The Holographic Principle

Severin Lüst

Outline:

- I Review
- II Entropy bounds from BHs
- III The holographic principle
- IV Entropy on light-like surfaces

Literature:

- gr-qc/9310026
- hep-th/9409089
- hep-th/0203101
- Susskind, Lindesay: An Introduction to Black Holes, Information and the String theory revolution: The Holographic Universe.

I Review

- Entropy of a BH:

$$\boxed{S = \frac{A}{4}},$$ where A is the area of the Schwarzschild horizon $\left(R_s = \frac{2MG}{c^2}\right)$

II Entropy bounds from BHs

- „naive" counting of states in QG

  × Introduce a cutoff of order $\sim M_{pl}$

  $\leftrightarrow$ lattice with spacing l_p

  × e.g.: spin lattice in a box of volume V:

$$N(V) = 2^n$$

$\uparrow$ dimensionality of Hilbert space

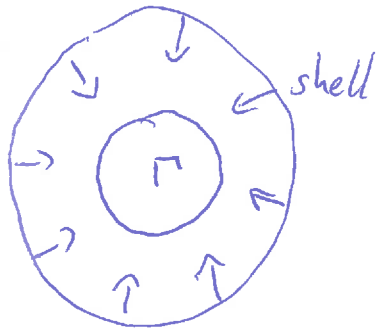
$\Rightarrow S_{max} \sim n \log 2 = V \cdot \frac{\log 2}{l_p^3}$ (maximal entropy)

× general local QFT: $\boxed{S \sim V}$

• Spherical entropy bound: the Susskind process:

× spherical region Γ filled with matter of mass

$$M \leq M_{BH}$$



$S_{initial} = S_{matter} + S_{shell}$

× collapse a shell of ~~extra~~ energy $M_{BH} - M$ into Γ

$\rightsquigarrow$ a BH will be created:

$S_{final} = S_{BH} = \frac{A}{4}$

× 2nd law of thermodynamics:

$$\boxed{S_{matter} < S_{BH} = \frac{A}{4}}$$

• Example: Gas with temperature T

$$\left.\begin{aligned} E &\sim Z R^3 T^4 \\ S &\sim Z R^3 T^3 \end{aligned}\right\} \Rightarrow S \sim Z^{1/4} R^{3/4} E^{3/4} \leq Z^{1/4} A^{3/4} < \frac{A}{4}$$

(notice: to have a valid geometric description, we need $A \gg 1$)

- Problem: local QFT has too many states

Increasing entropy $\longrightarrow$ Increasing energy $\longrightarrow$ BH

(in a finite spatial region entropy can't be generated by long wave length modes)

if energy density is too high

$\rightsquigarrow$ most of the states in QFT are too massive to be gravitational stable

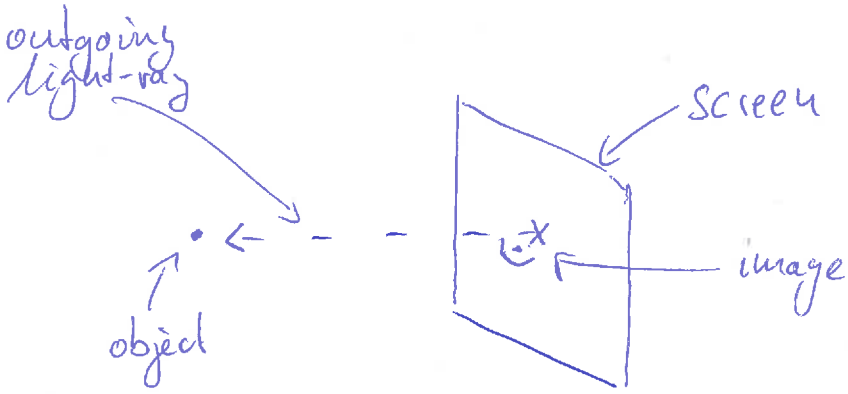
$\rightsquigarrow$ System with the highest possible energy density (BH) saturates the bound $S \leq \frac{A}{4}$

III The holographic principle

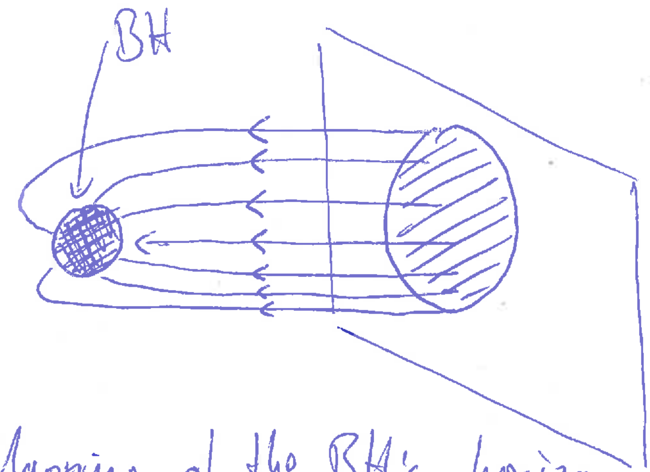
A region Γ with boundary area A is described by no more than $\frac{A}{4}$ degrees of freedom, so it should be possible to encode all the physics inside Γ on its boundary $\delta\Gamma$.

• Project all 3d physics to a far distant „viewing screen"

× static case: trace back light rays perpendicular to the screen



× Black Hole:



Mapping of the BH's horizon onto the screen.

× Entropy density $\sigma(x)$ on the screen:

Use focusing theorem from GR:

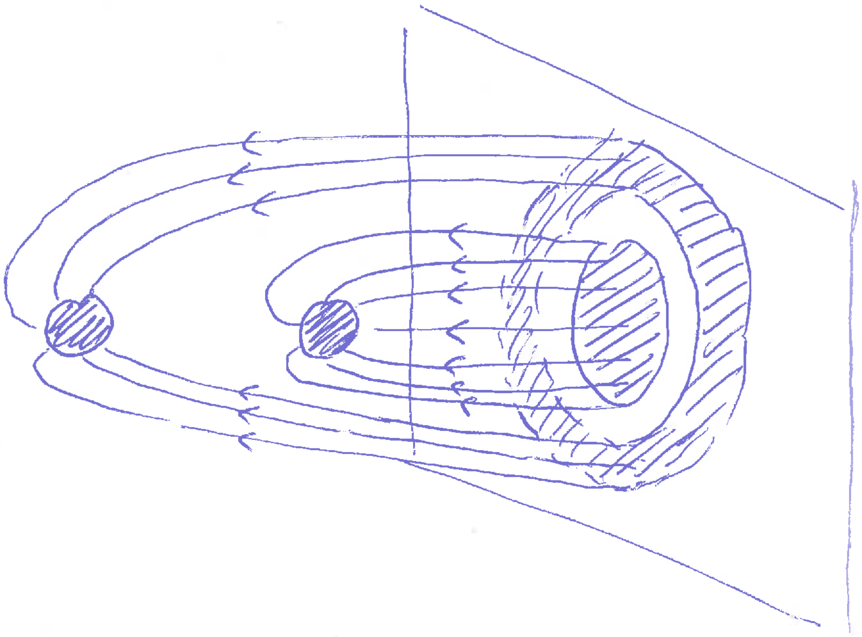
cross section of a bundle of light rays

$$\frac{d^2\alpha}{d\lambda^2} \leq 0$$

affine parameter

Close to the screen: $\frac{d\alpha}{d\lambda} \to 0$ $\Rightarrow$ $\boxed{\sigma(x) \leq \frac{1}{4}}$

× 2 BHs: Images do not overlap
⇝ no loss of information!



• <u>More general</u>: Asymptotically flat space-time:

× Introduce light-cone coordinates:

$$x^{\pm} = z \pm t, \quad x^{i}$$

× Screen: hypersurface $x^{-} = \text{const} \to \infty$

× holographic mapping:

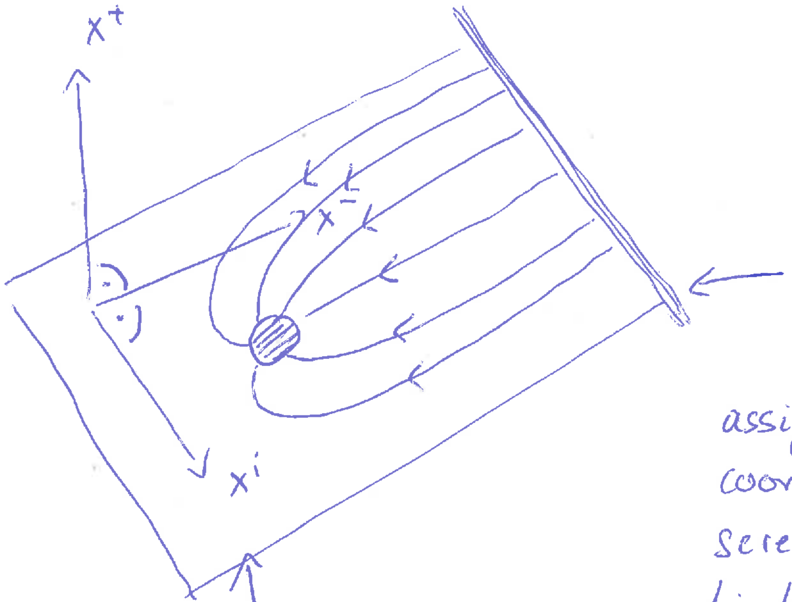
$$(x^{+}, x^{-}, x^{i}) \longrightarrow (x^{+}, x^{i})$$

time on the screen

spatial coordinates on the screen

× „light-sheet":

set of all light rays which lie for $x^{-} \to \infty$ in the same surface $x^{+} = \text{const}$.



assign a point the coordinates (x^+, x^i) on the screen, if it lies on a light ray which has asymptotic coordinates (x^+, x^i)

IV Entropy on light-like surfaces

(a covariant entropy bound)

- So far: $S(V) \leq \frac{A(\delta V)}{4}$

space-like volume

But: Not successful for less trivial examples (See: hep-th/0203101)

- Counterexample: FRW-cosmology

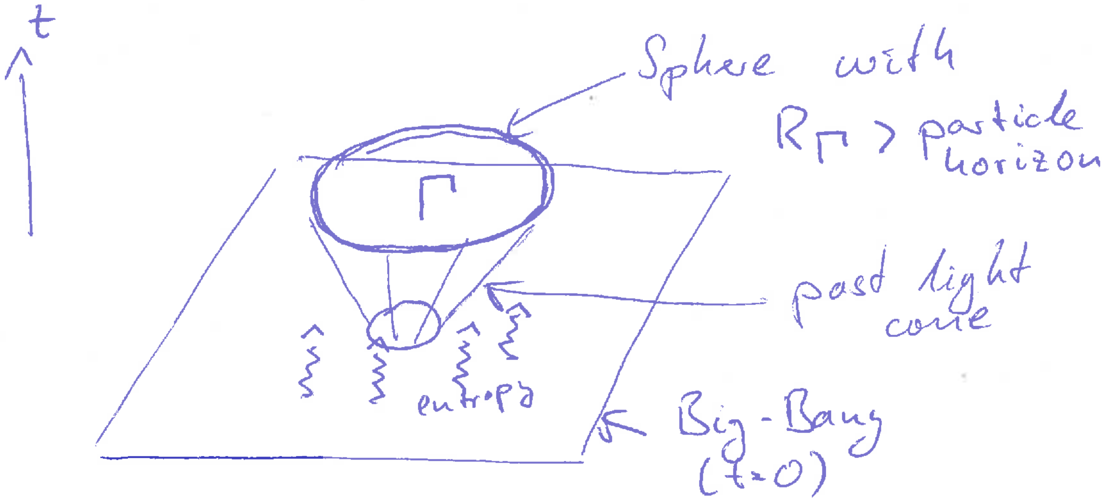
$\times$ $ds^2 = dt^2 - a^2(t)\, dx_i dx^i$

$= a^2(\eta)\left(d\eta^2 - dx_i dx^i\right)$

$\times$ uniform entropy-density:

$$s(\eta) = \frac{s}{a^3(\eta)}$$

⇝ for Γ large enough the entropy bound will be violated:



× entropy flux through Γ is not bounded

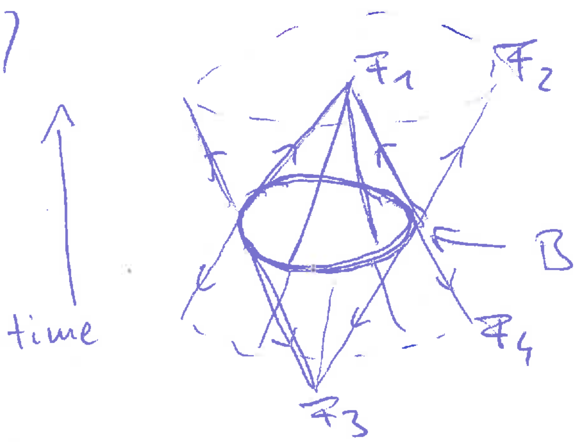
× But: Entropy flux through light cone is bounded!

- The example and the construction from section III suggest:
  Consider light-like hypersurfaces, instead of space-like regions!

- <u>Idea / Recipe:</u>

× Start with a codimension-2 surface B (the boundary ...)

× Follow light rays traveling away from B (in the future and in the past)



× obtain four null hypersurfaces $F_1, \dots, F_4$,
~~ending on~~ with boundary B

Problem: Not all of them are suitable to give an entropy bound ⇝ need a notion of inside/outside.

× select (at least) two hypersurfaces with negative expansion Θ (here: F_1, F_3)
⇝ light sheets

- The expansion Θ

„Following the light-rays, the area should shrink"

e.g. F_1:

$$A(B) \geq A(B')$$

- mathematically:

$$\boxed{\Theta(\lambda)\Big|_{\lambda=\lambda_0} \leq 0}$$

surface area

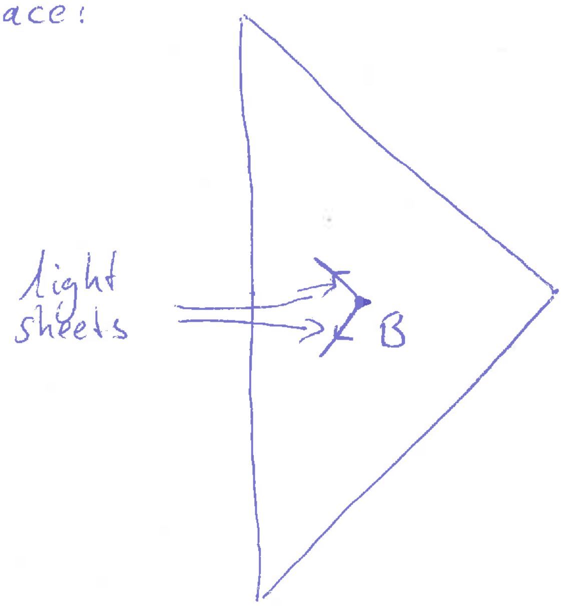
where the expansion Θ is defined as $\Theta(\lambda) = \frac{1}{A}\frac{dA}{d\lambda}$

- Entropy bound for light-sheets

If the hypersurface L is a light-sheet of B, then (i.e. $\Theta \leq 0$)

$$\boxed{S(L) \leq \frac{A(B)}{4}}$$

- Notation in Penrose-Diagrams,
  i.e. Minkowski-space:



- Three different situations:

  a) normal:  (for example in flat space-time)

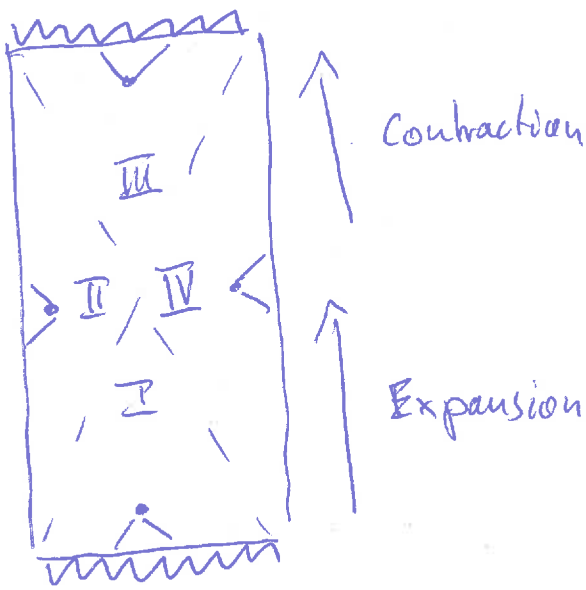
  b) anti-trapped:  (highly expanding space-time)

  c) trapped  (contracting universe, interior of BH, ..)

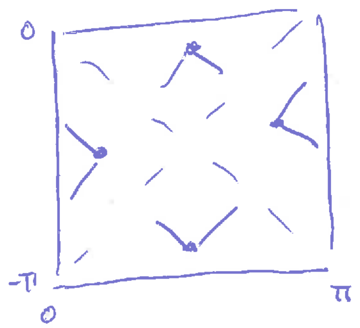
~~Matter do~~

- Examples

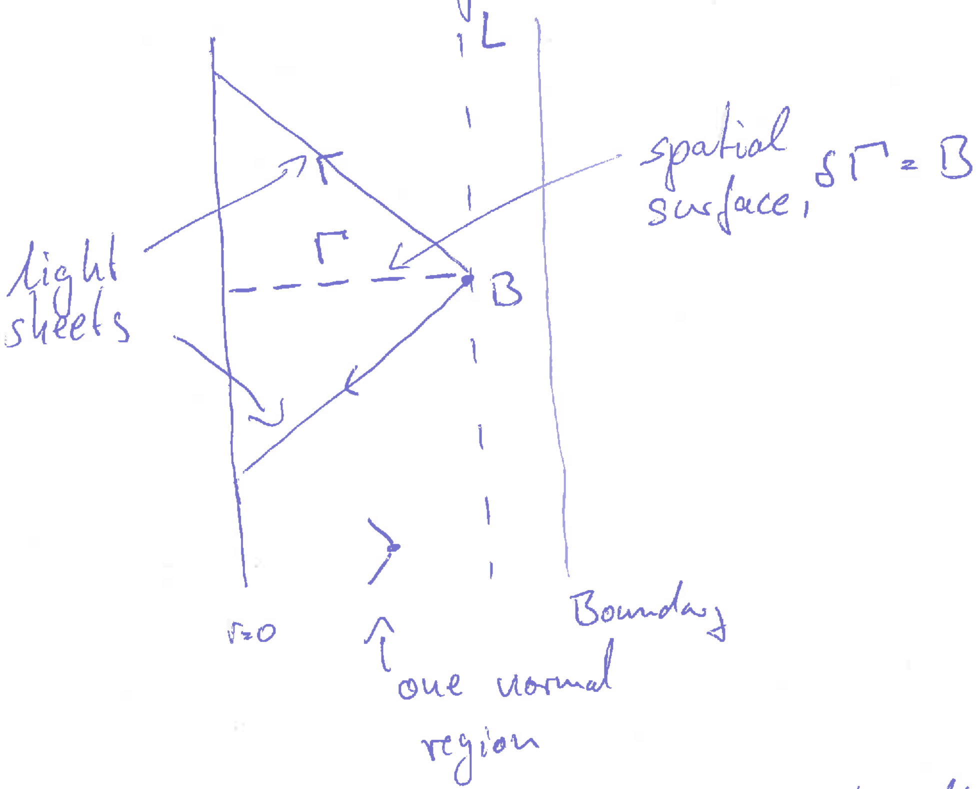
× Matter dominated universe:



× de Sitter cosmology $(\Lambda > 0)$



× Anti-de-Sitter space $(\Lambda < 0)$



$\rightsquigarrow$ the entropy on Γ is holographically bounded,

$$S(\Gamma) \leq \frac{A(B)}{4}$$

$\rightsquigarrow$ AdS can be foliated by space-like surfaces satisfying the holographic bound

$\rightsquigarrow$ all physics in the interior of L can be described in terms of a Hamiltonian acting on a Hilbert space of dimension

$$N_{\text{states}} = \exp\left(\frac{A(B)}{4}\right)$$

$\leftrightarrow$ corresponds to a QFT on the boundary $\mathcal{B}$

$\rightsquigarrow$ notice: L acts like an infrared cutoff

$\rightsquigarrow$ move L towards the boundary:

physics (QG) in the interior $\longleftrightarrow$ QFT on the boundary