Threshold Resummation for Polarized High-p_T Hadron Production at COMPASS

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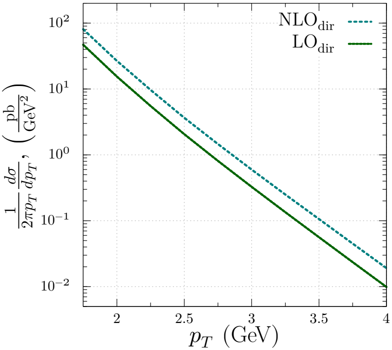
Outline

Motivation: COMPASS experiment $\mu N \to \mu' h X$



- aim of COMPASS experiment: nucleon's gluon helicity Δg
  $\to$ asymmetry A_{LL}: sensitive to Δg due to $\gamma g \to q\bar{q}$
- large p_T of observed hadron $\to$ use perturbative methods
- important corrections beyond NLO due to large logarithms at threshold coming from soft gluon radiation $\to$ **resummation** of these logarithms

goal: predictions for spin-dependent cross section and its longitudinal asymmetry

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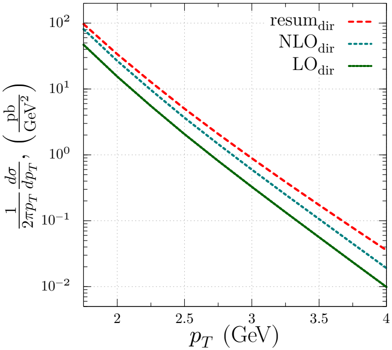

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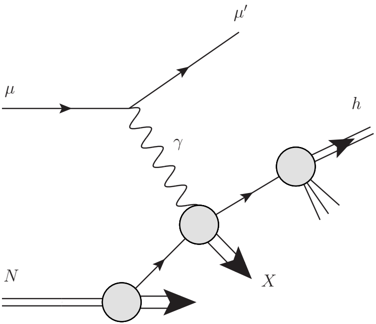
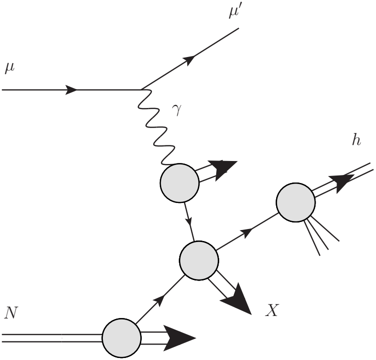
- direct and resolved photon contributions: $d\Delta\sigma = d\Delta\sigma_{dir} + d\Delta\sigma_{res}$



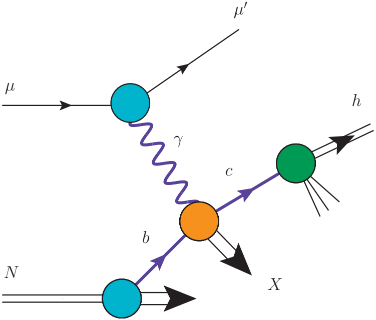


$\rightarrow$ so far: restricted ourselves to the resummation for the direct case

- 2 basic direct subprocesses: $\gamma g \to q\bar{q}$, $\gamma q \to qg$

Direct and resolved processes: $\mu N \to \mu' h X$

- direct and resolved photon contributions: $d\Delta\sigma = d\Delta\sigma_{dir} + d\Delta\sigma_{res}$



$\Delta f_{\gamma/\ell}, f_{b/N}$: parton distribution function (PDF)
$D_{h/c}$: parton-to-hadron fragmentation function (FF)
$\Delta d\hat{\sigma}_{\gamma b \to cX}$: partonic cross section

- **factorization:** separate large momentum-transfer process into nonperturbative long-distance and perturbative short-distance part

Factorization for $\ell\, N \to \ell'\, h^\pm\, X$

$$\frac{p_T^3 d\sigma}{dp_T\, d\eta} = \sum_{a,b,c} \int_{x_\ell^{min}}^1 dx_\ell \int_{x_n^{min}}^1 dx_n \int_x^1 dz \frac{\hat{x}_T^4 z^2}{8v}$$

$$\times \underbrace{\Delta f_{a/\ell}(x_\ell, \mu_{fi})\, \Delta f_{b/N}(x_n, \mu_{fi})}_{\text{PDFs, long-distance part}} \underbrace{D_{h/c}(z, \mu_{ff})}_{\text{FF, long-distance}} \underbrace{\frac{\hat{s} d\Delta\hat{\sigma}_{ab\to cX}}{dv\, dw}}_{\text{short-distance, pQCD}}$$

partonic variables

$v \equiv 1 + \frac{\hat{t}}{\hat{s}}$, $w \equiv \frac{-\hat{u}}{\hat{s}+\hat{t}}$,

$\hat{x}_T \equiv \frac{2p_T}{z\sqrt{x_\ell x_n S}}$, $\hat{\eta} = \eta + \frac{1}{2}\ln\frac{x_n}{x_\ell}$

cuts and kinematics at COMPASS

$\sqrt{S} = 17.4$ GeV, $Q^2_{max} = 1$ GeV,

$$0.2 \leq y \leq 0.9,$$
$$0.2 \leq z \leq 0.8,$$
$$-0.1 \leq \eta \leq 2.38$$

Known NLO result

$$\frac{\hat{s}\, d\Delta\hat{\sigma}^{(1)}_{\gamma b\to cd}(v,w)}{dv\, dw} = A(v)\delta(1-w) + B(v)\underbrace{\left(\frac{\ln(1-w)}{1-w}\right)_+}_{\to \ln^2 N\text{-terms}} + C(v)\underbrace{\left(\frac{1}{1-w}\right)_+}_{\to \ln N\text{-terms}} + F(v,w)$$

- origin of logs: soft gluon radiation
- at k-th order: $\alpha_s^k \left[\frac{\ln^m(1-w)}{1-w}\right]_+, \quad m \le 2k-1$; at threshold: $w \to 1$

threshold regime: phase space for radiation of additional gluons becomes small, allowing only soft and/or collinear gluons

$\Rightarrow$ **Use threshold resummation!**

Threshold resummation in a nutshell

- procedure to deal with infinitely many soft gluons, which lead to large logs

exponentiation formalism:

$$X = \exp Y$$

Y: subset of diagramms X, connected with a weight factor

need for phase space factorization:

phase space for multi-gluon emission contains energy-momentum conserving δ-functions connecting components of the gluons

$\rightarrow$ transform into Mellin moment space, here the phase space factorizes

Mellin transform:

$$f^N = \int_0^1 dz\, z^{N-1}\; f(z)$$

Mellin inversion:

$$f(z) = \frac{1}{2\pi i} \int_{\mathcal{C}} dN\, z^{-N} f^N$$

Large Logarithms in Mellin space

$$\alpha_s^k \left[\frac{\ln^m(1-z)}{1-z}\right]_+, \quad m \leq 2k-1 \quad \rightarrow \quad \alpha_s^k \ln^{m+1} N \quad \text{in Mellin space}$$

Reorganize large logarithms (note: $L \equiv \ln N$) :

$$d\sigma^N/\sigma_B^N = 1 + \sum_{k=1}^{\infty} \alpha_s^k \sum_{m=0}^{2k-1} \tilde{c}_{k,m} L^{m+1} = \exp\left\{\sum_{k=1}^{\infty} \alpha_s^k \sum_{m=0}^{k} c_{k,m} L^{m+1}\right\} C(\alpha_s)$$

$$= \exp\left(\underbrace{\underbrace{Lh^{(1)}(\alpha_s L)}_{LL} + h^{(2)}(\alpha_s L)}_{NLL} + \alpha_s h^{(3)}(\alpha_s L)\right) C(\alpha_s)$$

$\rightarrow$ Resummation: reorganize the perturbative structure;

NLL: 3 towers of logarithms

$\rightarrow$ Resummation: reorganize the perturbative structure;

LO	1

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NNLO	$\alpha_s^2 L^4$	$\alpha_s^2 L^3$	$\alpha_s^2 L^2$	...
N^3LO	$\alpha_s^3 L^6$	$\alpha_s^3 L^5$	$\alpha_s^3 L^4$	...
$\vdots$				

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$\vdots$				
N^kLO	$\alpha_s^k L^{2k}$	$\alpha_s^k L^{2k-1}$	$\alpha_s^k L^{2k-2}$	...

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⋮				
N^{k}LO	$\alpha_s^k L^{2k}$	$\alpha_s^k L^{2k-1}$	$\alpha_s^k L^{2k-2}$	...
	$\downarrow$			
	$L h^1(\alpha_s L)$			

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	↓	↓		
	$Lh^1(\alpha_s L)$	$h^2(\alpha_s L)$		...

The direct resummed contribution

take Mellin transform and then inverse:

$$\frac{p_T^3 d\sigma}{dp_T\, d\eta} = \sum_{b,c} \int_{x_\ell^{min}}^1 dx_n \int_{x_n^{min}}^1 dx_n \Delta f_{\gamma/\ell}(x_\ell, \mu_{fi})\, \Delta f_{b/N}(x_n, \mu_{fi})$$

$$\times \underbrace{\int_{\mathcal{C}} \frac{dN}{2\pi i} (x^2)^{-N}}_{\text{Mellin inverse}} \underbrace{D_{h/c}^{2N+3}(\mu_{ff})\, \Delta\tilde{w}^{2N}\,(\hat{\eta})}_{\text{in Mellin-space}}$$

$$\underbrace{\int_0^1 dx^2 (x^2)^{N-1}}_{\text{Mellin transform}} \int_x^1 dz \frac{\hat{x}_T^4 z^2}{8v} D_{h/c}(z, \mu_{ff}) \frac{\hat{s} d\Delta\hat{\sigma}_{ab\to cX}}{dv\, dw} \equiv D_{h/c}^{2N+3}(\mu_{ff})\, \tilde{w}^{2N}\,(\hat{\eta})$$

Mellin inversion: $f(x^2) = \int_{\mathcal{C}} \frac{dN}{2\pi i} (x^2)^{-N}\ f^N$

Mellin transform: $f^N = \int_0^1 dx^2\, (x^2)^{N-1}\ f(x^2)$

Direct NLL-resummed hard-scattering function

partonic resummed cross section in Mellin space factorizes near threshold at NLL:

$$\tilde{w}^{N,resum}_{\gamma b\to cd}(\hat{\eta}) = \left(1+\frac{\alpha_s}{\pi}\Delta C^{(1)}_{\gamma b\to cd}\right)\Delta_b^{(-\hat{t}/\hat{s})N}(\hat{s},\mu_{fi},\mu_r)\,\Delta_c^N(\hat{s},\mu_{ff},\mu_r)\,J_d^N(\hat{s})$$
$$\times\,\Delta\hat{\sigma}^{(0)}_{\gamma b\to cd}(N,\hat{\eta})\,\exp\left[\int_{\mu_r}^{\sqrt{\hat{s}}/N}\frac{d\mu'}{\mu'}2\mathrm{Re}\Gamma_{\gamma b\to cd}(\hat{\eta},\alpha_s(\mu'))\right]$$

$\Delta C^{(1)}_{\gamma b\to cd}$: match the resummed cross section to the NLO one

$\Delta_i^{N_i}$: soft gluon radiation collinear to partons b, c

J_d^N : soft and hard collinear emission off unobserved final-state parton

$\exp[\ldots]$: emission of soft-gluons at wide angles,

with $\Gamma_{\gamma b\to cd}$: rapidity-dependent soft anomalous dimensions

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$$\times\,\Delta\hat{\sigma}^{(0)}_{\gamma b\to cd}(N,\hat{\eta})\,\exp\left[\int_{\mu_r}^{\sqrt{\hat{s}}/N}\frac{d\mu'}{\mu'}2\mathrm{Re}\Gamma_{\gamma b\to cd}(\hat{\eta},\alpha_s(\mu'))\right]$$

$$\ln\Delta_i^N\left(\hat{s},\mu_f,\mu_r\right) = -\int_0^1 dz\frac{z^{N-1}-1}{1-z}\int_{(1-z)^2}^1\frac{dt}{t}A_i(\alpha_s(t\hat{s}))$$
$$-2\int_{\mu_r}^{\sqrt{\hat{s}}}\frac{d\mu'}{\mu'}\gamma_i(\alpha_s(\mu'^2)) + 2\int_{\mu_{fi}}^{\sqrt{\hat{s}}}\frac{d\mu'}{\mu'}\gamma_{ii}(N,\alpha_s(\mu'^2))$$

$$\ln J_d^N(\hat{s},\mu_r) = \int_0^1 dz\frac{z^{N-1}-1}{1-z}\left\{\int_{(1-z)^2}^{(1-z)}\frac{dt}{t}A_d(\alpha_s(t\hat{s})) - \gamma_d[\alpha_s((1-z)\hat{s})]\right\}$$
$$+2\int_{\mu_r}^{\sqrt{\hat{s}}}\frac{d\mu'}{\mu'}\gamma_d(\alpha_s(\mu'^2))$$

Direct NLL-resummed hard-scattering function

partonic resummed cross section in Mellin space factorizes near threshold at NLL:

$$\tilde{w}_{\gamma b \to cd}^{N,resum}(\hat{\eta}) = \left(1 + \frac{\alpha_s}{\pi} \Delta C_{\gamma b \to cd}^{(1)}\right) \Delta_b^{(-\hat{t}/\hat{s})N}(\hat{s}, \mu_{fi}, \mu_r)\, \Delta_c^N(\hat{s}, \mu_{ff}, \mu_r)\, J_d^N(\hat{s})$$
$$\times \Delta\hat{\sigma}_{\gamma b \to cd}^{(0)}(N, \hat{\eta}) \exp\left[\int_{\mu_r}^{\sqrt{\hat{s}}/N} \frac{d\mu'}{\mu'} 2\mathrm{Re}\Gamma_{\gamma b \to cd}(\hat{\eta}, \alpha_s(\mu'))\right]$$

$$\ln \Delta_i^N\left(\hat{s}, \mu_{fi}, \mu_r\right) = \ln N h_i^{(1)}(\lambda) + h_i^{(2)}\left(\lambda, \frac{\hat{s}}{\mu_r^2}, \frac{\hat{s}}{\mu_{fi}^2}\right)$$

$$\ln J_d^{2N}(\hat{s}) = \ln N f_d^{(1)}(\lambda) + f_d^{(2)}\left(\lambda, \frac{\hat{s}}{\mu_r^2}\right)$$

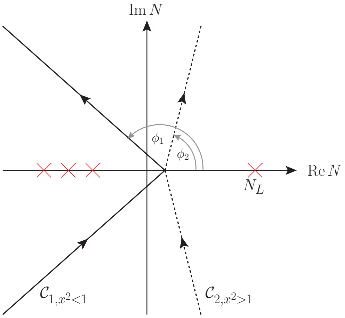
with:

$h_i^{(1)}$, $f_i^{(1)}$: collecting leading logarithmics $\alpha_s^k \ln^{k+1} N$
$h_i^{(2)}$, $f_i^{(2)}$: collecting next-to-leading logarithmics $\alpha_s^k \ln^k N$

Inverse Mellin transform, matching procedure

poles in the NLL expanded exponents
at $\lambda = 1/2, 1$ with $\lambda = \alpha_s b_0 \ln(N e^{\gamma_E})$
$\rightarrow$ at the left:
$N_L = \exp(1/(2\alpha_s b_0) - \gamma_E)$

contour: all poles to the left of integration contour, except N_L

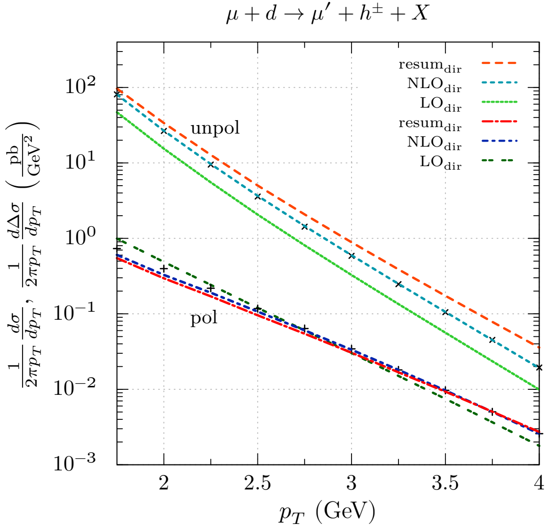


$$\frac{p_T^3 \Delta d\hat{\sigma}^{\text{matched}}}{dp_T\, d\eta} = \sum_{a=\gamma,b,c} \int_0^1 dx_l \int_0^1 dx_n\, \Delta f_{\gamma/l}(x_l, \mu_{fi}) \Delta f_{b/N}(x_n, \mu_{fi})$$

$$\times \int_{\mathcal{C}} \frac{dN}{2\pi i} (x^2)^{-N} D_{h/c}^{2N+3}(\mu_{ff}) \left[\Delta\tilde{w}_{\gamma b\to cd}^{2N,\text{resum}}(\hat{\eta}) - \Delta\tilde{w}_{\gamma b\to cd}^{2N,\text{resum}}(\hat{\eta})\Big|_{NLO} \right]$$

$$+ \frac{p_T^3\, d\Delta\sigma^{\text{NLO}}}{dp_T\, d\eta}$$

- default: $\mu_r = \mu_{fi} = \mu_{ff} = p_T$
- pol. PDF: DSSV14, unpol. PDF: MSTW, FF: DSS, unpol./pol. photonic PDFs: GRS/GRSV



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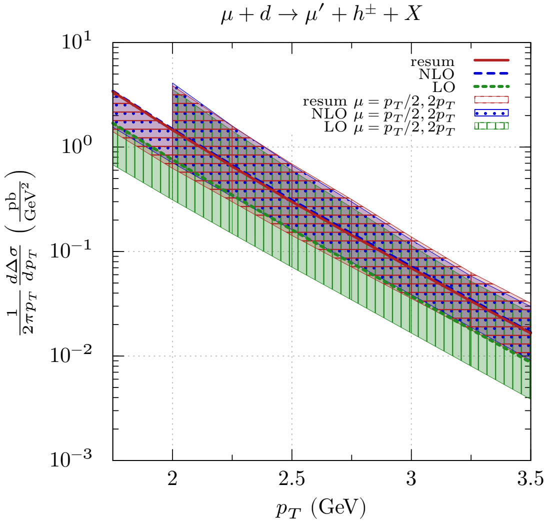
Results: Scale dependence

- include resolved-NLO contributions: $d\Delta\sigma_{\text{resum}} = d\Delta\sigma_{\text{dir,resum}} + d\Delta\sigma_{\text{res,NLO}}$
- vary the scale in the range $\frac{p_T}{2} \leq \mu \leq 2p_T$



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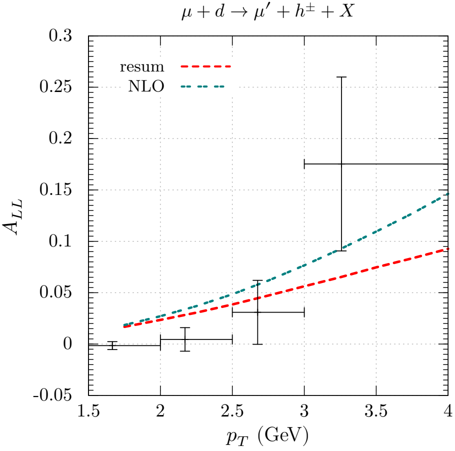
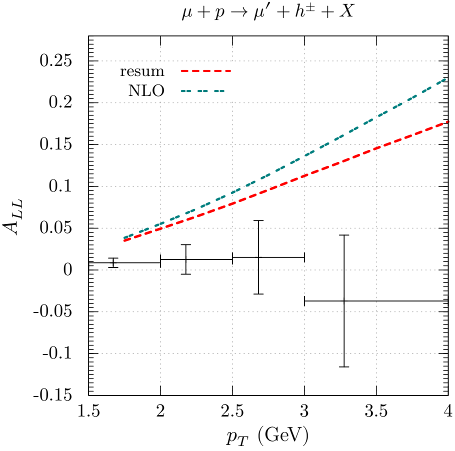
$$A_{LL,\text{resum}} = \frac{d\Delta\sigma}{d\sigma} = \frac{d\Delta\sigma_{\text{dir,resum}} + d\Delta\sigma_{\text{res,NLO}}}{d\sigma_{\text{dir,resum}} + d\sigma_{\text{res,NLO}}}$$





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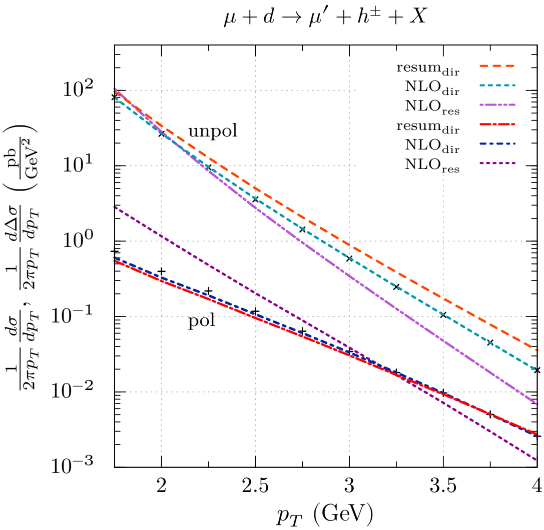
Conclusion and Outlook

- different size of resummation effects for $d\Delta\sigma$ and $d\sigma \rightarrow$ do not cancel in A_{LL}
- prediction for A_{LL} decrease when taking resummation into account
- scale-dependence remains large $\rightarrow$ better with NNLL?
- comparison with COMPASS: theoretical asymmetry fails to reproduce the proton data
- theory shows a **larger** spin asymmetry for the proton than for the deuteron, in contrast to data

What the future brings...

- $\rightarrow$ analytical study of the sign $A_{LL}^{\text{Prot}} - A_{LL}^{\text{Deut}}$
- extension with the resummed resolved calculation

including the NLO-resolved contributions:



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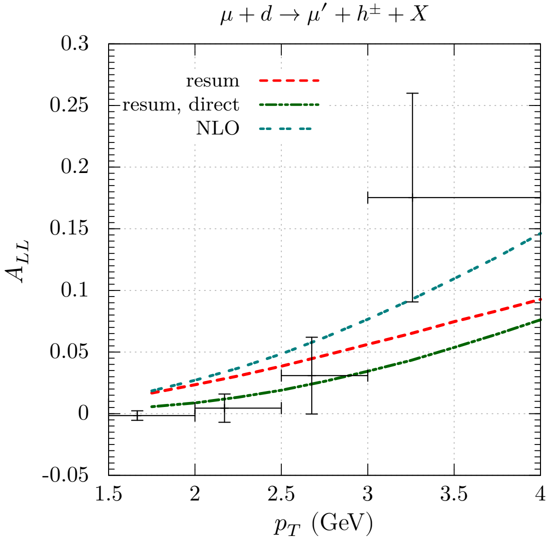
contribution of the resummed direct subprocesses to the asymmetry:



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