Single inclusive forward hadron production at next-to-leading order

Bertrand Ducloué
(University of Jyväskylä)

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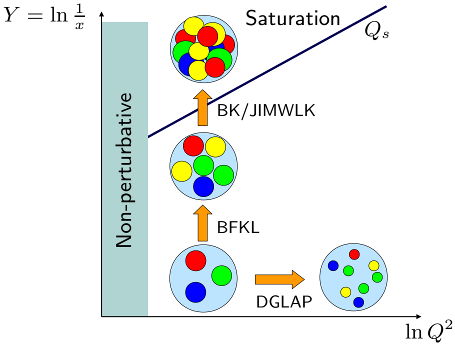
B. D., T. Lappi, Y. Zhu, arXiv:1604.00225 [hep-ph]

Our goal is to study QCD in the saturation regime

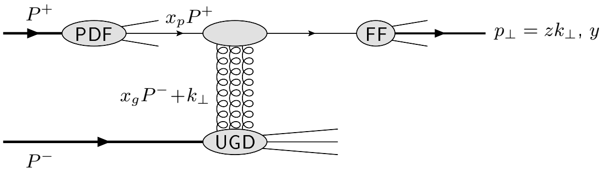


The production of forward particles is a crucial tool to probe small x values

Saturation effects should be enhanced by the higher densities in pA collisions

Here we study the inclusive production of a forward hadron in proton-nucleus collisions: $pA \to hX$

Single inclusive forward hadron production at LO in the $q \to q$ channel:



The values of x_p and x_g probed in the projectile and the target are given by $x_p = \frac{p_\perp}{\sqrt{s}} e^{y}$, $x_g = \frac{p_\perp}{\sqrt{s}} e^{-y}$

The dilute projectile is described in terms of well known collinear PDFs

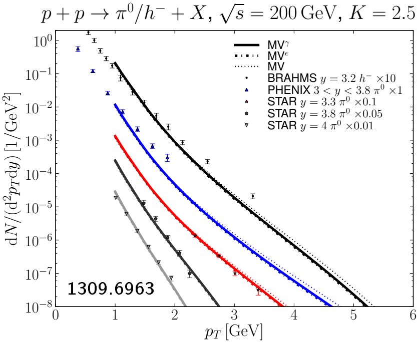
The dense target is described by an unintegrated gluon distribution $\mathcal{F}$, which is the Fourier-transform of the fundamental representation dipole correlator:

$$\mathcal{F}(k_\perp) = \int \frac{\mathrm{d}^2\mathbf{x}\mathrm{d}^2\mathbf{y}}{(2\pi)^2} e^{-ik_\perp\cdot(\mathbf{x}-\mathbf{y})} S(\mathbf{x},\mathbf{y}) \ , \quad S(\mathbf{x},\mathbf{y}) = \left\langle \frac{1}{N_c} \mathrm{Tr}\, U(\mathbf{x}) U^\dagger(\mathbf{y}) \right\rangle$$

The LO cross section reads $\frac{\mathrm{d}\sigma}{\mathrm{d}^2\mathbf{p}\mathrm{d}y} = \sum_q \int_\tau^1 \frac{\mathrm{d}z}{z^2} x_p q(x_p) \mathcal{F}(k_\perp) D_{h/q}(z)$,

where $D_{h/q}(z)$ are the fragmentation functions

Several LO calculations achieved a quite good description of experimental data, but often with rather large K factors to get the correct normalization

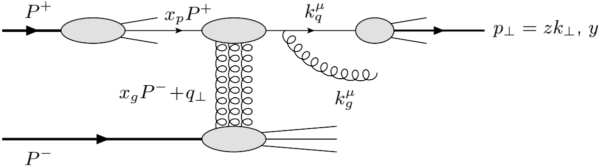


Lappi, Mäntysaari

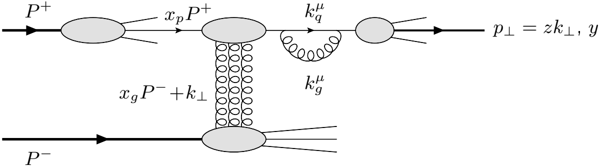
It is important to extend this framework beyond leading order to see for example if this normalization issue is due to missing higher order corrections

The expression for the NLO cross section has been computed by Chirilli, Xiao, Yuan

Example of real $q \to q$ contribution:

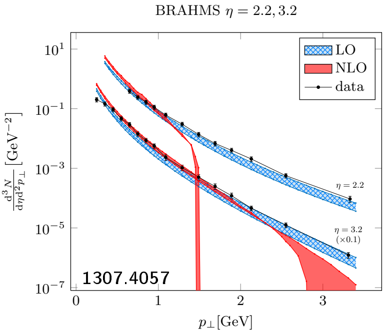


Example of virtual $q \to q$ contribution:



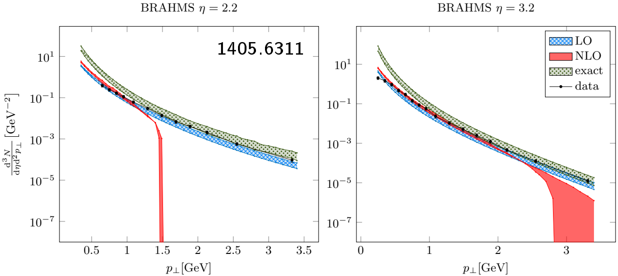
$1 - \xi = \frac{k_g^+}{x_p P^+}$ is the momentum fraction of the incoming quark carried by the gluon

First numerical implementation of the NLO cross section: Staśto, Xiao, Zaslavsky



The cross section becomes negative above some transverse momentum!

Several proposals to solve this issue. One of them: Staśto, Xiao, Yuan, Zaslavsky:
Matching to collinear factorization at large $p_\perp$
$\Leftrightarrow$ Large $k_\perp$ expansion + exact kinematics + kinematical constraint ($x_g \leq 1$)



In this case the cross section is positive at large $p_\perp$

Quite hard to understand why:

- The large $k_\perp$ expansion should not modify the behaviour
- The condition $x_g \leq 1$ should not play a significant role in a small x calculation

Motivations

The purpose of this work:

- Identify the origin of the negativity at large transverse momentum
- See if we can find a way to cure it

For this we make some simplifications

- We consider only the $q \to q$ channel
- We use a simple gaussian form for the dipole cross section

  Golec-Biernat and Wüsthoff (GBW) model: $S(\mathbf{r}) = e^{-\frac{\mathbf{r}^2 Q_s^2}{4}}$

Our goal is not (yet) to make predictions to compare to experimental data

The NLO cross section

The expression for the multiplicity at NLO reads

$$\begin{aligned}\frac{\mathrm{d}N^{pA\to hX}}{\mathrm{d}^2\mathbf{p}\,\mathrm{d}y_h} =& \int_\tau^1 \frac{\mathrm{d}z}{z^2} D_{h/q}(z) x_p q(x_p) \frac{\mathcal{S}^{(0)}(k_\perp)}{(2\pi)^2} && \leftarrow \text{LO term}\\ &+ \frac{\alpha_s}{2\pi^2}\int \frac{\mathrm{d}z}{z^2} D_{h/q}(z) \int_{\tau/z}^1 \mathrm{d}\xi \frac{1+\xi^2}{1-\xi}\frac{x_p}{\xi} q\left(\frac{x_p}{\xi}\right)\left\{C_F \mathcal{I}(k_\perp,\xi) + \frac{N_c}{2}\mathcal{J}(k_\perp,\xi)\right\} && \leftarrow \text{real NLO term}\\ &- \frac{\alpha_s}{2\pi^2}\int \frac{\mathrm{d}z}{z^2} D_{h/q}(z) \int_0^1 \mathrm{d}\xi \frac{1+\xi^2}{1-\xi} x_p q(x_p) \left\{C_F \mathcal{I}_v(k_\perp,\xi) + \frac{N_c}{2}\mathcal{J}_v(k_\perp,\xi)\right\} && \leftarrow \text{virtual NLO term}\end{aligned}$$

with

$$\mathcal{I}(k_\perp,\xi) = \int \frac{\mathrm{d}^2\mathbf{q}}{(2\pi)^2}\mathcal{S}(q_\perp)\left[\frac{\mathbf{k}-\mathbf{q}}{(\mathbf{k}-\mathbf{q})^2} - \frac{\mathbf{k}-\xi\mathbf{q}}{(\mathbf{k}-\xi\mathbf{q})^2}\right]^2$$

$$\mathcal{J}(k_\perp,\xi) = \int \frac{\mathrm{d}^2\mathbf{q}}{(2\pi)^2}\frac{2(\mathbf{k}-\xi\mathbf{q})\cdot(\mathbf{k}-\mathbf{q})}{(\mathbf{k}-\xi\mathbf{q})^2(\mathbf{k}-\mathbf{q})^2}\mathcal{S}(q_\perp) - \int \frac{\mathrm{d}^2\mathbf{q}}{(2\pi)^2}\frac{\mathrm{d}^2\mathbf{l}}{(2\pi)^2}\frac{2(\mathbf{k}-\xi\mathbf{q})\cdot(\mathbf{k}-\mathbf{l})}{(\mathbf{k}-\xi\mathbf{q})^2(\mathbf{k}-\mathbf{l})^2}\mathcal{S}(q_\perp)\mathcal{S}(l_\perp)$$

$$\mathcal{I}_v(k_\perp,\xi) = \mathcal{S}(k_\perp)\int \frac{\mathrm{d}^2\mathbf{q}}{(2\pi)^2}\left[\frac{\mathbf{k}-\mathbf{q}}{(\mathbf{k}-\mathbf{q})^2} - \frac{\xi\mathbf{k}-\mathbf{q}}{(\xi\mathbf{k}-\mathbf{q})^2}\right]^2$$

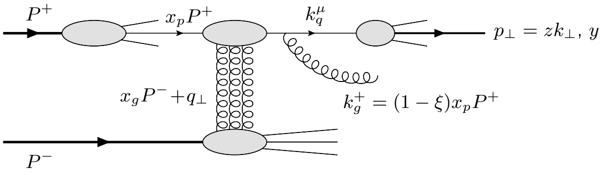
$$\mathcal{J}_v(k_\perp,\xi) = \mathcal{S}(k_\perp)\left[\int \frac{\mathrm{d}^2\mathbf{q}}{(2\pi)^2}\frac{2(\xi\mathbf{k}-\mathbf{q})\cdot(\mathbf{k}-\mathbf{q})}{(\xi\mathbf{k}-\mathbf{q})^2(\mathbf{k}-\mathbf{q})^2} - \int \frac{\mathrm{d}^2\mathbf{q}}{(2\pi)^2}\frac{\mathrm{d}^2\mathbf{l}}{(2\pi)^2}\frac{2(\xi\mathbf{k}-\mathbf{q})\cdot(\mathbf{l}-\mathbf{q})}{(\xi\mathbf{k}-\mathbf{q})^2(\mathbf{l}-\mathbf{q})^2}\mathcal{S}(l_\perp)\right]$$

Here and in the following we study the multiplicity which is related to the cross section by an integral over the impact parameter: $\frac{\mathrm{d}\sigma^{pA\to hX}}{\mathrm{d}^2\mathbf{p}\,\mathrm{d}y_h} = \int \mathrm{d}^2\mathbf{b}\frac{\mathrm{d}N^{pA\to hX}}{\mathrm{d}^2\mathbf{p}\,\mathrm{d}y_h}$

and we have defined $\mathcal{S}(k_\perp)$ such that $\mathcal{F}(k_\perp) = \int \frac{\mathrm{d}^2\mathbf{b}}{(2\pi)^2}\mathcal{S}(k_\perp)$

After summing the real and virtual contributions, two types of divergences remain in the NLO cross section:

- The collinear divergence
  - Occurs when the additional gluon is collinear to either the incoming or outgoing quark
  - Affects only the NLO corrections proportional to C_F
- The rapidity divergence
  - Occurs when $\xi \to 1 \Leftrightarrow$ the rapidity of the unobserved gluon $\to -\infty$ $\Leftrightarrow$ this gluon is collinear to the target
  - Affects only the NLO corrections proportional to N_c



For the collinear divergence we follow the same treatment as Chirilli, Xiao, Yuan:
Using dimensional regularization in $4-2\epsilon$ dimensions: $\int\frac{\mathrm{d}^2\mathbf{q}}{(2\pi)^2} \to \mu^{2\epsilon}\int\frac{\mathrm{d}^{2-2\epsilon}\mathbf{q}}{(2\pi)^{2-2\epsilon}}$,
the divergent part of the real C_F term reads

$$-\frac{1}{\hat{\epsilon}}\frac{\alpha_s}{2\pi}C_\mathsf{F}\int\frac{\mathrm{d}z}{z^2}D_{h/q}(z)\int_{\tau/z}^{1}\mathrm{d}\xi\frac{1+\xi^2}{1-\xi}\frac{x_p}{\xi}q\left(\frac{x_p}{\xi}\right)\left[\mathcal{F}(k_\perp)+\frac{1}{\xi^2}\mathcal{F}\left(\frac{k_\perp}{\xi}\right)\right]$$

And the divergent part of the virtual C_F term is

$$\frac{1}{\hat{\epsilon}}\frac{\alpha_s}{\pi}C_\mathsf{F}\int\frac{\mathrm{d}z}{z^2}D_{h/q}(z)x_p q\left(x_p\right)\int_0^1\mathrm{d}\xi\frac{1+\xi^2}{1-\xi}\mathcal{F}(k_\perp)$$

where $\frac{1}{\hat{\epsilon}}=\frac{1}{\epsilon}-\gamma_E+\ln 4\pi$.

These divergences can be factorized into the DGLAP evolution of the quark PDF $q(x)$ and the fragmentation function $D_{h/q}(z)$ in the $\overline{\text{MS}}$ scheme:

$$q(x,\mu)=q^{(0)}(x)-\frac{1}{\hat{\epsilon}}\frac{\alpha_s(\mu)}{2\pi}\int_x^1\frac{d\xi}{\xi}\mathcal{P}_{qq}(\xi)q\left(\frac{x}{\xi}\right)$$

$$D_{h/q}(z,\mu)=D_{h/q}^{(0)}(z)-\frac{1}{\hat{\epsilon}}\frac{\alpha_s(\mu)}{2\pi}\int_z^1\frac{d\xi}{\xi}\mathcal{P}_{qq}(\xi)D_{h/q}\left(\frac{z}{\xi}\right)$$

The N_c part of the NLO corrections is divergent when $\xi \to 1$

This corresponds to a gluon which is almost collinear to the target

Therefore it is natural to absorb this contribution in the gluon field of the target

Chirilli, Xiao, Yuan: define the renormalized gluon distribution of the target as

$$\mathcal{S}(k_\perp) = \mathcal{S}^{(0)}(k_\perp) + 2\alpha_s N_c \int_0^1 \frac{\mathrm{d}\xi}{1-\xi} \left[\mathcal{J}(k_\perp, 1) - \mathcal{J}_v(k_\perp, 1)\right]$$

In position space this can be written as

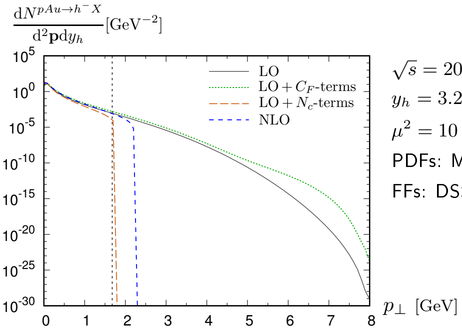
$$S(\mathbf{x}-\mathbf{y}) = S^{(0)}(\mathbf{x}-\mathbf{y}) - \frac{\alpha_s N_c}{2\pi^2} \int_0^1 \frac{\mathrm{d}\xi}{1-\xi} \int \mathrm{d}^2\mathbf{z} \frac{(\mathbf{x}-\mathbf{y})^2}{(\mathbf{x}-\mathbf{z})^2(\mathbf{y}-\mathbf{z})^2} \left[S(\mathbf{x}-\mathbf{y}) - S(\mathbf{x}-\mathbf{z})S(\mathbf{z}-\mathbf{y})\right]$$

or, if we differentiate with respect to Y,

$$\frac{\partial}{\partial Y} S(\mathbf{x}-\mathbf{y}) = -\frac{\alpha_s N_c}{2\pi^2} \int \mathrm{d}^2\mathbf{z} \frac{(\mathbf{x}-\mathbf{y})^2}{(\mathbf{x}-\mathbf{z})^2(\mathbf{y}-\mathbf{z})^2} \left[S(\mathbf{x}-\mathbf{y}) - S(\mathbf{x}-\mathbf{z})S(\mathbf{z}-\mathbf{y})\right]$$

Which is the well-known Balitsky-Kovchegov evolution equation for S

After the divergences have been subtracted, the multiplicity is finite...



$\sqrt{s} = 200$ GeV

$y_h = 3.2$

$\mu^2 = 10$ GeV^2

PDFs: MSTW2008 NLO

FFs: DSS NLO

...but negative above some $p_\perp$. This is similar to the results obtained when including all the channels (Stasto, Xiao, Zaslavsky)

At large $p_\perp$ the C_F term is positive $\to$ the negativity comes from the N_c term

The origin of the negativity

The fact that the N_c term is negative at large $p_\perp$ can be understood by looking at its large-$k_\perp$ limit:

$$\frac{N_c}{2}\frac{\alpha_s}{2\pi^2}\int\frac{dz}{z^2}D_{h/q}(z)\int_{\tau/z}^{\xi_f}\frac{d\xi}{(1-\xi)_+}\mathcal{K}(\xi)\ ,\quad \mathcal{K}(\xi)=(1+\xi^2)\frac{x_p}{\xi}q\left(\frac{x_p}{\xi}\right)\mathcal{J}(k_\perp,\xi)$$

At large $k_\perp$, $\mathcal{K}(\xi)$ behaves like $\mathcal{K}(\xi)\approx(1+\xi^2)\frac{x_p}{\xi}q\left(\frac{x_p}{\xi}\right)\frac{2\xi}{k_\perp^4}\int\frac{d^2\mathbf{q}}{(2\pi)^2}\mathbf{q}^2\mathcal{S}(q_\perp)$,

which is positive and generally increasing with ξ.

Therefore the plus-distribution will lead to a negative contribution.

This plus-distribution comes from the subtraction of the rapidity divergence

Let us come back to the renormalized UGD as defined by Chirilli, Xiao, Yuan:

$$\mathcal{S}(k_\perp)=\mathcal{S}^{(0)}(k_\perp)+2\alpha_s N_c\int_0^1\frac{d\xi}{1-\xi}\left[\mathcal{J}(k_\perp,1)-\mathcal{J}_v(k_\perp,1)\right]$$

The rapidity divergence occurs at $\xi=1$ so this point should be included in the subtraction term. But the choice of the lower limit is rather arbitrary

We propose to use

$$\mathcal{S}(k_\perp) = \mathcal{S}^{(0)}(k_\perp) + 2\alpha_s N_c \int_{\xi_f}^{1} \frac{d\xi}{1-\xi} \left[\mathcal{J}(k_\perp, 1) - \mathcal{J}_v(k_\perp, 1)\right]$$

where we have introduced $\xi_f \in [0:1[$ which plays the role of a (rapidity) factorization scale, arbitrary at this stage. It determines how much of the finite contribution is considered to be part of the evolution of the target

At large $k_\perp$ the N_c term now reads

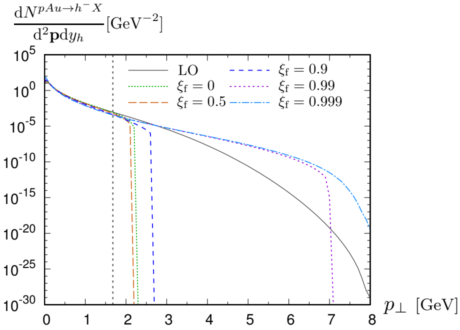
$$\frac{N_c}{2}\frac{\alpha_s}{2\pi^2}\int \frac{dz}{z^2} D_{h/q}(z) \left(\int_{\tau/z}^{\xi_f} \frac{d\xi}{1-\xi}\mathcal{K}(\xi) + \int_{\xi_f}^{1} \frac{d\xi}{1-\xi}\left[\mathcal{K}(\xi) - \mathcal{K}(1)\right] \right).$$

Since $\mathcal{K}(\xi)$ is positive and increases with ξ, the first term yields a positive contribution while the second one yields a negative contribution

If we increase ξ_f, we make the positive contribution larger and the negative contribution smaller $\rightarrow$ increase of the cross section

Like for other arbitrary scales, physical quantities should not depend on ξ_f

Multiplicity for several values of ξ_f between 0 and 1:

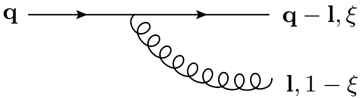


As expected, larger values of ξ_f lead to positive cross sections up to larger $p_\perp$

The results depend strongly on the choice of ξ_f

Here we have varied ξ_f in a very wide range. We need to fix it to a "physical" value and then vary it in a reasonable range to estimate the remaining uncertainty

We need a condition to specify which contributions will be part of the evolution of the target. Let us consider a typical NLO diagram:



The light cone energy introduced from the gluon emission is

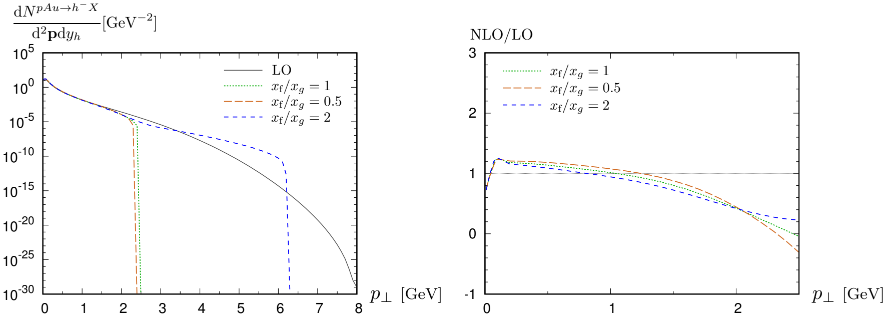
$$\Delta k^- = \frac{1}{2x_p P^+}\left[\frac{\mathbf{l}^2}{1-\xi}+\frac{(\mathbf{q}-\mathbf{l})^2}{\xi}-\mathbf{q}^2\right] = \frac{x_g P^-}{\mathbf{k}^2}\frac{(\mathbf{l}-(1-\xi)\mathbf{q})^2}{\xi(1-\xi)}$$

Here we decide to absorb fluctuations with Δk^- larger than a certain factorization scale x_{f} in the evolution of the target. At large $k_\perp$ this leads to

$$\Delta k^- \approx \frac{x_g P^-}{\mathbf{k}^2}\frac{Q_{\mathsf{s}}^2}{1-\xi} \geq x_{\mathsf{f}} P^- \Leftrightarrow 1-\xi \leq \frac{Q_{\mathsf{s}}^2}{\mathbf{k}^2}\frac{x_g}{x_{\mathsf{f}}} \Rightarrow \color{red}{\xi_{\mathsf{f}} = 1 - \frac{Q_{\mathsf{s}}^2}{\mathbf{k}^2}\frac{x_g}{x_{\mathsf{f}}}} \ ,$$

with a "natural" value $x_{\mathsf{f}} \sim x_g$. In practice we use $\xi_{\mathsf{f}} = \frac{k_\perp^2}{k_\perp^2 + \frac{x_g}{x_{\mathsf{f}}} Q_{\mathsf{s}}^2}$,
which has the same large $k_\perp$ behaviour and goes smoothly to $\xi_{\mathsf{f}} = 0$ at $k_\perp = 0$

Multiplicity for $\frac{x_f}{x_g} \in \{1, \frac{1}{2}, 2\}$:



At small $p_\perp$ the dependence of the cross section on $\frac{x_f}{x_g}$ is rather small

Values of $\frac{x_f}{x_g}$ in $[\frac{1}{2} : 2]$ still lead to negative cross sections at large $p_\perp$

However the $p_\perp$ value where this occurs depends strongly on this ratio

In particular a value of $\frac{x_f}{x_g} = 2$ extends significantly the range of positivity

Future improvements

These results may not seem very promising but they were obtained in a very simplistic approach.

Future directions that may lead to improvements:

- Implement the light cone ordering condition in an exact way in the transverse momentum integrals. For now we have used the external transverse scales $k_\perp$ and Q_s, which allows us to reuse many results of Chirilli, Xiao, Yuan
- Use a more physical dipole cross section
  The GBW model leads to simple analytical expressions. However in this model the NLO cross section is completely governed by the NLO corrections ($\sim k_\perp^{-4}$) at large $p_\perp$. A dipole cross section obtained by solving the Balitsky-Kovchegov equation should lead to a power-law behaviour of the LO contribution at large $p_\perp$ and so less sensitivity to the NLO corrections

Conclusions

We proposed to modify the subtraction procedure of the rapidity divergence to solve the issue of large negative NLO corrections at large $p_\perp$ in this process

- We introduced a rapidity factorization scale ξ_f
  - The NLO cross section at large $p_\perp$ is very sensitive to the choice of ξ_f
  - By increasing ξ_f it is possible to make the cross section positive up to arbitrarily large values of $p_\perp$
- We proposed to fix ξ_f by imposing light cone ordering
  - The cross section still becomes negative at some $p_\perp$ when ξ_f is varied in its "natural" range
  - The $p_\perp$ value at which this occurs changes a lot in this "natural" range

Directions for future work:

- Implement the light cone ordering condition in an exact way
- Use more physical dipole cross sections

These steps are necessary before drawing definitive conclusions since the NLO cross section at large $p_\perp$ can be very sensitive to small changes