The Information Paradox

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QM Basics

In the Schrödinger picture, have <u>unitary</u>

$$|\Psi(t)\rangle = U(t,t_0)|\Psi(t_0)\rangle$$

Given some pure state

$$|\Psi(t)\rangle = \sum_i c_i(t)|\Psi_i\rangle \quad \left(\begin{array}{c}\text{linear comb.}\\ \text{of eigenstates}\end{array}\right)$$

future evolution of $|\Psi(t)\rangle$ <u>precisely known</u>: $|\Psi(t_{future})\rangle$ may be predicted with certainty!

Now, consider two systems $|\Psi\rangle_A$, $|\phi\rangle_B$ which have been in contact and are no longer interacting.

Most general state in $H_A \otimes H_B$ is

$$|\Psi\rangle_{AB} = \sum_{ij} c_{ij}|i\rangle_A \otimes |j\rangle_B$$

This is a pure state!

Separable for $c_{ij} = c_i^A c_j^B$ (+)

inseparable for $c_{ij} \neq c_i^A c_j^B$ (*)

(*) <u>Quantum Entanglement</u>

<u>Impossible</u>, to define pure state for system A <u>or</u> B! Instead: Think density matrix!

$$\rho_A = \sum_i p_i |\Psi\rangle\langle\Psi| \,, \quad \rho_B = \sum_i p_i |\Psi\rangle\langle\Psi|$$

Measurement outcome $\langle A\rangle = Tr(A\rho_{A,B})$.

If, say H_B of $H_A \otimes H_B$ becomes <u>inaccessible</u> to observer, no pure state Ψ_A may be formulated and <u>mixed state</u> arises!

(pure state still exists in this scenario)

From QFT to Qubits...

equal time commutator

$$\langle \Omega | \phi(t,x)\phi(t,x') | \Omega \rangle \neq 0$$

Idea: Interpret correlation between different regions of spacetime as entanglement between two field excitations.

$\hookrightarrow$ $\boxed{\text{field excitation} \Rightarrow \text{qubit}}$

[Aside: $|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ has

$$\langle \psi | X_1 X_2 | \psi \rangle = 1 \quad , X_i \text{ is Pauli operator}$$]

Illustrative example: Vacuum $|\Omega\rangle$ in Rindler space

Recall generic way to obtain ground state

$$\langle \phi | \Omega \rangle \propto \lim_{T \to \infty} \langle \phi | e^{-HT} | \chi \rangle \propto \int \mathcal{D}\hat{\phi}\, e^{-I_E}$$

$|\chi\rangle$: generic state

I_E: Euclidean action

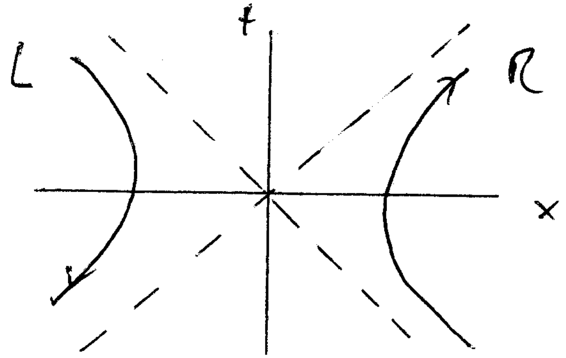
Rindler space has $\mathcal{H} = \mathcal{H}_L \otimes \mathcal{H}_R$ with ϕ_L, ϕ_R for $x<0$, $x>0$ respectively.

Aim: Find basis states in each factor to decompose the vacuum.

It may be shown (Harlow) that the vacuum may be cast as

$$\boxed{|\Omega\rangle \propto \sum_i e^{-\pi\omega_i} |i\rangle_L |i\rangle_R}$$

Immediately see thermal character & entanglement of basis states $|i\rangle_{L,R}$, i.e. the entanglement between subsystems in $\mathcal{H}_L$, $\mathcal{H}_R$ is manifest!



To the paradox...

Assume simple state

$$|\psi\rangle_{pair} = \frac{1}{\sqrt{2}}\left(|0\rangle_c|0\rangle_b + |1\rangle_c|1\rangle_b\right)$$

Considering matter, i.e. BH on spatial slice, have

$$|\psi\rangle \approx |\psi\rangle_M \otimes |\psi\rangle_{pair}$$

Locality allows for small departures from the above such as

$$|\psi\rangle = |\psi\rangle_M \otimes \left[\left(\tfrac{1}{\sqrt{2}}+\varepsilon\right)|0\rangle_c|0\rangle_b + \left(\tfrac{1}{\sqrt{2}}-\varepsilon\right)\ldots\right],$$

with $\varepsilon \ll 1$.

Recall Von-Neumann entropy:

$$S_{ent} = -\operatorname{tr}(\rho \ln \rho)$$

$\varepsilon = 0$: $S_{ent} = \ln 2$

$\varepsilon \neq 0$: $S_{ent} = \ln 2 - \mathcal{O}(\varepsilon^2)$

Or in other words $\frac{S_{ent}}{\ln 2} - 1 \ll 0$.

Leading order process

Start with

$$|\psi\rangle \approx |\psi\rangle_M \otimes |\psi\rangle_{pair}, \quad S_{ent} = \ln 2$$

"no" evolution

Next step:

$$|\psi\rangle \approx |\psi\rangle_M \otimes \underbrace{|\psi\rangle_{pair} \otimes |\psi\rangle'_{pair}}$$

Entanglement of $\{b_1, b_2\}$ with $\{c_1, c_2\}$ is $S_{ent} = 2\ln 2$.

For N steps:

$$|\psi\rangle \longrightarrow |\psi\rangle_M \otimes \bigotimes_{i=0}^{N} |\psi\rangle^i_{pair}$$

It follows that

$$\boxed{S_{ent} = N \ln 2}$$

Recall from Laura's talk, that B.H. will eventually <u>evaporate</u>!

This means, that $\{c_i\}$ quanta are "gone"!

$\{b_i\}$ quanta have $S_{ent} = N \ln 2$, but nothing left to be entangled with ?

I) $\Longrightarrow \mathcal{H} = \mathcal{H}_{u,c} \otimes \mathcal{H}_b \longrightarrow \mathcal{H}_b$

Final state <u>cannot</u> be described by some state vector but only by density matrix!

We are left with <u>mixed states</u>!

II) Have remaining object within bounds ($m \lesssim m_r$, $l \leq l_r$), i.e. remnant.

This object has to have

$$S_{ent} = N \ln 2 \sim \mathcal{O}(N)$$

$\hookrightarrow$ $\boxed{\text{unbounded degeneracy}}$

I) <u>Loss of unitarity</u>, as $|\Psi\rangle_{pure} \to \rho_{mixed}$

II) OK with QM, but unbounded degeneracy seems odd...

Also, remnant needs to hold <u>more</u> S_{ent}, than B.H. of comparable M!

<u>Deformations of leading order result</u>

Consider:

$$\underbrace{|\Psi\rangle = |\Psi\rangle'_{u} \otimes |\Psi\rangle^{N+1}_{pair}}_{|\Psi\rangle_u \underset{\text{iso}}{\otimes} |\Psi\rangle^N_{pair}} \quad \text{at } t_{N+1}$$

leading order

Now, however, assume new state at t_{n+1} to be spanned by

$$S^{(1)} = \frac{1}{\sqrt{2}}\left(|0\rangle_{c_{n+1}}|0\rangle_{b_{n+1}} + |1\rangle_{c_{n+1}}|1\rangle_{b_{n+1}}\right)$$

$$S^{(2)} = \frac{1}{\sqrt{2}}\left(|0\rangle_{c_{n+1}}|0\rangle_{b_{n+1}} - |1\rangle_{c_{n+1}}|1\rangle_{b_{n+1}}\right)$$

At t_n, have $|\Psi\rangle = |\Psi\rangle_M'$. Choose orthonormal basis χ_n for $\{b\}$ outside and ψ_n for $\{c_i\}$ inside horizon, so that

$$|\Psi\rangle = \sum_i c_i \psi_i \chi_i$$

For which $\rho_{ij} = |c_i|^2 \delta_{ij}$ <u>for $\{b_i\}$</u>

and $\quad S_{ent} = -\sum_i |c_i|^2 \ln |c_i|^2$ at t_n.

Now evolve to $t_n \to t_{n+1}$:

$$\chi_i \to \chi_i$$

$$\psi_i \to \psi_i^{(1)} S^{(1)} + \psi_i^{(2)} S^{(2)}$$

where ψ_i evolves to tensor product of $\psi_i^{(i)}$ and the $S^{(i)}$ of newly created pair.

Unitarity: $\|\psi_i^{(1)}\|^2 + \|\psi_i^{(2)}\|^2 = 1$

<u>Leading order:</u> $\boxed{\psi_i^{(1)} = \psi_i, \quad \psi_i^{(2)} = 0}$

Then at t_{n+1} have

$$|\Psi\rangle = \sum_i c_i \left[\psi_i^{(1)} S^{(1)} + \psi_i^{(2)} S^{(2)}\right] \chi_i$$

recast as

$$= S^{(1)} \Lambda^{(1)} + S^{(2)} \Lambda^{(2)}$$

<u>Define:</u> corrections are small when

$$\boxed{\|\Lambda^{(2)}\| < \epsilon, \quad \epsilon << 1} \qquad (*)$$

<u>Entropy bounds...</u>

Seek to show that if $(*)$ holds,

$\boxed{\Delta S > 0}$ for every step?

Then next-to leading order corrections can only alter Hawking state if they are $O(1)$!

<u>Idea:</u> b_{n+1}, c_{n+1} may weakly interact with $(M, \{c\})$, creating entanglement not present in leading order Hawking state.

Write $p = (c_{n+1}, b_{n+1})$

At t_n, have $S_{ent} \equiv S_0 = S_{\{b\}}$, which will remain S_0 as earlier $\{b_i\}$ may not be influenced by new pair!

Seek to show that

$$S(\{b\}, b_{n+1}) > S_0 - 2\varepsilon, \text{ i.e.}$$

$$\Delta S > 0 \text{ for } \varepsilon \ll 1.$$

a) If $\varepsilon \ll 1$, $\boxed{S(p) \equiv -\mathrm{tr}\,\rho_p \ln \rho_p < \varepsilon}$

(Proof: Start with $\rho_p = \begin{pmatrix} \lambda^{(p)}_{11} & \lambda^{(p)}_{12} \\ \lambda_{21} & \lambda_{22} \end{pmatrix}$

and use Schwartz inequality along with $\rho = \frac{1}{2}I + |\vec{a}|\sigma_3$)

b) $\boxed{S(\{b\} + p) \geq S_0 - \varepsilon}$

(Proof: $S(A+B) \geq |S(A) - S(B)|$
$A = \{b\},\ B = p$)

c) $\boxed{S_{n+1} > \ln 2 - \varepsilon}$

(Proof: evaluate ρ ...)

Thus $\boxed{\underbrace{S(\{b\}}_{\text{up to } t_n} + \underbrace{b_{n+1}) > S_0}_{\{b\}} + \underbrace{\ln 2}_{b_{n+1}} - 2\varepsilon}$

(Proof $S(A+B) + S(B+C) \geq S(A) + S(C)$
$A \to \{b\},\ B = b_{n+1},\ C = c_{n+1}$
$S(\{b\} + b_{n+1}) + S(p) \geq S(\{b\}) + S(c_{n+1})$)

<u>Concluding:</u>

If we assume:

(i) local Hamiltonian evolution

(ii) traditional black hole

$|\psi\rangle_{mode} = q_0|0\rangle + q_1|1\rangle + q_2|2\rangle \ldots$

(A) If $\sum_{i>0} |q_i|^2 \sim 1$: No vacuum at r_s!

$\hookrightarrow$ violation of (ii)

(B) So $\underline{\underline{\sum_{i>0} |q_i|^2 < \epsilon'}}$, $\epsilon' \ll 1$ (i.e. $S^{(1)}$)

And $\boxed{S_{ent} \sim N \ln 2, \; \Delta S > 0}$

If BH evaporates away, have $\{b_i\}$ entangled with nothing, otherwise need remnant with unbounded degeneracy!

If the state of Hawking radiation has to be a pure state with no entanglement, then the evolution of low energy modes at the horizon has to be altered by order unity!

<u>References:</u>

- Harlow, 1409.1231
- Mathur, 0909.1038