Quantum field theory of leptogenesis

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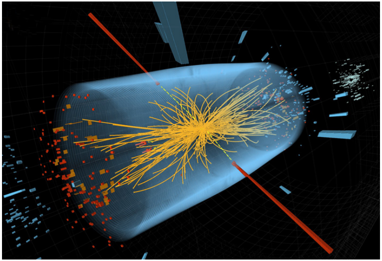
Physics beyond the Standard Model

all SM particles
have been found



beyond SM physics required for

- neutrino masses
- baryon asymmetry
- dark matter
- solution to the strong CP problem

Baryon asymmetry of the Universe

net baryon density

$n_B \equiv n_b - n_{\bar{b}}$

baryon to photon ratio

$\eta \equiv n_B/n_\gamma$

Big Bang Nucleosynthesis

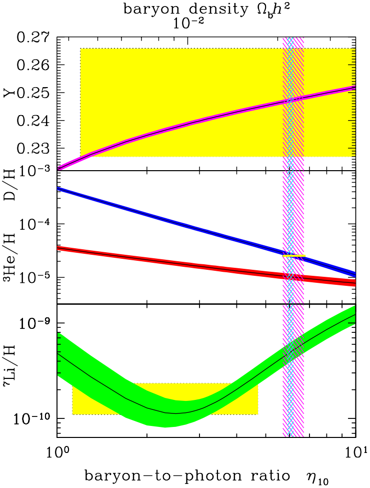
(1 MeV $\gtrsim T \gtrsim$ 10 keV) $\rightsquigarrow$

$5.7 < (\eta \times 10^{10}) < 6.7$

Cosmic Microwave Background

($T \sim$ 0.25 eV)

$\eta \times 10^{10} = 6.04 \pm 0.08$ [Planck]



[Particle Data Group]

Baryon asymmetry of the Universe

why is $\eta \neq 0$?

initial condition?

not if there was inflation

asymmetry can be dynamically generated if there is

1. baryon number violation
2. C and CP violation
3. non-equilbrium

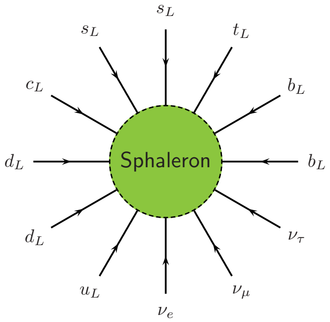
Baryon + lepton number violation

$B - L$ is conserved in the Standard Model

$B + L$ is not [t'Hooft]

$B + L$ violation unsuppressed for $T \gtrsim$ 160 GeV

'sphaleron' processes



[Fig. from Buchmüller, Di Bari, Plümacher]

Lepton asymmetry $\leftrightarrow$ Baryon asymmetry

Neutrino masses

$\Delta m^2_{\rm solar} \simeq 7.6 \times 10^{-5}$eV, $\qquad \Delta m^2_{\rm atmospheric} \simeq 2.4 \times 10^{-3}$eV

add right-handed (sterile) neutrinos $N = N^c$:

$$\mathscr{L}_N = \frac{i}{2}\overline{N}\partial\!\!\!/ N - \frac{1}{2}\overline{N}MN - \left(\overline{N}h\widetilde{\varphi}^\dagger \ell + \text{ h.c. }\right)$$

$M \gg hv \Rightarrow$ see-saw formula for light ν mass matrix

$$m_\nu = hM^{-1}h^T v^2$$

$m_\nu \sim$ 0.1 eV:

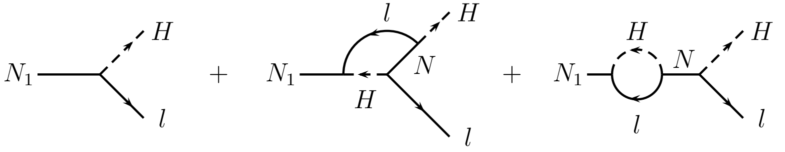
$$m_e/v < h < 1 \qquad \Leftrightarrow \qquad \text{TeV} \lesssim M \lesssim M_{\rm GUT}$$

Baryogenesis through leptogenesis

[Fukugita, Yanagida (1986)]

Majorana masses $M_{ij} \to$ lepton number violation

complex Yukawa couplings $h_{ij} \to$ CP-violation



decay rates $\Gamma(N \to \ell\varphi) \neq \Gamma(N \to \bar{\ell}\bar{\varphi})$

$\Gamma \lesssim H \to$ non-equilibrium

Leptogenesis scenarios

thermal leptogenesis (non-resonant) [Buchmüller, Plümacher '97]

- asymmetry from sterile neutrino decay

- no fine tuning

- close to thermal equilibrium

- M_1, $T_{\rm RH} > 10^{10}$GeV

Leptogenesis scenarios

asymmetry from **production** of sterile N_i [Akhmedov, Smirnov, Rubakov]

- far from equilibrium

- M_2, $M_3 \gtrsim$ MeV [Canetti, Drewes, Shaposhnikov '13]

- thermal effects $\rightarrow$ no fine tuned mass degeneracy needed

  [Drewes, Garbrecht]

Traditional approach to leptogenesis

Boltzmann equations for phase space densities $f_a(t, \mathbf{p})$

$$D_t f_a = \ \mathsf{Coll}_a[f]$$

collision term (for leptons)

$$\mathsf{Coll}_\ell[f] = \int_{\mathbf{p}_i} (2\pi)^4 \delta(p_\ell + p_{\overline{\varphi}} - p_N)$$
$$\times \Big[|\mathcal{M}|^2_{N\to\ell\overline{\varphi}}\, f_N(1 - f_\ell)(1 + f_{\overline{\phi}}) - |\mathcal{M}|^2_{\ell\overline{\varphi}\to N}\, f_\ell f_{\overline{\varphi}}(1 - f_N) \Big] + \cdots$$

moments of Boltzmann equations

$\rightsquigarrow$ rate equations for number densities

Quantum theory of leptogenesis

"A shortcoming of this approach is that the Boltzmann equations are classical equations for the time evolution of phase space distribution functions. On the contrary, the involved collision terms are S-matrix elements which involve quantum interferences of different amplitudes in a crucial manner. Clearly, a full quantum mechanical treatment is highly desirable. This is also required in order to justify the use of the Boltzmann equations and to determine the size of corrections."

"Recent progress in leptogenesis" (2001)

Quantum theory of leptogenesis

Schwinger-Dyson/Kadanoff-Baym equations for
non-equilibrium Green functions [Buchmüller, Fredenhagen '00]

Two strategies:

1. Find approximate solutions to Kadanoff-Baym equations
   [Anisimov, Buchmüller, Drewes, Mendizabel]
2. Derive rate equations from Kadanoff-Baym equations
   [Garny, Hohenegger, Kartavtsev, Lindner, Frossard]

important results for

resonant leptogenesis

flavored leptogenesis [Beneke, Fidler, Garbrecht, Herranen, Schwaller]

Effective field theory approach

[DB, M. Laine, M. Sangel, M. Wörmann]

identify slow and fast variables based on equilibration rate γ:

$\gamma \gg H$ fast, in thermal equilibrium

spectator processes [Buchmüller, Plümacher 01]

$\gamma \sim H$ slow, interesting non-equilibrium dynamics

$\gamma \ll H$ practically conserved

write effective equations of motion for slow variables

$$D_t X_a = -\gamma_{ab} X_b$$

valid - on time scales $\gtrsim \gamma^{-1}$

- to all orders in Standard Model couplings

Example: Non-relativistic unflavored leptogenesis

$T \sim M_1 \ll M_2, M_3$, only N_1 present in plasma

N_1 decays into single charged lepton flavor

non-relativistic approximation: neglect motion of N_1

slow variables: $n_N \equiv n_{N_1}$, n_{B-L}

rates $\gamma \sim h^2 T \quad \ll \quad g^n T$ SM equilibration rates

effective equations of motion

$$D_t n_N = -\gamma_{\rm eq} \left(n_N - n_{N,\rm eq} \right) + \gamma'_{\rm asym}\, n_{B-L}$$

$$D_t n_{B-L} = \gamma_{\rm asym} \left(n_N - n_{N,\rm eq} \right) - \gamma_{\rm wash}\, n_{B-L}$$

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Effective field theory approach

coefficients

$$\gamma_{ab} = \gamma_{ab}(T)$$

- determined by short time physics
- only depend on temperature T and chemical potentials μ of practically conserved charges
- radiative corrections can be systematically computed

computation = ‘two step procedure’

1. compute coefficients from short time/distance physics
2. solve effective equations of motion

Step 1: γ_{ab} from correlation of thermal fluctuations

effective eqs. of motion for thermal fluctuations

$$\dot{X}_a = -\gamma_{ab} X_b + \xi_a$$

Langevin equation

Gaussian noise ξ

$$\langle \xi_a(t) \xi_b(t') \rangle \propto \delta(t - t')$$

real time correlation function

$$\langle X_a(t) X_c(0) \rangle = (e^{-\gamma t})_{ab} \langle X_b X_c \rangle = (e^{-\gamma t} \Xi)_{ab}$$

Step 1: Correlations from finite temperature QFT

real time correlation function from Langevin equation:

$$\langle X_a(t) X_c(0) \rangle = (e^{-\gamma t})_{ab} \langle X_b X_c \rangle$$

real time correlation function in quantum field theory:

$$\frac{1}{2} \Big\langle \{ X_a(t), X_c(0) \} \Big\rangle$$

match at

$$t_{\rm UV} \ll t \ll \gamma^{-1}, \qquad \omega_{\rm UV} \gg \omega \gg \gamma$$

where - both descriptions are valid

- one can expand in γ

$\rightsquigarrow \gamma_{ab}$, valid to all orders in SM couplings

Washout rate

$$D_t n_{_N} = -\gamma_{\rm eq}\left(n_{_N} - n_{_{N,\rm eq}}\right)$$

$$D_t n_{_{B-L}} = \gamma_{\rm asym}\left(n_{_N} - n_{_{N,\rm eq}}\right) - \gamma_{\rm wash}\, n_{_{B-L}}$$

Washout rate

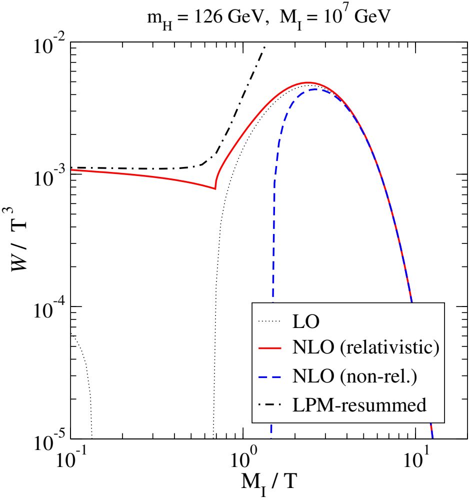
at leading order in h: [DB, M. Laine]

$$\gamma_{ab} = \frac{1}{2V} \lim_{\omega\to 0} \frac{1}{\omega} \int \mathrm{d}t \, e^{i\omega t} \Big\langle \Big[\dot{X}_a(t), \dot{X}_c(0) \Big] \Big\rangle_0 \left(\Xi^{-1}\right)_{cb}$$

matrix of susceptibilities $\Xi_{ab} \equiv \frac{1}{TV} \langle X_a X_b \rangle$

similar to Kubo relation for transport coefficients (conductivity, viscosity, ...)

Washout rate



integral over spectral function

spectral function results

ultra-relativistic ($M_I \lesssim g^2 T$):

complete LO [D Besak, DB]

relativistic ($M_I \sim T$):

NLO [M Laine]

non-relativistic ($M_I \gg T$):

NLO

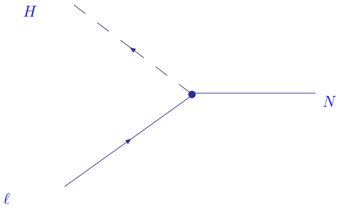
[A Salvio et al., Laine, Y Schröder]

Equilibration rate

for phase space density $f_N(\mathbf{k})$

$$D_t f_N = -\gamma_{\rm eq}(\mathbf{k})\Big(f_N - f_N^{\rm eq}\Big)$$

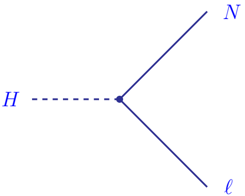
N-equilibration: thermal mass effects



inverse decay

high temperature $T \gtrsim M_N$:

- close to light cone
- nearly collinear



thermal mass effect:

$m_H > m_N \Rightarrow$

Higgs decay

[Giudice et al.]

Equilibration rate from quantum field theory

at LO in N-Yukawa coupling, all orders in SM couplings:

$$\gamma_{\rm eq}(\mathbf{k}) = \text{ Im Tr}\,(\not{k}\Sigma(k))$$

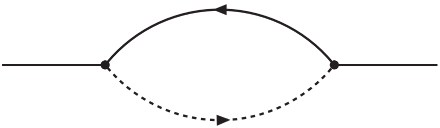
with N-selfenergy

$$\Sigma(x) = 2\langle J(x)\bar{J}(0)\rangle$$

for interaction

$$\mathscr{L}_{\rm int} = \bar{N}_1 J + \text{ h.c. }, \qquad J \equiv \sum_i h_{1i}\tilde{\varphi}^\dagger \ell_i$$

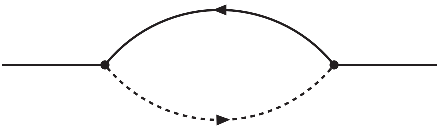
Equilibration at high T



without gauge interactions:

cut $\rightarrow$ inverse decay, H-decay

Equilibration at high T



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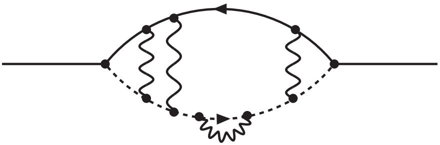
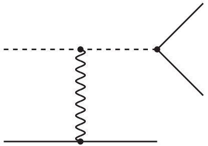


multiple soft gauge interaction
resummation necessary

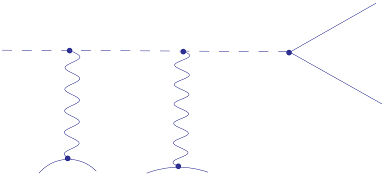
Soft gauge interactions



collinear enhancement

compensates additional vertices

$\rightsquigarrow$ leading order contribution



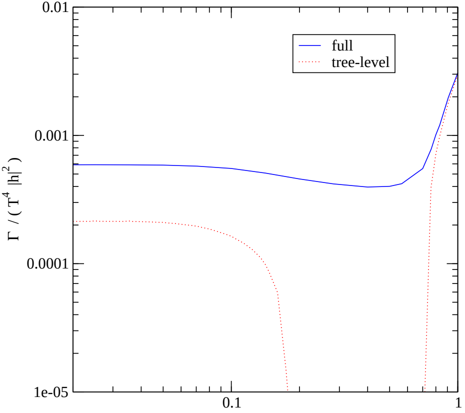
multiple soft scattering unsuppressed

leading order contribution

Landau-Pomeranchuk-Migdal effect

[Anisimov, Besak, DB]

Thermal masses + soft gauge interactions



soft gauge interactions

$\rightsquigarrow$

factor 3 enhancement

at high T

[Anisimov, Besak, DB]

Summary and outlook

lot of theoretical progress in leptogenesis

effective field theory approach

coefficients from real time finite temperature QFT, to all orders in SM couplings

NLO and NNLO for washout rate

N-equilibration: thermal mass effects, soft gauge interactions

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CP-asymmetry ?

error bars for leptogenesis ?

CP asymmetry in ultrarelativistic regime ?

Viel Glück und viel Segen
auf all Deinen Wegen!

Happy Birthday!

Washout rate

integrate out N_I at leading order $\Rightarrow$

$$\gamma_{ab} = -\frac{1}{2}\sum_I \int_{\mathbf{k}} \frac{f_{\rm F}'(E_I)}{2E_I}\, h_{Ii}\ \mathsf{tr}\Big[\not{k} \Big(T^\ell_a \big[\, \widetilde{\rho}(k) + \widetilde{\rho}(-k) \,\big] T^\ell_c + T^\ell_c \big[\, \widetilde{\rho}(k) + \widetilde{\rho}(-k) \,\big] T^\ell_a \Big)_{ij} \Big] h^*_{Ij}\, \big(\Xi^{-1}\big)_{cb}$$

with spectral function

$$\widetilde{\rho}_{ij\alpha\beta}(k) \;\equiv \int_x e^{ik\cdot x} \Big\langle \Big\{ (\widetilde{\varphi}^\dagger \ell_{i\alpha})(x), (\overline{\ell}_{j\beta}\, \widetilde{\varphi})(0) \Big\} \Big\rangle_0$$

Washout rate

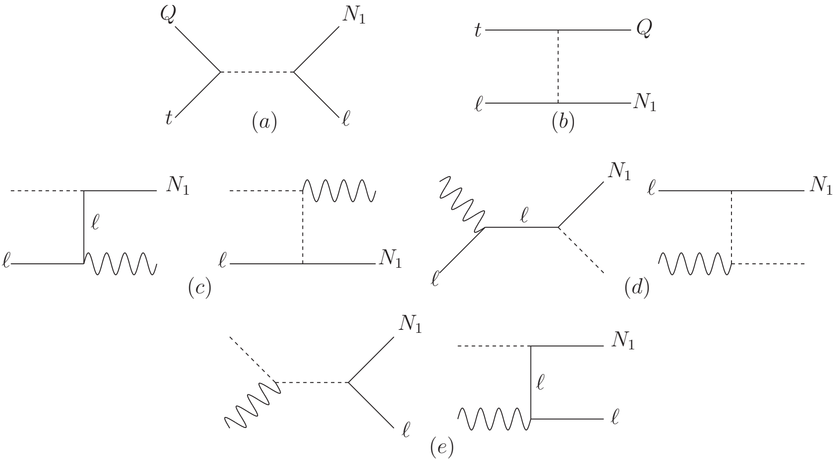
corrections to spectral functions (non-relativistic and relativistic) $= O(g^2)$

corrections to susceptibilities $= O(g)$, infrared effect

leading corrections from 'simple' thermodynamics

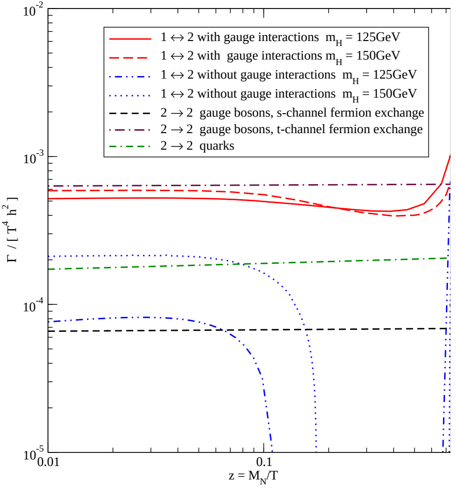
complete $O(g^2)$ computed [DB, M. Sangel]
corrections $\leq 4\%$, mostly from QCD

N-production: $2 \to 2$ scattering



Besak, DB

Complete equilibration rate



gauge interactions dominate