Next-to-leading order Balitsky-Kovchegov equation with resummation

DIS 2016

Heikki Mäntysaari

Based on: T. Lappi, H.M., arXiv:1601.06598, arXiv:1502.02400

Brookhaven National Laboratory

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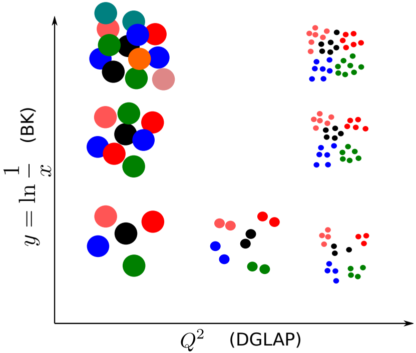
QCD evolution at small x



- CGC: QCD at high energies
- QCD favors soft gluon emission $\Rightarrow$ parton density grows at small x
- Saturation at high densities

Evolution in x (energy, rapidity) from QCD:

- QCD dynamics included in dipole-target amplitude $N(r_T, x)$
- Evolution in x: BFKL/BK/JIMWLK

CGC phenomenology

Non-perturbative input

Fit initial condition for the dipole amplitude to DIS data

Evolve dipole amplitude

Balitsky-Kovchegov equation LO and NLO (no solution so far)

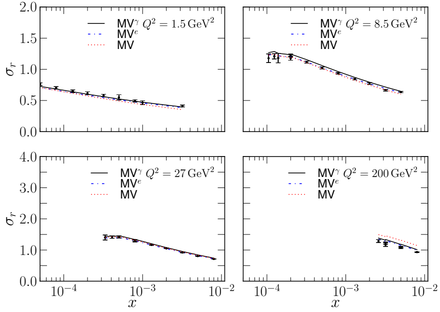
- Current state of the art: LO + running coupling corrections

Calculate observables

- DIS (LO and NLO)
- Single inclusive spectra (LO and NLO)
- Two-particle correlations (LO)
- ...

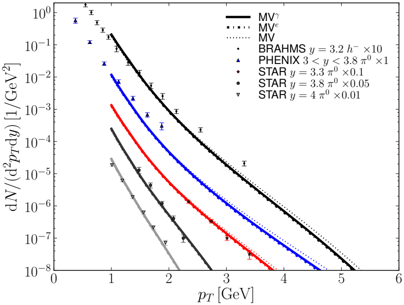
Successful phenomenology at LO

Structure functions



T. Lappi, H.M., arXiv:1309.6963

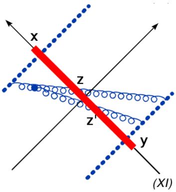
Single inclusive at RHIC pp

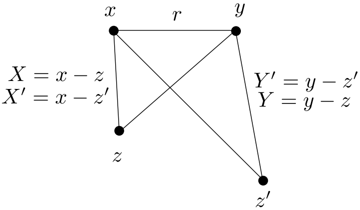


T. Lappi, H.M., arXiv:1309.6963

- Next step: upgrade the saturation picture to NLO accuracy

NLO BK equation





$$\otimes = \int_{zz'}$$

Balitsky, Chirilli, arXiv:0710.4330 + mean field, large-N_c:

$$\partial_y S(r) = \frac{\alpha_s}{2\pi^2} K_1 \otimes [S(X)S(Y) - S(r)]$$
$$+ \frac{\alpha_s^2 N_c^2}{8\pi^4} K_2 \otimes [S(X)S(z-z')S(Y') - S(X)S(Y)]$$
$$+ \frac{\alpha_s^2 N_f N_c}{8\pi^4} K_f \otimes S(Y)[S(X') - S(X)]$$

$S = 1 - N$

NLO BK equation, a closer look

$$K_1 = \frac{r^2}{X^2 Y^2}\left[1 + \frac{\alpha_s N_c}{4\pi}\left(\frac{\beta}{N_c}\ln r^2\mu^2 - \frac{\beta}{N_c}\frac{X^2 - Y^2}{r^2}\ln\frac{X^2}{Y^2} + \frac{67}{9} - \frac{\pi^2}{9} - 2\ln\frac{X^2}{r^2}\ln\frac{Y^2}{r^2}\right)\right]$$

$$K_2 = -\frac{2}{(z-z')^4} + \left[\frac{X^2 Y'^2 + X'^2 Y^2 - 4r^2(z-z')^2}{(z-z')^4(X^2 Y'^2 - X'^2 Y^2)} + \frac{r^4}{X^2 Y'^2(X^2 Y'^2 - X'^2 Y^2)} + \frac{r^2}{X^2 Y'^2 (z-z')^2}\right]\ln\frac{X^2 Y'^2}{X'^2 Y^2}$$

- Leading order
- Running coupling part ($\sim \beta$): Balitsky prescription in numerics
- Non-conformal double log (diverges at $r \to 0$)

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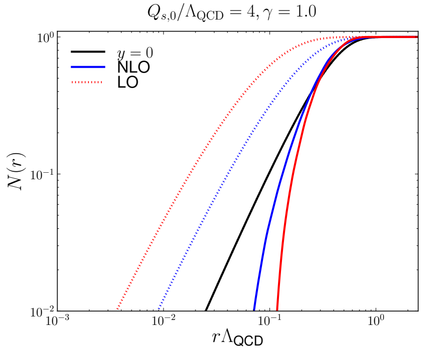
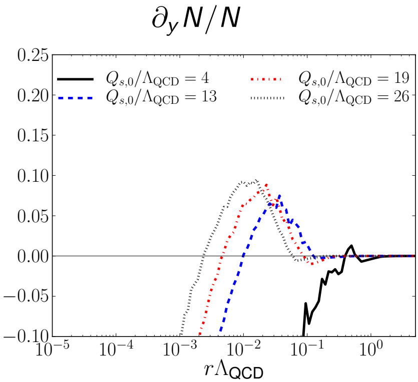
Direct numerical solution:





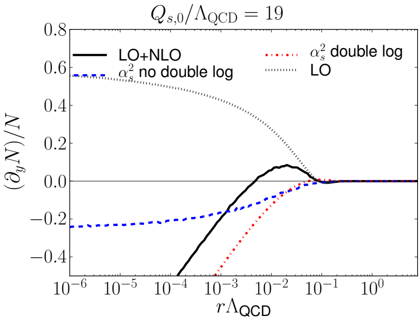
T. Lappi, H.M., arXiv:1502.02400

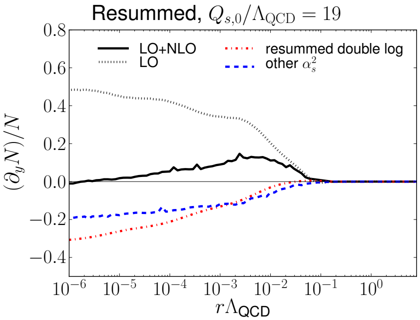
- Double log $\sim \ln X^2/r^2 \ln Y^2/r^2$ drives the amplitude negative
- Negative evolution speed ($\sim$ decreasing UGD when decreasing x)

Resummations: double logs

Large double logarithmic corrections:
Iancu, Madrigal, Mueller, Soyez, Triantafyllopoulos, arXiv:1502.05642

- Resum gluon emissions that are strongly ordered in momentum
- Removes double log $\ln X^2/r^2 \ln Y^2/r^2$, multiplies LO kernel by an oscillatory factor K_{resum}.
- Partially cures the problem





Iancu, Madrigal, Mueller, Soyez, Triantafyllopoulos, arXiv:1507.03651

- Large single transverse logarithm $\sim \alpha_s \ln 1/rQ_s$, resum at leading log accuracy
- Source at NLO: $q \to qg$ and $g \to gg$ splittings that are strongly ordered in transverse size

Effect: LO kernel is multiplied by a factor

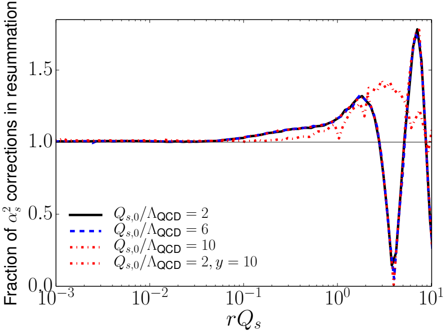
$$K_{\mathrm{STL}} = \exp\left\{ -\frac{\alpha_s N_c A_1}{\pi} \left| \ln \frac{C_{\mathrm{sub}} r^2}{\min\{X^2, Y^2\}} \right| \right\}$$

α_s^2 contribution is also included in K_2 exactly, subtract it from K_{STL}.

Subtraction

Resumming logs $\sim \alpha_s \ln 1/rQ_s$ does not fix the constant C_{sub} inside the logarithm in $KSTL$.

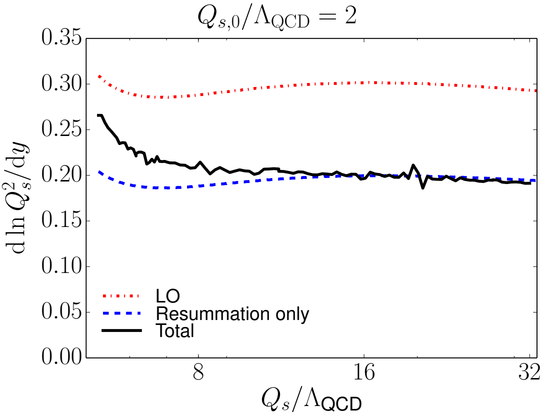
- Fix C_{sub} to capture as much as possible of the NLO corrections



T. Lappi, H.M., arXiv:1601.06598

$C_{\text{sub}} \approx 0.65$

Results: evolution speed

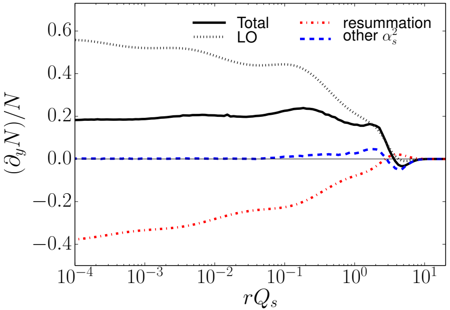


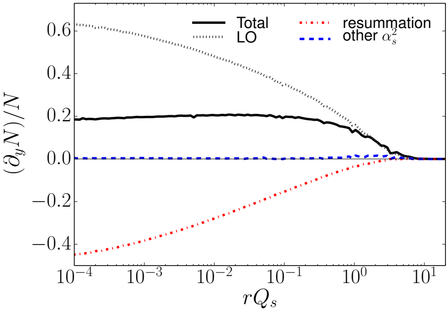
T. Lappi, H.M., arXiv:1601.06598

- Resummation only is good approximation only at very large Q_s
- NLO corrections to the small-x evolution are important
- Higher order corrections are negative: good for phenomenology

Results: contributions to the evolution speed

$y = 0$ $y = 10$



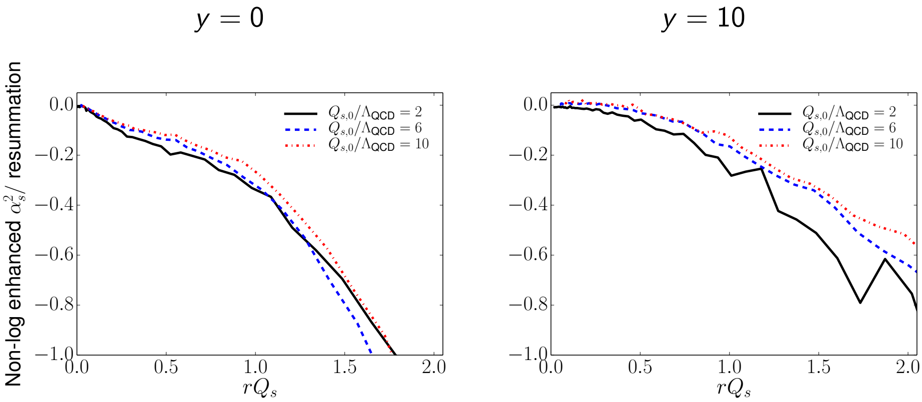


T. Lappi, H.M., arXiv:1601.06598

- Fixed α_s^2 corrections are important around $r \sim 1/Q_s$
- At large rapidities (saturation scales) resummation captures higher order corrections accurately
- Resummation introduces oscillations that are washed out

Closer look around $r \sim 1/Q_s$

Non-log enhanced NLO contribution compared to leading $\log 1/r$.



- Pure α_s^2 terms are important at $r \sim 1/Q_s$

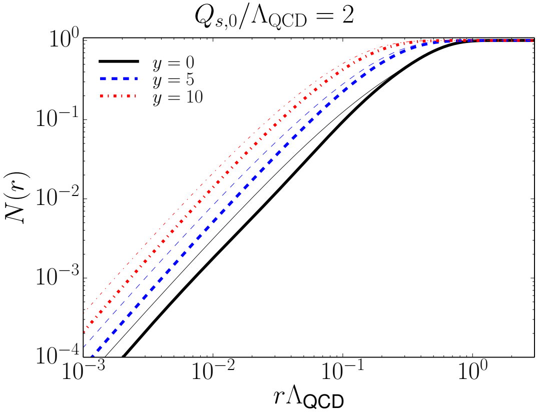
Conclusions

- First solution to the NLO BK equation with
  - Resummation of large double and single logarithms
  - Full NLO contributions
- Unphysical features of fixed α_{s}^2 NLO BK equation are cured by including both resummations
- Fixed order α_{s}^2 contributions are important at moderate Q_s (and numerically demanding)
- At large Q_s the resummation alone is a good approximation

BACKUPS

Results: amplitude

NLO BK with resummations and all terms of the order α_s^2
Positive evolution speed and amplitude!



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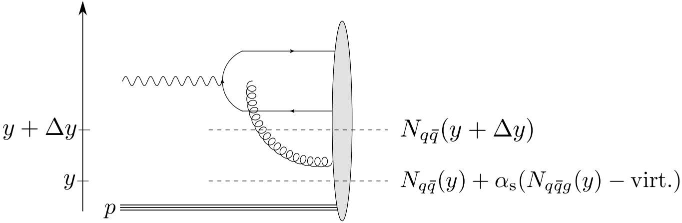
Thin lines: initial condition not resummed

Resummed NLO BK

$$\partial_y S(r) = \frac{\alpha_s}{2\pi^2} K_{\text{Resum}} K_{\text{STL}} K_1 \otimes [S(X)S(Y) - S(r)]$$
$$- \frac{\alpha_s}{2\pi^2} K_1 K_{\text{sub}} \otimes [S(X)S(Y) - S(r)]$$
$$+ \frac{\alpha_s^2 N_c^2}{8\pi^4} K_2 \otimes [S(X)S(z-z')S(Y') - S(X)S(Y)]$$
$$+ \frac{\alpha_s^2 N_f N_c}{8\pi^4} K_f \otimes S(Y)[S(X') - S(X)]$$

- K_{sub}: α_s part of K_{STL}, removes double counting

CGC evolution: BK



Consider γ^* − *hadron* scattering

- $q\bar{q}$ dipole emits a gluon
- The gluon can be counted as a part of
  - γ^* wave function
  - Target wave function
- Renormalization group equation for the dipole-target amplitude N