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Gluon transverse momentum dependent correlators in polarized high energy processes

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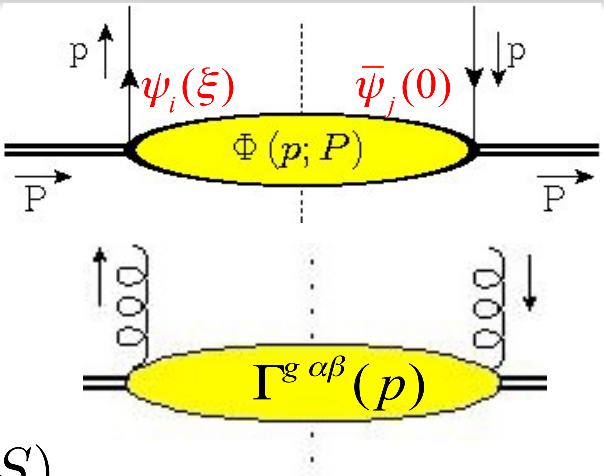
European Research Council

Introduction



- PDFs and TMDs to incorporate hadron structure

$$u_i(k)\overline{u}_j(k) \sim (\not{k})_{ij} \Longrightarrow \Phi_{ij}(k;P,S)$$
$$\epsilon^{\alpha}(k)\epsilon^{\beta *}(k) \sim -g_T^{\alpha\beta} \Longrightarrow \Gamma^{\alpha\beta}(k;P,S)$$

- Employed at high energies

$$k^{\mu} = xP^{\mu} + k_T^{\mu} + (k^2 - k_T^2)n^{\mu}$$
$$\Longrightarrow \ \Phi(x,k_T;P,S) \quad \text{and} \quad \Gamma^{\alpha\beta}(x,k_T;P,S)$$

- Polarized targets provide opportunities and challenges

$$M\,S^{\mu} = S_L\,P^{\mu} + M\,S_T^{\mu} + M^2\,S_L\,n^{\mu}$$
$$M^2\,S^{\mu\nu} = -S_{LL}\,P^{\mu}P^{\nu} + \tfrac{1}{2}P^{\{\mu}S_{LT}^{\nu\}} + \tfrac{1}{2}M^2\,S_{TT}^{\mu\nu} + \ldots$$

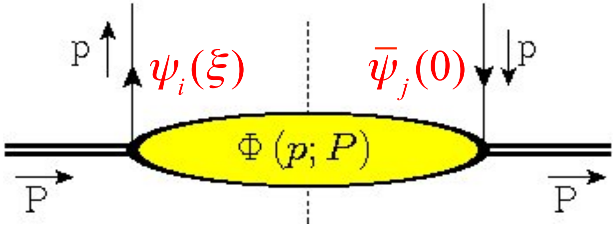
- At high energies x linked to scaling variables (e.g. x = $Q^2/2P.q$) and convolutions of transverse momenta are linked to azimuthal asymmetries (noncollinearity) requiring semi-inclusivity and/or polarization

- Operator structure PDFs and TMDs can be embedded in a field theoretical framework via Operator Product Expansion (OPE), connecting Mellin moments and transverse moments of (TMD) PDFs with particular QCD matrix elements of operators (spin and twist expansion, gluonic pole matrix elements)

Matrix elements for TMDs

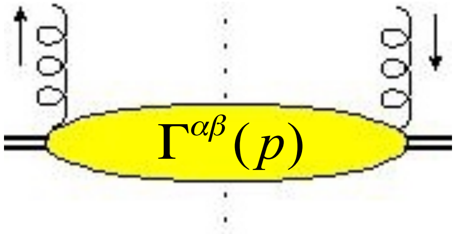
- quark-quark

$$\Phi_{ij}(x,p_T;n) = \int \frac{d\,\xi{\cdot}P\,d^2\xi_T}{(2\pi)^3}\,e^{ip\cdot\xi}\langle P,S|\overline{\psi}_j(0)\,\psi_i(\xi)|P,S\rangle\,\big|_{\xi\cdot n=0}$$

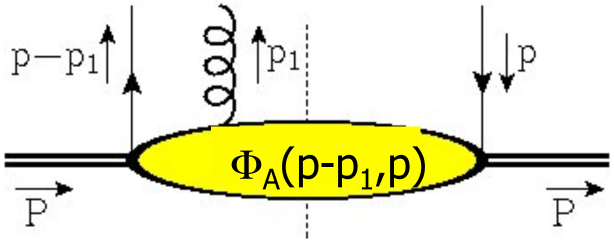


- gluon-gluon

$$\Gamma^{\mu\nu}(x,p_T;n) = \int \frac{d\,\xi{\cdot}P\,d^2\xi_T}{(2\pi)^3}\,e^{ip\cdot\xi}\,\langle P,S|\,F^{n\mu}(0)\,F^{n\nu}(\xi)\,|P,S\rangle\big|_{\xi\cdot n=0}$$



- quark-gluon-quark



$$\Phi^{\alpha}_{D;ij}(p-p_1,p_1\,|\,p) = \int \frac{d^4\xi\,d^4\eta}{(2\pi)^8}\,e^{i(p-p_1).\xi+ip_1.\eta}\left\langle P\left|\overline{\psi}_j(0)D^{\alpha}(\eta)\,\psi_i(\xi)\right|P\right\rangle$$

$$\Phi^{\alpha}_{F;ij}(p-p_1,p_1\,|\,p) = \int \frac{d^4\xi\,d^4\eta}{(2\pi)^8}\,e^{i(p-p_1).\xi+ip_1.\eta}\left\langle P\left|\overline{\psi}_j(0)F^{n\alpha}(\eta)\,\psi_i(\xi)\right|P\right\rangle$$

TMDs and color gauge invariance

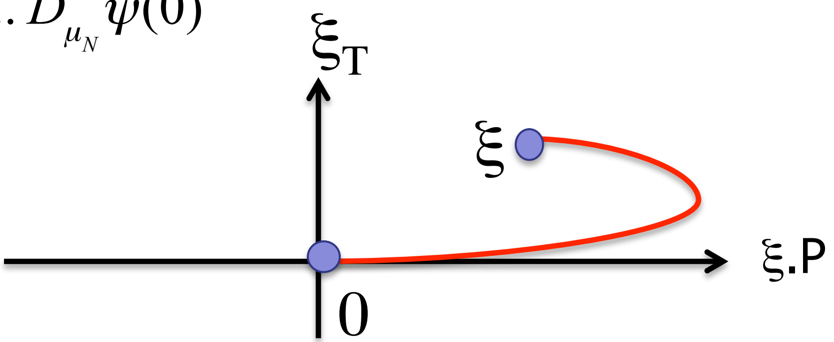
- Gauge invariance in a non-local situation requires a gauge link U(0,ξ)

$$\bar{\psi}(0)\psi(\xi) = \sum_n \frac{1}{n!}\xi^{\mu_1}\ldots\xi^{\mu_N}\bar{\psi}(0)\partial_{\mu_1}\ldots\partial_{\mu_N}\psi(0)$$

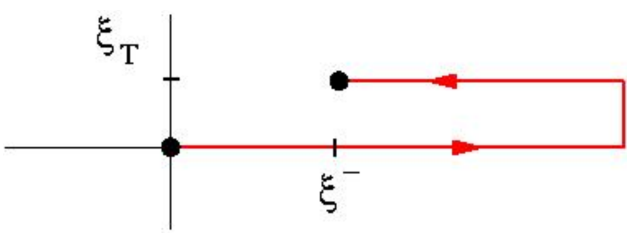
$$U(0,\xi) = \mathcal{P}\ \exp\left(-ig\int_0^\xi ds^\mu A_\mu\right)$$

$$\bar{\psi}(0)U(0,\xi)\psi(\xi) = \sum_n \frac{1}{n!}\xi^{\mu_1}\ldots\xi^{\mu_N}\bar{\psi}(0)D_{\mu_1}\ldots D_{\mu_N}\psi(0)$$

- Introduces path dependence for $\Phi(x,p_T)$



- 'Dominant' paths: along lightcone connected at lightcone infinity (staples)



- Reduces to 'straight line' for $\Phi(x)$

$$\Phi^{[U]}(x,p_T) \;\Rightarrow\; \Phi(x)$$

Non-universality because of process dependent gauge links

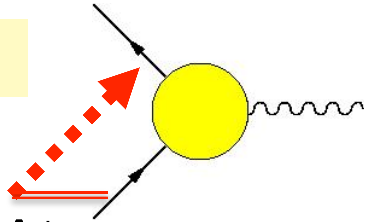
$$\Phi_{ij}^{q[C]}(x,p_T;n) = \int \frac{d(\xi.P)\,d^2\xi_T}{(2\pi)^3}\, e^{i\,p.\xi} \left\langle P \middle| \bar{\psi}_j(0)\, U_{[0,\xi]}^{[C]}\, \psi_i(\xi) \middle| P \right\rangle_{\xi.n=0}$$

TMD

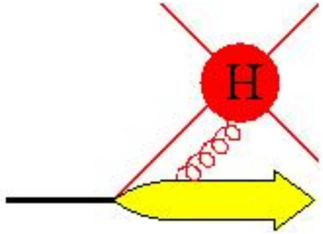
path dependent gauge link

- Gauge links associated with dimension zero (not suppressed!) collinear $A^n = A^+$ gluons, leading for TMD correlators to process-dependence:

DY



... A^+ ...
(resummation)



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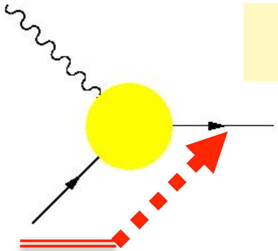
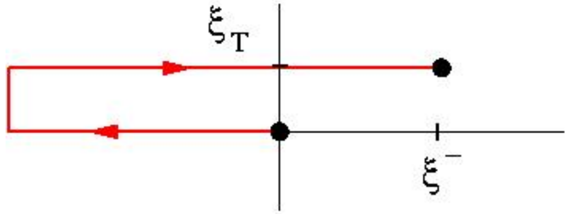
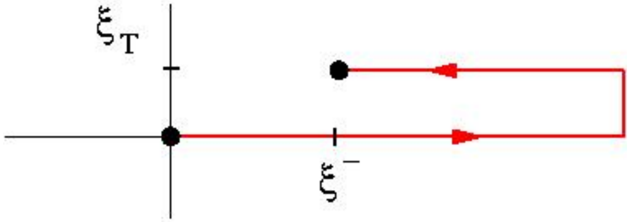


... A^+ ...

$\Phi^{[-]}$



Time reversal



$\Phi^{[+]}$

Belitsky, Ji, Yuan, 2003; Boer, M, Pijlman, 2003

Non-universality because of process dependent gauge links

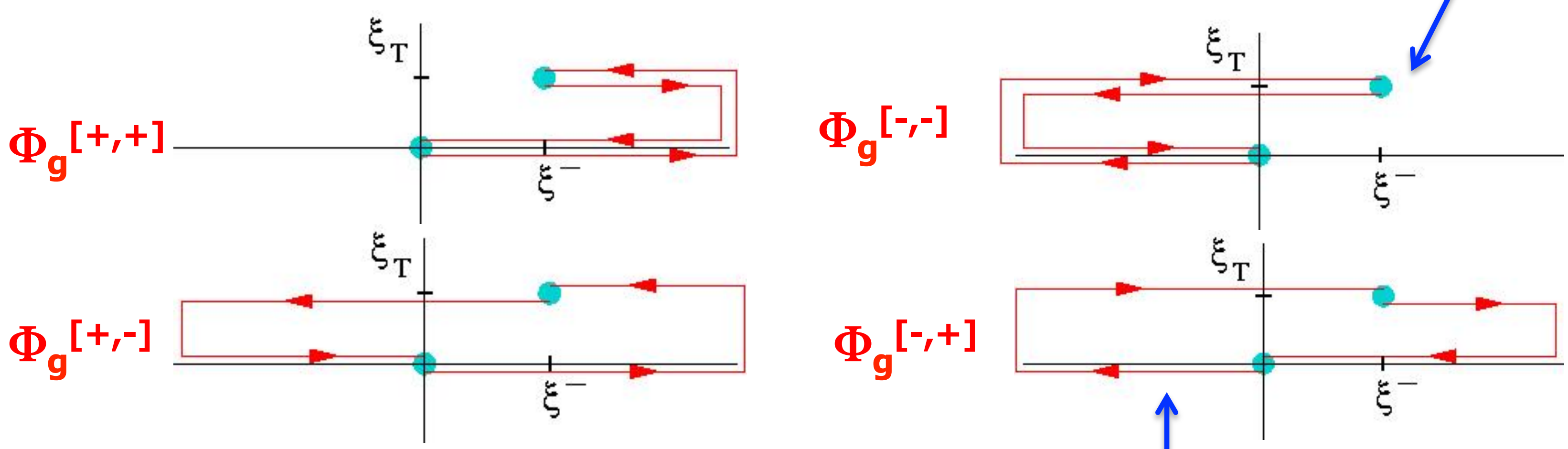
$$\Phi_g^{\alpha\beta[C,C']}(x,p_T;n) = \int \frac{d(\xi.P)d^2\xi_T}{(2\pi)^3} e^{i\,p.\xi} \left\langle P \middle| U_{[\xi,0]}^{[C]} F^{n\alpha}(0) U_{[0,\xi]}^{[C']} F^{n\beta}(\xi) \middle| P \right\rangle_{\xi.n=0}$$

- The TMD gluon correlators contain two links, which can have different paths. Note that standard field displacement involves C = C’

$$F^{\alpha\beta}(\xi) \rightarrow U_{[\eta,\xi]}^{[C]} F^{\alpha\beta}(\xi) U_{[\xi,\eta]}^{[C]}$$

- Basic (simplest) gauge links for gluon TMD correlators:

gg → H



in gg → $Q\bar{Q}$

Bomhof, M, Pijlman, 2006; Dominguez, Xiao, Yuan, 2011

Matrix elements for TMDs

- quark-quark

$$\Phi_{ij}^{[U]}(x,p_T;n) = \int \frac{d\,\xi{\cdot}P\,d^2\xi_T}{(2\pi)^3}\,e^{ip\cdot\xi}\langle P,S|\overline{\psi}_j(0)\,U_{[0,\xi]}\psi_i(\xi)|P,S\rangle\,\big|_{\xi\cdot n=0}$$

- gluon-gluon

$$\Gamma^{[U,U']\,\mu\nu}(x,p_T;n) = \int \frac{d\,\xi{\cdot}P\,d^2\xi_T}{(2\pi)^3}\;e^{ip\cdot\xi}\,\langle P,S|\,F^{n\mu}(0)\,U_{[0,\xi]}\,F^{n\nu}(\xi)\,U'_{[\xi,0]}\,|P,S\rangle\big|_{\xi\cdot n=0}$$

- … and even single Wilson loop correlator

$$\delta(x)\,\Gamma_0^{[U,U']}(p_T;n) = \int \frac{d\,\xi{\cdot}P\,d^2\xi_T}{(2\pi)^3}\;e^{ip\cdot\xi}\,\langle P,S|U_{[0,\xi]}\,U'_{[\xi,0]}\,|P,S\rangle\big|_{\xi\cdot n=0}$$

Quark correlator

- Unpolarized target

$$\Phi^{[U]}(x,k_T) = \left\{ f_1^{[U]}(x,k_T^2) + i\, h_1^{\perp\,[U]}(x,k_T^2)\,\frac{\not{k}_T}{M} \right\} \frac{\not{P}}{2}$$

- Vector polarized target

$$\Phi_L^{[U]}(x,k_T) = \left\{ S_L g_1^{[U]}(x,k_T^2)\gamma_5 + S_L h_{1L}^{\perp\,[U]}(x,k_T^2)\,\frac{\gamma_5 \not{k}_T}{M} \right\} \frac{\not{P}}{2}$$

$$\Phi_T^{[U]}(x,k_T) = \left\{ g_{1T}^{[U]}(x,k_T^2)\,\frac{k_T\cdot S_T}{M} \right.$$

$$\left. + h_1^{[U]}(x,k_T^2)\gamma_5 \not{S}_T + h_{1T}^{\perp\,[U]}(x,k_T^2)\,\frac{k_T^{\alpha\beta} S_{T\alpha}\gamma_\beta\gamma_5}{M^2} \right\} \frac{\not{P}}{2}$$

- Surviving in collinear correlators Φ(x) and including flavor index

$$f_1^q(x) \equiv q(x) \quad g_1^q(x) = \Delta q(x) \quad h_1^q(x) = \delta q(x)$$

- Note: be careful with use of h_{1T} and non-traceless tensor with $k_T.S_T$ since h_{1T} is not a TMD of definite rank!

Definite rank TMDs

- Expansion in constant tensors in transverse momentum space

$$g_T^{\mu\nu} = g^{\mu\nu} - P^{\{\mu}n^{\nu\}} \qquad \epsilon_T^{\mu\nu} = \epsilon^{Pn\mu\nu} = \epsilon^{-+\mu\nu}$$

- … or traceless symmetric tensors (of definite rank)

$$k_T^i$$

$$k_T^{ij} = k_T^i k_T^j - \tfrac{1}{2} k_T^2\, g_T^{ij}$$

$$k_T^{ijk} = k_T^i k_T^j k_T^k - \tfrac{1}{4} k_T^2 \left(g_T^{ij} k_T^k + g_T^{ik} k_T^j + g_T^{jk} k_T^i\right)$$

- Simple azimuthal behavior: $k_T^{i_1\ldots i_m} \longleftrightarrow |k_T|\, e^{\pm i m\varphi}$

functions showing up in cos(mϕ) or sin(mϕ) asymmetries (wrt e.g. ϕ_T)

- Simple Bessel transform to b-space (relevant for evolution):

$$F_m(x,k_T) = \int_0^\infty bdb\ J_m(k_T b)\, F_m(x,b)$$

$$F_m(x,b) = \int_0^\infty k_T\, dk_T\ J_m(k_T b)\, F_m(x,k_T)$$

Structure of quark (8) **TMD PDFs** in spin ½ target

- 8 TMDs F…(x,k_T^2)

TARGET SPIN \ PARTON SPIN: **QUARKS**	γ^+	$\gamma^+\gamma_5$	$\gamma^+\gamma^\alpha\gamma_5$
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1 $h_{1T}^\perp$

- Integrated (collinear) correlator: only circled ones survive
- Collinear functions are spin-spin correlations
- TMDs also momentum-spin correlations (spin-orbit) including also T-odd (single-spin) functions (appearing in single-spin asymmetries)
- Existence of T-odd functions because of gauge link dependence!

Structure of quark **TMD PDFs** in spin 1 target

		PARTON SPIN	
QUARKS	γ^+	$\gamma^+\gamma_5$	$\gamma^+\gamma^\alpha\gamma_5$
TARGET SPIN			
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1 $h_{1T}^\perp$
LL	f_{1LL}		$h_{1LL}^\perp$
LT	f_{1LT}	g_{1LT}	h_{1LT} $h_{1LT}^\perp$
TT	f_{1TT}	g_{1TT}	h_{1TT} $h_{1TT}^\perp$

Hoodbhoy, Jaffe & Manohar, NP B312 (1988) 571: introduction of $f_{1LL} = b_1$

Bacchetta & M, PRD 62 (2000) 114004; h_{1LT} first introduced as T-odd PDF

X. Ji, PRD 49 (1994) 114; introduction of $H_{1LT} \equiv \hat{h}_{\bar{1}}$ (PFF)

Gluon correlators

- Unpolarized target

$$\Gamma^{ij[U]}(x,k_T) = \frac{x}{2}\left\{-g_T^{ij}\,f_1^{[U]}(x,k_T^2) + \frac{k_T^{ij}}{M^2}\,h_1^{\perp\,[U]}(x,k_T^2)\right\}$$

- Vector polarized target

$$\Gamma_L^{ij[U]}(x,k_T) = \frac{x}{2}\left\{i\epsilon_T^{ij}\,S_L g_1^{[U]}(x,k_T^2) + \frac{\epsilon_{T\,\alpha}^{\{i}k_T^{j\}\alpha}}{M^2}\,S_L h_{1L}^{\perp\,[U]}(x,k_T^2)\right\}$$

$$\Gamma_T^{ij[U]}(x,k_T) = \frac{x}{2}\left\{\frac{g_T^{ij}\epsilon_T^{kS_T}}{M}\,f_{1T}^{\perp[U]}(x,k_T^2) - \frac{i\epsilon_T^{ij}k_T{\cdot}S_T}{M}\,g_{1T}^{[U]}(x,k_T^2)\right.$$
$$\left. - \frac{\epsilon_T^{k\{i}S_T^{j\}} + \epsilon_T^{S_T\{i}k_T^{j\}}}{4M}\,h_1(x,k_T^2) - \frac{\epsilon_{T\,\alpha}^{\{i}k_T^{j\}\alpha S_T}}{2M^3}\,h_{1T}^{\perp}(x,k_T^2)\right\}$$

Gluon correlators

- Tensor polarized target

$$\Gamma_{LL}^{ij[U]]}(x,k_T) = \frac{x}{2}\Big\{ -g_T^{ij} S_{LL}\, f_{1LL}^{[U]}(x,k_T^2) + \frac{k_T^{ij}}{M^2}\, S_{LL} h_{1LL}^{\perp[U]}(x,k_T^2)\Big\}$$

$$\Gamma_{LT}^{ij[U]}(x,k_T) = \frac{x}{2}\Big\{ -g_T^{ij}\frac{k_T\!\cdot\! S_{LT}}{M}\, f_{1LT}^{[U]}(x,k_T^2) + i\epsilon_T^{ij}\frac{\epsilon_T^{S_{LT}k}}{M}\, g_{1LT}^{[U]}(x,k_T^2)$$

$$+ \frac{S_{LT}^{\{i}k_T^{j\}}}{M} h_{1LT}^{[U]}(x,k_T^2) + \frac{k_T^{ij\alpha} S_{LT\alpha}}{M^3} h_{1LT}^{\perp[U]}(x,k_T^2)\Big\}$$

$$\Gamma_{TT}^{ij[U]}(x,k_T) = \frac{x}{2}\Big\{ -g_T^{ij}\frac{k_T^{\alpha\beta} S_{TT\alpha\beta}}{M^2}\, f_{1TT}^{[U]}(x,k_T^2) + i\epsilon_T^{ij}\frac{\epsilon_{T\,\gamma}^{\beta} k_T^{\gamma\alpha} S_{TT\alpha\beta}}{M^2}\, g_{1TT}^{[U]}(x,k_T^2)$$

$$+ S_{TT}^{ij}\, h_{1TT}^{[U]}(x,k_T^2) + \frac{S_{TT\,\alpha}^{\{i} k_T^{j\}\alpha}}{M^2}\, h_{1TT}^{\perp[U]}(x,k_T^2)$$

$$+ \frac{k_T^{ij\alpha\beta} S_{TT\alpha\beta}}{M^4}\, h_{1TT}^{\perp\perp[U]}(x,k_T^2)\Big\}$$

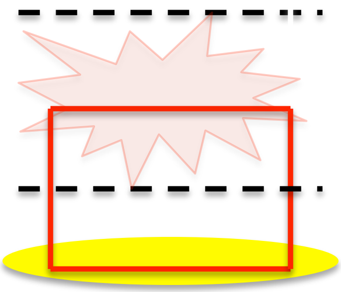
Structure of gluon **TMD PDFs** in spin 1 target

	PARTON SPIN		
GLUONS	$-g_T^{\alpha\beta}$	$\varepsilon_T^{\alpha\beta}$	$p_T^{\alpha\beta}, \ldots$
U	f_1^g		$h_1^{\perp g}$
L		g_1^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	$h_1^g \quad h_{1T}^{\perp g}$
LL	f_{1LL}^g		$h_{1LL}^{\perp g}$
LT	f_{1LT}^g	g_{1LT}^g	$h_{1LT}^g \quad h_{1LT}^{\perp g}$
TT	f_{1TT}^g	g_{1TT}^g	$h_{1TT}^g \quad h_{1TT}^{\perp g} \quad h_{1TT}^{\perp\perp g}$

TARGET SPIN

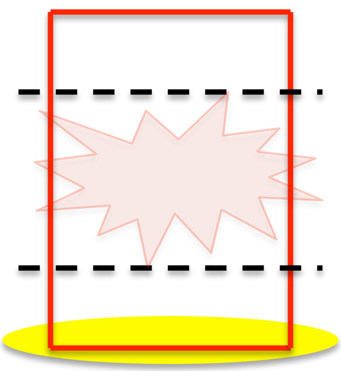
For Gluon PDFs in spin 1 target two collinear PDFs: linear pol. of gluons reflected in S_{TT}

Simplest color flow classes for quarks (in lower hadron)



$\gamma q \to q$

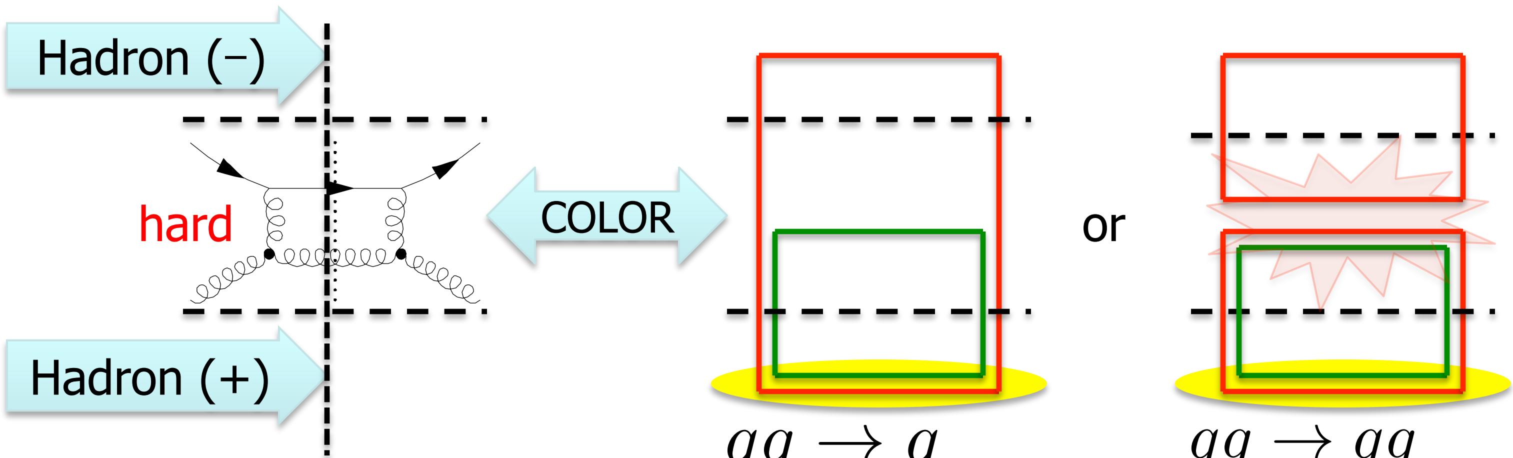
$$\langle|\overline{\psi}(0)\, U^{[+]}_{[0,\xi]}\, \psi(\xi)|\rangle$$



$\bar{q}q \to \gamma$

$$\langle|\overline{\psi}(0)\, U^{[-]}_{[0,\xi]}\, \psi(\xi)|\rangle$$

Color flow and gauge-link

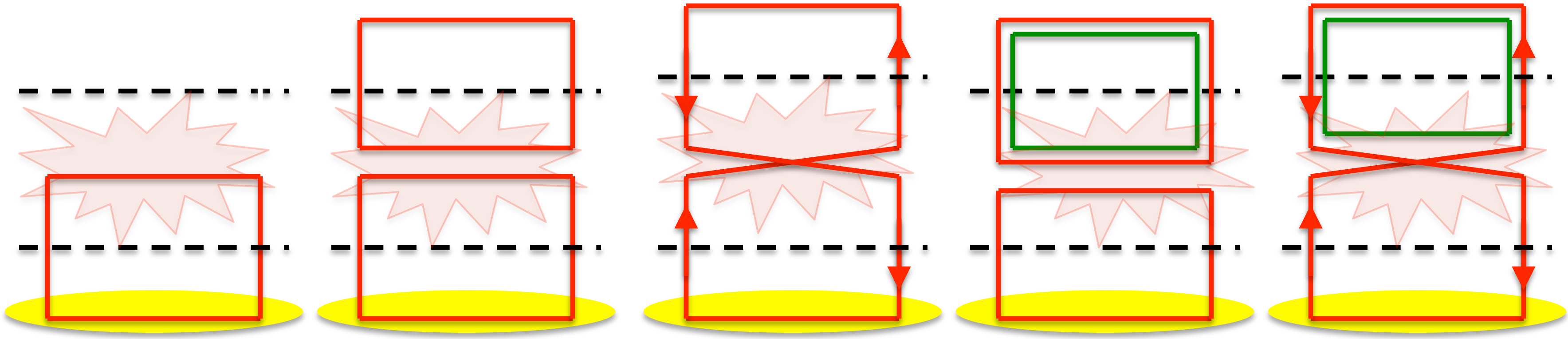


$$qg \to q \qquad qg \to qg$$

$$\langle | F(0)\, U^{[+]}_{[0,\xi]}\, F(\xi)\, U^{[-]}_{[\xi,0]} | \rangle$$

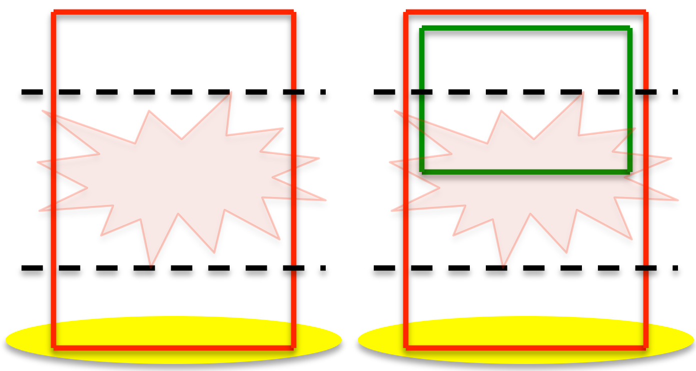
$$\langle | F(0)\, U^{[+]}_{[0,\xi]}\, F(\xi)\, U^{[+]}_{[\xi,0]} U^{(\square)} | \rangle$$

Color flow classes for quarks (in lower hadron)



$\gamma q \to q$ $\quad$ $qq \to qq$ $\quad$ $qq \to qq$ $\quad$ $gq \to gq$ $\quad$ $gq \to gq$

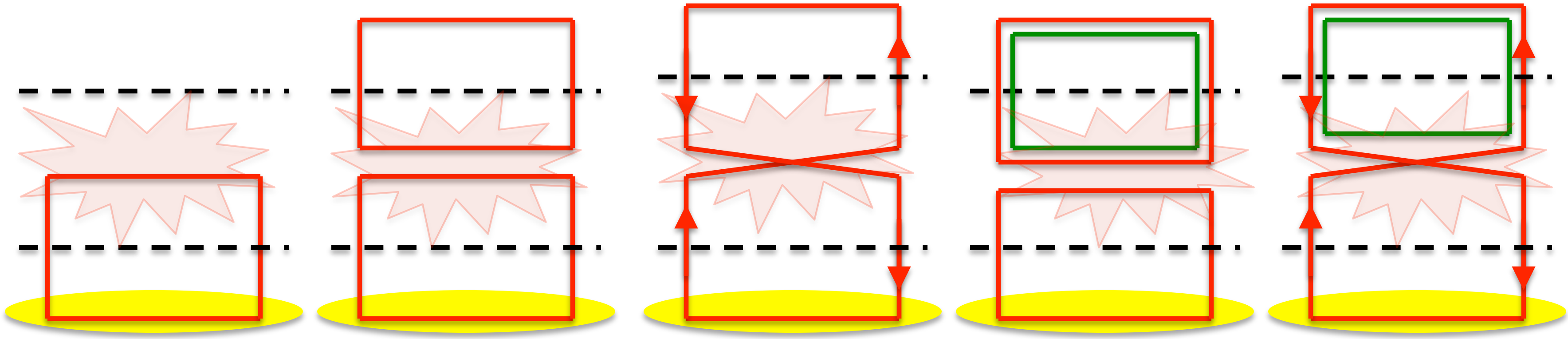
$\langle|\overline{\psi}(0)\, U_{[0,\xi]}^{[+]}\, \psi(\xi)|\rangle$ $\quad$ $\bar{q}q \to g \text{ or } \bar{q}q$



$\bar{q}q \to \gamma$ $\quad$ $gq \to q$

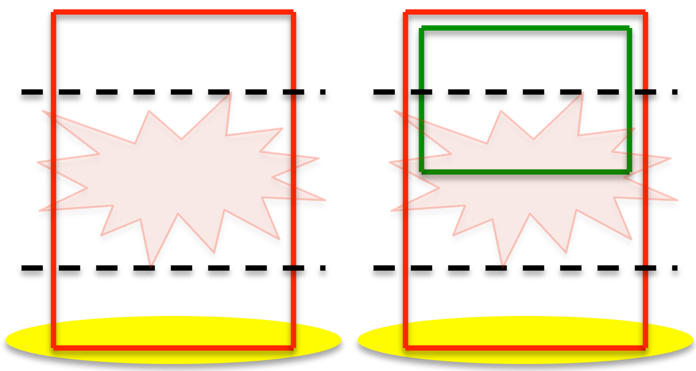
$\langle|\overline{\psi}(0)\, U_{[0,\xi]}^{[-]}\, \psi(\xi)|\rangle$

Color flow classes for quarks or diffractive (in lower hadron)



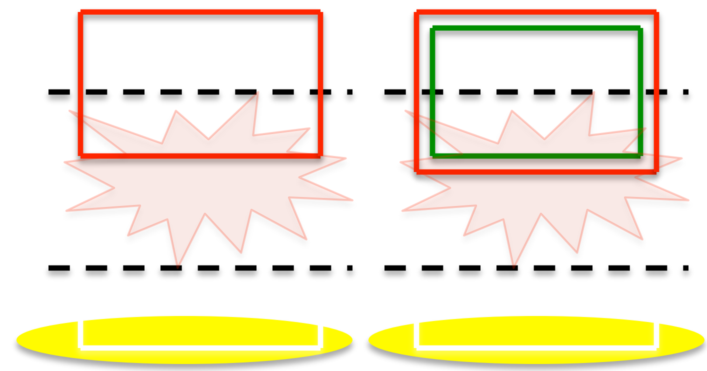
$\gamma q \to q$ $\quad$ $qq \to qq$ $\quad$ $qq \to qq$ $\quad$ $gq \to gq$ $\quad$ $gq \to gq$

$\langle|\overline{\psi}(0)\,U^{[+]}_{[0,\xi]}\,\psi(\xi)|\rangle$ $\quad$ $\bar{q}q \to g$ or $\bar{q}q$



$\bar{q}q \to \gamma$ $\quad$ $gq \to q$

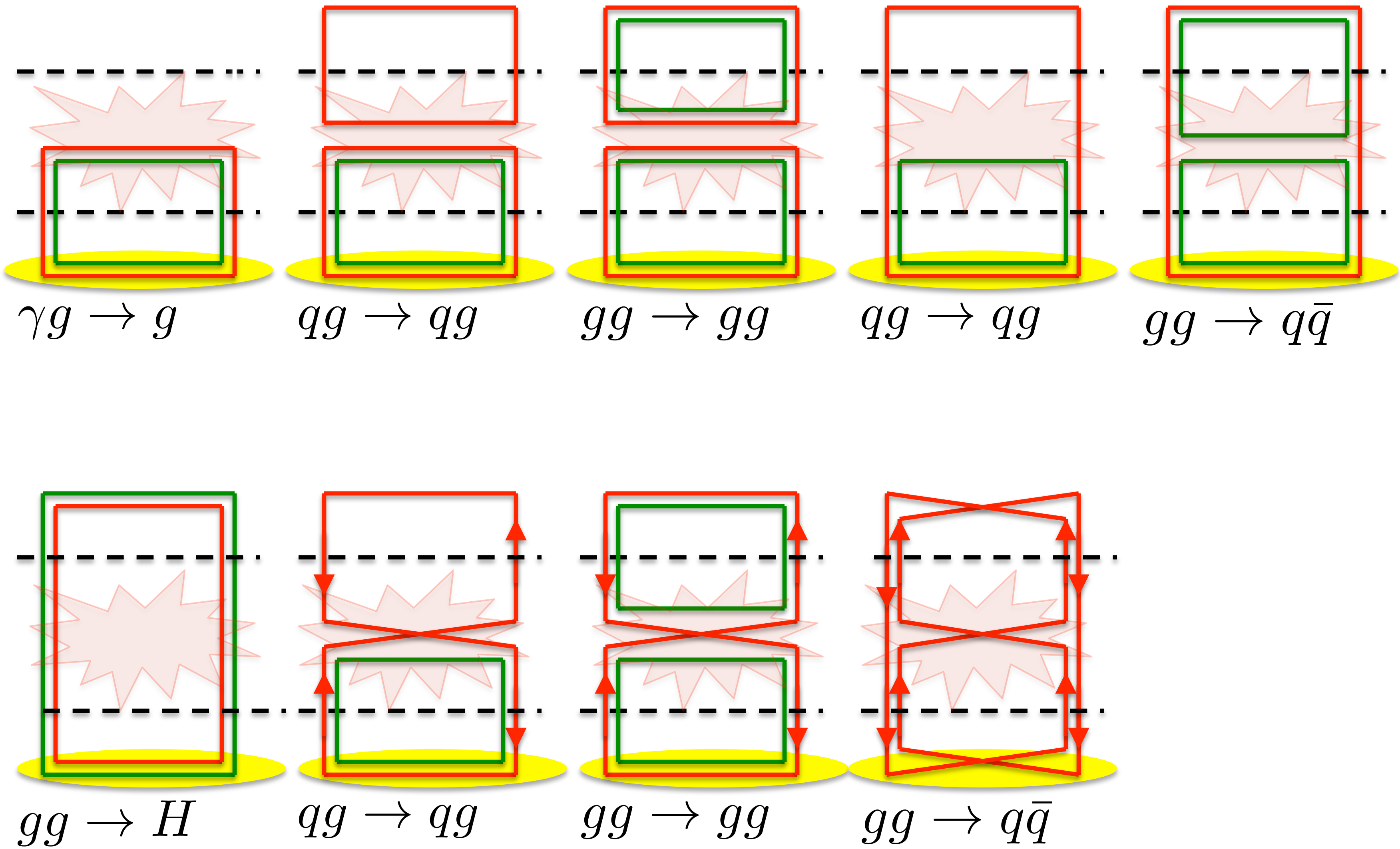
$\langle|\overline{\psi}(0)\,U^{[-]}_{[0,\xi]}\,\psi(\xi)|\rangle$



$q(gg) \to q$ $\quad$ $g(gg) \to g$

$\langle|U^{[+]}_{[0,\xi]}\,U^{[-]}_{[\xi,0]}|\rangle = \langle|U^{[\square]}|\rangle$

Color flow classes for gluons (in lower hadron)



Operator structure in collinear case (reminder)

- Collinear functions and x-moments

$$\Phi^q(x) = \int \frac{d(\xi.P)}{(2\pi)} e^{i\,p.\xi} \left\langle P \middle| \bar{\psi}(0) U^{[n]}_{[0,\xi]} \psi(\xi) \middle| P \right\rangle_{\xi.n=\xi_T=0}$$

$$x^{N-1}\Phi^q(x) = \int \frac{d(\xi.P)}{(2\pi)} e^{i\,p.\xi} \left\langle P \middle| \bar{\psi}(0) (\partial^n_\xi)^{N-1} U^{[n]}_{[0,\xi]} \psi(\xi) \middle| P \right\rangle_{\xi.n=\xi_T=0}$$

x = p.n

$$= \int \frac{d(\xi.P)}{(2\pi)} e^{i\,p.\xi} \left\langle P \middle| \bar{\psi}(0) U^{[n]}_{[0,\xi]} (D^n_\xi)^{N-1} \psi(\xi) \middle| P \right\rangle_{\xi.n=\xi_T=0}$$

- Moments correspond to local matrix elements of operators that all have the same twist since dim(D^n) = 0

$$\Phi^{(N)} = \left\langle P \middle| \bar{\psi}(0) (D^n)^{N-1} \psi(0) \middle| P \right\rangle$$

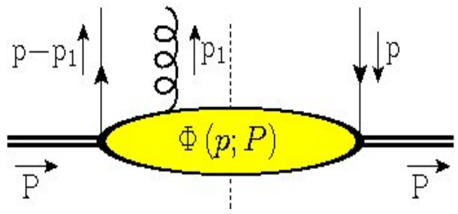
- Moments are particularly useful because their anomalous dimensions can be rigorously calculated and these can be Mellin transformed into the splitting functions that govern the QCD evolution.

Transverse moments → operator structure of TMD PDFs

- Operator analysis for [U] dependence (e.g. [+] or [-]) TMD functions: in analogy to Mellin moments consider transverse moments → role for quark-gluon m.e.

$$p_T^\alpha \Phi^{[\pm]}(x,p_T;n) = \int \frac{d(\xi.P)\,d^2\xi_T}{(2\pi)^3}\, e^{i\,p.\xi} \left\langle P \middle| \overline{\psi}(0)\, U_{[0,\pm\infty]}\, iD_T^\alpha\, U_{[\pm\infty,\xi]}\, \psi(\xi) \middle| P \right\rangle_{\xi.n=0}$$

calculable



- $$\boxed{\int dp_T\; p_T^\alpha \Phi^{[U]}(x,p_T;n) = \tilde{\Phi}_\partial^\alpha(x) + C_G^{[U]}\,\Phi_G^\alpha(x)}$$

T-even

T-odd

$$\boxed{\tilde{\Phi}_\partial^\alpha(x) = \Phi_D^\alpha(x) - \Phi_A^\alpha(x)}$$

T-even (gauge-invariant derivative)

$$\Phi_A^\alpha(x) = PV\int \frac{dx_1}{x_1}\, \Phi_F^{n\alpha}(x-x_1,x_1\,|\,x)$$

$$\Phi_D^\alpha(x) = \int dx_1\, \Phi_D^\alpha(x-x_1,x_1\,|\,x)$$

$$\boxed{\Phi_G^\alpha(x) = \pi\, \Phi_F^{n\alpha}(x,0\,|\,x)}$$

T-odd (soft-gluon or gluonic pole, ETQS m.e.)

Efremov, Teryaev; Qiu, Sterman;
Brodsky, Hwang, Schmidt; Boer, Teryaev; Bomhof, Pijlman, M

Gluonic pole factors are calculable

- $C_G^{[U]}$ calculable gluonic pole factors (quarks)

U	$U^{[\pm]}$	$U^{[+]}\,U^{[\square]}$	$\frac{1}{N_c}\,\mathrm{Tr}_c(U^{[\square]})\,U^{[+]}$
$\Phi^{[U]}$	$\Phi^{[\pm]}$	$\Phi^{[+\square]}$	$\Phi^{[(\square)+]}$
$C_G^{[U]}$	± 1	3	1
$C_{GG,1}^{[U]}$	1	9	1
$C_{GG,2}^{[U]}$	0	0	4

Buffing, Mukherjee, M, PRD86 (2012) 074030, ArXiv 1207.3221
Buffing, Mukherjee, M, PRD88 (2013) 054027, ArXiv 1306.5897

Operator classification of TMDs according to rank

factor	TMD PDF RANK			
	0	1	2	3
1	$\Phi(x)$	$\tilde{\Phi}_{\partial}(x)$	$\tilde{\Phi}_{\partial\partial}(x)$	$\tilde{\Phi}_{\partial\partial\partial}(x)$
$C_{G,c}^{[U]}$		$\tilde{\Phi}_{G,c}(x)$	$\tilde{\Phi}_{\{G\partial\},c}(x)$	$\tilde{\Phi}_{\{G\partial\partial\},c}(x)$
$C_{GG,c}^{[U]}$			$\tilde{\Phi}_{GG,c}(x)$	$\tilde{\Phi}_{\{GG\partial\},c}(x)$
$C_{GGG,c}^{[U]}$				$\tilde{\Phi}_{GGG,c}(x)$

- Transverse moments to be used as coefficients in $\Phi(x,p_T^2)$

Buffing, Mukherjee, M, PRD86 (2012) 074030, ArXiv 1207.3221

Buffing, Mukherjee, M, PRD88 (2013) 054027, ArXiv 1306.5897

Operator classification of TMDs (including trace terms)

factor	TMD PDF RANK			
	0	1	2	3
1	$\Phi(x,p_T^2)$	$\tilde{\Phi}_{\partial}(x,p_T^2)$	$\tilde{\Phi}_{\partial\partial}(x,p_T^2)$	$\tilde{\Phi}_{\partial\partial\partial}(x,p_T^2)$
$C_{G,c}^{[U]}$		$\Phi_{G,c}(x,p_T^2)$	$\tilde{\Phi}_{\{G\partial\},c}(x,p_T^2)$	$\tilde{\Phi}_{\{G\partial\partial\},c}(x,p_T^2)$
$C_{GG,c}^{[U]}$			$\Phi_{GG,c}(x,p_T^2)$	$\tilde{\Phi}_{\{GG\partial\},c}(x,p_T^2)$
$C_{GGG,c}^{[U]}$				$\Phi_{GGG,c}(x,p_T^2)$

- Transverse moments to be used as coefficients in $\Phi(x,p_T^2)$
- Trace terms involving $\partial\cdot\partial$ are responsible for $f_1(x) \rightarrow f_1(x,p_T^2)$

Boer, Buffing, M, JHEP08 (2015) 053, arXiv:1503.03760

Operator classification of quark TMDs (polarized nucleon)

factor	QUARK TMD RANK FOR VECTOR POL. (SPIN ½) HADRON			
	0	1	2	3
1	$f_1 \quad g_1 \quad h_1$	$g_{1T} \quad h_{1L}^{\perp}$	$h_{1T}^{\perp[\partial\partial]}$	
$C_G^{[U]}$		$h_1^{\perp} \quad f_{1T}^{\perp}$		
$C_{GG,c}^{[U]}$			$h_{1T}^{\perp[GG1]} \quad h_{1T}^{\perp[GG2]}$	

Three pretzelocities:

$$[\partial\partial]:\ \bar{\psi}\partial\partial\psi = Tr_c\left[\partial\partial\psi\bar{\psi}\right]$$

$$[GG\ 1]:\ Tr_c\left[GG\psi\bar{\psi}\right]$$

$$[GG\ 2]:\ Tr_c\left[GG\right]Tr_c\left[\psi\bar{\psi}\right]$$

Process dependence also for (T-even) pretzelocity,

$$h_{1T}^{\perp[U]} = h_{1T}^{\perp[\partial\partial]} + C_{GG,1}^{[U]} h_{1T}^{\perp[GG1]} + C_{GG,2}^{[U]} h_{1T}^{\perp[GG2]}$$

Buffing, Mukherjee, M, PRD86 (2012) 074030, ArXiv 1207.3221

Operator classification of TMDs (including trace terms)

factor	TMD PDF RANK			
	0	1	2	3
1	$\tilde{\Phi}(x, p_T^2)$	$\tilde{\Phi}_{\partial}$	$\tilde{\Phi}_{\partial\partial}$	$\tilde{\Phi}_{\partial\partial\partial}$
$C_{G,c}^{[U]}$	$\delta\tilde{\Phi}_{\{G.\partial\},c}$	$\tilde{\Phi}_{G,c}$	$\tilde{\Phi}_{\{G\partial\},c}$	$\tilde{\Phi}_{\{G\partial\partial\},c}$
$C_{GG,c}^{[U]}$	$\delta\tilde{\Phi}_{G.G,c}$	$\delta\tilde{\Phi}_{\{G.G\partial\},c}$	$\tilde{\Phi}_{GG,c}$	$\tilde{\Phi}_{\{GG\partial\},c}$
$C_{GGG,c}^{[U]}$	$\delta\tilde{\Phi}_{\{G.GG.\partial\},c}$	$\delta\tilde{\Phi}_{G.GG,c}$	$\delta\tilde{\Phi}_{\{G.GG\partial\},c}$	$\tilde{\Phi}_{GGG,c}$

- Transverse moments to be used as coefficients in $\Phi(x,p_T^2)$
- Trace terms involving $\partial\cdot\partial$ are responsible for $f_1(x) \rightarrow f_1(x,p_T^2)$
- Trace terms involving G.G are responsible for new functions (not necessarily new structures)
- Trace terms affect p_T width (note $\delta\Phi_{G.G}(x) = 0$, etc.

Boer, Buffing, M, JHEP08 (2015) 053, arXiv:1503.03760

Operator classification of quark TMDs (including trace terms)

factor	QUARK TMD RANK FOR VECTOR POL. (SPIN ½) HADRON			
	0	1	2	3
1	$f_1 \quad g_1 \quad h_1$	$g_{1T}^{[\partial]} \quad h_{1L}^{\perp[\partial]}$	$h_{1T}^{\perp[\partial\partial]}$	
$C_{G,c}^{[U]}$		$h_1^{\perp[G]} \quad f_{1T}^{\perp[G]}$		
$C_{GG,c}^{[U]}$	$\delta f_1^{[GGc]} \ldots$	…	$h_{1T}^{\perp[GG1]} \quad h_{1T}^{\perp[GG2]}$	
$C_{GGG,c}^{[U]}$		…		

Process dependence in p_T dependence of TMDs due to gluonic pole operators (e.g. affecting $\langle p_T^2 \rangle$)

$$f_1^{[U]}(x, p_T^2) = f_1 + C_{GG,c}^{[U]} \delta f_1^{[GGc]} \quad \text{with } \delta f_1^{[GG\,c]}(x) = 0$$

Boer, Buffing, M, JHEP08 (2015) 053, arXiv:1503.03760

Classifying Polarized Quark TMDs (including tensor pol)

factor	**QUARK TMD RANK FOR VECTOR POL. (SPIN ½) HADRON**			
	0	1	2	3
1	$f_1 \quad g_1 \quad h_1$	$g_{1T}^{[\partial]} \quad h_{1L}^{\perp[\partial]}$	$h_{1T}^{\perp[\partial\partial]}$	
$C_{G,c}^{[U]}$		$h_1^{\perp[G]} \quad f_{1T}^{\perp[G]}$		
$C_{GG,c}^{[U]}$	$\delta f_1^{[GGc]} \ldots$	$\ldots$	$h_{1T}^{\perp[GG1]} \quad h_{1T}^{\perp[GG2]}$	
$C_{GGG,c}^{[U]}$		$\ldots$		

factor	**QUARK TMD RANK FOR TENSOR POL. (SPIN 1) HADRON**			
	0	1	2	3
1	f_{1LL}	$f_{1LT}^{[\partial]}$	$f_{1TT}^{[\partial\partial]}$	
$C_{G}^{[U]}$	$\delta h_{1LT}^{[\partial.G]}$	$h_{1LL}^{\perp[G]} \quad g_{1LT}^{[G]} \quad h_{1TT}^{[G]}$	$h_{1LT}^{\perp[\partial G]} \quad g_{1TT}^{[\partial G]}$	$h_{1TT}^{\perp[\partial\partial G]}$
$C_{GG,c}^{[U]}$	$\ldots$	$\ldots$	$f_{1TT}^{[GGc]}$	
$C_{GGG,c}^{[U]}$	$\ldots$	$\ldots$	$\ldots$	$h_{1TT}^{\perp[GGGc]}$

$$\delta h_{1LT}^{[U]}(x,p_T^2) = C_G^{[U]}\,\delta h_{1LT}^{[\partial.G]}(x,p_T^2) \quad with \quad \delta h_{1LT}^{[\partial.G]}(x) = 0$$

Operator classification of gluon TMDs

factor	GLUON TMD PDF RANK FOR SPIN ½ HADRON				
	0	1	2	3	
1	$f_1 \quad g_1$	$g_{1T}^{[\partial]}$	$h_1^{\perp[\partial\partial]}$		
$C_{G,c}^{[U]}$		$f_{1T}^{\perp[Gc]} \quad h_1^{[Gc]}$	$h_{1L}^{\perp[\partial Gc]}$	$h_{1T}^{\perp[\partial\partial Gc]}$	
$C_{GG,c}^{[U]}$	$\delta f_1^{[GGc]} \ldots$	...	$h_1^{\perp[GGc]}$		
$C_{GGG,c}^{[U]}$		...	...	$h_{1T}^{\perp[GGGc]}$	

factor	ADDITIONAL PDFs FOR TENSOR POL. SPIN 1 HADRON				
	0	1	2	3	4
1	$f_{1LL} \quad h_{1TT}$	$f_{1LT}^{[\partial]} \quad h_{1LT}^{[\partial]}$	$f_{1TT}^{[\partial\partial]} \; h_{1LL}^{\perp[\partial\partial]} \; h_{1TT}^{\perp[\partial\partial]}$	$h_{1LT}^{\perp[\partial\partial\partial]}$	$h_{1TT}^{\perp\perp[\partial\partial\partial\partial]}$
$C_{G,c}^{[U]}$		$g_{1LT}^{[Gc]}$	$g_{1TT}^{\perp[\partial Gc]}$		
$C_{GG,c}^{[U]}$	...	...	$f_{1TT}^{\perp[GGc]} \; h_{1LL}^{\perp[GGc]} \; h_{1TT}^{\perp[GGc]}$	$h_{1LT}^{\perp[\partial GG]}$	$h_{1TT}^{\perp\perp[\partial\partial GGc]}$
$C_{GGG,c}^{[U]}$		...	...		
$C_{GGGG,c}^{[U]}$	...	...	...	...	$h_{1TT}^{\perp\perp[GGGGc]}$

Small x physics in terms of TMDs

- The single Wilson-loop correlator Γ_0

$$\Gamma_0(x,k_T) = \frac{1}{2M^2}\left\{e(k_T^2) - \frac{\epsilon^{kS_T}}{M}\,e_T(k_T^2)\right\}$$

factor	GLUON TMD PDF RANK FOR UNPOL. AND SPIN ½ HADRON			
	0	1	2	3
1	e			
$C_{G,c}^{[U]}$		$e_T^{\perp[Gc]}$		
$C_{GG,c}^{[U]}$	$\delta e^{[GGc]}$			
$C_{GGG,c}^{[U]}$		$\delta e_T^{\perp[G.GGc]}$		

- Note limit x → 0 for gluon TMDs linked to gluonic pole m.e. of Γ_0

$$\pi^2\,\Gamma^{\alpha\beta\,[U,U']}(0,p_T) = C_{GG}^{[U,U']}\Gamma_{0\,GG}^{\alpha\beta}(p_T)$$

Small x physics in terms of TMDs

- Note limit x → 0 for gluon TMDs linked to gluonic pole m.e. of Γ_0

$$\pi^2\,\Gamma^{\alpha\beta\,[U,U']}(0,p_T) = C_{GG}^{[U,U']}\Gamma_{0\,GG}^{\alpha\beta}(p_T)$$

- At small x only two structures for unpolarized and transversely polarized nucleons with same effects as pomeron & odderon structure

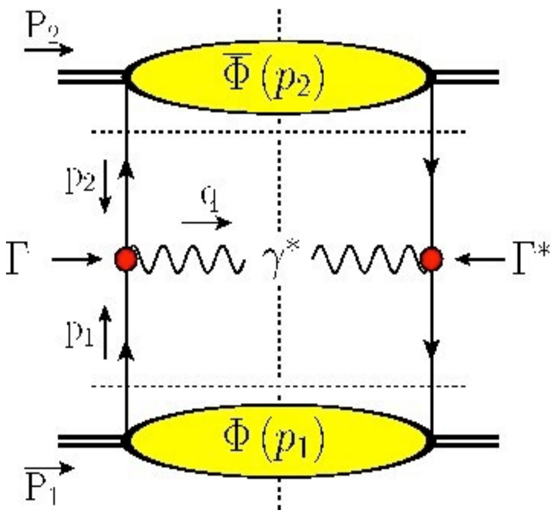
$$f_1(x,k_T^2) \longrightarrow C_{GG,c}^{[U]}\,\delta e^{[G.G\,c]}(k_T^2)$$

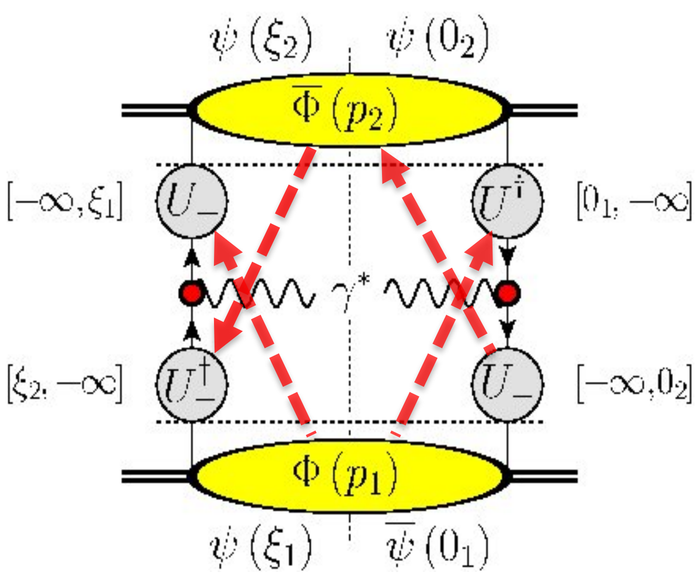
$$C_{G,c}^{[U]}\,f_{1T}^{\perp[Gc]}(x,k_T^2) \longrightarrow C_{GGG,c}^{[U]}\delta e_T^{[G.GG\,c]}(k_T^2)$$

$$C_{G,c}^{[U]}\,h_1^{[Gc]}(x,k_T^2) \longrightarrow C_{GGG,c}^{[U]}\delta e_T^{[G.GG\,c]}(k_T^2)$$

D. Boer, M.G. Echevarria, PJM, J. Zhou, PRL 116 (2016) 122001, ArXiv 1511.03485

Two initial state hadrons (e.g. DY)





$$\cancel{\begin{aligned} d\sigma_{\rm DY} &\sim \mathrm{Tr}_c\left[\Phi(x_1,p_{1T})\Gamma^*\overline{\Phi}(x_2,p_{2T})\Gamma\right] \\ &= \frac{1}{N_c}\Phi(x_1,p_{1T})\Gamma^*\overline{\Phi}(x_2,p_{2T})\Gamma, \end{aligned}}$$

- Complications if the transverse momentum of two initial state hadrons is involved, resulting for DY at measured Q_T in

$$\begin{aligned} d\sigma_{\rm DY} &= \mathrm{Tr}_c\Big[U_-^\dagger[p_2]\Phi(x_1,p_{1T})U_-[p_2]\Gamma^* \\ &\qquad \times U_-^\dagger[p_1]\overline{\Phi}(x_2,p_{2T})U_-[p_1]\Gamma\Big] \\ &\neq \frac{1}{N_c}\,\Phi^{[-]}(x_1,p_{1T})\Gamma^*\overline{\Phi}^{[-^\dagger]}(x_2,p_{2T})\Gamma, \end{aligned}$$

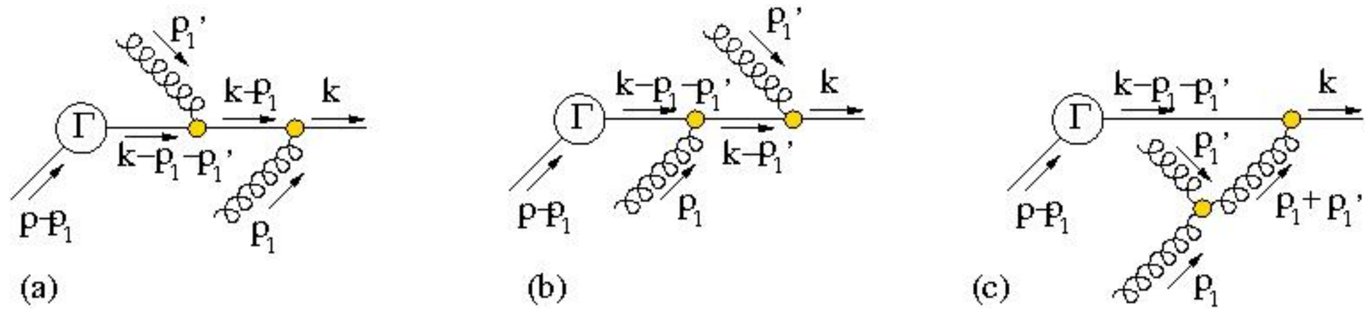
- This leads to color factors just as for twist-3 squared in collinear DY

$$\begin{aligned} \sigma_{DY}(x_1,x_2,q_T) &= \frac{1}{N_c} f_1(x_1,p_{1T})\otimes \overline{f}_1(x_2,p_{2T}) \\ &\quad - \frac{1}{N_c}\frac{1}{N_c^2-1} h_1^\perp(x_1,p_{1T})\otimes \overline{h}_1^\perp(x_2,p_{2T})\cos(2\varphi) \end{aligned}$$

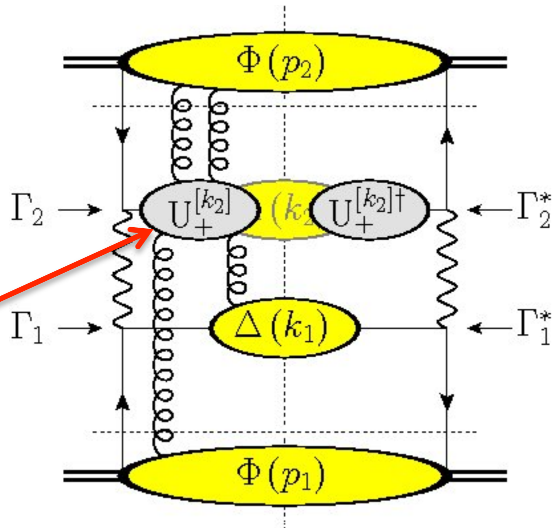
Buffing, M, PRL 112 (2014), 092002

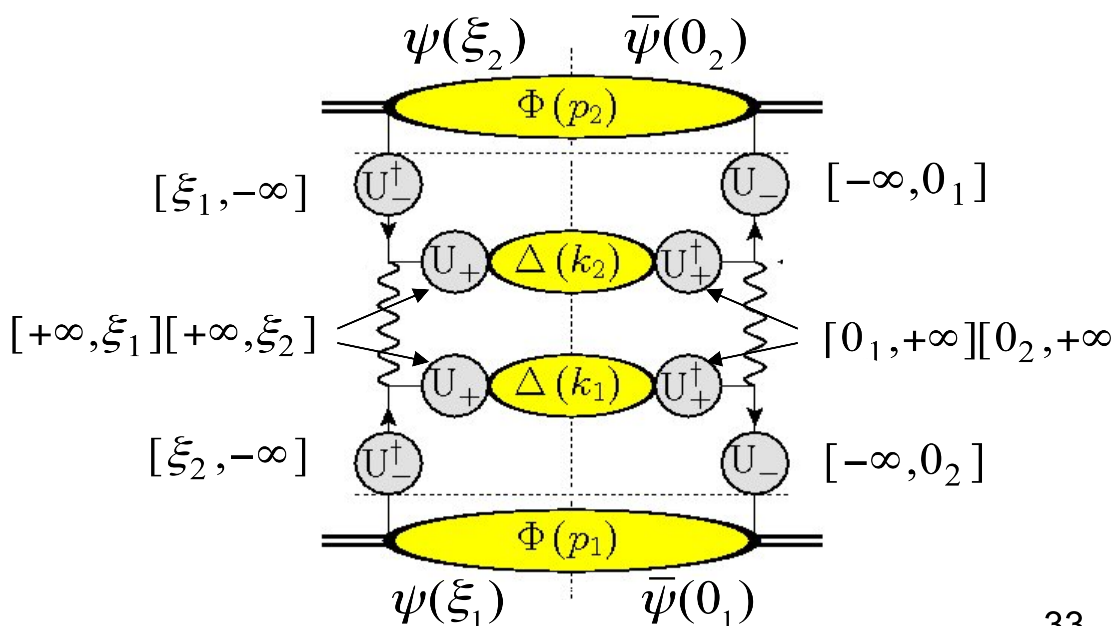
Entanglement in processes with two initial state hadrons

- Resummation of collinear gluons coupling onto external lines contribute to gauge links



- …. leading to entangled situation (Rogers, M), breaking universality





- Gauge knots (Buffing, M)

Conclusions and outlook

- TMDs extend collinear PDFs to novel TMD PDF functions
- Although operator structure including ETQS matrix elements ultimately has same operators basis as collinear approach, it is a physically relevant combination/resummation of higher twist operators that governs transverse structure (definite rank linked to azimuthal structure)
- Transverse structure for PDFs (in contrast to PFFs) requires careful study of process dependence linked to color flow in hard process, determining gauge links
- Like spin, the transverse structure does offer valuable tools, but you need to know how to use them!