# APFEL

## Recent Developments

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# Recap on APFEL

- APFEL is a **public** library for PDF evolution:
  - up to NNLO in QCD combined to LO QED corrections.
  - FFN and VFN schemes available.
  - Pole and $\overline{\text{MS}}$ heavy-quark masses.
  - Module for the fast computation of DIS NC and CC observables up to NNLO in different mass schemes (ZM-VFNS, FFNS and FONLL).
  - interfaces to FORTRAN, C/C++ and Python.
  - Amazing web interface available on http://apfel.mi.infn.it.
  - APFEL is available from http://apfel.hepforge.org/.
- Interfaced to **xFitter** and **Alpos**.
- Used for the next generation of the **NNPDF** fits.

# Intrinsic Charm

- Introducing an intrinsic charm (IC) component in the context of a GM-VFNS like FONLL (or ACOT, or TR) requires some care:
  - **relax** the assumption of **pure perturbative generation** of heavy quarks at the thresholds,
  - take into account **charm-initiated diagrams** both in the **massive** and in the massless sectors [arXiv:1510.00009].

- A full formulation of the FONLL scheme in the presence of IC has **recently** been **achieved** [arXiv:1510.02491]:
  - interestingly, it has been found that **FONLL** with IC is **equivalent** to full **ACOT** to all orders, while the standard FONLL (w/o IC) is instead equivalent to S-ACOT.
  - Implemented in APFEL up to NLO both in the **NC** and **CC** sector and **benchmarked** against the public `massiveDISsFuntion` code (`https://www.ge.infn.it/~bonvini/massivedis/`).

# Intrinsic Charm

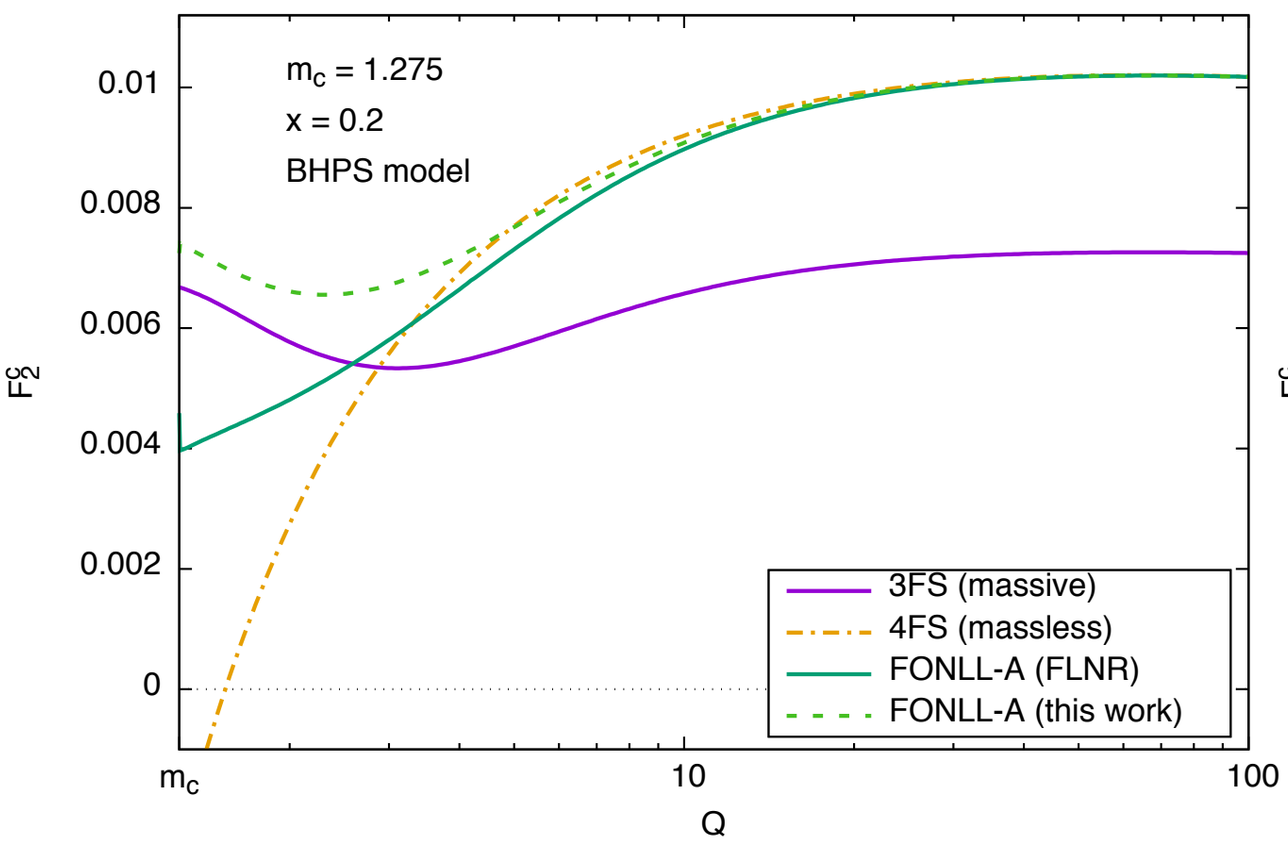
- Consider realistic models:
  - BHPS model:

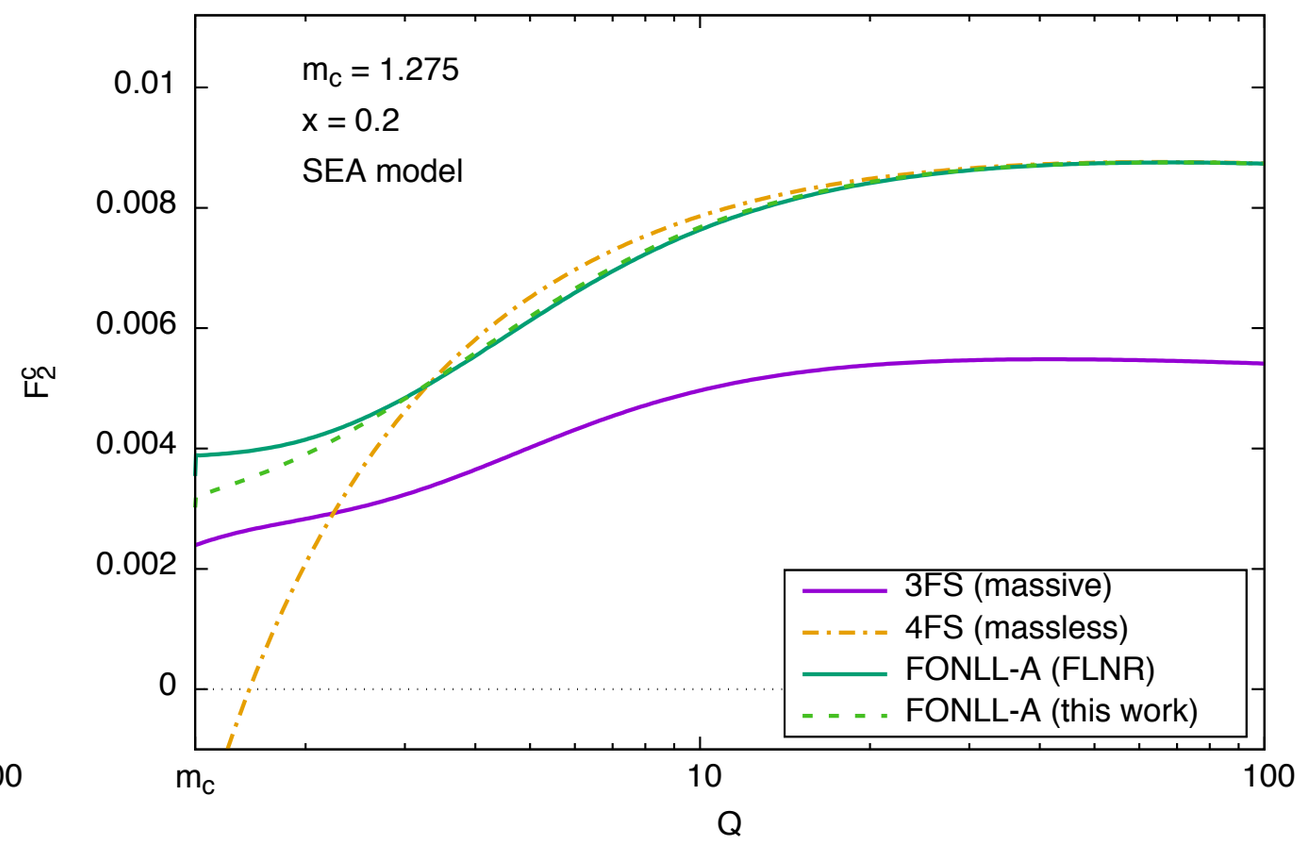
$$f_c^{(3)}(x) = f_{\overline{c}}^{(3)}(x) = Ax^2\left[6x(1+x)\ln x + (1-x)(1+10x+x^2)\right]$$

  - SEA model:

$$f_c^{(3)}(x) = f_{\overline{c}}^{(3)}(x) = Ax^{-1.25}(1-x)^3$$

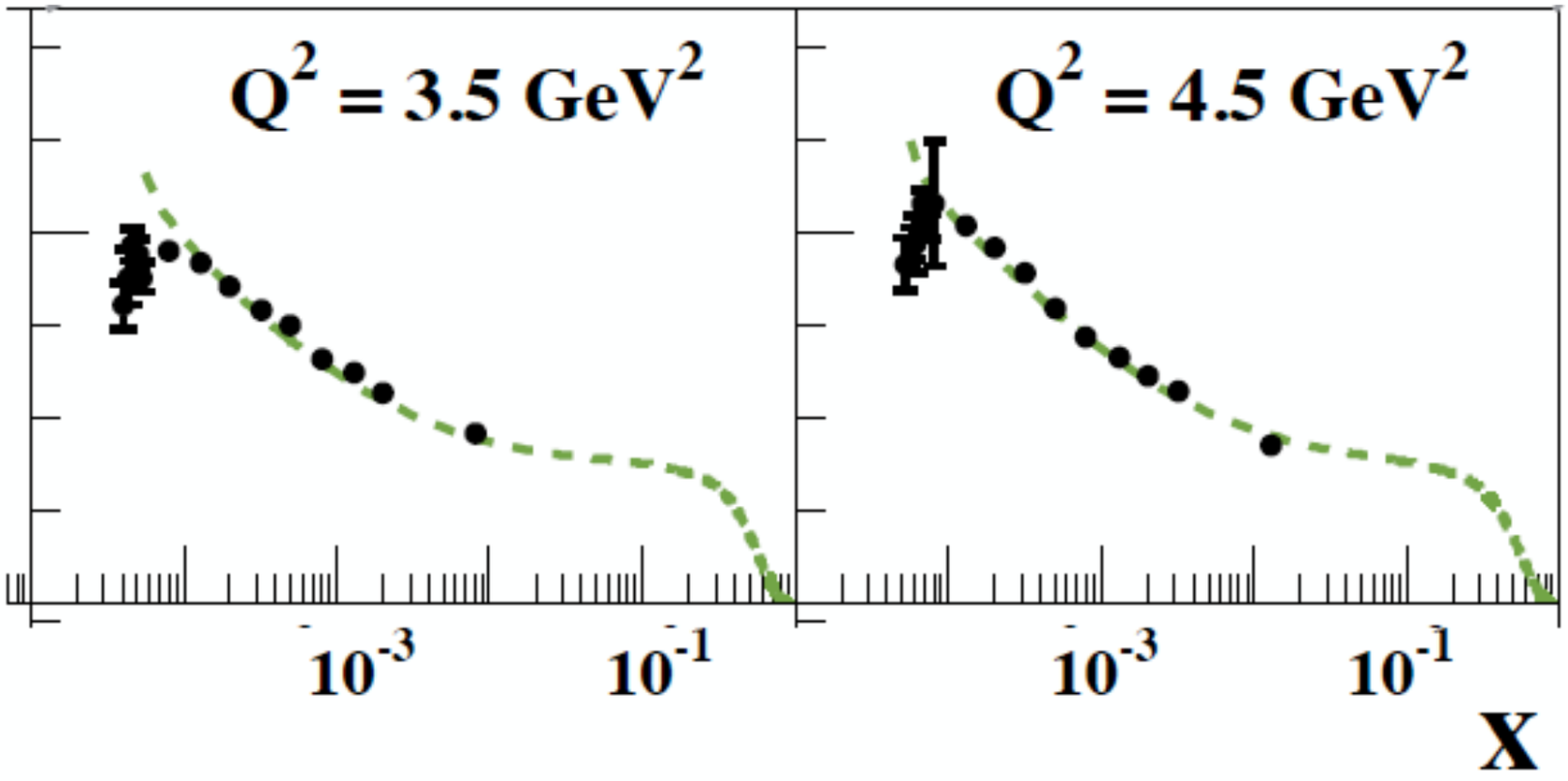
- $A$ determined requiring the charm to carry 0.5% of the momentum



# Small-$x$ Resummation

- Some **tension** between fixed-order predictions and data in the low-$x$ region reached by HERA:




From Eram Rizvi talk at QCD@LHC14

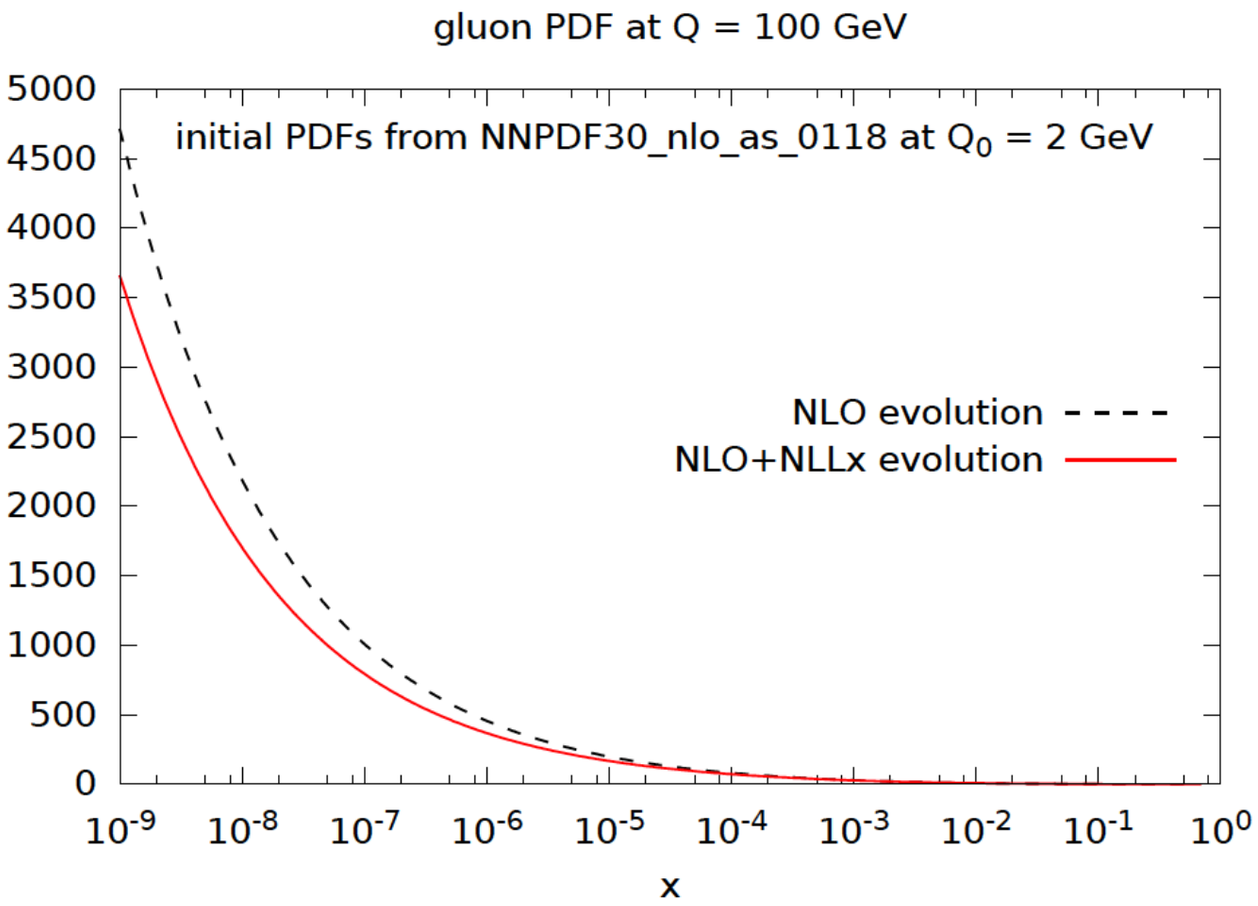
- A similar effect was observed some time ago in the NNPDF framework by F. Caola *et al.* [arXiv:1007.5405].

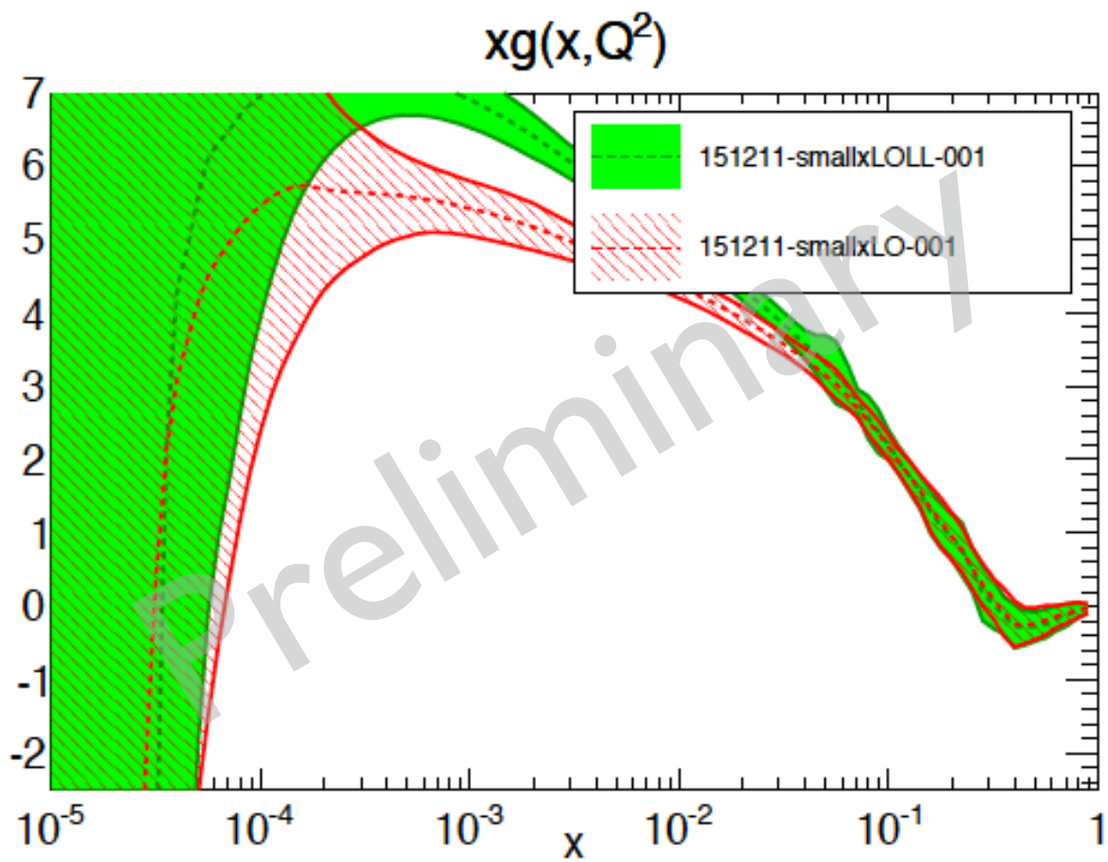
- Suggestion of the need for **small-$x$ resummation**.

# Small-$x$ Resummation

- In collaboration with Marco Bonvini, quite some work has been done to interface the **HELL** code to APFEL:
  - HELL implements small-$x$ **resummed splitting functions** up to **NLL** accuracy based on the ABF approach [arXiv:0802.0032].
  - it will soon implement also small-$x$ resummed **DIS coefficient functions** (Marco Bonvini, Simone Marzani and Tiziano Peraro are presently working on that).
- The actual **interface** is **already in place** and fully operative.
- As a proof of concept, in NNPDF we attempted PDF fits with small-$x$ resummed evolution obtaining **encouraging results**.
- A fully consistent PDF fit would require resummed **coefficient functions** which should be available in HELL soon.

# Small-$x$ Resummation



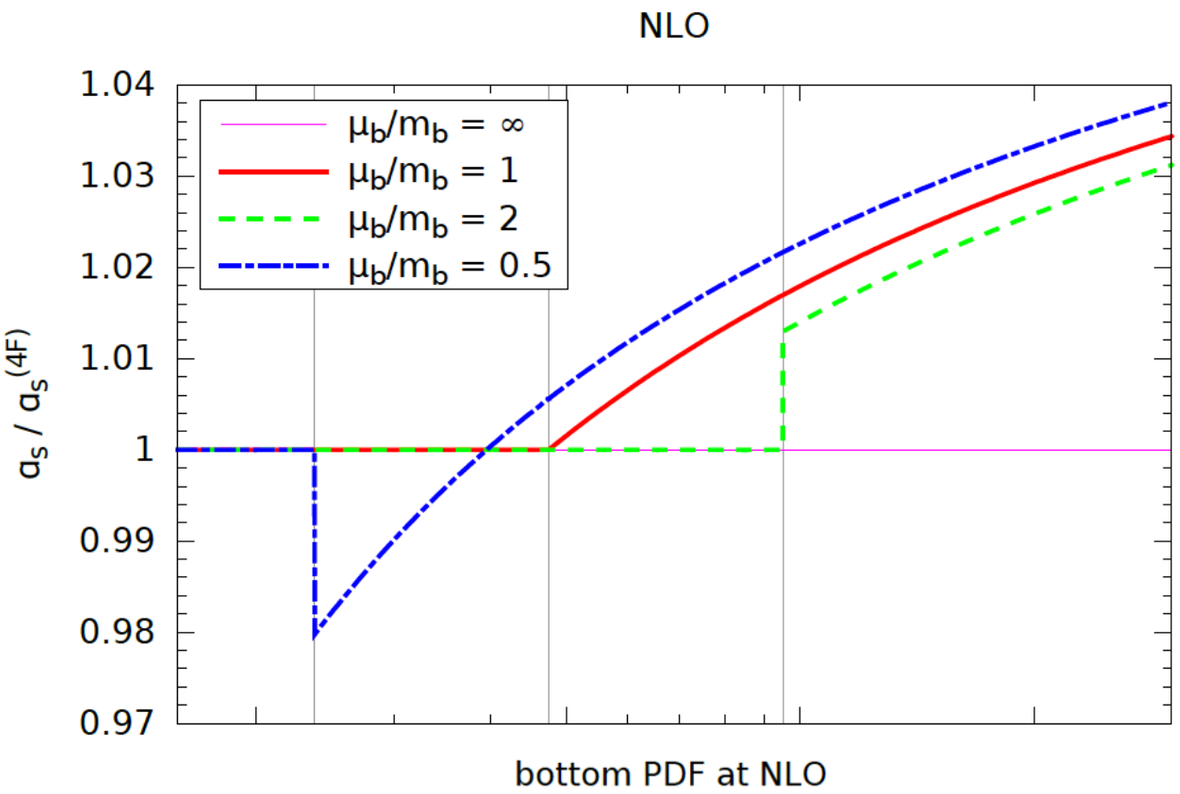


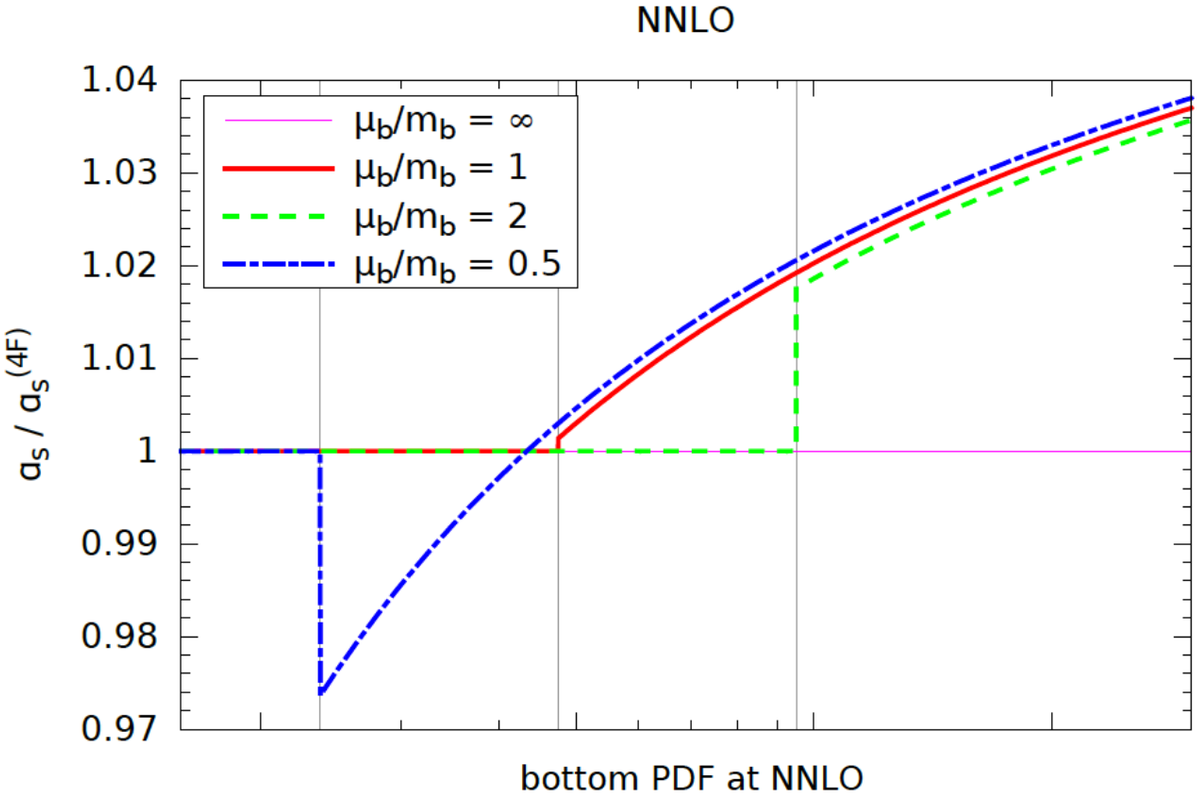

- **Enhancement** of the fitted **gluon PDF** at small values of $x$ as consequence of the relative suppression of the resummed evolution.
- **Compensation** expected when also resummed **coefficient functions** will be introduced.
- Other PDFs mostly unchanged.

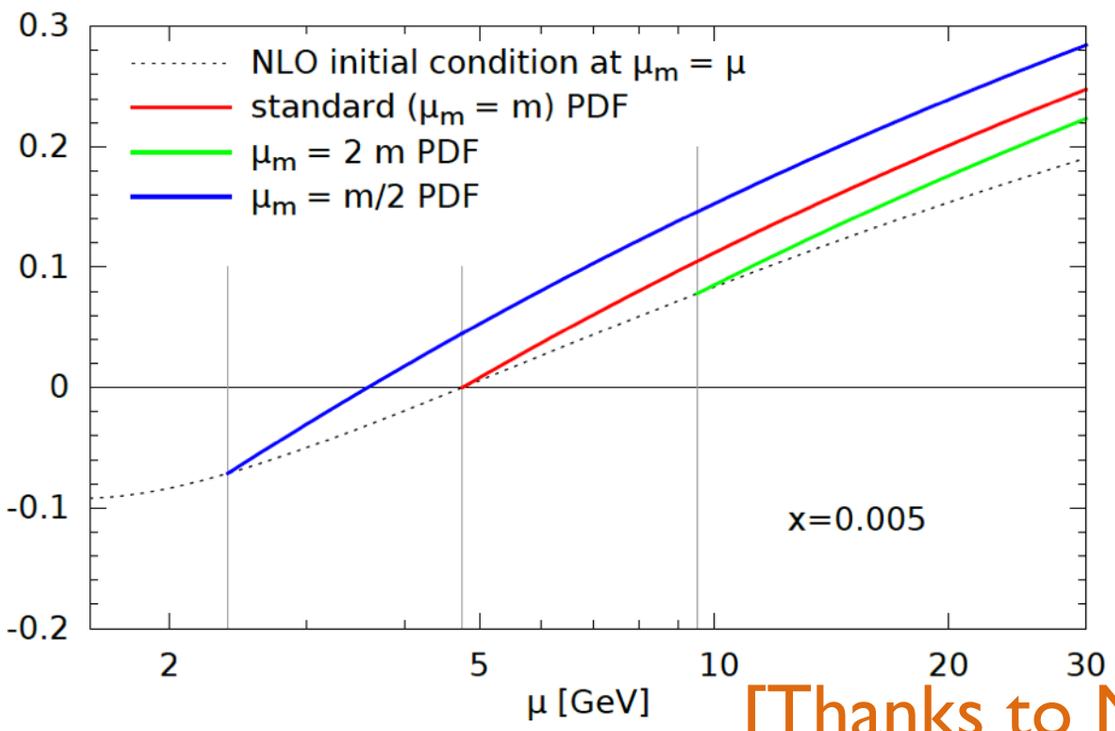
# Displaced Heavy-Quark Thresholds

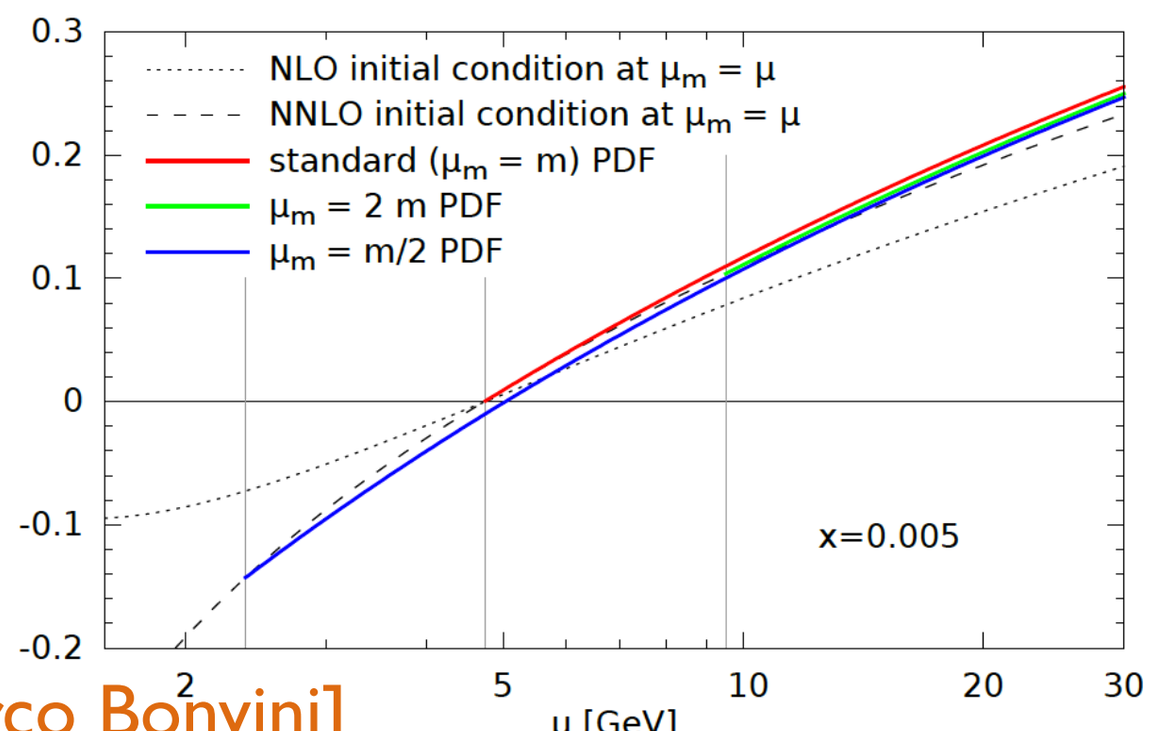
- The implementation of the VFNS evolution both for PDFs and $\alpha_s$ requires **matching** factorization schemes differing in the number of active flavours:
  - the scale at which two consecutive factorization schemes are matched are usually referred to as **heavy-quark thresholds**.
  - Heavy-quark thresholds are usually (and for convenience) identified with the heavy quark masses by means of the so-called **matching conditions** presently know up to $O(\alpha_s^2)$ [hep-ph/9612398].
  - However, heavy-quark thresholds are actually free parameters and can be chosen **arbitrarily**.
  - If masses and thresholds are taken to be different, the matching conditions need to be "generalized" including **logarithmic terms**.
- APFEL now implements the possibility to set masses and thresholds to different values in a consistent way both in the pole mass and in the $\overline{\text{MS}}$ renormalization schemes.

# Displaced Heavy-Quark Thresholds




[Thanks to Marco Bonvini]

# APFELgrid

## *A fast(er) interface for PDF fits*

- While being an extremely useful tool, **APPLgrid** might not be appropriate to be directly employed in a global PDF fit where usually **thousands of iterations** are needed:
  - Need to calculate PDF and $\alpha_s$ **evolution in real time**.
  - **Not particularly fast** convolution.
    - many tables need to be loaded with the concrete risk of **exceeding the memory limit** (pretty common on clusters).

- In the NNPDF collaboration we developed **APFELgrid** which, starting from an existing APPLgrid, combines PDF evolution from APFEL to the hard cross sections producing *derived* interpolation tables (FK tables):

| Observable | APPLgrid | FK | optimized FK |
|---|---|---|---|
| $W^+$ production | 1.03 ms | 0.41 ms (2.5x) | 0.32 ms (3.2x) |
| Inclusive jet production | 2.45 ms | 20.1 $\mu$s (120x) | 6.57 $\mu$s (370x) |

- APFELgrid will soon be made **public in APFEL.** [thanks to N. Harthland]

# Other Recent Developments

- **Polarized DGLAP evolution** up to NNLO [arXiv:1409.5131].

- **Time-like evolution** and computation of **SIA structure functions** up to NNLO (getting ready to fit fragmentation functions in the NNPDF framework).

- Independent factorization and renormalization **scale variations** both in the DIS structure functions and in the evolution.

# In the Pipeline

- Full **QED NLO corrections** to the PDF and $\alpha_s$ evolution (including the mixed QCD-QED corrections) [arXiv:1512.00612].

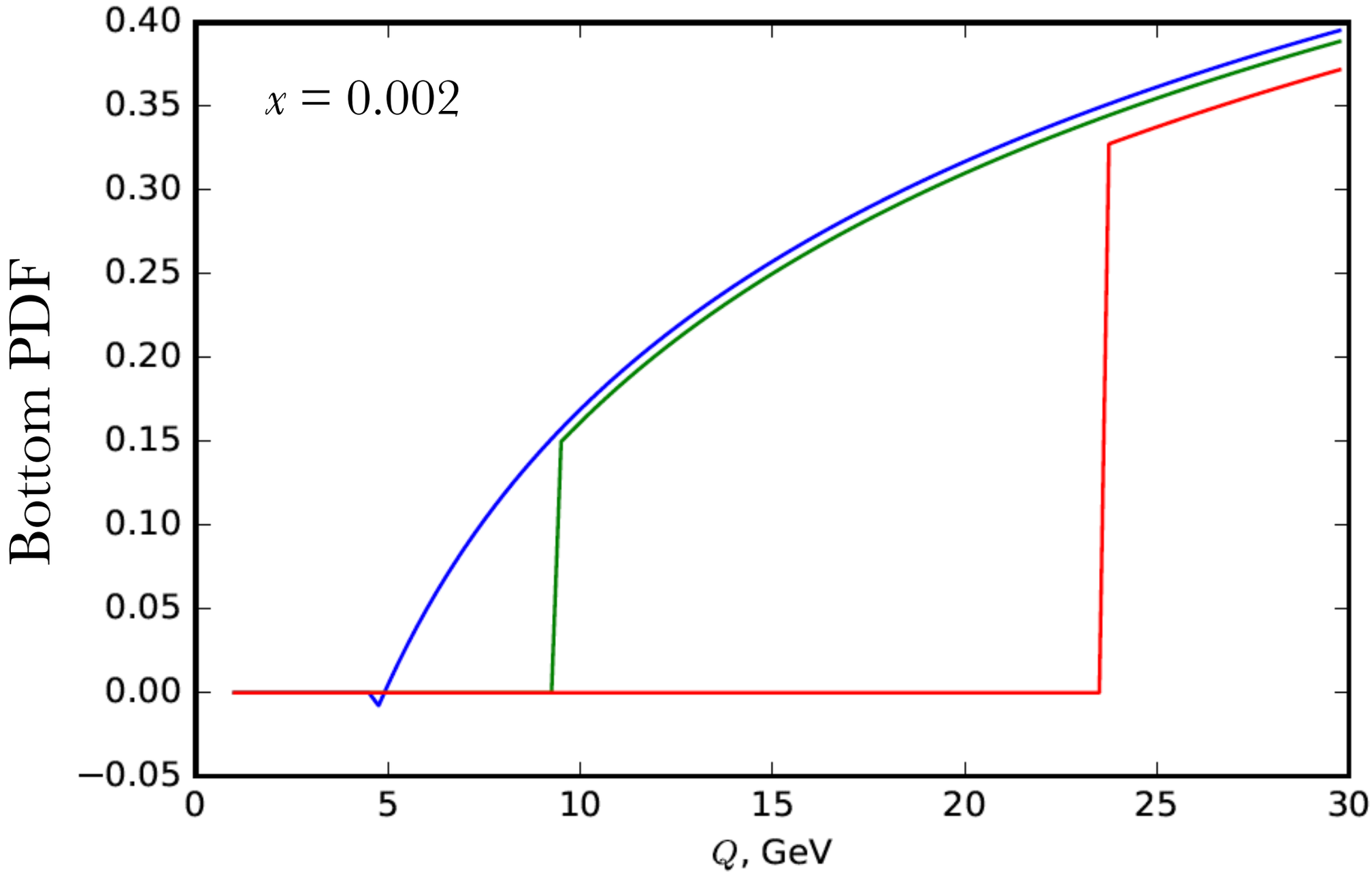
- Inclusion of **semi-inclusive DIS** cross sections.

- Inclusion of the **photon-initiated channels in DIS**.

- Implementation of the **polarized structure functions**.

# Backup Slides

# Displaced Heavy-Quark Thresholds




[Thanks to Sasha Glazov]

# Tools for Determining FFs

- A faithful determination of fragmentation functions (FFs) is extremely important to study the universality of the QCD factorization theorem.

- The inclusive hadron measurements at the LHC, sensibly extending the previous kinematical coverage, are particularly useful for studying the FFs.

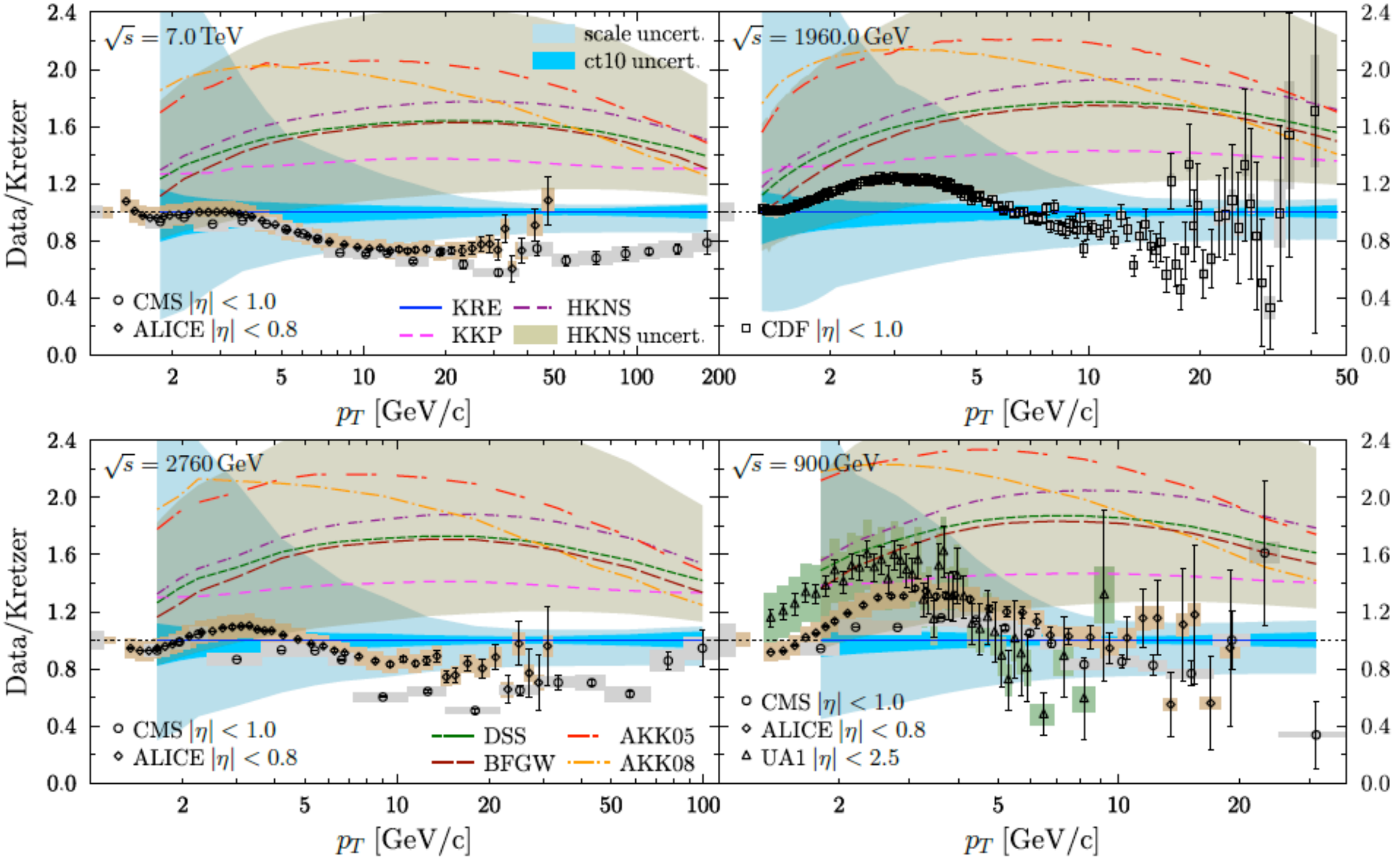
- Moreover, a good knowledge of FFs is functional to the determination of the **polarized PDFs**.

- The **spread between the different FFs** present on the market is currently very large.

- In addition, none of the existing FF sets can reproduce the experimental results optimally.

# Tools for Determining FFs

- Inclusive charge-hadron spectrum:




d'Enterria *et al.* [arXiv:1311.1415]

# Tools for Determining FFs

- APFEL implements the time-like evolution:
  - up to NLO in the VFNS,
  - up to NNLO in the FFNS (NNLO matching conditions missing).
- In collaboration with E. Nocera and S. Carrazza, we have performed a careful **benchmark** of the time-like evolution:
  - we are in contact with the people who calculated the time-like splitting functions: A. Mitov, S.O. Moch, A. Vogt.
- Single-inclusive $e^+e^-$ annihilation (SIA) structure functions also implemented in APFEL up to NNLO:
  - partial benchmark against DSS implementation.
- APFEL can now effectively be used to fit FFs.

# Improvements
## *A New Fast Evolution*

- In the previous versions of APFEL the DGLAP evolution equations were written in terms of the **evolution operator**:

$$\mu^2 \frac{\partial}{\partial \mu^2} M_{ij}(\mu, \mu_0) = P_{ik}(\mu) \otimes M_{kj}(\mu, \mu_0) \quad \text{with} \quad f_i(\mu) = M_{ij}(\mu, \mu_0) \otimes f_j(\mu_0)$$

- This may be convenient because the evolution operator can be evaluated once and for all and convoluted with any initial PDF set.

- On the other hand, this requires solving numerically a big coupled system of ODEs, therefore it can be slow.

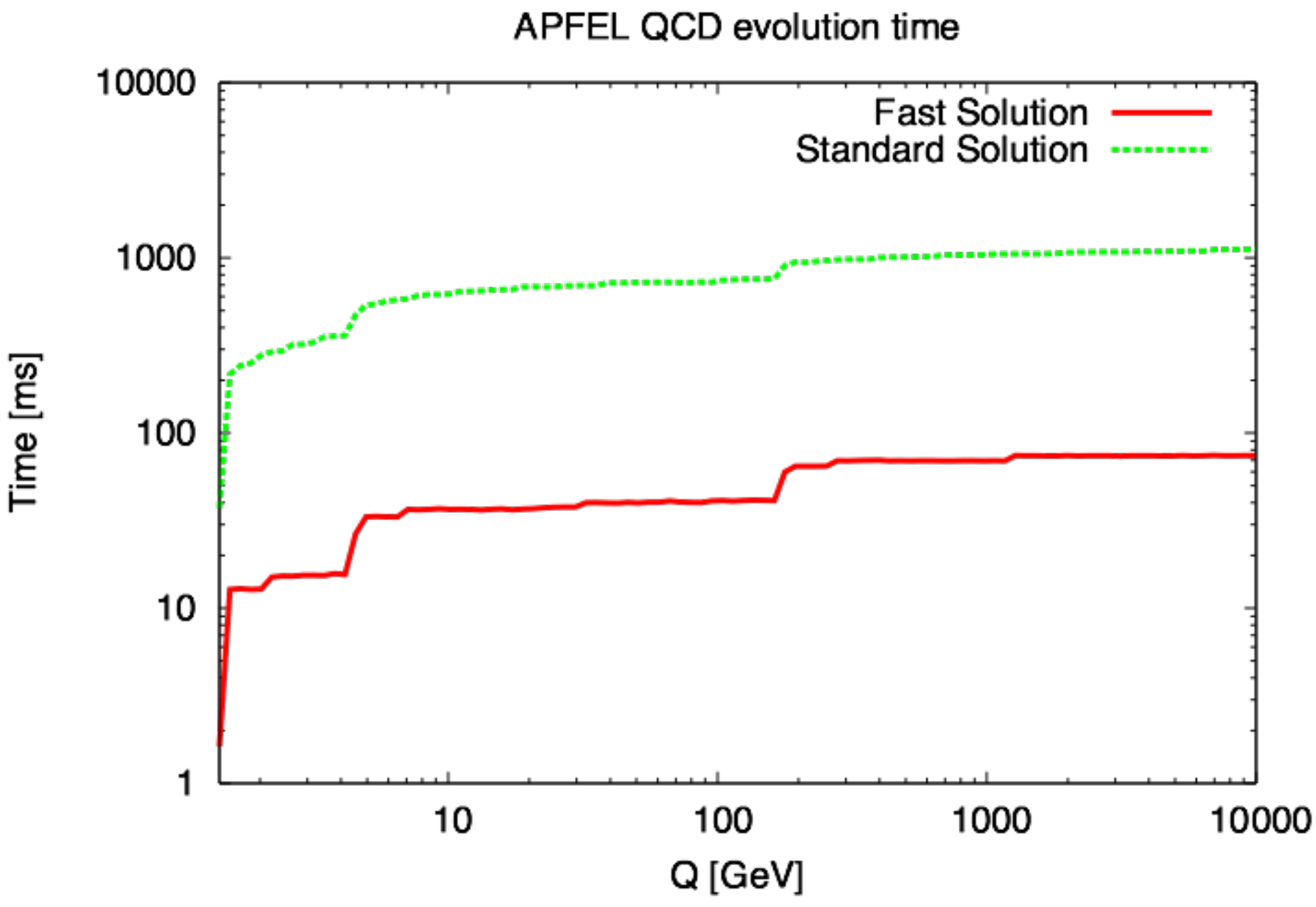
- Alternatively, one can directly solve the DGLAP equations in terms of **PDFs**:

$$\mu^2 \frac{\partial}{\partial \mu^2} f_i(\mu) = P_{ij}(\mu) \otimes f_j(\mu)$$

- This requires the solution of a much smaller system of equations and is consequently much faster.

# Improvements
## *A New Fast Evolution*

- Comparison between old (operatorial) and new (in terms of PDFs) solution for the QCD evolution:



# Improvements
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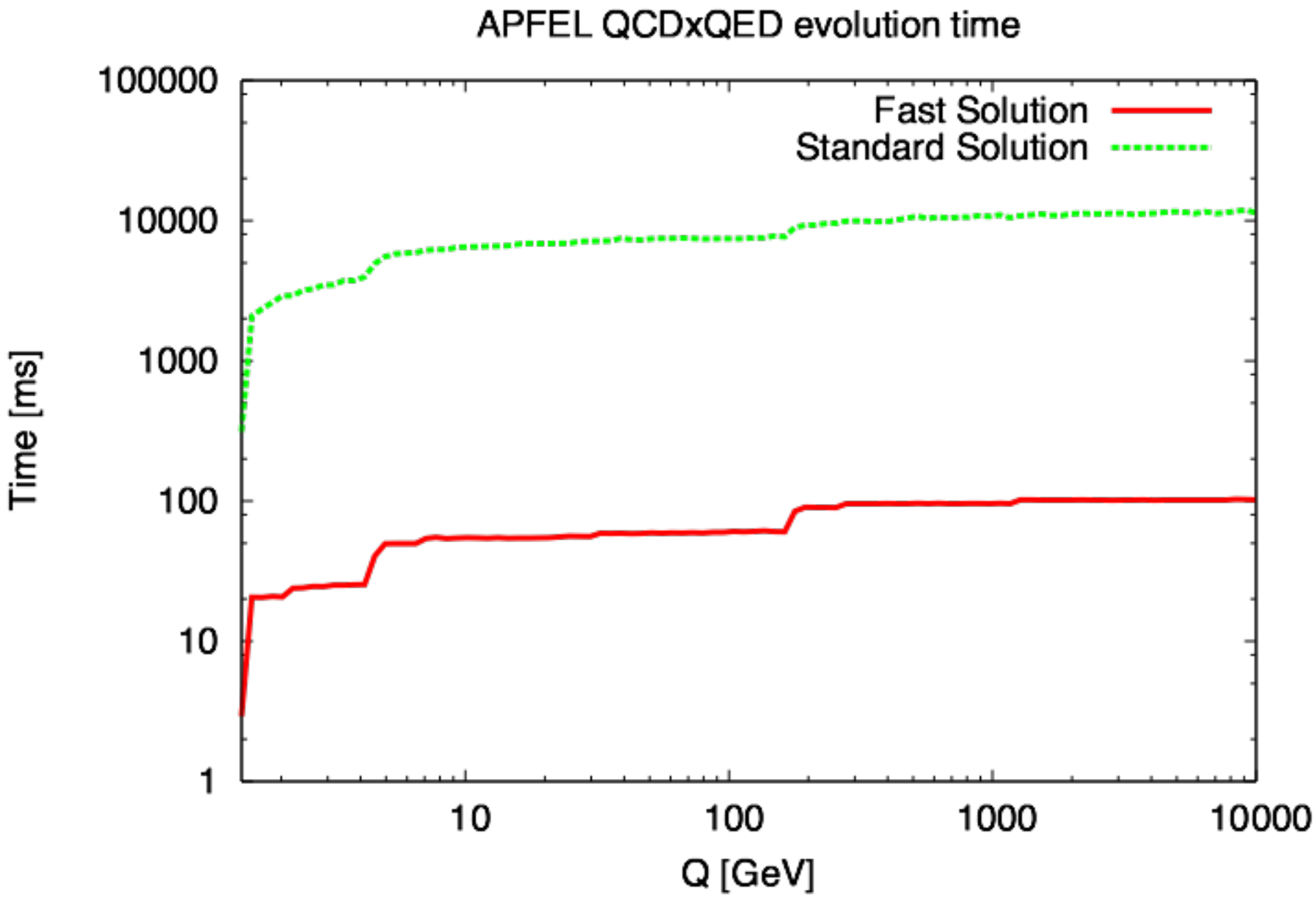
- Comparison between old (operatorial) and new (in terms of PDFs) solution for the QCD+QED evolution:



# Improvements

## *A New QCD+QED Evolution*

- In the previous versions of APFEL the QCD+QED evolution was performed by combining the **separate** QCD and QED evolution:
  - we showed that the differences, of a few % at most, with the standard implementations which evolve contemporaneously in QCD and QED were due to **subleading terms in $\alpha$**.
- We have now implemented a new evolution basis which allows a **simultaneous diagonalization** of the QCD+QED evolution matrix:

$$
\begin{aligned}
&1)\ g \\
&2)\ \gamma \\
&3)\ \Sigma = \Sigma_u + \Sigma_d \\
&4)\ \Delta_\Sigma = \Sigma_u - \Sigma_d \\
&5)\ T_1^u = u^+ - c^+ \\
&6)\ T_2^u = u^+ + c^+ - 2t^+ \\
&7)\ T_1^d = d^+ - s^+ \\
&8)\ T_2^d = d^+ + s^+ - 2b^+
\end{aligned}
\qquad
\begin{aligned}
&9)\ V = V_u + V_d \\
&10)\ \Delta_V = V_u - V_d \\
&11)\ V_1^u = u^- - c^- \\
&12)\ V_2^u = u^- + c^- - 2t^- \\
&13)\ V_1^d = d^- - s^- \\
&14)\ V_2^d = d^- + s^- - 2b^-
\end{aligned}
$$

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*Coupled*

$$
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\end{aligned}
\qquad
\begin{aligned}
&9)\ V = V_u + V_d \\
&10)\ \Delta_V = V_u - V_d
\end{aligned}
$$

*Decoupled*

$$
\begin{aligned}
&5)\ T_1^u = u^+ - c^+ \\
&6)\ T_2^u = u^+ + c^+ - 2t^+ \\
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\end{aligned}
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&14)\ V_2^d = d^- + s^- - 2b^-
\end{aligned}
$$

- This new basis is also suitable for an easy implementation of the mixed **higher order corrections** to the evolution.