deconfinement transition

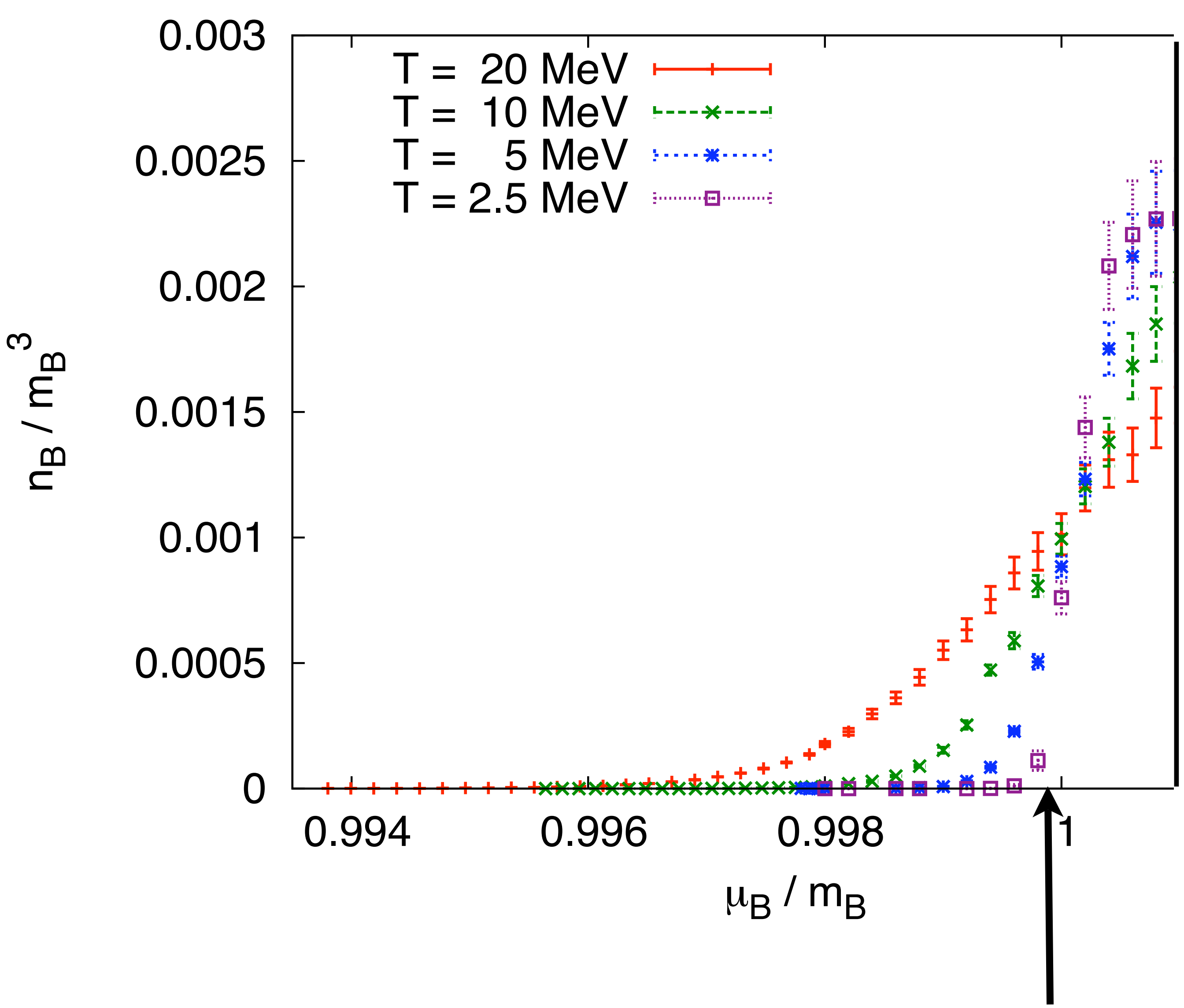




Cold and dense, interacting: onset to nuclear matter

continuum extrapolated

$m_\pi = 20 \text{ GeV}$



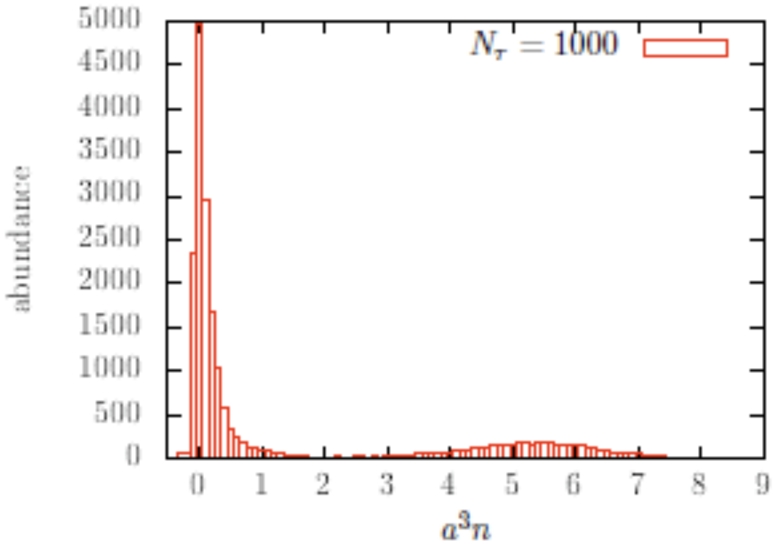
Effect of binding between baryons: $\mu_c < m_B$

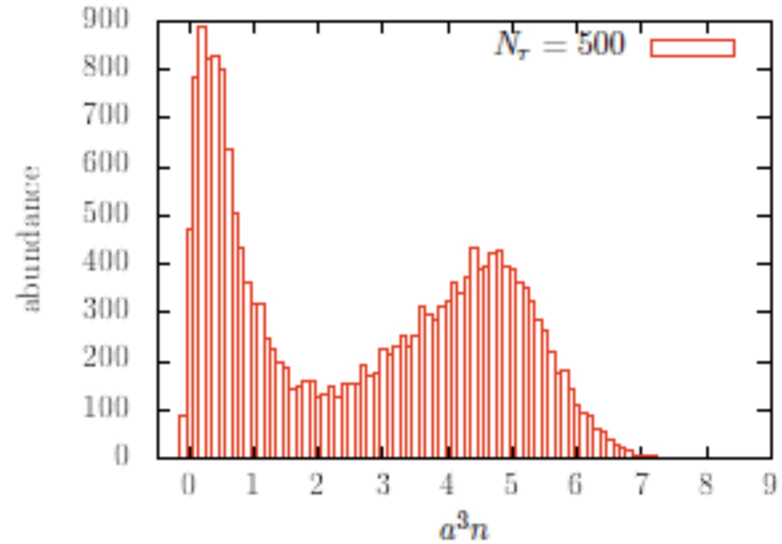
$\frac{\mu}{T} \sim 4000$

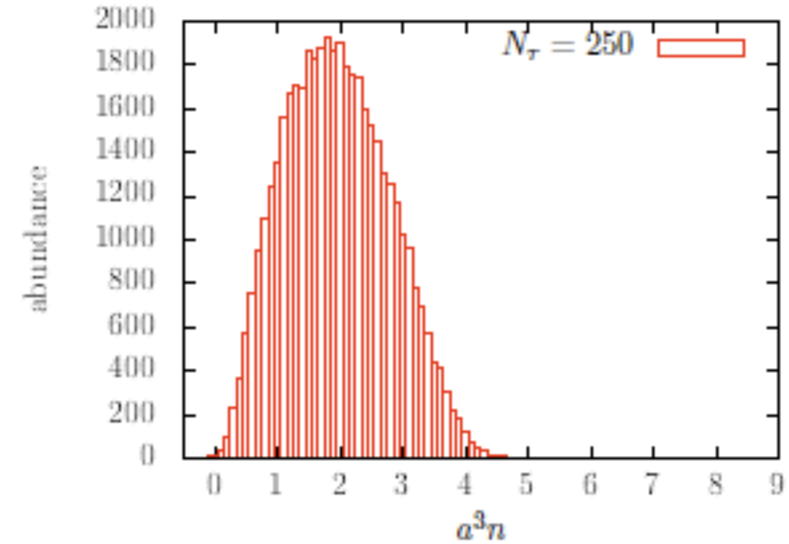
Binding energy per nucleon: $\epsilon = \frac{\mu_c - m_B}{m_B} \sim 10^{-3}$

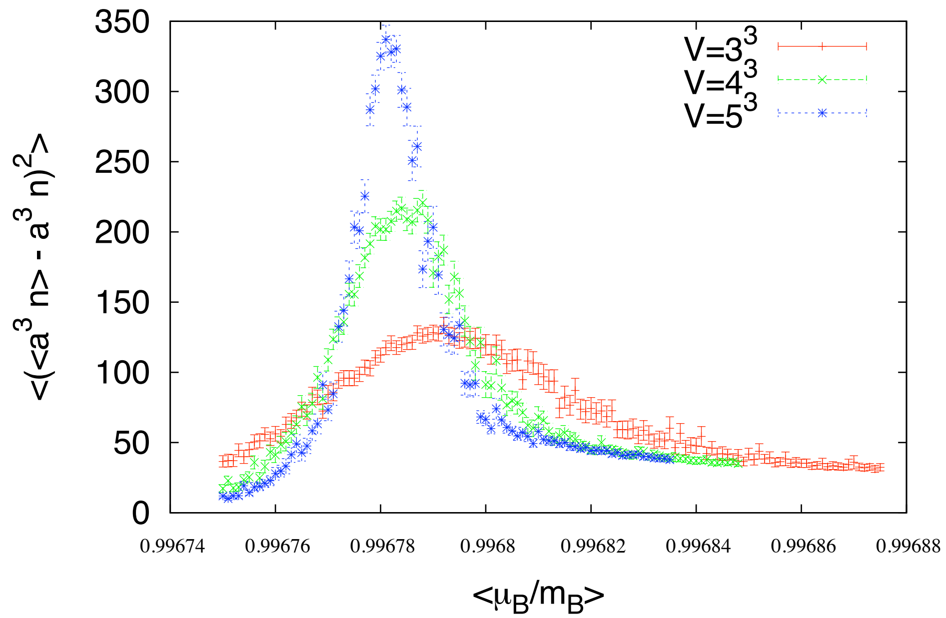
Transition is smooth crossover: $T > T_c \sim \epsilon\, m_B$

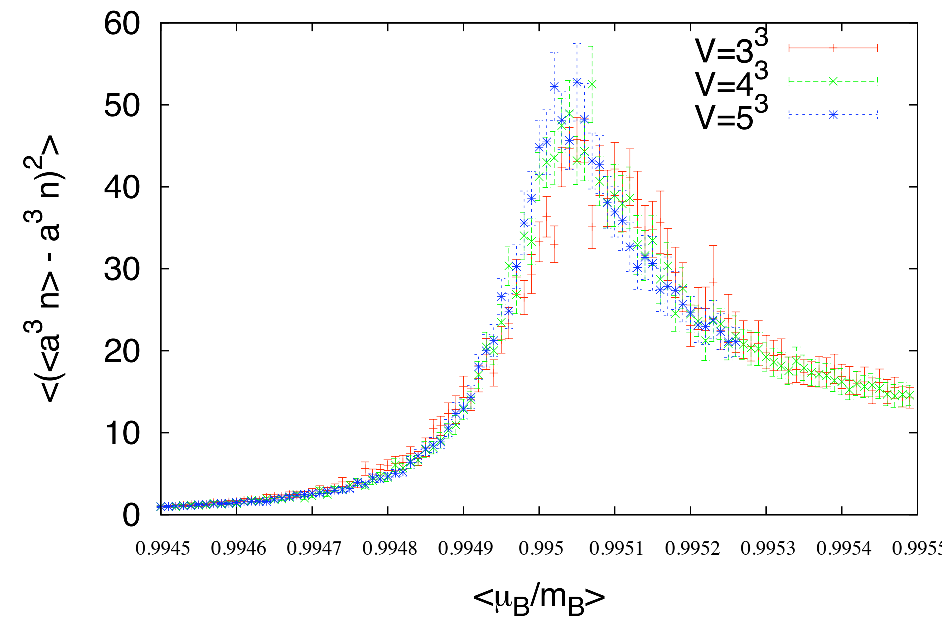
Liquid gas transition: first order + endpoint









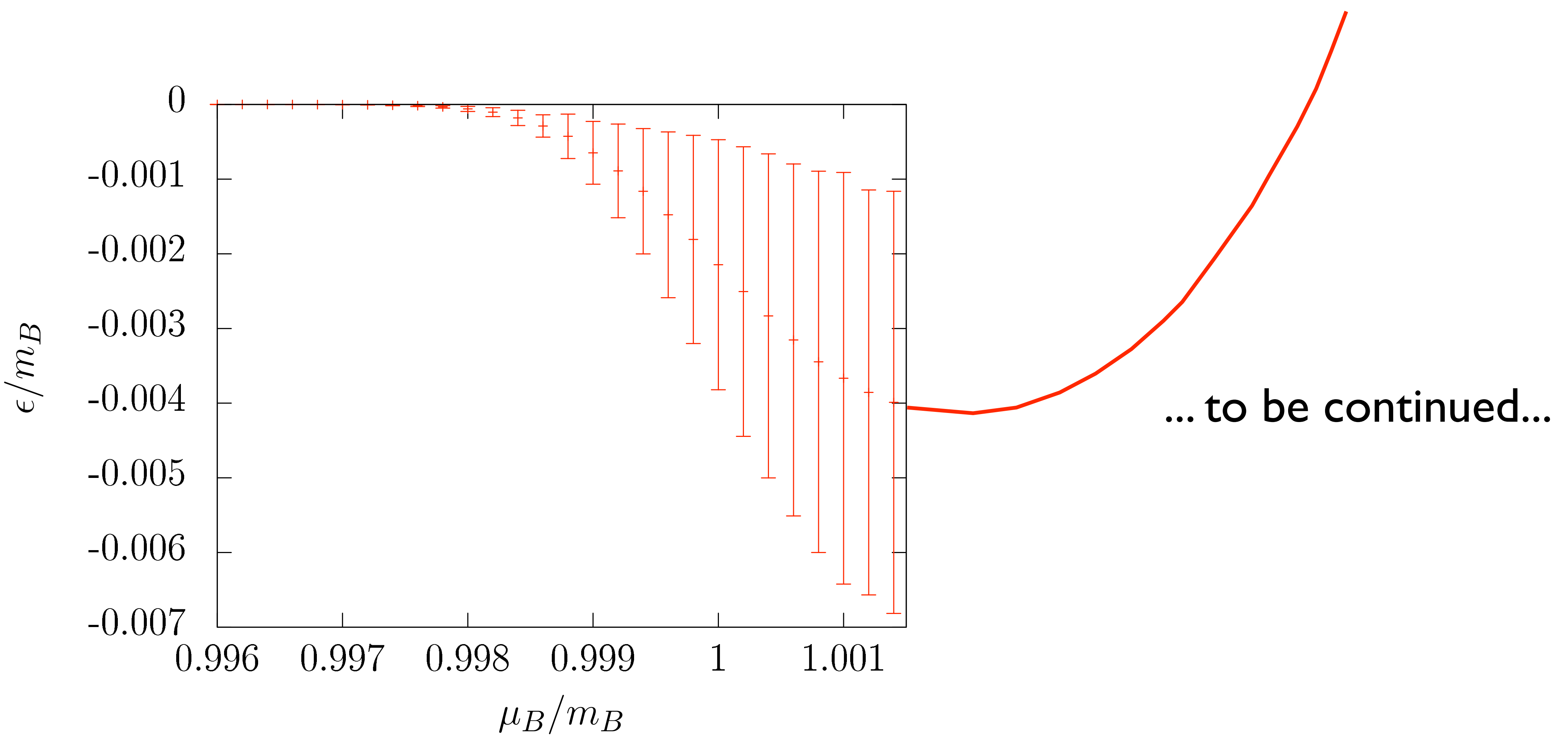


- Coexistence of vacuum and finite density phase: 1st order
- If the temperature $T = \frac{1}{aN_\tau}$ or the quark mass is raised this changes to a crossover

nuclear liquid gas transition!!!

Binding energy per nucleon

$$\epsilon = \frac{e - n_B m_B}{n_B m_B} = \frac{e}{n_B m_B} - 1$$



Minimum: access to nucl. binding energy, nucl. saturation density!

$\epsilon \sim 10^{-3}$ consistent with the location of the onset transition

Cold and dense QCD: static strong coupling limit

For T=0 (at finite density) anti-fermions decouple $N_f = 1, h_1 = C, h_2 = 0$

$$C_f \equiv (2\kappa_f e^{a\mu_f})^{N_\tau} = e^{(\mu_f - m_f)/T}, \; \bar{C}_f(\mu_f) = C_f(-\mu_f)$$

$$Z(\beta=0) = \left[\prod_f \int \mathrm{d}W \left(1 + C_f L + C_f^2 L^* + C_f^3\right)^2 \right]^{N_s^3}$$

$$\stackrel{T\to 0}{\longrightarrow} \left[1 + 4C^{N_c} + C^{2N_c}\right]^{N_s^3}$$

Free gas of baryons!

Quarkyonic?

$$n = \frac{T}{V}\frac{\partial}{\partial\mu}\ln Z = \frac{1}{a^3}\frac{4N_c C^{N_c} + 2N_c C^{2N_c}}{1 + 4C^{N_c} + C^{2N_c}}$$

$$\lim_{\mu\to\infty}(a^3 n) = 2N_c$$

Sivler blaze property + saturation!

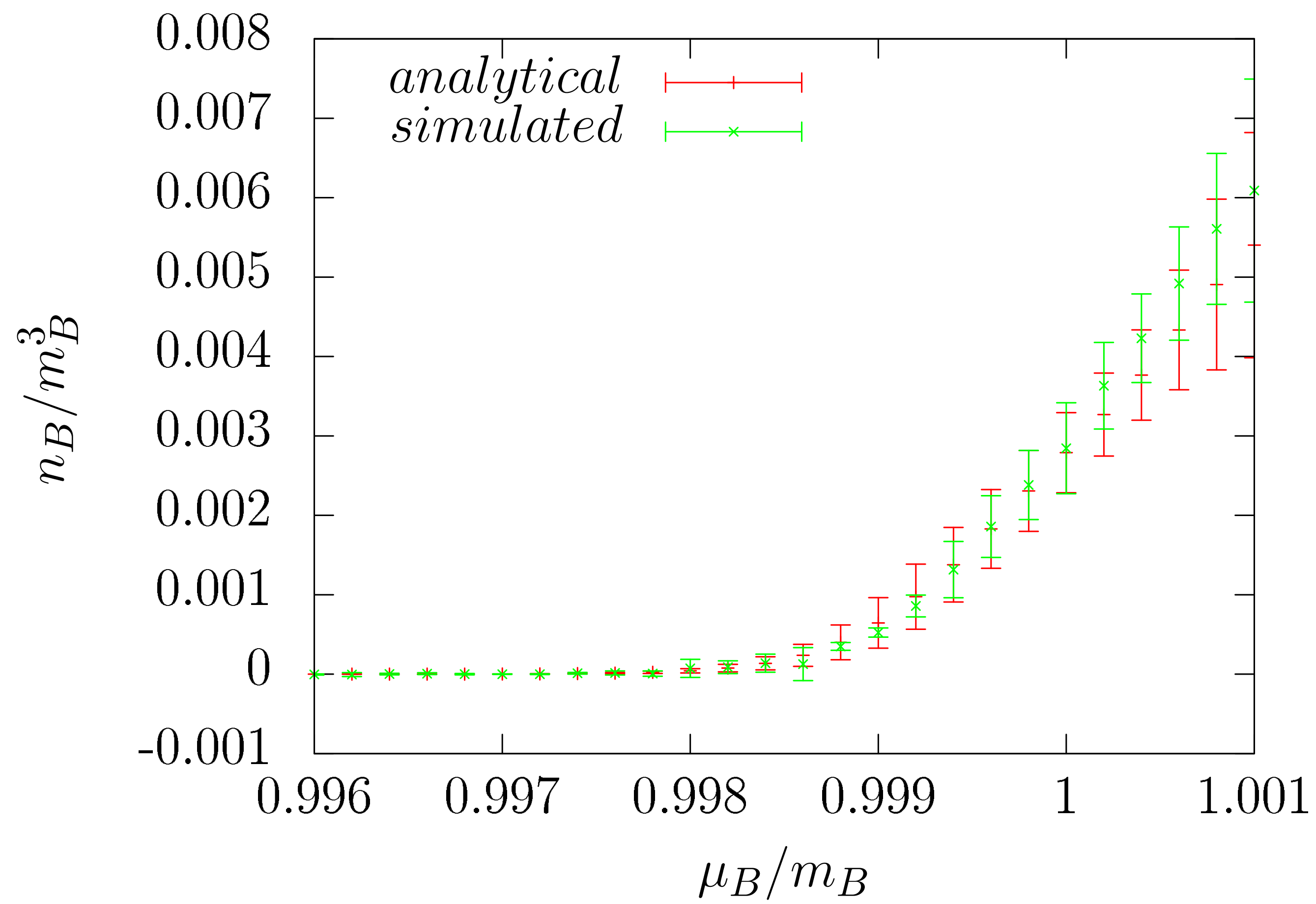
$$\lim_{T\to 0} a^3 n = \begin{cases} 0, & \mu < m \\ 2N_c, & \mu > m \end{cases}$$

$N_f = 2$

$$z_0 = (1 + 4h_d^3 + h_d^6) + (6h_d^2 + 4h_d^5)h_u + (6h_d + 10h_d^4)h_u^2 + (4 + 20h_d^3 + 4h_d^6)h_u^3 + (10h_d^2 + 6h_d^5)h_u^4 + (4h_d + 6h_d^4)h_u^5 + (1 + 4h_d^3 + h_d^6)h_u^6 \; . \quad (3.11)$$

Perturbation theory also possible!

- Effective couplings small
- Linked cluster expansion in effective couplings
- =expansion about static, strong coupling limit



Binding energy per nucleon:

$$\epsilon = -\frac{4}{3}\frac{1}{a^3 n_B}\left(\frac{z_3}{z_0}\right)^2 \kappa^2 = -\frac{1}{3}\frac{1}{a^3 n_B}\left(\frac{z_3}{z_0}\right)^2 e^{-am_M} \quad + \quad \ldots$$

The effective lattice theory approach II

- Two-step treatment:

  I. Calculate effective theory analytically
  II. Simulate effective theory

de Forcrand, Langelage, O.P., Unger
Phys.Rev.Lett. 113 (2014) 152002

- Step I.: integrate over gauge links in strong coupling expansion, leave fermions

$$Z_{\mathrm{QCD}} = \int d\psi d\bar{\psi} dU\, e^{S_F+S_G} = \int d\psi d\bar{\psi} Z_F \left\langle e^{S_G} \right\rangle_{Z_F}$$

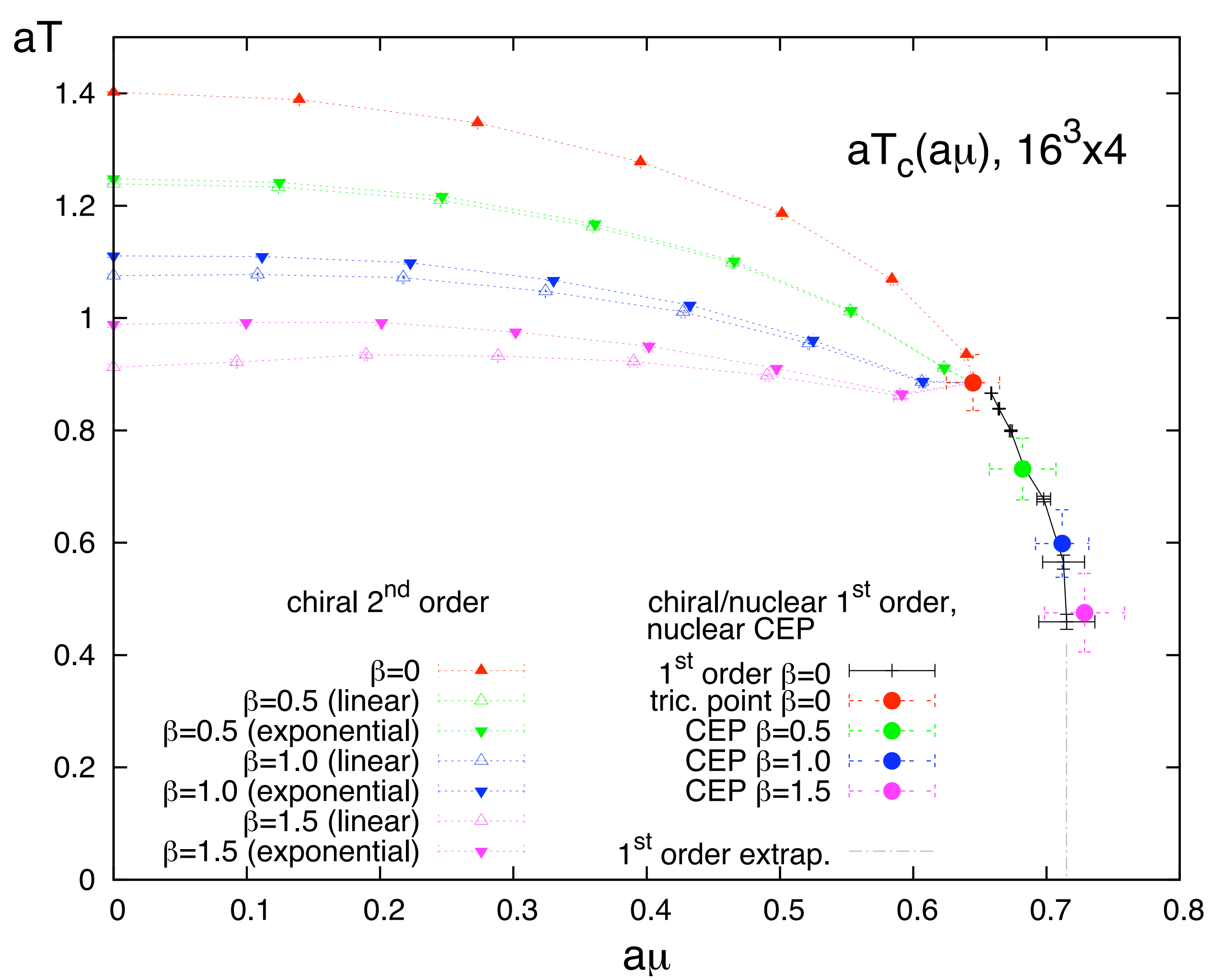
$$\left\langle e^{S_G} \right\rangle_{Z_F} \simeq 1 + \langle S_G \rangle_{Z_F} = 1 + \frac{\beta}{2N_c} \sum_P \left\langle \mathrm{tr}[U_P + U_P^\dagger] \right\rangle_{Z_F} \qquad Z_F(\psi,\bar{\psi}) = \int dU\, e^{S_F}$$

- Result: 4d "polymer" model of QCD (hadronic degrees of freedom!)
  Valid for all quark masses (also m=0!), at strong coupling (very coarse lattices)

- Step II: sign problem milder: Monte Carlo with worm algorithm

- Numerical simulations without fermion matrix inversion, very cheap!

Chiral QCD at strong coupling



- LO gauge correction included, simulation by worm algorithm
- Chiral phase transition with 2nd order and 1st order line meeting in tricritical point
- Nuclear liquid gas transition on top of first order chiral one at strong coupling

Conclusions

- LQCD for vacuum physics mature and precise, discovery tool
- Finite density QCD enormous challenge, but urgently needed
- QCD description of nuclear densities now possible for

  -heavy quarks near continuum

  -chiral quarks on coarse lattices
- Can this be pushed far enough to cover light quarks near the continuum?