Variable flavor number schemes (VFNS) for multiscale processes in QCD

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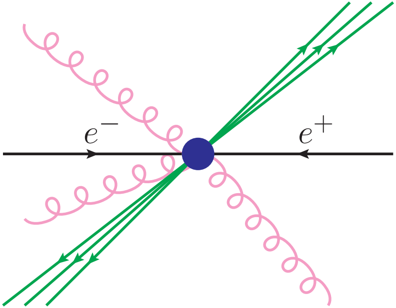
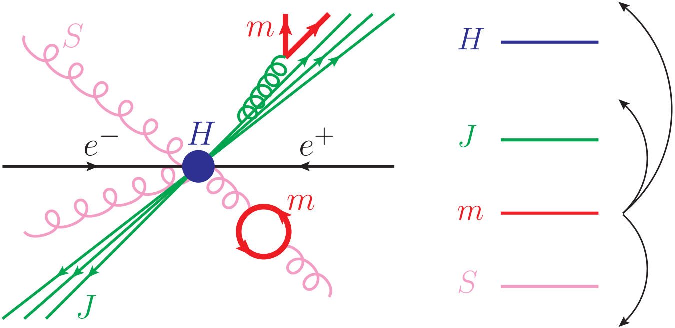
Motivation

- nowadays: QCD at colliders is precision physics
- understanding of quark mass effects relevant for many processes, e.g.
  $\rightarrow$ heavy flavor initiated processes at hadron colliders
  $\rightarrow$ measurement of the top quark mass
  $\rightarrow$ event shapes at low c.m. energies , ...
- typical collider event with QCD radiation (e^+e^--collision):



Motivation

- aim: full quark mass dependence ($m \gg \Lambda_{\mathrm{QCD}}$) for jet observables
  $\rightarrow$ new: systematic treatment of virtual and real **secondary** massive quarks



- main emphasis: development of factorization setup

Outline

1. Prelude: Renormalization schemes
2. VFNS for the hadronic R-ratio
3. VFNS for DIS in the classical region $x \sim 1$
4. VFNS for DIS in the endpoint region $x \to 1$
5. VFNS for event shapes in the dijet region
6. Summary

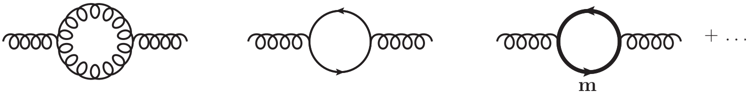
Outline

Renormalization of the strong coupling

- consider QCD with n_l massless and 1 massive quark (fields q_i, Q)

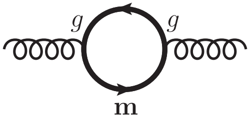
$$\mathcal{L}_{\rm QCD} = -\frac{1}{4} F^a_{\mu\nu} F_a^{\mu\nu} + \sum_{i=1}^{n_l} \bar{q}_i (i\partial\!\!\!/ + gA\!\!\!/) q_i + \bar{Q}(i\partial\!\!\!/ + gA\!\!\!/ - m) Q$$

renormalization of "bare" strong coupling g, e.g.



default for massless partons:
dimensional regularization ($d = 4 - 2\epsilon$ with $\epsilon \to 0$) with $\overline{\rm MS}$ renormalization

Renormalization of the strong coupling: Massive quark contributions



→ vacuum polarization $\Pi(q^2)$, reads for $q^2 = 0$

$$\Pi(0) = \frac{\alpha_s T_F}{3\pi}\left[\frac{1}{\epsilon} + \ln\left(\frac{\mu^2}{m^2}\right) - \gamma_E + \ln(4\pi) + \mathcal{O}(\epsilon)\right]$$

Renormalization for $\alpha_s \equiv g^2/4\pi$:

$$\alpha_s = \mu^{2\epsilon} Z_\alpha^{\overline{\mathrm{MS}}} \alpha_s^{\overline{\mathrm{MS}}}(\mu) = \mu^{2\epsilon} Z_\alpha^{\mathrm{OS}} \alpha_s^{\mathrm{OS}}(\mu) = \ldots$$

→ $\overline{\mathrm{MS}}$-type renormalization: $Z_\alpha^{\overline{\mathrm{MS}}} = 1 + \frac{\alpha_s^{\overline{\mathrm{MS}}} T_F}{3\pi}\frac{1}{\epsilon} + \mathrm{const} + \ldots$

→ OS (on-shell) renormalization: $Z_\alpha^{\mathrm{OS}} = 1 + \Pi(0) + \ldots$

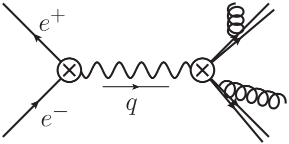
Anomalous dimensions for resummation of logarithms (RGE)

$$\beta^{\overline{\mathrm{MS}}} = \frac{d\alpha_s^{\overline{\mathrm{MS}}}}{d\ln\mu^2} + \epsilon\alpha_s^{\overline{\mathrm{MS}}} = -\mu^{2\epsilon}\alpha_s^{\overline{\mathrm{MS}}}\frac{d\ln Z_\alpha^{\overline{\mathrm{MS}}}}{d\ln\mu^2} = \beta^{(n_l+1)} \quad \rightarrow \alpha_s^{\overline{\mathrm{MS}}} \equiv \alpha_s^{(n_l+1)}(\mu)$$

$$\beta^{\mathrm{OS}} = \frac{d\alpha_s^{\mathrm{OS}}}{d\ln\mu^2} + \epsilon\alpha_s^{\mathrm{OS}} = -\mu^{2\epsilon}\alpha_s^{\mathrm{OS}}\frac{d\ln Z_\alpha^{\mathrm{OS}}}{d\ln\mu^2} = \beta^{(n_l)} \quad \rightarrow \alpha_s^{\mathrm{OS}} \equiv \alpha_s^{(n_l)}(\mu)$$

Outline

Hadronic R-ratio for massless quark production



$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} \sim \mathrm{Im}\left[-\mathrm{i}\int \mathrm{d}x\, \mathrm{e}^{-\mathrm{i}qx}\langle 0|\mathrm{T}\left[\mathrm{j}^\mu(x)\mathrm{j}_\mu(0)\right]|\rangle\right]$$

- one relevant scale: c.m. energy $q^2 = Q^2$
- current conservation
  $\to$ UV divergences only related to strong coupling & field redefinitions
  $\to$ only running structure: α_s
- perturbative expansion (with $\overline{\mathrm{MS}}$-renormalized α_s with n_f light flavors)

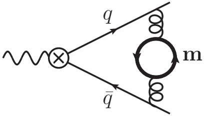
$$R_{n_f}[\alpha_s^{(n_f)}] = N_c \sum e_q^2 \left\{1 + \frac{\alpha_s^{(n_f)}(\mu)}{4\pi} r_1 + \left(\frac{\alpha_s^{(n_f)}(\mu)}{4\pi}\right)^2 \left[r_2^{(n_f)} - \beta_0 r_1 \ln\left(\frac{Q^2}{\mu^2}\right)\right]\right\}$$

$\to$ log minimized for $\mu \sim Q$

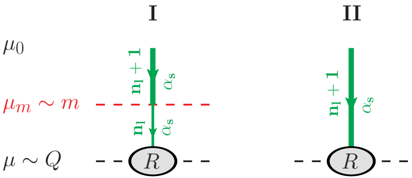
Massive quark contributions

$$R_{n_f}[\alpha_s^{(n_f)}] = N_c \sum e_q^2 \left\{ 1 + \frac{\alpha_s^{(n_f)}(\mu)}{4\pi} r_1 + \left(\frac{\alpha_s^{(n_f)}(\mu)}{4\pi} \right)^2 \left[r_2^{(n_f)} - \beta_0 r_1 \ln\left(\frac{Q^2}{\mu^2} \right) \right] \right\}$$

- virtual massive quark effects:



- aims for a VFNS:
  - $\to$ resummation of all large logarithms $\ln\left(\frac{Q}{m}\right)$
  - $\to$ correct limits: decoupling for $m \to \infty$ + massless limit for $m \to 0$
  - $\to$ continuous description with full mass dependence for arbitrary $m \leftrightarrow Q$
  - $\Rightarrow$ use of proper renormalization schemes: CWZ-scheme [Collins, Wilczek, Zee (1978)]



- I. OS renormalization for massive quark contributions to α_s: $\alpha_s^{(n_l)}(\mu \sim Q)$

$$R_{n_l,m}[\alpha_s^{(n_l)}(\mu)] \stackrel{m \gg Q}{\longrightarrow} R_{n_l}[\alpha_s^{(n_l)}(\mu)] + \mathcal{O}\left(\frac{Q^2}{m^2}\right) \checkmark$$

$$R_{n_l,m}[\alpha_s^{(n_l)}(\mu)] \stackrel{m \ll Q}{\longrightarrow} R_{n_l+1}[\alpha_s^{(n_l)}(\mu)] + \left(\frac{\alpha_s^{(n_l)}(\mu)}{4\pi}\right)^2 \left(\beta_0^{(n_l+1)} - \beta_0^{(n_l)}\right) r_1 \ln\left(\frac{m^2}{\mu^2}\right) ↯$$

$\Rightarrow$ appropriate for $m \gtrsim Q$

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$$R_{n_l,m}[\alpha_s^{(n_l+1)}(\mu)] \stackrel{m \gg Q}{\longrightarrow} R_{n_l}[\alpha_s^{(n_l+1)}(\mu)] - \left(\frac{\alpha_s^{(n_l+1)}(\mu)}{4\pi}\right)^2 \left(\beta_0^{(n_l+1)} - \beta_0^{(n_l)}\right) r_1 \ln\left(\frac{m^2}{\mu^2}\right) \quad ↯$$

$\Rightarrow$ appropriate for $m \lesssim Q$

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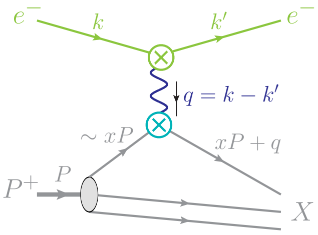
$$R_{n_l,m}[\alpha_s^{(n_l+1)}(\mu)] \overset{m \gg Q}{\longrightarrow} R_{n_l}[\alpha_s^{(n_l+1)}(\mu)] - \left(\frac{\alpha_s^{(n_l+1)}(\mu)}{4\pi}\right)^2 \left(\beta_0^{(n_l+1)} - \beta_0^{(n_l)}\right) r_1 \ln\left(\frac{m^2}{\mu^2}\right) \quad ϟ$$

$\Rightarrow$ appropriate for $m \lesssim Q$

- for arbitrary $m \leftrightarrow Q$: use I. for $m \gtrsim Q$ and II. for $m \lesssim Q$
  $\Rightarrow$ matching at $\mu \sim Q \sim m$ = decoupling relation for α_s
  $\Rightarrow$ exact mass-dependence + correct limiting behavior

Outline

Deep inelastic scattering: $e^- P^+ \to e^- X$



- two relevant scales: $q^2 = -Q^2$, $\Lambda_{\rm QCD} \sim M_P$
- $x = \frac{Q^2}{2P\cdot q}$: $0 \leq x \leq 1$, classical region: $1 - x \sim \mathcal{O}(1)$

$$\frac{d\sigma}{dQ^2 dx} \sim W_{\mu\nu} L^{\mu\nu}$$

$\to$ leptonic tensor $L^{\mu\nu}$: purely electromagnetic (up to higher orders in $\alpha_{\rm em}$)
$\to$ hadronic tensor $W_{\mu\nu} \leftrightarrow$ structure functions $F_{1,2}$

$$W_{\mu\nu} = \left(-g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) F_1(x, Q^2) + \frac{1}{P\cdot q}\left(P_\mu + \frac{q_\mu}{2x}\right)\left(P_\nu + \frac{q_\nu}{2x}\right) F_2(x, Q^2)$$

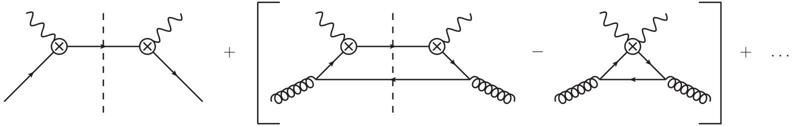
Factorization in DIS

factorization = separation of quantum fluctuations at different energy scales
$\rightarrow$ conveniently achieved with EFTs

Factorization theorem for massless quarks: [Collins, Soper, Sterman (1988); Bauer et al. (2002)]

$$F_{1,2} \sim \sum_{i=q,\bar{q}} \sum_{j=q,\bar{q},g} H_{ij}(\mu_H) \otimes U^{\Phi}_{jk}(\mu_H, \mu_\Phi) \otimes \Phi_{k/P}(\mu_\Phi) + \mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{Q^2}\right)$$

$\rightarrow$ hard function $H_{ij}(\mu_H \sim Q)$: ratio between full QCD and low-energy description



$\rightarrow$ low-energy parton distribution function (PDF) $\Phi_{j/P}(\mu_\Phi \sim \Lambda_{\mathrm{QCD}})$

$$\Phi_{k/P}(x, \mu_\Phi) = \langle P^+ | \mathcal{O}_k(x, \mu_\Phi) | P^+ \rangle$$

$\rightarrow$ logs $\ln(\frac{\mu_H}{\mu_\Phi})$ resummed via RG factor $U^{\Phi}_{jk}(\mu_H, \mu_\Phi)$ (implicit in the following)

Mass effects in DIS

Factorization theorem for massless quarks:

$$F_{1,2} \sim \sum_{i=q,\bar{q}} \sum_{j=q,\bar{q},g} H_{ij}(\mu_H) \otimes \Phi_{j/P}(\mu_\Phi)$$

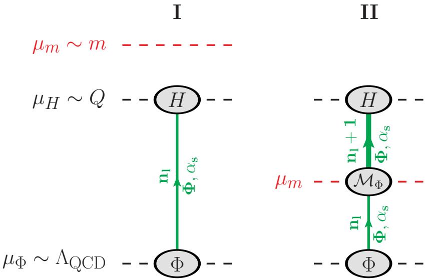
How to incorporate heavy quark mass effects ($m \gg \Lambda_{\mathrm{QCD}}$)?

$\rightarrow$ resummation of all logarithms $\ln\left(\frac{m}{\Lambda_{\mathrm{QCD}}}\right)$, $\ln\left(\frac{Q}{m}\right)$

$\rightarrow$ correct limits for H_{ij}: decoupling + massless limit

$\rightarrow$ continuous description for arbitrary masses

$\Rightarrow$ first achieved in the ACOT scheme [Aivazis, Collins, Olness, Tung (1994)]

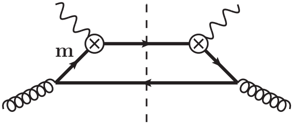


Massive quark corrections for $m \gtrsim Q$

$$m \gtrsim Q: \quad \boxed{F_{1,2} = \sum_{i=q,\bar{q},Q,\bar{Q}} \sum_{j=q,\bar{q},g} H_{ij}^{(n_l)}(\mu_H) \otimes \Phi_{j/P}(\mu_\Phi)}$$

use OS renormalization = low-momentum subtraction for PDFs and α_s

- evolution always with n_l flavors
- massive quark contributions in low-energy theory vanish
- only full QCD contributions to $H_{ij}^{(n_l)}$, e.g. at one-loop to $H_{Qg}^{(n_l)}$:



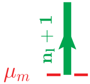
→ for $m \gg Q$: automatic decoupling $\checkmark$

→ for $m \ll Q$: unresummed logarithms $\sim \ln(m^2/Q^2)$ ↯

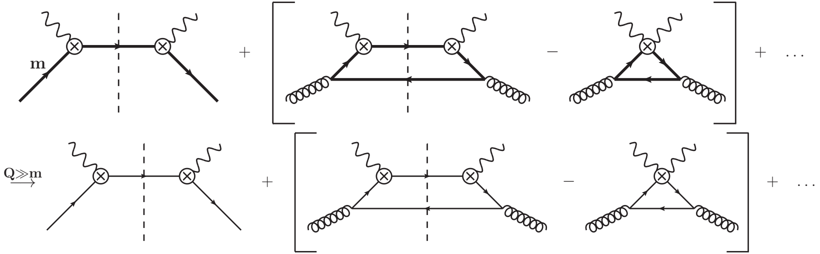
Massive quark corrections for $m \lesssim Q$

$$m \lesssim Q: \quad \boxed{F_{1,2} \sim \sum_{i=q,\bar{q},Q,\bar{Q}} \sum_{j=q,\bar{q},Q,\bar{Q},g} \sum_{k=q,\bar{q},g} H_{ij}^{(n_l+1)}(\mu_H) \otimes \mathcal{M}_{jk}^{\phi}(\mu_m) \otimes \Phi_{k/P}(\mu_\Phi)}$$

use $\overline{\mathrm{MS}}$ renormalization above the mass scale for PDFs and α_s



- evolution with $n_l + 1$ flavors above μ_m
- now massive quark contributions from full QCD and EFT to $H_{ij}^{(n_l+1)}$, at one loop:

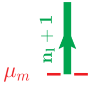


→ for $m \ll Q$: mass logarithms resummed, correct massless limit for $H_{ij}^{(n_l+1)}$ √

Massive quark corrections for $m \lesssim Q$

$$m \lesssim Q: \quad \boxed{F_{1,2} \sim \sum_{i=q,\bar{q},Q,\bar{Q}} \; \sum_{j=q,\bar{q},Q,\bar{Q},g} \; \sum_{k=q,\bar{q},g} H_{ij}^{(n_l+1)}(\mu_H) \otimes \mathcal{M}_{jk}^{\phi}(\mu_m) \otimes \Phi_{k/P}(\mu_\Phi)}$$

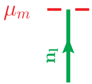
use $\overline{\mathrm{MS}}$ renormalization above the mass scale for PDFs and α_s



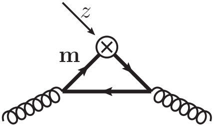
- evolution with $n_l + 1$ flavors above μ_m
- massive quark contributions from full QCD and EFT to $H_{ij}^{(n_l+1)}$

  $\rightarrow$ for $m \ll Q$: mass logarithms resummed, correct massless limit for $H_{ij}^{(n_l+1)}$ $\surd$

use OS renormalization below the mass scale for PDFs and α_s

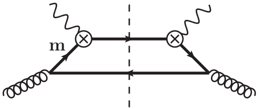


- evolution with n_l flavors below μ_m
- scheme change $\leftrightarrow$ PDF matching $\mathcal{M}_{ij}^{\phi}$, e.g. at one-loop for $\mathcal{M}_{Qg}^{\phi} = \langle g | \mathcal{O}_Q | g \rangle$



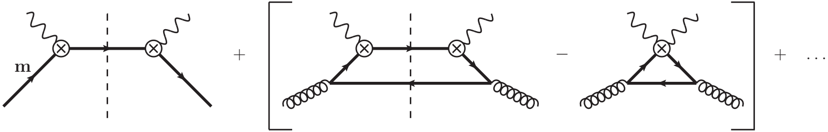
- I. $m \gtrsim Q$: $F_{1,2} = \sum_{i=q,\bar{q},Q,\bar{Q}} \sum_{j=q,\bar{q},g} H_{ij}^{(n_l)}(\mu_H) \otimes \Phi_{j/P}(\mu_\Phi)$

  $\rightarrow$ massive contributions to $H_{ij}^{(n_l)}$ at one-loop

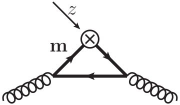


- II. $m \lesssim Q$: $F_{1,2} \sim \sum_{i=q,\bar{q},Q,\bar{Q}} \sum_{j=q,\bar{q},Q,\bar{Q},g} \sum_{k=q,\bar{q},g} H_{ij}^{(n_l+1)}(\mu_H) \otimes \mathcal{M}_{jk}^{\phi}(\mu_m) \otimes \Phi_{k/P}(\mu_\Phi)$

  $\rightarrow$ massive contributions to $H_{ij}^{(n_l+1)}$ at one-loop



  $\rightarrow$ massive contributions to PDF matching $\mathcal{M}_{ij}^{\phi}$ at one-loop

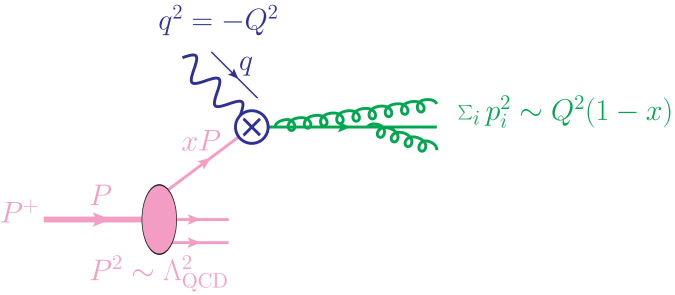


$\Rightarrow$ continuous transition to I at $\mathcal{O}(\alpha_s)$ $\surd$

Outline

Scales for $x \to 1$

- $x \to 1$: nontrivial factorization setup $\to$ interesting as a showcase for concepts
- use factorization theorem for $x \sim \mathcal{O}(1)$?
  unresummed logarithms in H_{ij}: $\ln\left(\frac{Q^2(1-x)}{Q^2}\right) = \ln(1-x)$ ↯
  $\leftrightarrow$ additional scale: final state jet invariant mass $\sum_i p_i^2 = s \sim Q^2(1-x)$
- here: $1 - x \gg \Lambda^2_{\mathrm{QCD}}/Q^2 \to s \gg \Lambda^2_{\mathrm{QCD}}$



Massless factorization theorem for $x \to 1$

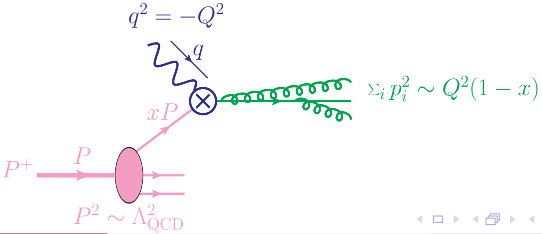
Factorization theorem for massless quarks:

$$F_{1,2} \sim \sum_{i=q,\bar{q}} H_{\rm DIS}(\mu_H)\, J_{\rm DIS}(\mu_J) \otimes \Phi_{i/P}(\mu_\Phi)\, [1 + \mathcal{O}(1-x)]$$

[Sterman (1987); Manohar (2003); Becher, Neubert, Pecjak (2006), . . .]

Ingredients:

- at $\mu_H \sim Q$: hard function $H_{\rm DIS}(\mu_H) = |C(\mu_H)|^2$
  $\to C(\mu_H)$: current matching between full QCD and low-energy description
- at $\mu_J \sim Q\sqrt{1-x}$: final state jet function $J_{\rm DIS}(\mu_J)$
  $\to$ jet rate in terms of its invariant mass (nonlocal!)
- at $\mu_\Phi \sim \Lambda_{\rm QCD}$: endpoint PDF $\Phi_{q/P}(\mu_\Phi)$



Massive quark effects

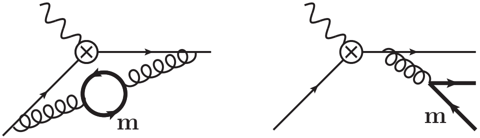
Factorization theorem for massless quarks:

$$F_{1,2} \sim \sum_{i=q,\bar{q}} H_{\mathrm{DIS}}(\mu_H)\, J_{\mathrm{DIS}}(\mu_J) \otimes \Phi_{i/P}(\mu_\Phi)$$

- note: only flavor-diagonal contributions in low-energy theory

$\sim \frac{1}{1-x}$ $\qquad\qquad$ ~ 1

- for massive quarks: massive threshold corrections also flavor-diagonal
  ⇒ no generation of massive quarks as initial state of the hard interaction
  ⇒ only "secondary" massive corrections to light quark inititated processes



Massive quark effects

Factorization theorem for massless quarks:

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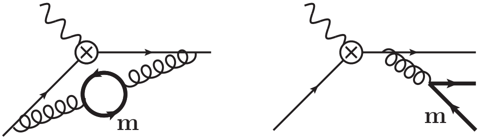
- only “secondary“ massive corrections to light quark inititated processes



- aim: factorization setup with secondary massive quarks incorporating
  - $\rightarrow$ summation of large logarithms
  - $\rightarrow$ correct limits for $H_{\rm DIS}$, $J_{\rm DIS}$
  - $\rightarrow$ continuous behavior in between with full singular mass-dependence

$\Rightarrow$ achieved by proper renormalization conditions

[Gritschacher, Hoang, Jemos, Mateu, P.P. (2013,2014)]

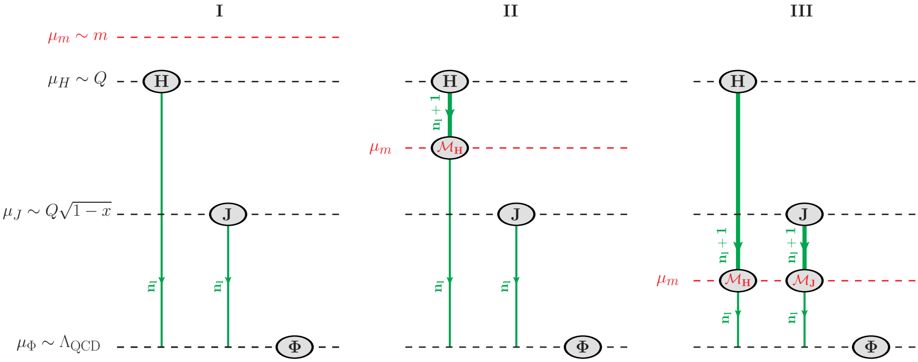
[Hoang, P.P., Samitz (in preparation)]

Mass factorization: Overview

scaling hierarchies for a heavy quark ($m \gg \Lambda_{\rm QCD}$) in the endpoint region ($1 - x \ll 1$):

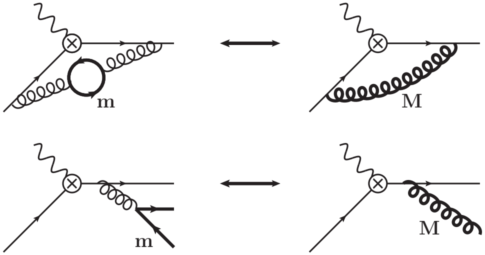
$$\text{I.}\; m > Q\,, \quad \text{II.}\; Q > m > Q\sqrt{1-x}\,, \quad \text{III.}\; Q\sqrt{1-x} > m > \Lambda_{\rm QCD}\,,$$

here: top-down evolution $\to$ final renormalization scale $\mu = \mu_\Phi$

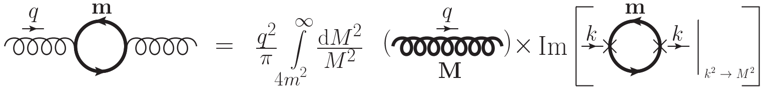


Computation of secondary massive quark effects

- massive quark corrections at $\mathcal{O}(\alpha_s^2 C_F T_F) \leftrightarrow$ "massive gluon" corrections at $\mathcal{O}(\alpha_s)$



- connection: dispersion relations



$\to$ generic scaling $M \sim m$ is inherited, same field theoretic setup

$\Rightarrow$ hard, jet functions + perturbative PDF at $\mathcal{O}(\alpha_s^2 C_F T_F)$ $\surd$

Massive threshold corrections

Example: threshold correction in jet sector

bare jet function:

$$J^{\text{bare}} = Z_J^{OS} \otimes J^{\text{OS}} = Z_J^{\overline{\text{MS}}} \otimes J^{\overline{\text{MS}}}$$

in OS renormalization:

$$J^{\text{OS}}(s, m, \mu) = J^{(n_l)}(s, \mu) + \theta(s - 4m^2)\delta J_m^{\text{real}}(s, m) \xrightarrow{m \gg s} J^{(n_l)}(s, \mu)$$

in $\overline{\text{MS}}$ renormalization:

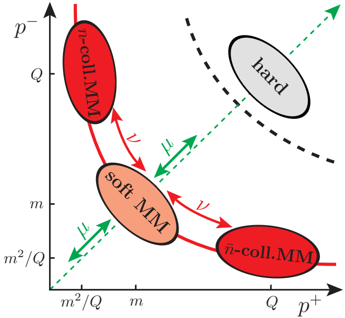
$$J^{\overline{\text{MS}}}(s, m, \mu) = J^{(n_l+1)}(s, \mu) + \delta J_m^{\text{dist}}(s, m, \mu) + \theta(s - 4m^2)\delta J_m^{\text{real}}(s, m) \xrightarrow{m \ll s} J^{(n_l+1)}(s, \mu)$$

$$\boxed{\Rightarrow \mathcal{M}_J(s, m, \mu) = J^{\text{OS}}(s, m, \mu) \otimes (J^{\overline{\text{MS}}}(s, m, \mu))^{-1}}$$

$\rightarrow$ matching condition directly related to jet function
$\rightarrow$ continuity by construction

Rapidity logarithms

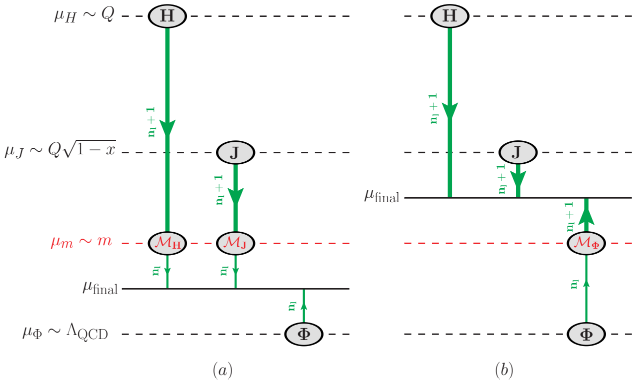
- note: large logarithms present in threshold corrections
  $\rightarrow$ in $\mathcal{M}_H$: $\ln\left(\frac{Q^2}{m^2}\right)$, in $\mathcal{M}_J$: $\ln\left(\frac{Q^2(1-x)}{m^2}\right)$, in $\mathcal{M}_\Phi$: $\ln(1-x)$
- related to separation of mass-shell fluctuations in rapidity, e.g. for $\mathcal{M}_H$:



- resummation via "rapidity RGE" [Chiu, Jain, Neill, Rothstein (2012)]
  $\leftrightarrow$ alternative: collinear anomaly [Becher, Neubert (2011)]

$\Rightarrow$ rapidity logarithms exponentiate

Consistency conditions for $Q\sqrt{1-x} > m > \Lambda_{\text{QCD}}$



physical cross section independent of $\mu_{\text{final}} \to$ (a) and (b) equivalent
$\to$ relation between evolution factors

$$U_H^{(n_f)} \times U_J^{(n_f)} = \left(U_\Phi^{(n_f)}\right)^{-1} \quad \text{for } n_f = n_l, n_l+1$$

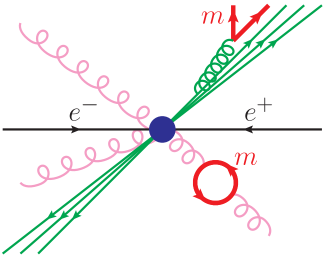
$\to$ relation between massive threshold contributions

$$\boxed{\mathcal{M}_H \times \mathcal{M}_J = \mathcal{M}_\Phi}$$

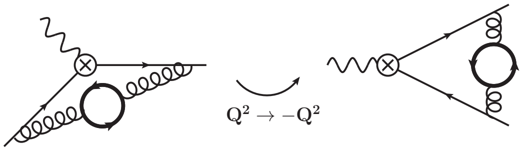
Outline

Event shapes

- goal: VFNS for differential distributions in e^+e^--collisions

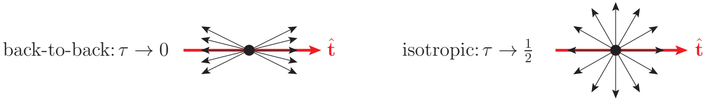


- in fact: similar to DIS (crossed process + universal ingredients)

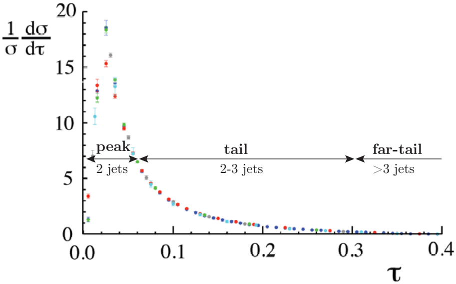


Event shapes: Thrust

- event shape variables: geometric description of final state kinematics
- thrust: $\tau \equiv 1 - \max_{\hat{\mathbf{t}}} \frac{\sum_i |\hat{\mathbf{t}} \cdot \vec{p}_i|}{\sum_i E_i} \in [0, \frac{1}{2}]$



- thrust distribution from LEP data ($e^+e^- \to jets$)



→ peak region ($\tau \sim \Lambda_{QCD}/Q$): typical jet scale $s = Q\Lambda_{QCD}$
→ tail region ($\tau \gg \Lambda_{QCD}/Q$): typical jet scale $s = Q^2\tau$

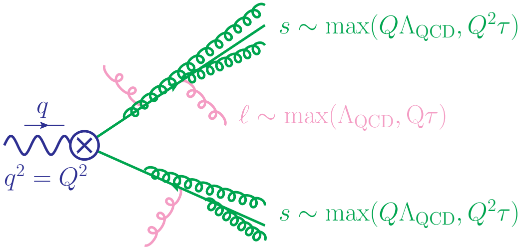
Factorization theorem for massless quarks

Massless factorization theorem for $\tau \ll 1$:

$$\frac{d\sigma}{d\tau} \sim H_\tau(\mu_H)\, J_\tau(\mu_J) \otimes S_\tau(\mu_S)\, [1 + \mathcal{O}(\tau)]$$

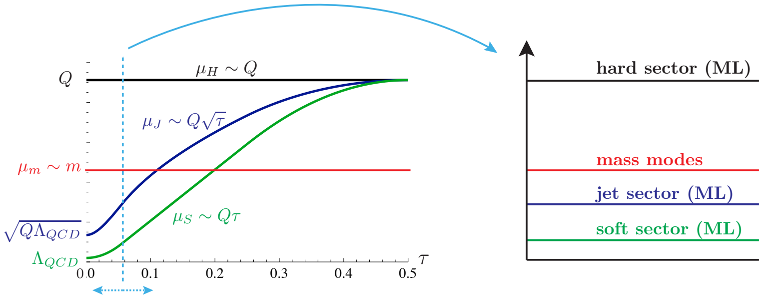
[Berger, Kucs, Sterman (2003); Fleming et al. (2007); Bauer et al. (2008), . . .]

- compared to DIS: $H_\tau = H_{\rm DIS}(Q^2 \to -Q^2)$, $J_\tau \to J_{\rm DIS} \otimes J_{\rm DIS}$
- main difference concerns soft physics: $S_\tau \leftrightarrow \Phi_{i/P}$
  $\to$ in tail region ($\tau \gg \Lambda_{\rm QCD}/Q$): $\mu_S \sim Q\tau \gg \Lambda_{\rm QCD}$



Scale hierarchies with massive quarks

- profile functions: Parametrization of renormalization scales in terms of thrust
  $\rightarrow$ continuous transition between peak, tail and far-tail region
- include massive quark effects $\rightarrow$ scales and hierarchies:



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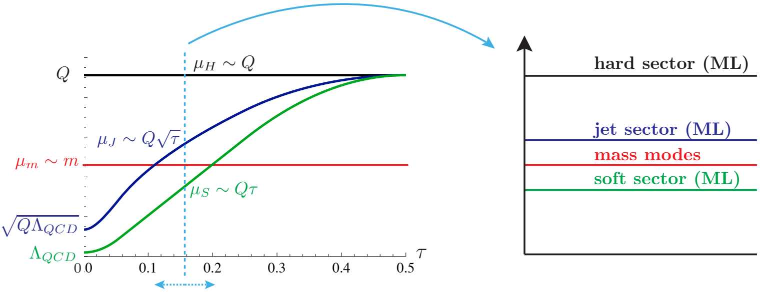

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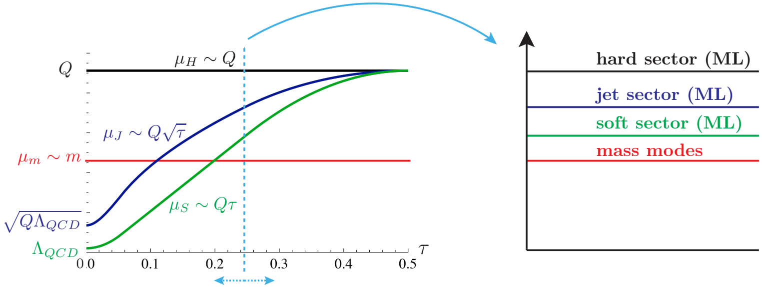
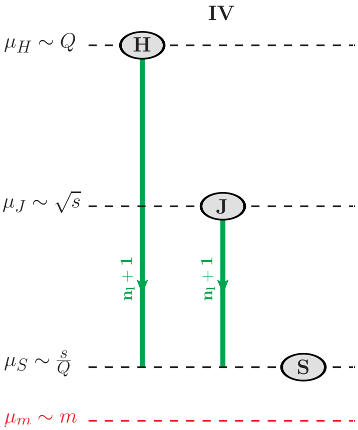


Setup for secondary massive quarks

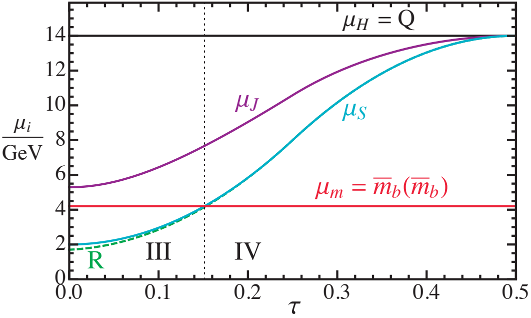
- Setup for event shapes $\approx$ Setup for DIS (similar factorization theorems)
- now: additional hierarchy possible $m < Q\tau \sim \mu_S$
  $\rightarrow$ $\overline{\text{MS}}$ renormalization for all structures
  $\rightarrow$ massive contributions to soft function [Gritschacher, Hoang, Jemos, P.P. (2013)]



Analysis of secondary massive bottom effects

[Gritschacher, Hoang, Jemos, Mateu, P.P. (2014)]

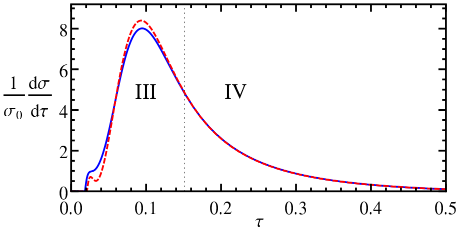
- analysis for $Q = 14, 22, 35$ GeV $\leftrightarrow$ bottom mass effects relevant
- ingredients for analysis at N^{3}LL in the dijet region $\tau \ll 1$ $\surd$
- profile functions for $Q = 14$ GeV:



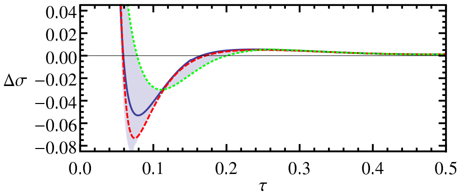
Secondary massive bottom effects for $Q = 14$ GeV

comparison between massless and massive thrust distribution
ML: $n_l = 5$, M: $n_l = 4$ & massive b ($m_b = 4.2$ GeV)

massive vs. massless



relative deviation massive vs. massless
$\mu_m = m$, $\mu_m = m/2$, $\mu_m = 2m$



Outline

Summary & Outlook

- understanding of massive quark effects is important in precision QCD
- VFNS cover different hierarchies between the mass scale and the kinematic scales
- proper renormalization schemes allow for log-resummation and correct limits
- our contribution: VFNS with final state jets
  $\rightarrow$ DIS for $x \rightarrow 1$: setup
  $\rightarrow$ thrust distribution for $\tau \rightarrow 0$: setup + numerical analysis
- Applications:
  $\rightarrow$ analysis of low-energy LEP data ($Q = 14, 22, 35$ GeV)
  $\rightarrow$ top quark mass determination from reconstruction
  $\rightarrow$ VFNS for $pp \rightarrow X + N$ jets

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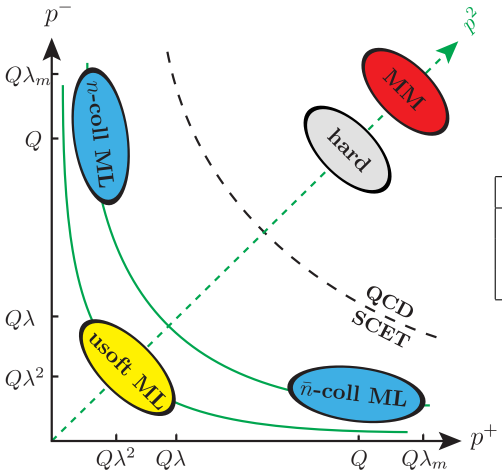
Thank you!

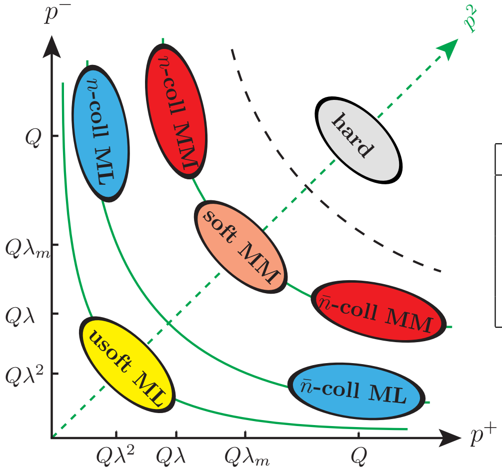
Outline

Scenario I. $\lambda_m > 1 > \lambda > \lambda^2$



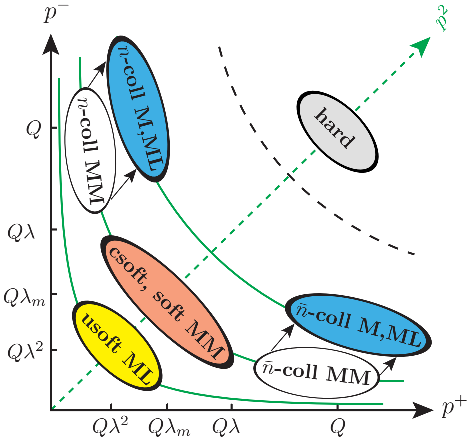
mode	$p^\mu = (+, -, \perp)$	p^2
hard	$Q(1, 1, 1)$	Q^2
n-coll ML	$Q(\lambda^2, 1, \lambda)$	$Q^2\lambda^2$
usoft ML	$Q(\lambda^2, \lambda^2, \lambda^2)$	$Q^2\lambda^4$

Scenario II. $1 > \lambda_m > \lambda > \lambda^2$



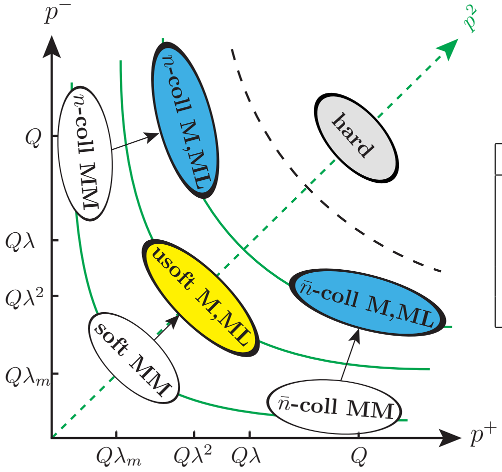
mode	$p^\mu = (+, -, \perp)$	p^2
hard	$Q(1, 1, 1)$	Q^2
n-coll MM	$Q(\lambda_m^2, 1, \lambda_m)$	m^2
soft MM	$Q(\lambda_m, \lambda_m, \lambda_m)$	m^2
n-coll ML	$Q(\lambda^2, 1, \lambda)$	$Q^2\lambda^2$
usoft ML	$Q(\lambda^2, \lambda^2, \lambda^2)$	$Q^2\lambda^4$

Scenario III. $1 > \lambda > \lambda_m > \lambda^2$



mode	$p^\mu = (+, -, \perp)$	p^2
hard	$Q(1, 1, 1)$	Q^2
n-coll M	$Q(\lambda^2, 1, \lambda)$	$Q^2\lambda^2$
n-coll ML	$Q(\lambda^2, 1, \lambda)$	$Q^2\lambda^2$
n-csoft MM	$Q(\lambda^2, \frac{\lambda_m^2}{\lambda^2}, \frac{\lambda_m}{\lambda})$	m^2
soft MM	$Q(\lambda_m, \lambda_m, \lambda_m)$	m^2
usoft ML	$Q(\lambda^2, \lambda^2, \lambda^2)$	$Q^2\lambda^4$

Scenario IV. $1 > \lambda > \lambda^2 > \lambda_m$



mode	$p^\mu = (+, -, \perp)$	p^2
hard	$Q(1,1,1)$	Q^2
n-coll M	$Q(\lambda^2, 1, \lambda)$	$Q^2\lambda^2$
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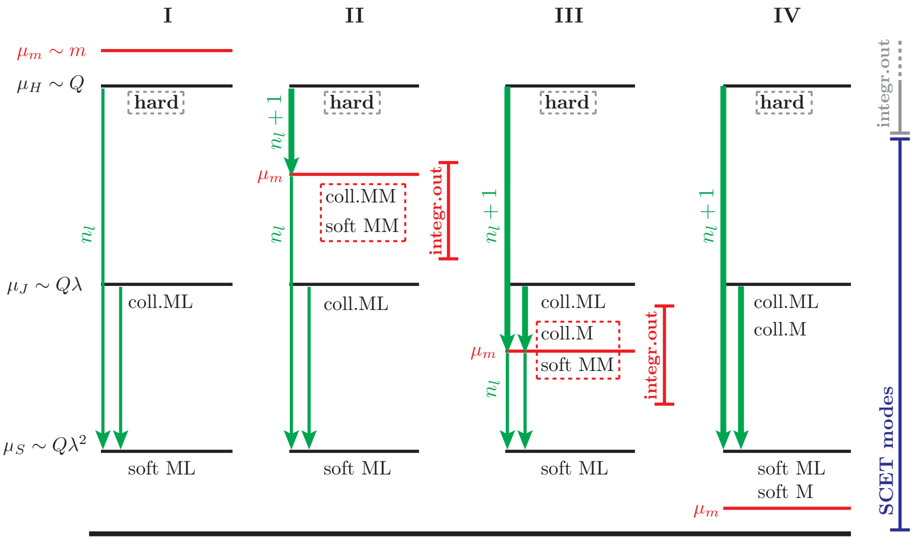
Mass mode setup: Summary



MM = mass-mode, ML = massless, M = massive

Factorization theorems

- I. $m > Q$: massive quark integrated out when matching to SCET

$$\boxed{\frac{d\sigma}{d\tau} \sim H_l^{(n_l)}(\mu_H) U_H^{(n_l)}(\mu_H, \mu_\Phi)\, J^{(n_l)}(\mu_J) \otimes U_J^{(n_l)}(\mu_J, \mu_\Phi) \otimes \Phi^{(n_l)}(\mu_\Phi)}$$

modification of hard matching coefficient due to massive quark
$\rightarrow$ use OS renormalization for current
$\Rightarrow$ decoupling for $m \gg Q$, but mass-singularities for $m \rightarrow 0$

Factorization theorems

- I. $m > Q$: massive quark integrated out when matching to SCET

$$\frac{d\sigma}{d\tau} \sim H_I^{(n_l)}(\mu_H) U_H^{(n_l)}(\mu_H, \mu_\Phi)\, J^{(n_l)}(\mu_J) \otimes U_J^{(n_l)}(\mu_J, \mu_\Phi) \otimes \Phi^{(n_l)}(\mu_\Phi)$$

- II. $Q > m > Q\sqrt{1-x}$: virtual mass mode contributions in SCET

$$\frac{d\sigma}{d\tau} \sim H_{II}^{(n_l+1)}(\mu_H) U_H^{(n_l+1)}(\mu_H, \mu_m)\, \mathcal{M}_H(\mu_m) U_H^{(n_l)}(\mu_m, \mu_\Phi) \times J^{(n_l)}(\mu_J) \otimes U_J^{(n_l)}(\mu_J, \mu_\Phi) \otimes \Phi^{(n_l)}(\mu_\Phi)$$

modification of hard matching coefficient, subtractions due to SCET diagrams
$\rightarrow$ use $\overline{\text{MS}}$ renormalization for hard current matching
$\Rightarrow$ correct massless limit for $m \ll Q$
additional current mass mode matching contribution $\mathcal{M}_H$ at μ_m

Factorization theorems

- I. $m > Q$: massive quark integrated out when matching to SCET

$$\frac{d\sigma}{d\tau} \sim H_I^{(n_l)}(\mu_H) U_H^{(n_l)}(\mu_H, \mu_\Phi)\, J^{(n_l)}(\mu_J) \otimes U_J^{(n_l)}(\mu_J, \mu_\Phi) \otimes \Phi^{(n_l)}(\mu_\Phi)$$

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- III. $Q\sqrt{1-x} > m > \Lambda_{\rm QCD}$: virtual and real mass mode contributions in SCET

$$\frac{d\sigma}{d\tau} \sim H_{II}^{(n_l+1)}(\mu_H) U_H^{(n_l+1)}(\mu_H, \mu_m)\, \mathcal{M}_H(\mu_m) U_H^{(n_l)}(\mu_m, \mu_\Phi) \times J^{(n_l+1)}(\mu_J) \otimes U_J^{(n_l+1)}(\mu_J, \mu_m) \otimes \mathcal{M}_J(\mu_m) \otimes U_J^{(n_l)}(\mu_m, \mu_\Phi) \otimes \Phi^{(n_l)}(\mu_\Phi)$$

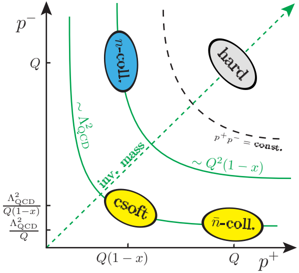
modification of the jet function at μ_J due to massive quark
$\rightarrow$ use $\overline{\rm MS}$ renormalization $\Rightarrow$ correct massless limit for $m \ll Q\sqrt{1-x}$

$$J^{(n_l+1)}(s, m, \mu_J) = J_0^{(n_l+1)}(s, \mu_J) + \delta J_m^{\rm dist}(s, m, \mu_J) + \theta(s - 4m^2)\delta J_m^{\rm real}(s, m)$$

additional jet mass mode matching contribution $\mathcal{M}_J$ at μ_m

Massless mode setup and factorization

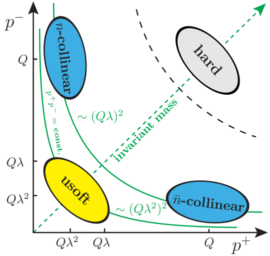
- DIS



Factorization theorem for $1 - x \ll 1$:

$$F_1 \sim \sum_{i=q} H_{\rm DIS}(\mu_H) J_{\rm DIS}(\mu_J) \otimes \underbrace{S_{\rm DIS}(\mu_\Phi) \otimes f_{i/P}(\mu_\Phi)}_{=\Phi_{i/P}(\mu_\Phi)}$$

- Thrust



Factorization theorem for $\tau \ll 1$:

$$\frac{d\sigma}{d\tau} \sim H_\tau(\mu_H) J_\tau(\mu_J) \otimes S_\tau(\mu_S)$$

Plots: secondary massive bottom quarks

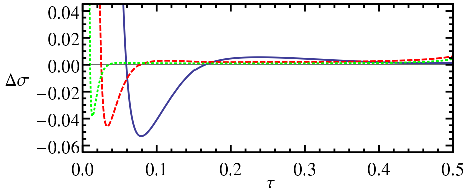


Figure : $Q = 14$ GeV (blue, solid), $Q = 35$ GeV (red, dashed) and $Q = m_Z$ (green, dotted).

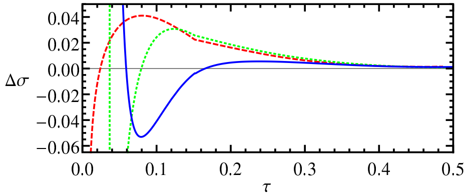


Figure : $Q = 14$ GeV: partonic (red, dashed), incl. the nonperturbative soft model function (green, dotted) and + gap formalism = default (blue, solid).

Plots: secondary massive top quarks

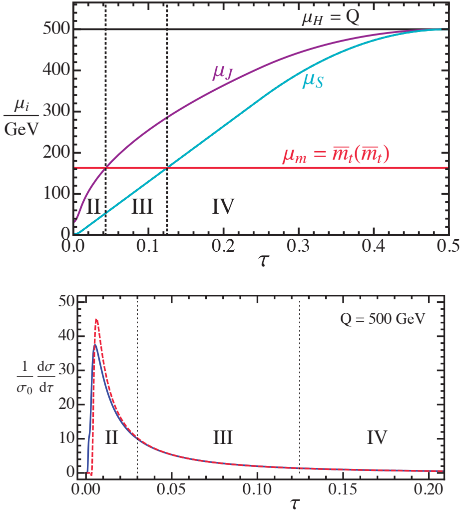


Figure : $Q = 500$ GeV: massive (blue, solid) vs. massless (red, dashed).

Plots: secondary massive top quarks

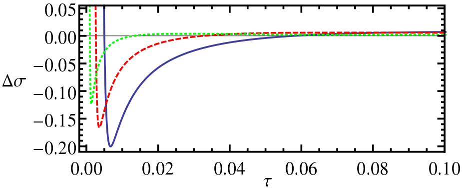


Figure : $Q = 500$ GeV (blue, solid), $Q = 1000$ GeV (red, dashed) and $Q = 3000$ GeV (green, dotted).

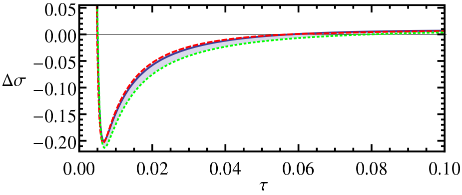


Figure : $Q = 500$ GeV: $\mu_m = m_t$ (blue, solid), $\mu_m = m_t/2$ (red, dashed) and $\mu_m = 2m_t$ (green, dotted).