Stability structure of the GLR equation

UNIVERSITY OF CAMPINAS

Guillermo Gambini, Giorgio Torrieri

Conselho Nacional de Desenvolvimento Científico e Tecnológico

Instituto de Física Gleb Wataghin

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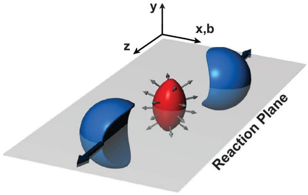
Elliptic flow v_2 (Harmonic flow v_n)

Well established in AA and recentrly found in pA even pp collisions



$$\frac{dN}{dp_T dy d\phi} = \frac{dN}{dp_T dy}\left[1 + \sum_{n=1}^{\infty} 2 v_n(p_T) cos(\phi - \phi_{0n})\right]$$

- Temperature, Radial flow
- Anisotropic flow affects the spatial orientation of particle momenta. The most dominant contribution to it is called **elliptic flow** v_2

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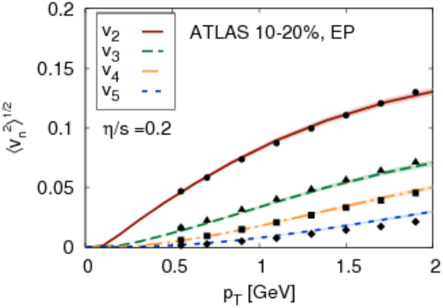
v_2 usual interpretation: Hydrodynamics

Hot QCD matter behaves like a "perfect fluid", initial anisotropies in the collision area produce anisotropies in the collective flow of matter.

Hydro simulations fit impressively all lower v_n at RHIC, LHC.

Phys.Rev.Lett. 110.012302



This is reasonable, but **scalings** in energy and system size of v_2 look remarkable **simple** compared to the Hydrodinamical picture.

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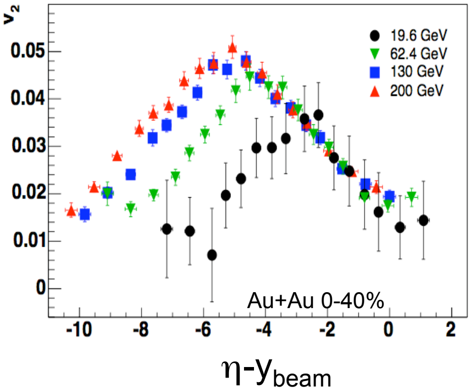
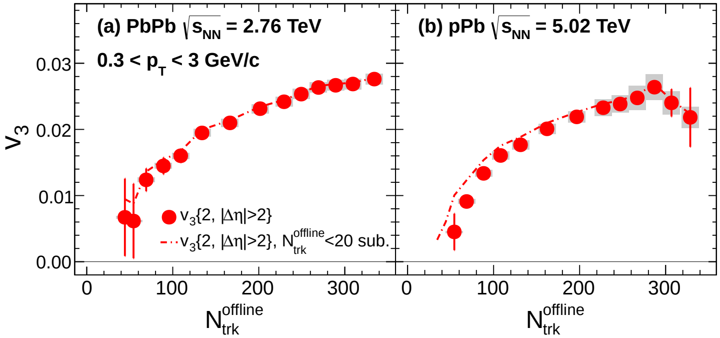
Elliptic flow v_2 scales with rapidity



Dependence similar to limiting fragmentation and Bjorken scaling

- PHOBOS collaboration,arXiv:0907.4719v3

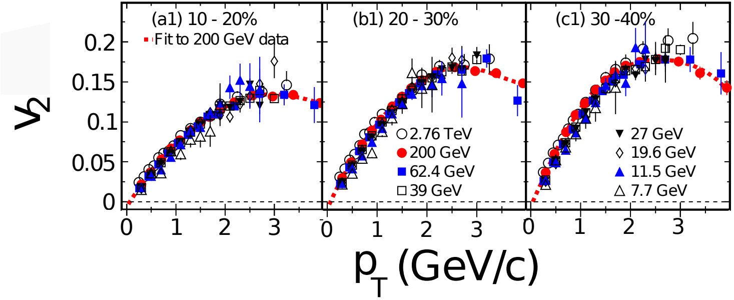
Elliptic flow v_2 scales with system size



No explicit dependence on system size/lifetime apart from eccentricity

- CMS Collaboration arXiv:1305.0609v3
- PHENIX Collaboration Phys. Rev. Lett. 111(2013) no.21,212301

Elliptic flow v_2 scales with energy



$v_2 = eccentricity \times \mathrm{F}(p_T)$, F is universal, and all that changes with energy is $< pT >$

- STAR Collaboration, arXiv:1210.4607v1
- BRAHMS Collaboration arXiv:0907.4742v2

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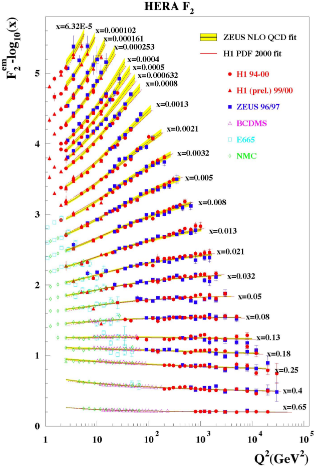
Bjorken scaling and its deviations

Structure functions depend on the scale they are measured; i.e. x and Q^2. In the perturbative limit dependence on Q^2 is subleading.

v_2 looks like Bjorken scaling

As $p_T \sim Q$ and $\eta \sim ln(\frac{1}{x})$, then scaling of elliptic flow in HIC may resemble Bjorken scaling when adding an angular dependence on the structure functions.



Also, structure functions should be azimuthally symmetric, but a non-linear evolution can introduce instabilities.

The Gribov-Levin-Ryskin & Mueller-Qiu (GLR-MQ) equation

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The GLR-MQ evolution equation

In the dense parton limit, the equation that governs the evolution of parton distribution functions inside hadrons is thought to be given by

$$\underbrace{\frac{Q}{2}\frac{\partial}{\partial Q}\frac{\partial xG(x,Q^2)}{\partial ln(1/x)} = \frac{\alpha_s N_c}{\pi} xG(x,Q^2)}_{BFKL} - \underbrace{\frac{\alpha_s^2 N_c \pi}{2 C_F S_\perp}\frac{1}{Q^2}[xG(x,Q^2)]^2}_{non-linear\ \ term}$$

where
x: Bjorken's x
Q^2: the photon's virtuality

BFKL breaks Froissart's bound at low x. GLR corrects this *divergence problem* adding a non-linear term.

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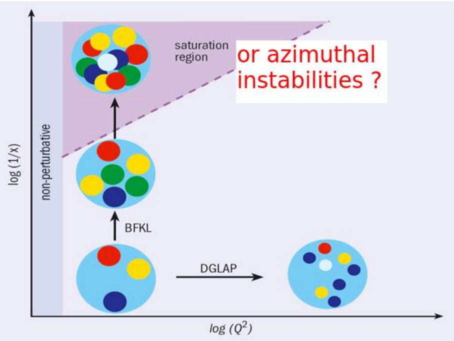
If we assume azimuthally symmetric evolution, then we get **saturation** $Q_s(x)$

$$0 = \frac{\alpha_s N_c}{\pi} xG(x, Q^2) - \frac{\alpha_s^2 N_c \pi}{2 C_F S_\perp} \frac{1}{Q^2} [xG(x, Q^2)]^2$$



but non-linear 2+1 differential equation *can have instabilities*.

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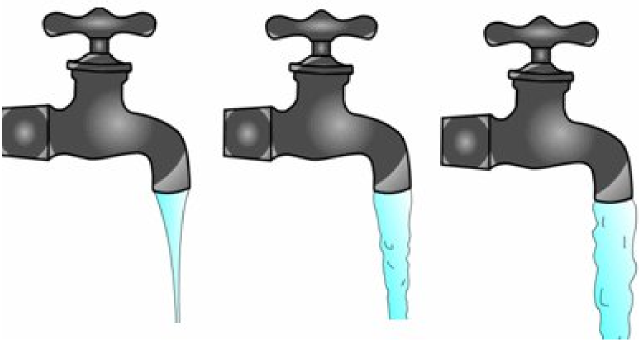

Under a non-linear evolution, at higher energies (smaller x), instabilities can break azimuthal symmetry. **How can we create such instabilities?**
Many non-linear 2D differential equation has this behaviour.

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The angular dependence proposal

Adding an angular dependence modifies the GLR-MQ equation the following way

$$\frac{xQ}{2}\left(\frac{\partial}{\partial Q}+\frac{1}{Q}\frac{\partial}{\partial\phi}\right)\frac{\partial}{\partial x}[xG(x,Q^2,\phi)]=\frac{\alpha_s N_c}{\pi}xG(x,Q^2,\phi)$$

$$-\frac{\alpha_s^2 N_c\pi}{2C_F S_\perp}\frac{1}{Q^2}[xG(x,Q^2,\phi)]^2$$

As a solution, we try

$$G(x,Q^2,\phi)=G_0(x,Q^2)\left(1+\sum_{n=1}^{\infty}u_n(x,Q^2)cos(n\phi+\beta_n)\right),$$

$G_0(x,Q^2)$ is the azimuthally symmetric solution (i.e. saturation)

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Equations for the Fourier coefficients

Working on the limiting case $Q << Q_s(x)$, we insert our proposal $\mathrm{G}(x, Q^2, \phi)$ into our equation and an infinite set of equations

$$(2\lambda+1)\frac{Q}{2}\frac{\partial u_n(x,Q^2)}{\partial Q} + \frac{Q}{2}x\frac{\partial^2 u_n(x,Q^2)}{\partial Q \partial x} = \frac{\alpha_s N_c}{\pi} u_n(x,Q^2)$$

$$+ \frac{N_c \pi}{2 C_F S_\perp \alpha_s^2}\frac{1}{Q^2}x^{2\lambda+1}\Bigg[2u_n(x,Q^2)$$

$$+\frac{1}{2}\sum_k^{n-1} u_k(x,Q^2)u_{n-k}(x,Q^2)cos(\beta_n - \beta_k - \beta_{n-k})$$

$$+\sum_k u_k(x,Q^2)u_{n+k}(x,Q^2)cos(\beta_n + \beta_k - \beta_{n+k})\Bigg]$$

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The polynomial solution

As a first attempt to solve this equation we try the following ansatz

$$u_n(x \to 0, Q^2) = \epsilon_{n,2}\delta_{n,2} \sum_{k=0}^{\infty} A_k \frac{(Bx^C)^k}{k!} Q^{D-2k}$$

which has wrong boundary conditions, but the <u>right scaling</u>

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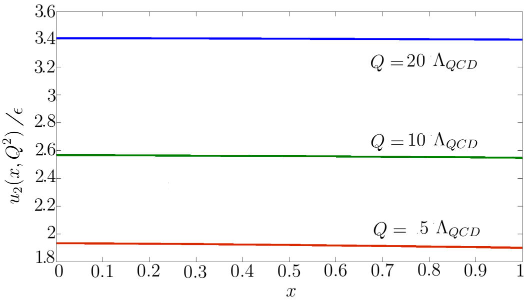
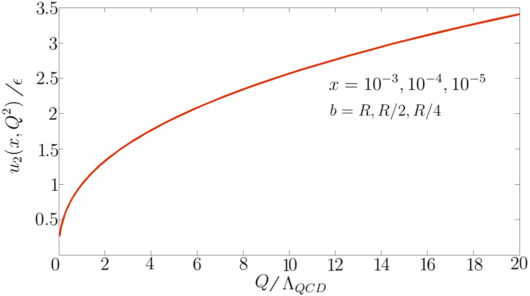


$Q = 20\ \Lambda_{QCD}$
$Q = 10\ \Lambda_{QCD}$
$Q = 5\ \Lambda_{QCD}$
$u_2(x, Q^2)/\epsilon$
x



$x = 10^{-3}, 10^{-4}, 10^{-5}$
$b = R, R/2, R/4$
$u_2(x, Q^2)/\epsilon$
Q/Λ_{QCD}

The linearized solution

We want to relate u_2 to v_2, and the latter is the dominant harmonic, so we set as a initial configuration

$$u_n(x, Q^2) \to \delta_{n,2} u_n(x, Q^2) = x^{-C} f(x^C/Q^2),$$

and we get a **Bessel equation** with pure imaginary order $(i\nu)$, giving us for our Gluon Distribution Function $xG(x, Q^2, \phi)$:

$$\frac{1}{\alpha_s^4}\left(x^{2\lambda+1}+[A(z)cos(\nu \ln z)+B(z)sin(\nu \ln z)]cos(2\phi+\beta_2)+...\right)$$

Here A(z) and B(z) are convergent series (arXiv:0910.0365, 2009), and $z = z(x^C/Q^2)$.

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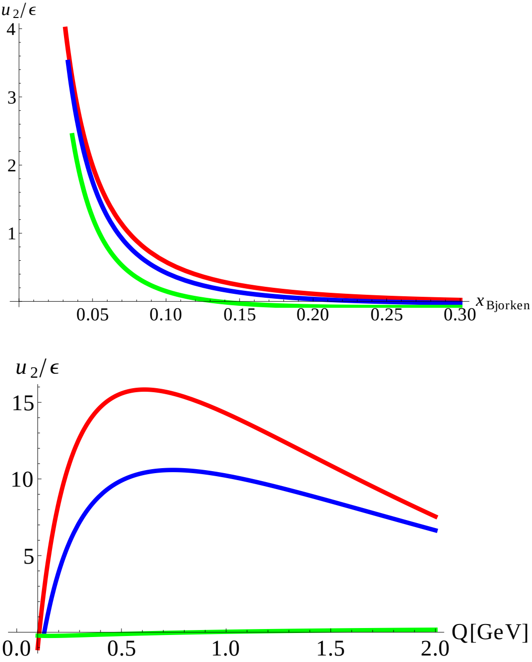

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k_T-factorization

Usually collinear approximation (Parton Model). To account for incident parton transverse momenta we use the k_T-factorization

$$\frac{dN}{dp_T\, d\phi\, d(x_A + x_B)} \sim \frac{1}{p_T^2} \int k dk d\theta f_A(x_A, k, \theta) f_B(x_B, p_T - k, \pi + \phi - \theta)$$

Still a poor approximation (neglects fragmentation), we use it since **scaling** is the most important characteristic of v_2 under study at this moment

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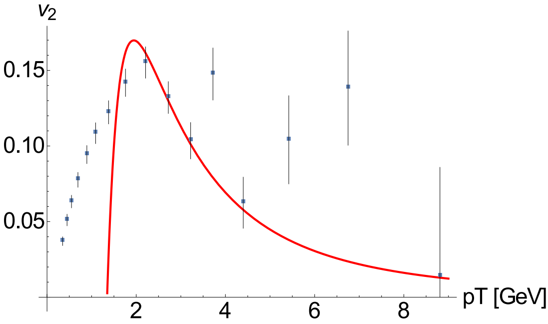
Inserting our gluon distribution function in the k_T-factorization formula we get (data from CMS collaboration, PRL109 022301 (2012), 50-60%)



$$v_2(p_T) \propto \frac{\Theta(\phi = 0) - \Theta(\phi = \pi/2)}{2(\Theta(\phi = 0) + \Theta(\phi = \pi/2))}$$

with

$$\Theta(\phi) = \int k dk d\theta f_A(x_A, k, \theta) f_B(x_B, p_T - k, \pi + \phi - \theta)$$

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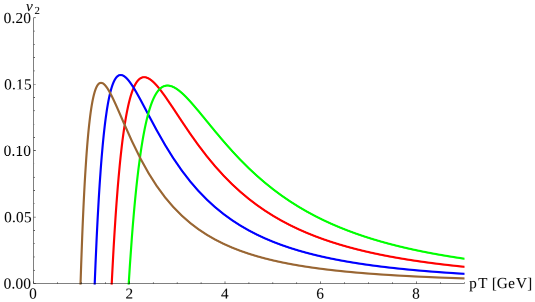
We are still exploring **scaling** of our solutions.



- red - $\sqrt{s}$ =2760 GeV
- blue - $\sqrt{s}$ =2160 GeV
- green - $\sqrt{s}$ =3360 GeV
- brown - $\sqrt{s}$ =1660 GeV

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- v_2 scaling similar to scaling of parton distribution functions. Could they be azimuthally asymmetric?
- Instabilities in the non-linear regime?
- If our model is right, perhaps one should see azimuthal correlations extended in rapidity in eA collisions at the Electron Ion Collider (EIC)
- Work in progress to test this hypothesis

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