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The high energy radiation pattern from BFKLex

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Outline

- Short introduction to the iterative solution of the Balitsky-Fadin-Kuraev-Lipatov (BFKL) equation
- The collinear double logarithms in the BFKL gluon Green's function
- The actual implementation in BFKLex, a BFKL based Monte Carlo code in C++ (G.C, A. Sabio Vera)
- The anatomy of the gluonic ladder and new observables
- Conclusions and Outlook

Brief introduction (LL)

- BFKL formalism: in high energy scattering terms like $\alpha_s^n \log^n(s) \sim \alpha_s^n (y_A - y_B)^n$ need to be resummed.
- $$\sigma(Q_1, Q_2, Y) = \int d^2\vec{k}_A d^2\vec{k}_B \underbrace{\phi_A(Q_1, \vec{k}_A)\, \phi_B(Q_2, \vec{k}_B)}_{\text{PROCESS-DEPENDENT}} \underbrace{f(\vec{k}_A, \vec{k}_B, Y)}_{\text{UNIVERSAL}}$$
- $\phi_{A,B}(Q_{1,2}, \vec{k}_{A,B})$ are the process-dependent impact factors
- The gluon Green's function $f(\vec{k}_A, \vec{k}_B, Y)$ is universal and depends on the scales $\vec{k}_{A,B}$ and the energy $\sim e^{Y/2}$

Brief introduction (LL)

- The main goal is to have a way to calculate the gluon Green's function (GGF).

- The GGF is the solution to the BFKL equation. Use the iterative form:

- $$f = e^{\omega\left(\vec{k}_A\right)Y}\left\{\delta^{(2)}\left(\vec{k}_A-\vec{k}_B\right)+\sum_{n=1}^{\infty}\prod_{i=1}^{n}\frac{\alpha_s N_c}{\pi}\int d^2\vec{k}_i\frac{\theta\left(k_i^2-\lambda^2\right)}{\pi k_i^2}\right.$$
$$\left.\int_0^{y_{i-1}}dy_i e^{\left(\omega\left(\vec{k}_A+\sum_{l=1}^{i}\vec{k}_l\right)-\omega\left(\vec{k}_A+\sum_{l=1}^{i-1}\vec{k}_l\right)\right)y_i}\delta^{(2)}\left(\vec{k}_A+\sum_{l=1}^{n}\vec{k}_l-\vec{k}_B\right)\right\}$$

$\omega\left(\vec{q}\right) = -\frac{\alpha_s N_c}{\pi}\log\frac{q^2}{\lambda^2}$ is the gluon Regge trajectory

From LO to NLO

- At NLL terms like $\alpha_s^{n-1} \log^n(s)$ need to be resummed
- The BFKL kernel changes, the real part becomes much more complicated. Dominant term:

$$\theta\left(k_i^2-\lambda^2\right) \to \theta\left(k_i^2-\lambda^2\right) - \underbrace{\frac{\bar{\alpha}_s}{4} \ln^2\left(\frac{\vec{k}_A^2}{\left(\vec{k}_A+\vec{k}_i\right)^2}\right)}_{\text{NLL}}$$

- These collinear logarithms are an issue, they need to be resummed as well.

Dealing with the collinear logarithms

- Remove from the NLO BFKL kernel the double log:

$$-\frac{\bar{\alpha}_s^2}{4}\frac{1}{(\vec{q}-\vec{k})^2}\ln^2\left(\frac{q^2}{k^2}\right)$$

- Replace it with:

$$\left\{\left(\frac{q^2}{k^2}\right)^{-b\,\bar{\alpha}_s\frac{|k-q|}{k-q}}\sqrt{\frac{2(\bar{\alpha}_s+a\,\bar{\alpha}_s^2)}{\ln^2\left(\frac{q^2}{k^2}\right)}}J_1\left(\sqrt{2(\bar{\alpha}_s+a\,\bar{\alpha}_s^2)\ln^2\left(\frac{q^2}{k^2}\right)}\right)\right.$$
$$\left.-\bar{\alpha}_s-a\,\bar{\alpha}_s^2+b\,\bar{\alpha}_s^2\frac{|k-q|}{k-q}\ln\left(\frac{q^2}{k^2}\right)\right\}\frac{1}{(\vec{q}-\vec{k})^2}$$

$$a=\frac{5}{12}\frac{\beta_0}{N_c}-\frac{13}{36}\frac{n_f}{N_c^3}-\frac{55}{36}\text{ and } b=-\frac{1}{8}\frac{\beta_0}{N_c}-\frac{1}{6}\frac{n_f}{N_c^3}-\frac{11}{12}$$

Implementation in the BFKLex

- We work at NLO: NLO kernel with the additional resummation of the double logs (NLO+Double Logs)

- In particular we use the scale invariant part of the NLO BFKL kernel, $\beta_0 = 0$ and $n_f = 0$

- We use the $\mathrm{anti}\text{-}k_t$ jet algorithm in the FastJet implementation (Cacciari, Salam, Soyez)
  The jet radius is set to R=0.7

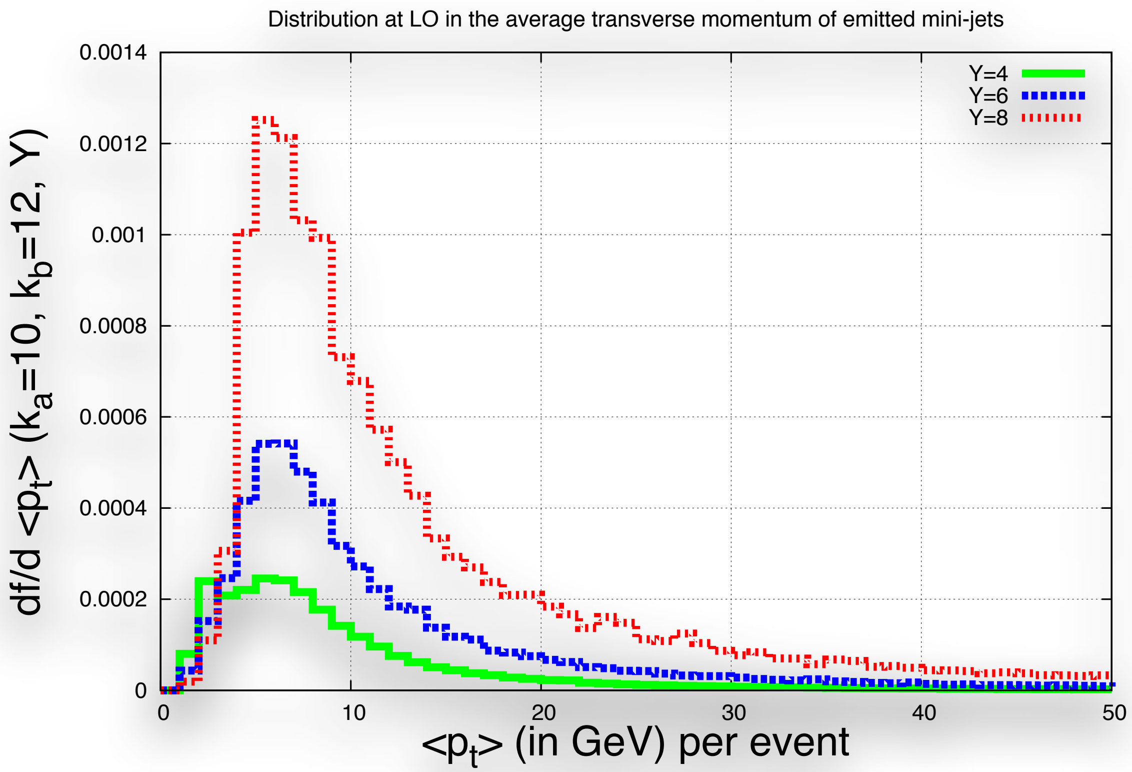
Three characteristics of the jet radiation pattern in the multi-Regge kinematics

- Introduce three quantities related to the jet activity along the ladder. These characterize uniquely the event (but not fully).

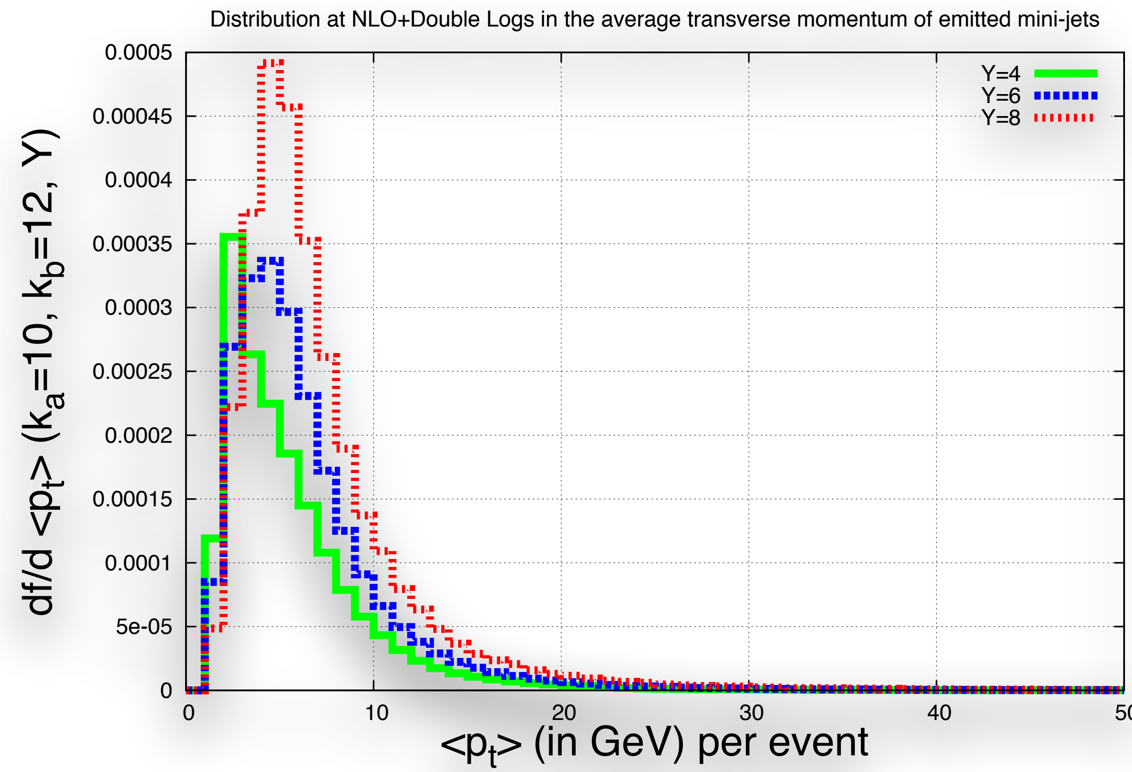
- average p_t $$\langle p_t \rangle = \frac{1}{N}\sum_{i=1}^{N} |k_i|$$
- average azimuthal angle $$\langle \phi \rangle = \frac{1}{N}\sum_{i=1}^{N} \phi_i$$
- rapidity ratio between subsequent jets $$\langle \mathcal{R}_y \rangle = \frac{1}{N+1}\sum_{i=1}^{N+1} \frac{y_i}{y_{i-1}}$$

$$y_0 = y_a,\ \ y_{N+1} = y_b = 0 \text{ and } y_{i-1} > y_i$$

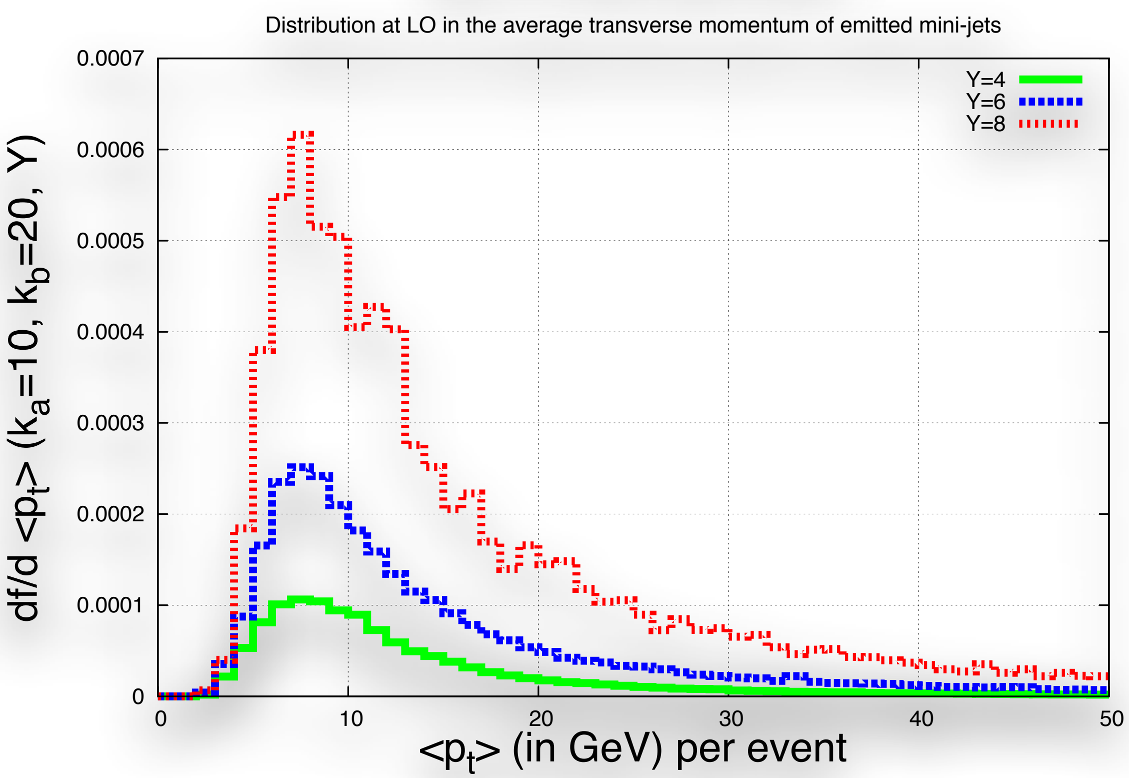
Average p_t



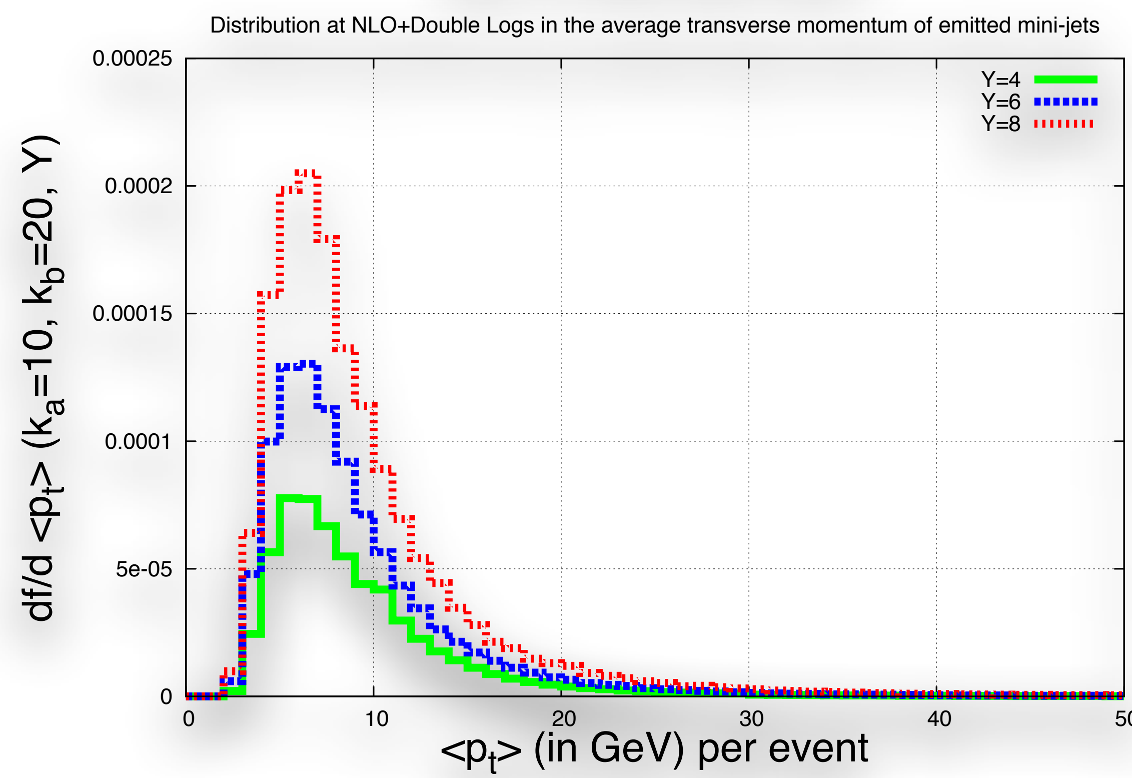
Distribution at LO in the average transverse momentum of emitted mini-jets
df/d <pt> (ka=10, kb=12, Y)
<pt> (in GeV) per event
Y=4
Y=6
Y=8



Distribution at NLO+Double Logs in the average transverse momentum of emitted mini-jets
df/d <pt> (ka=10, kb=12, Y)
<pt> (in GeV) per event
Y=4
Y=6
Y=8

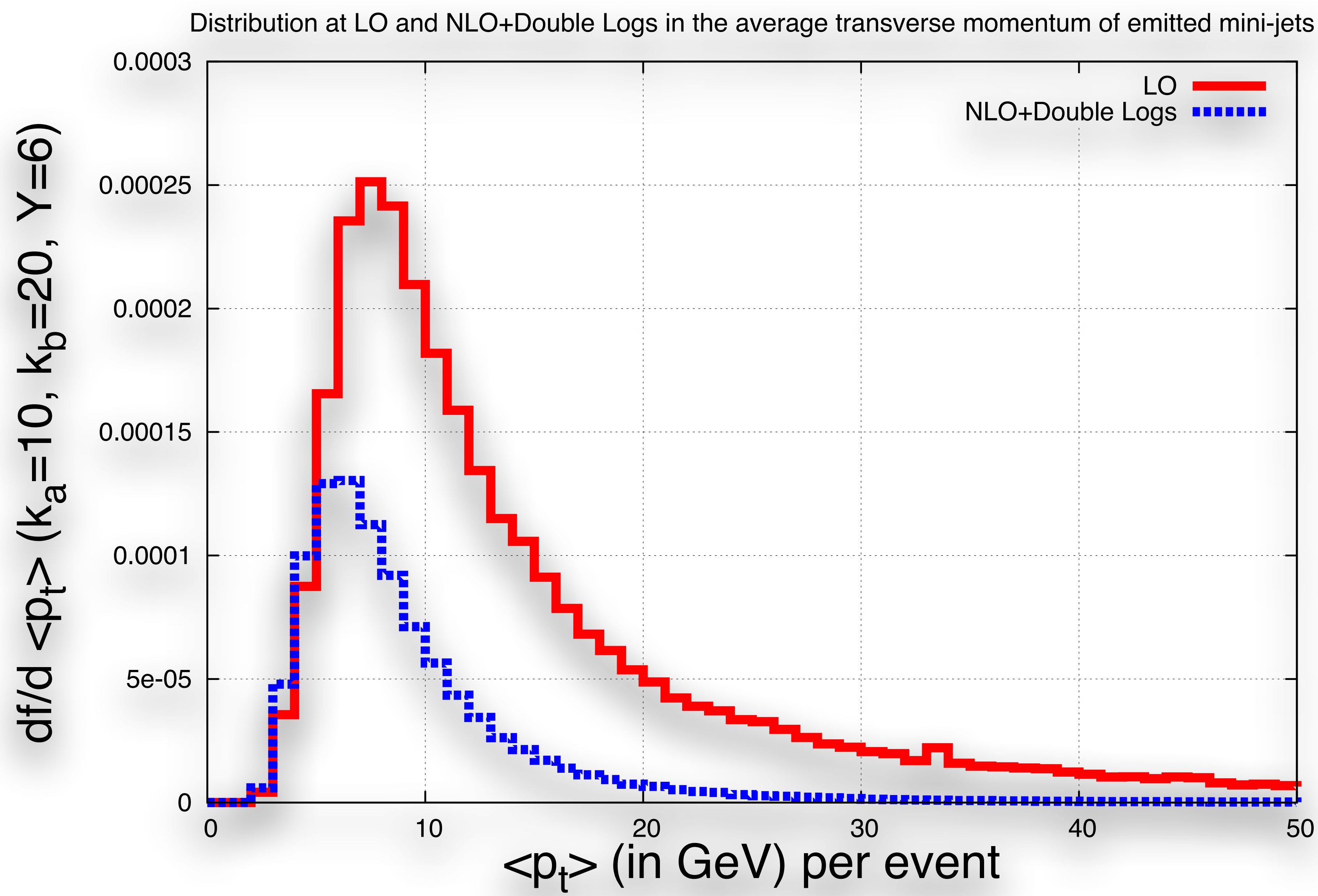


Distribution at LO in the average transverse momentum of emitted mini-jets
df/d <pt> (ka=10, kb=20, Y)
<pt> (in GeV) per event
Y=4
Y=6
Y=8



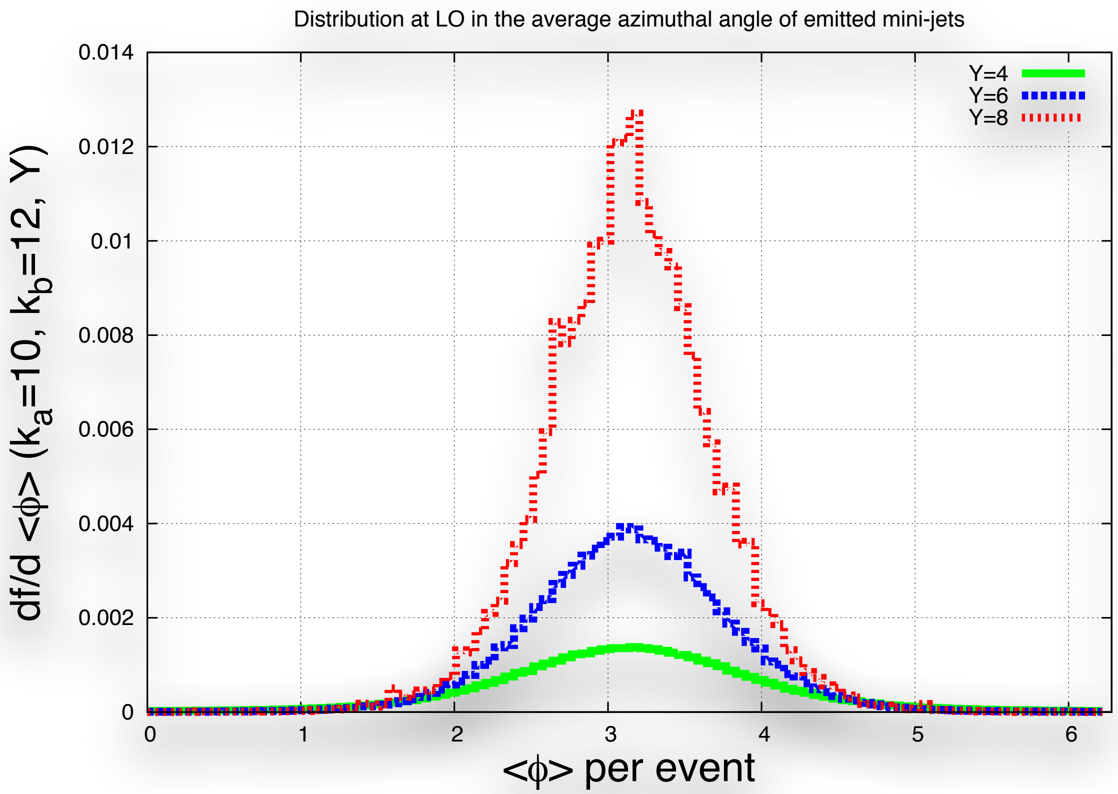
Distribution at NLO+Double Logs in the average transverse momentum of emitted mini-jets
df/d <pt> (ka=10, kb=20, Y)
<pt> (in GeV) per event
Y=4
Y=6
Y=8

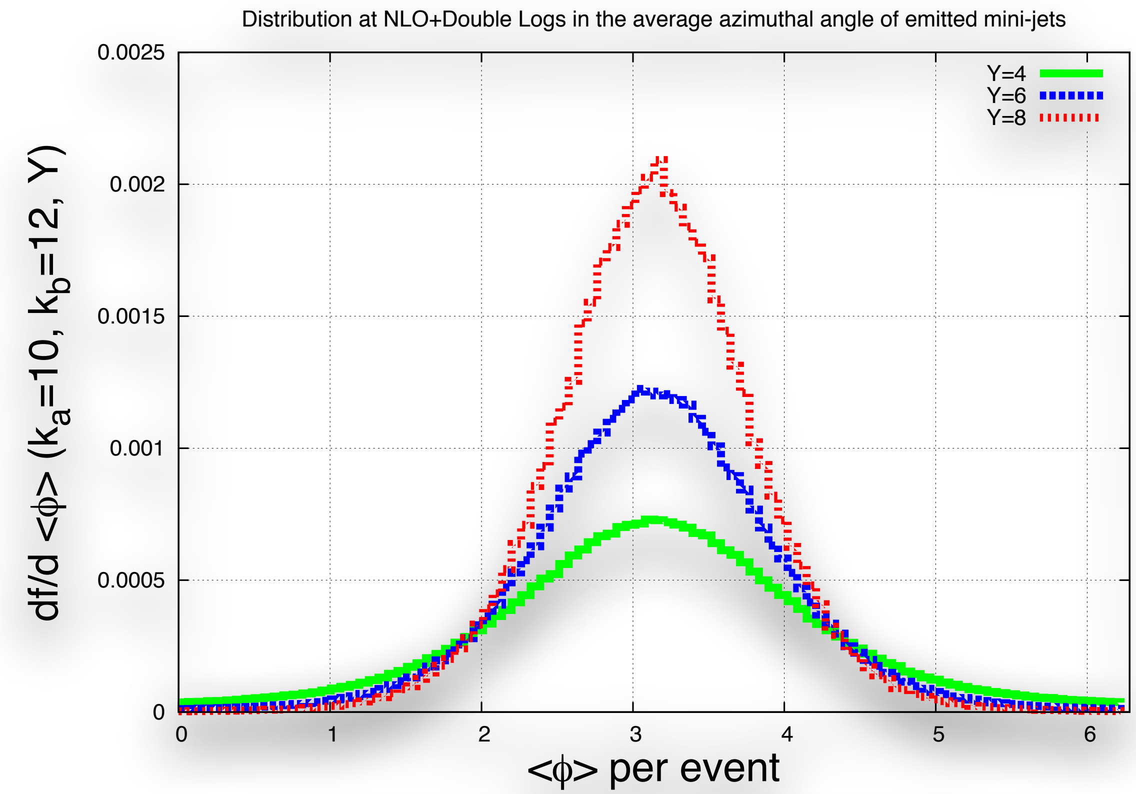
Average p_t

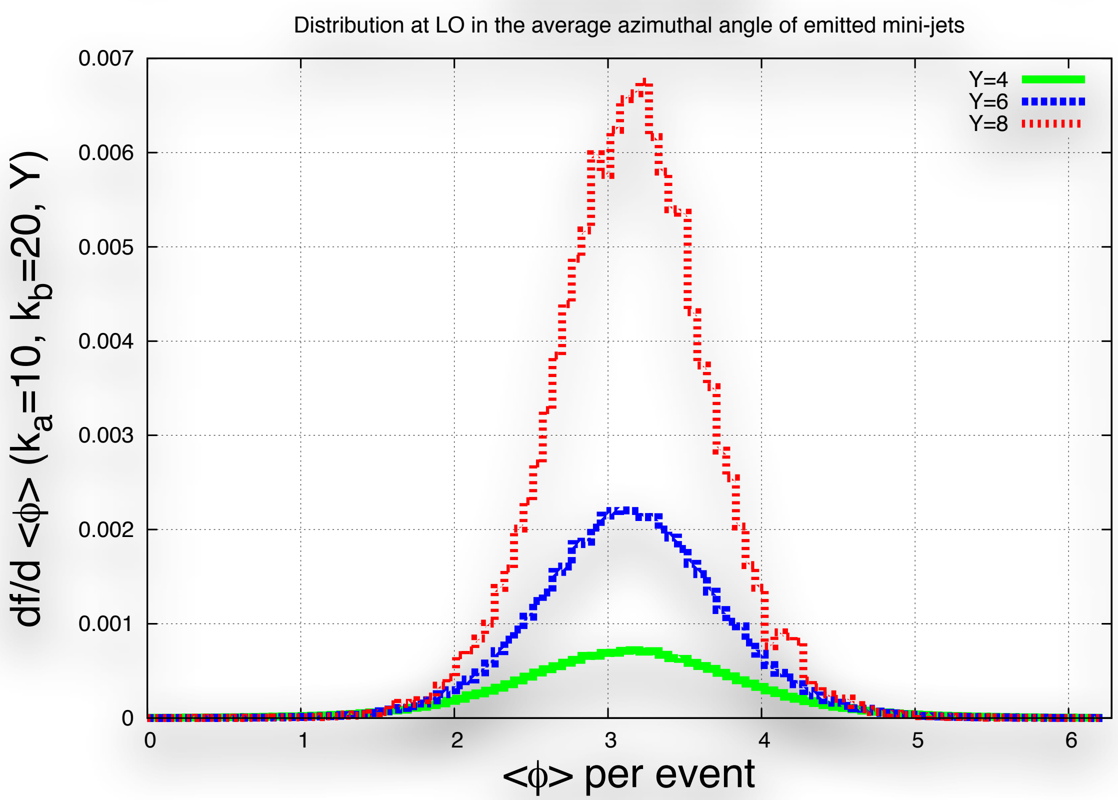


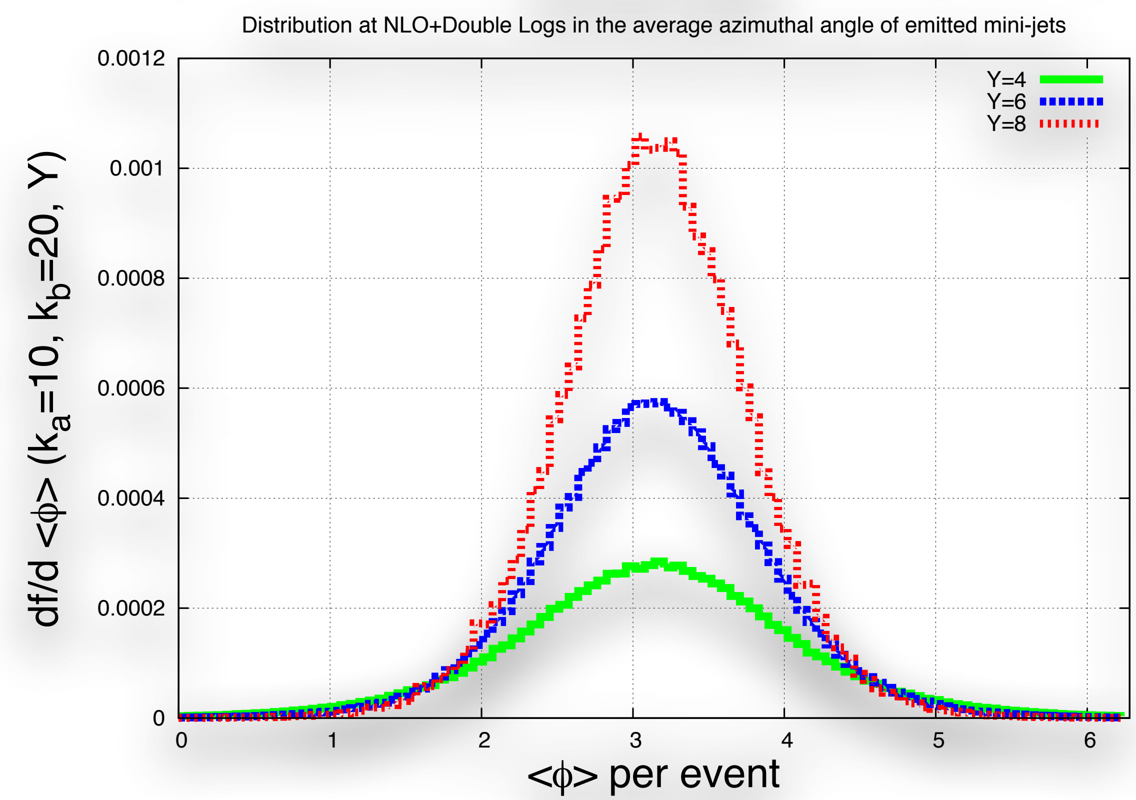
Distribution at LO and NLO+Double Logs in the average transverse momentum of emitted mini-jets
LO
NLO+Double Logs
df/d <pt> (ka=10, kb=20, Y=6)
<pt> (in GeV) per event
0.0003
0.00025
0.0002
0.00015
0.0001
5e-05
0
0
10
20
30
40
50

Average azimuthal angle

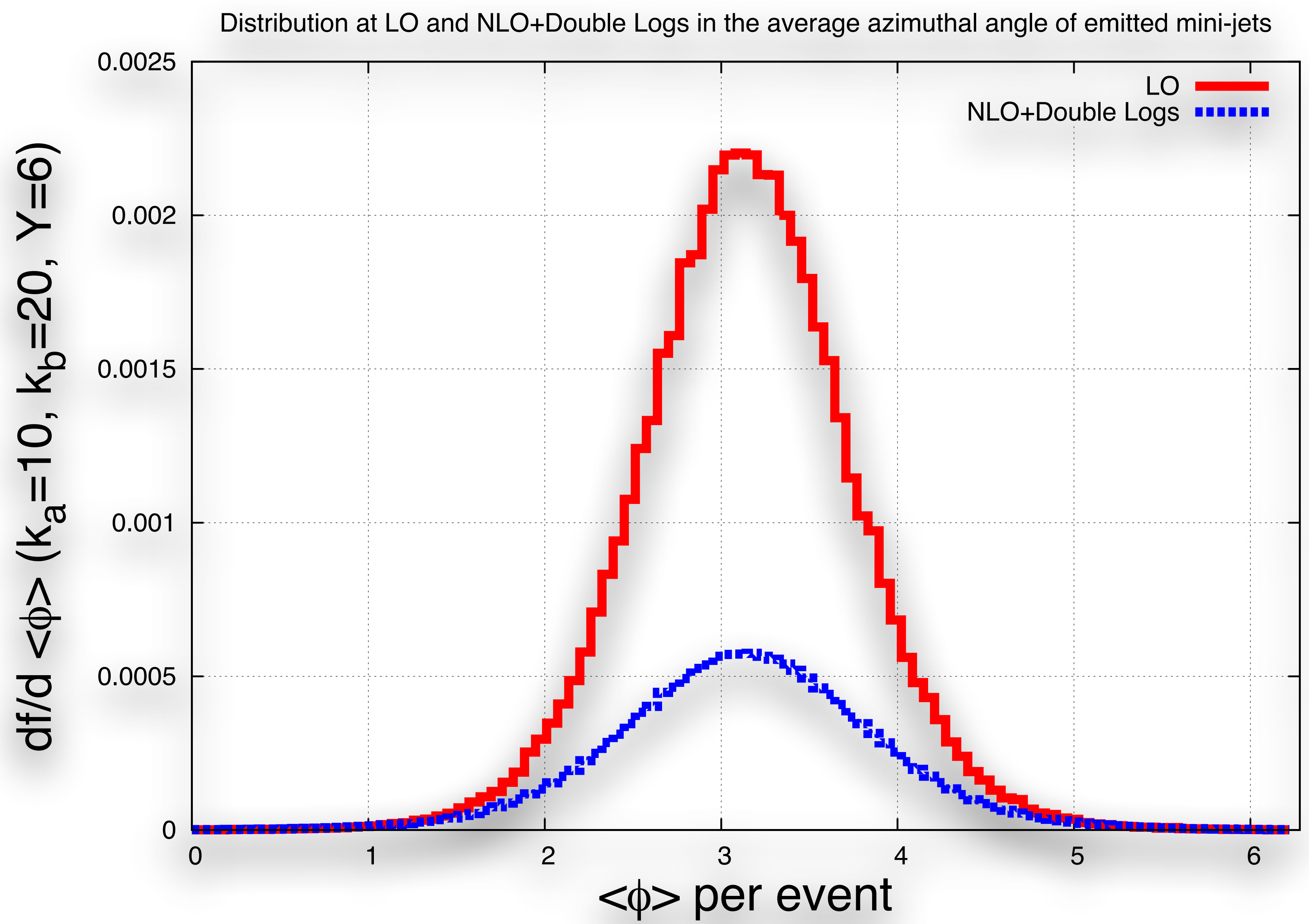








Average azimuthal angle

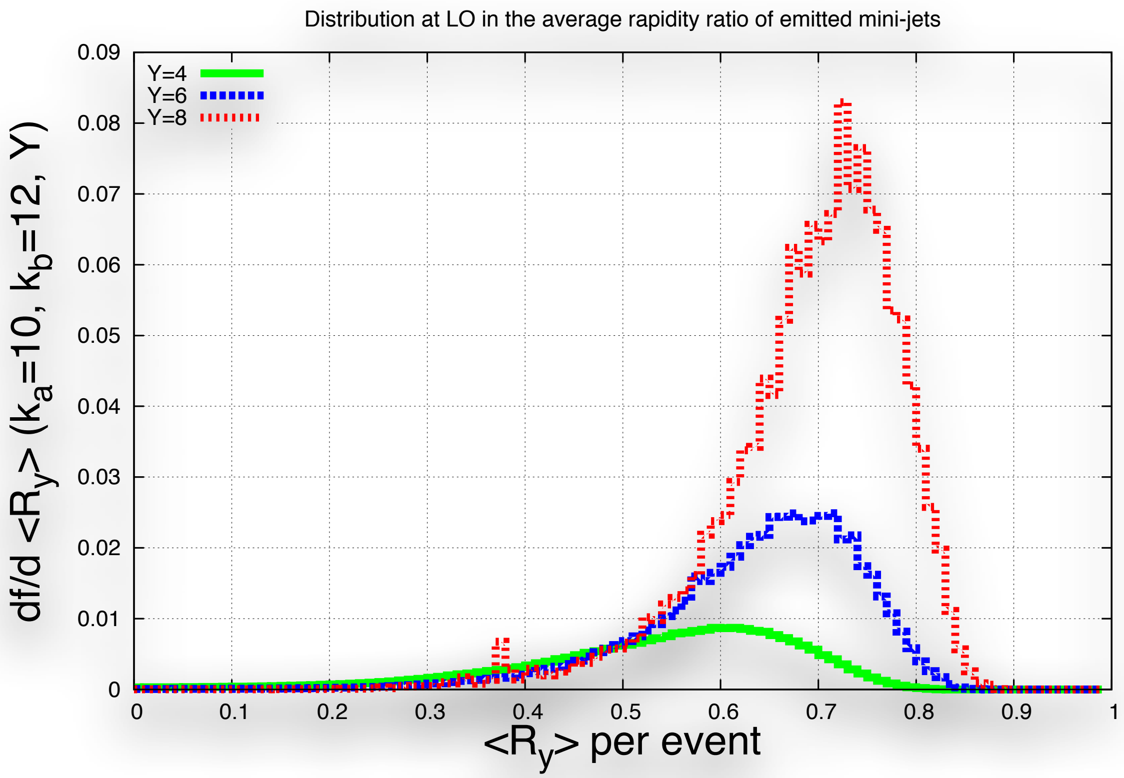


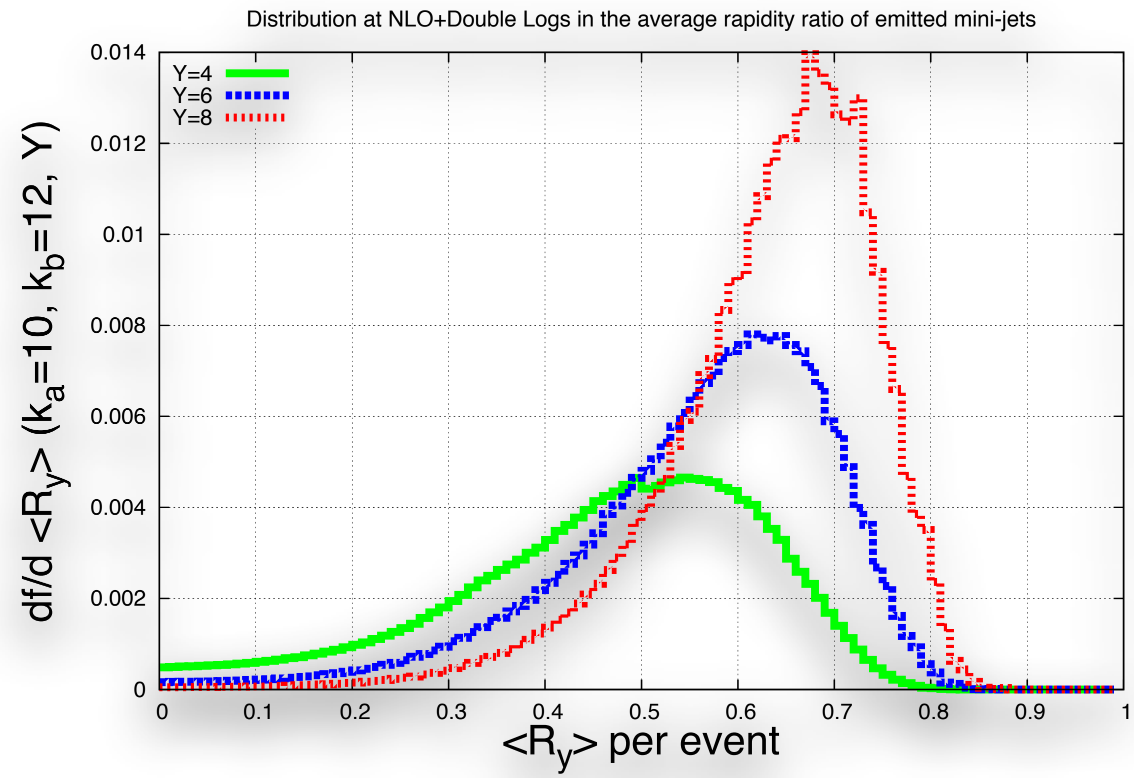
Average rapidity ratio for a typical multi-Regge kinematics event

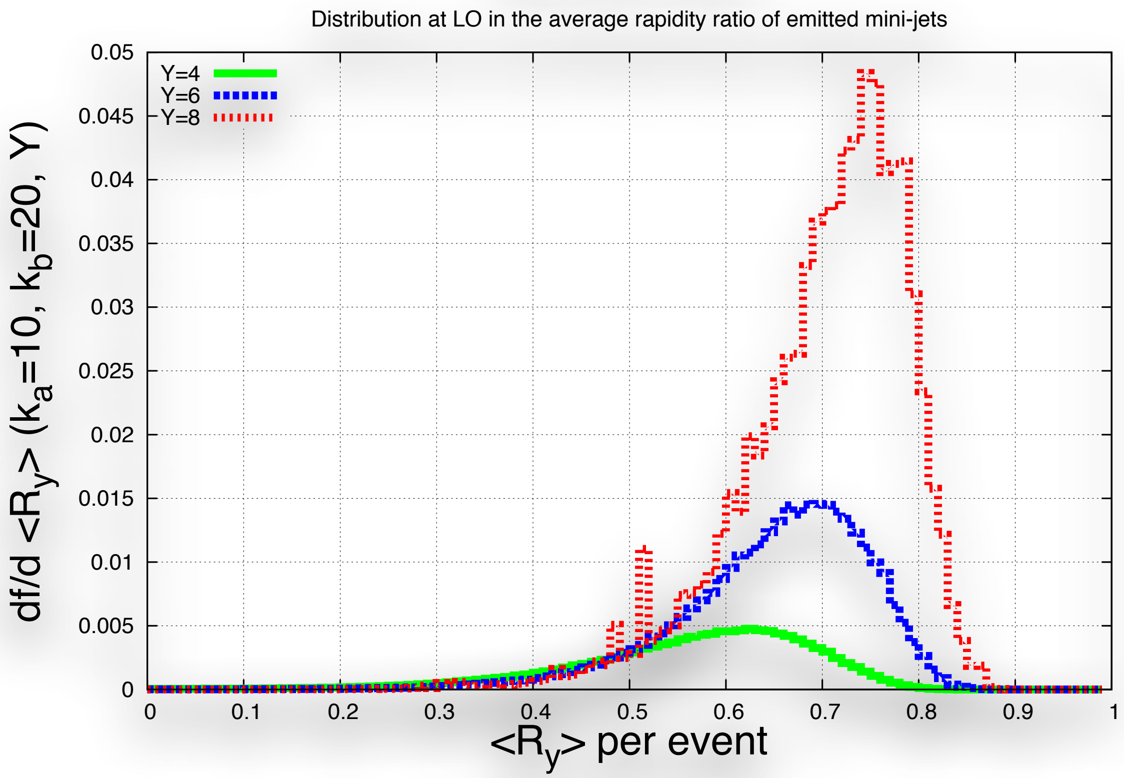
- In a typical multi-Regge kinematics event we expect the total rapidity interval Y to be populated by N emissions equally spaced in rapidity intervals of length Δ such that $\mathrm{Y} = (\mathrm{N}+1)\,\Delta$. Then, the i-th emission is at $(\mathrm{N}+1-i)\,\Delta$ and

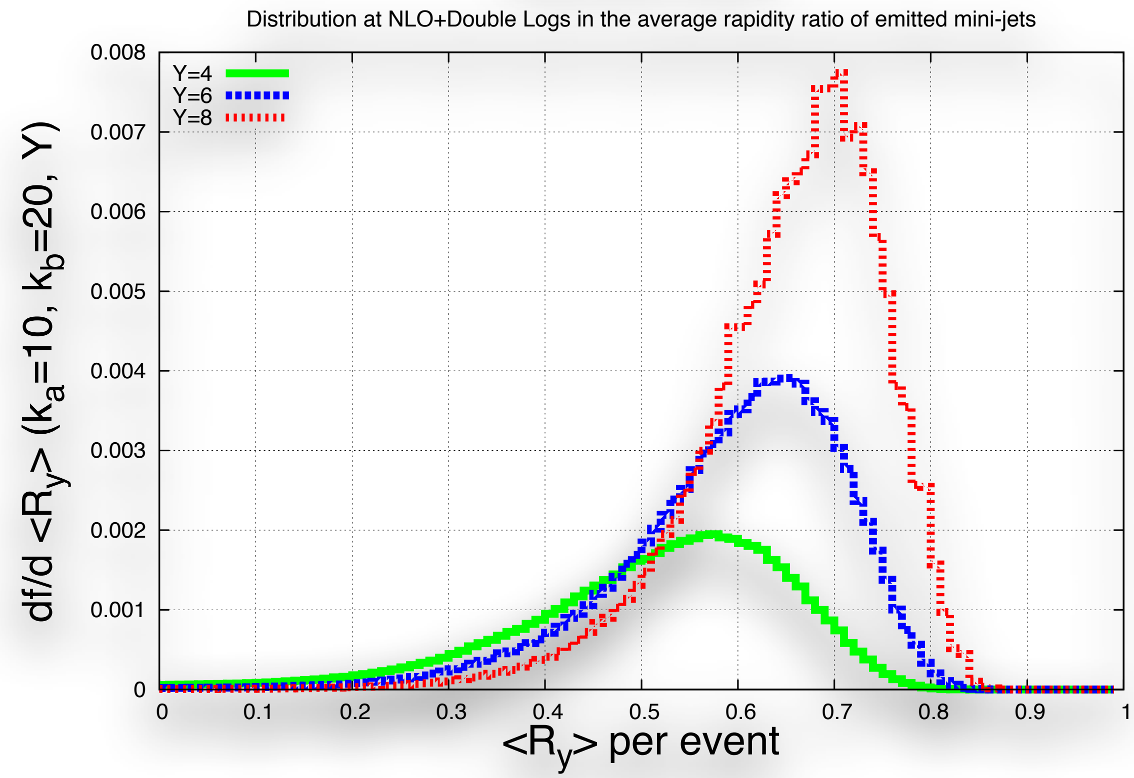
$$
\begin{aligned}
\langle \mathcal{R}_y \rangle_{\mathrm{MRK}} &= \frac{1}{N+1} \sum_{i=1}^{N} \frac{i}{i+1} \;=\; \frac{N+1-\psi(N+2)+\psi(1)}{N+1} \\
&= 1 + \frac{\Delta}{Y} \left(\psi(1) - \psi\left(1 + \frac{Y}{\Delta}\right) \right) \\
&\simeq 1 + \frac{\Delta}{Y} \left(\psi(1) + \ln \frac{\Delta}{Y} \right) + \cdots
\end{aligned}
$$

Average rapidity ratio

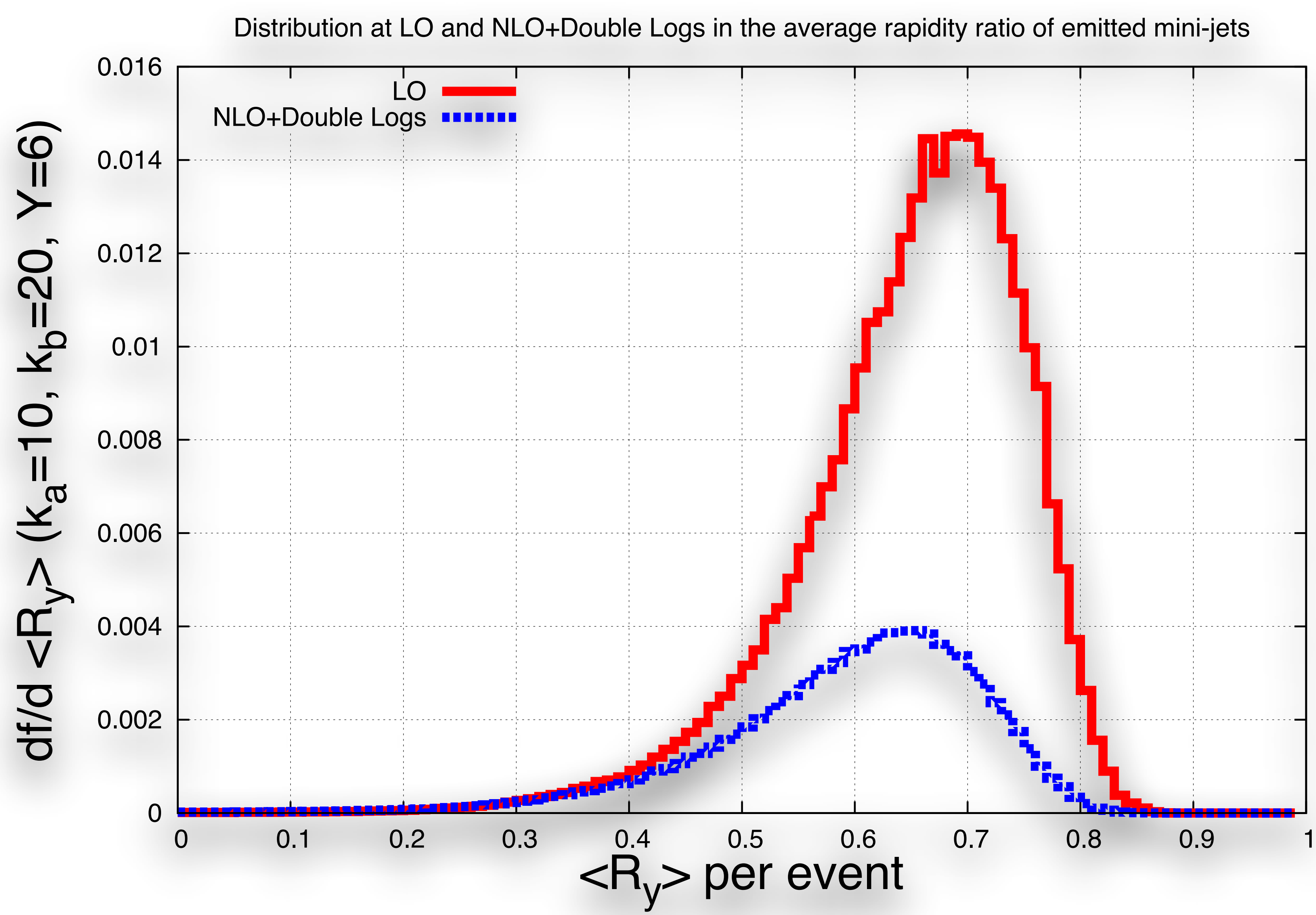








Average rapidity ratio



Conclusions

- BFKLex is a MC tool with the NLO BFKL collinearly improved kernel implemented
- One can apply all sorts of kinematical cuts to pin down situation where the multi-Regge kinematics should be dominant
- Newly proposed observables can be studied in a fully differential way and new BFKL probes should emerge