Inclusive four-jet production: a study of Multi-Regge kinematics and BFKL observables

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based on

[F. Caporale, F.G. C., G. Chachamis, A. Sabio Vera (2016)]

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Outline

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Motivation

So far, search for BFKL effects had these general drawbacks:

- ◇ too low $\sqrt{s}$ or rapidity intervals among tagged particels in the final state
- ◇ too inclusive oblservables, other approaches can fit them

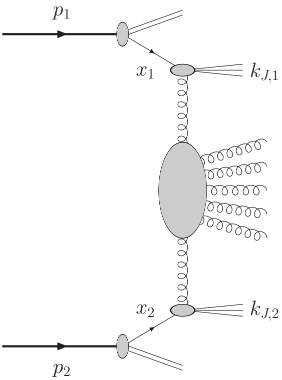
Advent of LHC:

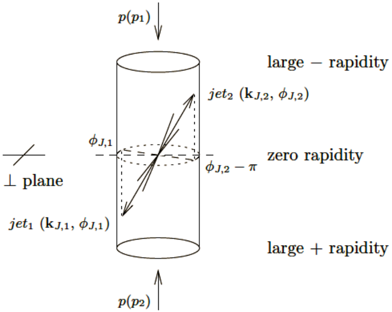
- → higher energies, larger rapidity gaps, abundance of data produced and being produced
- → unique opportunity to disentangle BFKL applicability region
- → new, suitable BFKL observables needed

Last years: **Mueller–Navelet jets** → theory vs experiment

[B. Ducloué, L. Szymanowski, S. Wallon (2014)]
[F. Caporale, D.Yu Ivanov, B. Murdaca, A. Papa (2014)]

Mueller–Navelet jets





Pictures from
[D. Colferai, F. Schwennsen, L. Szymanowski, S. Wallon (2010)]

...large jet transverse momenta: $\vec{k}_{J,1}^2 \sim \vec{k}_{J,2}^2 \gg \Lambda_{\rm QCD}^2$

...large rapidity gap between jets (high energies) $\Rightarrow$ $\Delta y = \ln \frac{x_{J,1} x_{J,2} s}{|\vec{k}_{J,1}||\vec{k}_{J,2}|}$

Looking for new observables

- BFKL feature: factorization between transverse and longitudinal (rapidities) degrees of freedom
- Usual "growth with energy" signal mainly probes the longitudinal degrees of freedom
- Mueller–Navelet correlation momenta mainly probe one of the transverse components, the azimuthal angles

! We would like to study observables for which the p_T (any p_T along the BFKL ladder) enters the game...

- ...to probe not only the general properties of the BFKL ladder, but also "to peek into the interior"...
- ...by studying azimuthal decorrelations where the p_T of extra particles introduces a new dependence...

...multi-jet production!

[R. Maciula, A. Szczurek (2014, 2015)]
[K. Kutak, R. Maciula, M. Serino, A. Szczurek, A. van Hameren (2016)]

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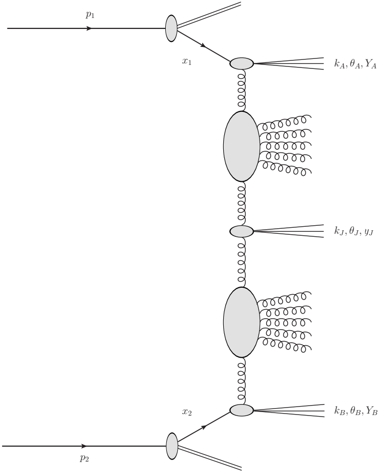
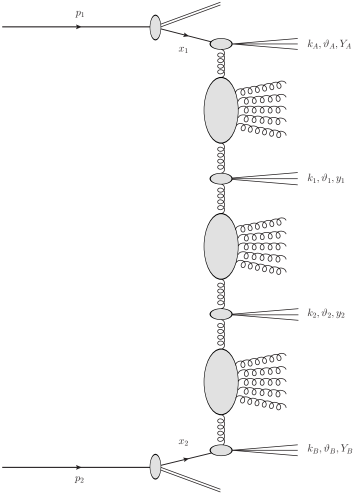
Three- and four-jet production





[F. Caporale, G. Chachamis, B. Murdaca, A. Sabio Vera (2015)]
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[F. Caporale, F.G. C., G. Chachamis, D. Gordo Gómez, A. Sabio Vera (in progr.)]

Outline

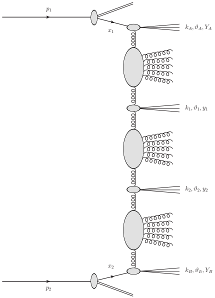
The partonic cross section

Starting point: differential partonic cross-section (no PDFs)

$$\frac{d^6\sigma^{4-\text{jet}}\left(\vec{k}_A, \vec{k}_B, Y_A - Y_B\right)}{d^2\vec{k}_1 dy_1 d^2\vec{k}_2 dy_2} = \frac{\bar{\alpha}_s\left(\mu_R\right)^2}{\pi^2 k_1^2 k_2^2} \int d^2\vec{p}_A \int d^2\vec{p}_B \int d^2\vec{p}_1 \int d^2\vec{p}_2$$

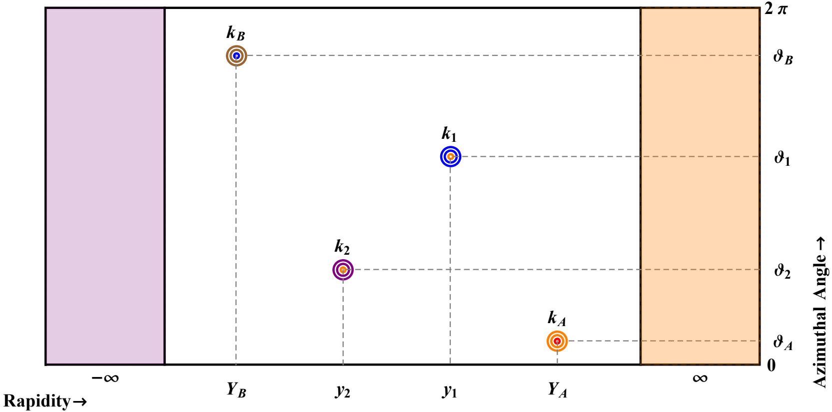
$$\delta^{(2)}\left(\vec{p}_A + \vec{k}_1 - \vec{p}_1\right) \delta^{(2)}\left(\vec{p}_B - \vec{k}_2 - \vec{p}_2\right)$$

$$\varphi\left(\vec{k}_A, \vec{p}_A, Y_A - y_1\right) \varphi\left(\vec{p}_1, \vec{p}_2, y_1 - y_2\right) \varphi\left(\vec{p}_B, \vec{k}_B, y_2 - Y_B\right)$$



- *Multi-Regge kinematics* rapidity ordering: $Y_B < y_2 < y_1 < Y_A$
- k_1^2, k_2^2 lie above the experimental resolution scale
- φ is the LO BFKL gluon Green function
- $\bar{\alpha}_s = \alpha_s N_c/\pi$

A four-jet primitive lego-plot



Outline

First step - *À la* Mueller–Navelet observables

Integrate over the azimuthal angles of the two central jets and over the difference in azimuthal angle between the two forward jets $\Delta\theta = \vartheta_A - \vartheta_B - \pi$ ...

$\rightarrow$ ...to define:

$$\int_0^{2\pi} d\Delta\theta \cos\left(M\Delta\theta\right) \int_0^{2\pi} d\vartheta_1 \int_0^{2\pi} d\vartheta_2 \frac{d^6\sigma^{4-\text{jet}}\left(\vec{k_A}, \vec{k_B}, Y_A - Y_B\right)}{dk_1 dy_1 d\vartheta_1 dk_2 d\vartheta_2 dy_2}$$

$$= \frac{4\bar{\alpha}_s\left(\mu_R\right)^2}{k_1 k_2}\left(e^{iM\pi}\,\tilde{\Omega}_M(\vec{k_A}, \vec{k_B}, Y_A, Y_B, \vec{k_1}, \vec{k_2}, y_1, y_2) + c.c.\right)$$

where

$$\tilde{\Omega}_n(\vec{k_A}, \vec{k_B}, Y_A, Y_B, \vec{k_1}, \vec{k_2}, y_1, y_2) = \int_0^{+\infty} dp_A\, p_A \int_0^{+\infty} dp_B\, p_B \int_0^{2\pi} d\phi_A \int_0^{2\pi} d\phi_B$$

$$\frac{\left(p_A + k_1 e^{-i\phi_A}\right)^n \left(p_B - k_2 e^{i\phi_B}\right)^n}{\sqrt{\left(p_A^2 + k_1^2 + 2p_A k_1 \cos\phi_A\right)^n}\sqrt{\left(p_B^2 + k_2^2 - 2p_B k_2 \cos\phi_B\right)^n}}$$

$$\varphi_n\left(|\vec{k_A}|, |\vec{p_A}|, Y_A - y_1\right)\varphi_n\left(|\vec{p_B}|, |\vec{k_B}|, y_2 - Y_B\right)$$

$$\varphi_n\left(\sqrt{p_A^2 + k_1^2 + 2p_A k_1 \cos\phi_A}, \sqrt{p_B^2 + k_2^2 - 2p_B k_2 \cos\phi_B}, y_1 - y_2\right)$$

and

$$\varphi_n\left(|p|, |q|, Y\right) = \int_0^{\infty} d\nu \cos\left(\nu \ln\frac{p^2}{q^2}\right)\frac{e^{\bar{\alpha}_s \chi_{|n|}(\nu) Y}}{\pi^2\sqrt{p^2 q^2}},$$

First step - *À la* Mueller–Navelet observables

Integrate over the azimuthal angles of the two central jets and over the difference in azimuthal angle between the two forward jets $\Delta\theta = \vartheta_A - \vartheta_B - \pi$ ...

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$$= \frac{4\bar{\alpha}_s\left(\mu_R\right)^2}{k_1 k_2}\left(e^{iM\pi}\,\tilde{\Omega}_M(\vec{k_A}, \vec{k_B}, Y_A, Y_B, \vec{k_1}, \vec{k_2}, y_1, y_2) + c.c.\right)$$

Associated experimental observable:

$$\langle\cos(M(\vartheta_A - \vartheta_B - \pi))\rangle = \frac{\int_0^{2\pi} d\Delta\theta \cos(M\Delta\theta) \int_0^{2\pi} d\vartheta_1 \int_0^{2\pi} d\vartheta_2 \frac{d^6\sigma^{4-\mathrm{jet}}}{dk_1 dy_1 d\vartheta_1 dk_2 d\vartheta_2 dy_2}}{\int_0^{2\pi} d\Delta\theta \int_0^{2\pi} d\vartheta_1 \int_0^{2\pi} d\vartheta_2 \frac{d^6\sigma^{4-\mathrm{jet}}}{dk_1 dy_1 d\vartheta_1 dk_2 d\vartheta_2 dy_2}}$$

Then, take the ratios:

$$\mathcal{R}_N^M = \frac{\langle\cos(M(\vartheta_A - \vartheta_B - \pi))\rangle}{\langle\cos(N(\vartheta_A - \vartheta_B - \pi))\rangle}$$

Second step - Generalized azimuthal coefficients

By far more interesting, it is to integrate over all angles after using the projections on the three azimuthal angle differences indicated below...

→ ...to define:

$$\mathcal{C}_{MNL} = \int_0^{2\pi} d\vartheta_A \int_0^{2\pi} d\vartheta_B \int_0^{2\pi} d\vartheta_1 \int_0^{2\pi} d\vartheta_2 \ \cos\left(M\left(\vartheta_A - \vartheta_1 - \pi\right)\right)$$

$$\cos\left(N\left(\vartheta_1 - \vartheta_2 - \pi\right)\right)\cos\left(L\left(\vartheta_2 - \vartheta_B - \pi\right)\right) \frac{d^6\sigma^{4-\text{jet}}\left(\vec{k_A}, \vec{k_B}, Y_A - Y_B\right)}{dk_1 dy_1 d\vartheta_1 dk_2 d\vartheta_2 dy_2}$$

$$= \frac{2\pi^2 \bar{\alpha}_s\left(\mu_R\right)^2}{k_1 k_2} (-1)^{M+N+L} \left(\tilde{\Omega}_{M,N,L} + \tilde{\Omega}_{M,N,-L} + \tilde{\Omega}_{M,-N,L} \right.$$

$$\left. + \tilde{\Omega}_{M,-N,-L} + \tilde{\Omega}_{-M,N,L} + \tilde{\Omega}_{-M,N,-L} + \tilde{\Omega}_{-M,-N,L} + \tilde{\Omega}_{-M,-N,-L}\right)$$

with

$$\tilde{\Omega}_{m,n,l} = \int_0^{+\infty} dp_A\, p_A \int_0^{+\infty} dp_B\, p_B \int_0^{2\pi} d\phi_A \int_0^{2\pi} d\phi_B$$

$$\frac{e^{-im\phi_A}\, e^{il\phi_B} \left(p_A e^{i\phi_A} + k_1\right)^n \left(p_B e^{-i\phi_B} - k_2\right)^n}{\sqrt{\left(p_A^2 + k_1^2 + 2p_A k_1 \cos\phi_A\right)^n} \sqrt{\left(p_B^2 + k_2^2 - 2p_B k_2 \cos\phi_B\right)^n}}$$

$$\varphi_m\left(|\vec{k_A}|, |\vec{p_A}|, Y_A - y_1\right) \varphi_l\left(|\vec{p_B}|, |\vec{k_B}|, y_2 - Y_B\right)$$

$$\varphi_n\left(\sqrt{p_A^2 + k_1^2 + 2p_A k_1 \cos\phi_A}, \sqrt{p_B^2 + k_2^2 - 2p_B k_2 \cos\phi_B}, y_1 - y_2\right)$$

Second step - Generalized azimuthal correlations

By far more interesting, it is to integrate over all angles after using the projections on the three azimuthal angle differences indicated below...

→ ...to define:

$$\mathcal{C}_{MNL} = \int_0^{2\pi} d\vartheta_A \int_0^{2\pi} d\vartheta_B \int_0^{2\pi} d\vartheta_1 \int_0^{2\pi} d\vartheta_2 \ \cos\left(M\left(\vartheta_A - \vartheta_1 - \pi\right)\right) \cos\left(N\left(\vartheta_1 - \vartheta_2 - \pi\right)\right) \cos\left(L\left(\vartheta_2 - \vartheta_B - \pi\right)\right) \frac{d^6\sigma^{4-\text{jet}}\left(\vec{k}_A, \vec{k}_B, Y_A - Y_B\right)}{dk_1 dy_1 d\vartheta_1 dk_2 d\vartheta_2 dy_2}$$

Main observables: **generalized azimuthal correlation momenta**

$$\mathcal{R}^{MNL}_{PQR} = \frac{C_{MNL}}{C_{PRQ}} = \frac{\langle \cos(M(\vartheta_A - \vartheta_1 - \pi)) \cos(N(\vartheta_1 - \vartheta_2 - \pi)) \cos(L(\vartheta_2 - \vartheta_B - \pi)) \rangle}{\langle \cos(P(\vartheta_A - \vartheta_1 - \pi)) \cos(Q(\vartheta_1 - \vartheta_2 - \pi)) \cos(R(\vartheta_2 - \vartheta_B - \pi)) \rangle}$$

Outline

Observables and kinematics

- **Observables:**

$$\mathcal{R}^{MNL}_{PQR} = \frac{C_{MNL}}{C_{PRQ}} = \frac{\langle \cos(M(\vartheta_A - \vartheta_1 - \pi)) \cos(N(\vartheta_1 - \vartheta_2 - \pi)) \cos(L(\vartheta_2 - \vartheta_B - \pi)) \rangle}{\langle \cos(P(\vartheta_A - \vartheta_1 - \pi)) \cos(Q(\vartheta_1 - \vartheta_2 - \pi)) \cos(R(\vartheta_2 - \vartheta_B - \pi)) \rangle}$$

◇ remove the contribution from the zero conformal spin

$\xrightarrow{to}$ drastically reduce the dependence on collinear configurations

study $\mathcal{R}^{MNL}_{PQR}$ with integer $M, N, L, P, Q, R > 0$

- **Kinematic settings:**

◇ **1.** $k_A = 40$ GeV; $k_B = 50$ GeV; quasi − symmetric
2. $k_A = 30$ GeV; $k_B = 60$ GeV; different

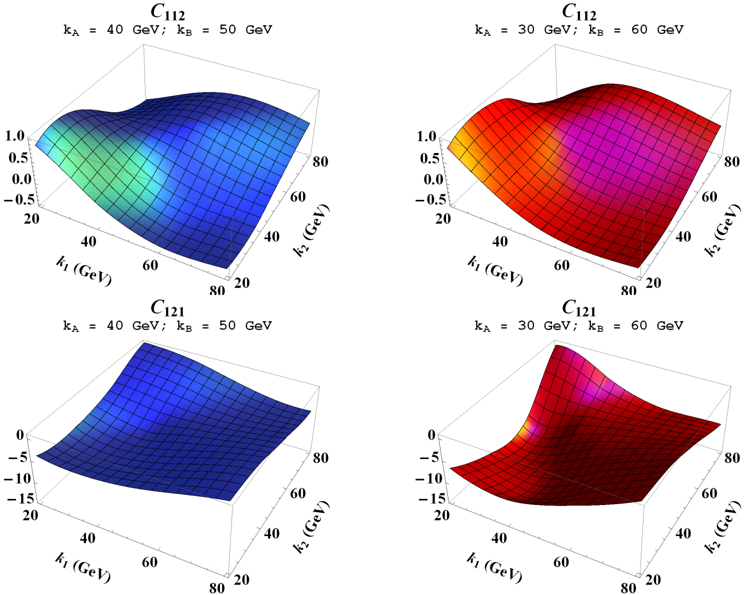
◇ $Y_A - y_1 = y_1 - y_2 = y_2 - Y_B = 3;$ $Y_A - Y_B = 9$

◇ 20 GeV $\leq k_1, k_2 \leq$ 80 GeV

- **Numerical tools:**

◇ Fortran & Mathematica

$\mathcal{C}_{MNL}$ vs $k_{1,2}$



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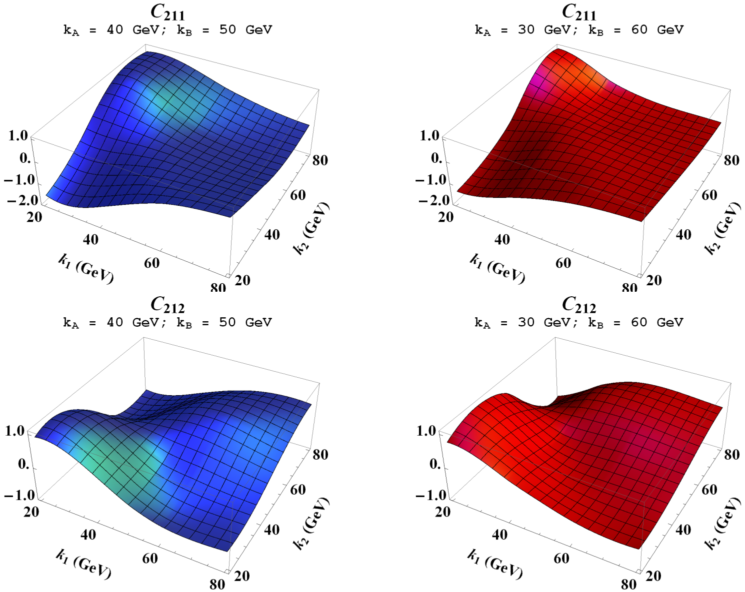
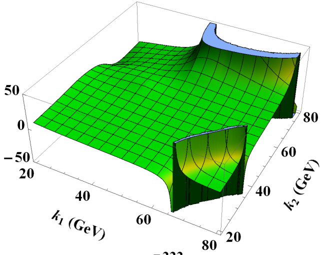


$\mathcal{R}^{MNL}_{PQR}$ vs $k_{1,2}$

$\mathcal{R}^{111}_{122}$
k_A = 40 GeV; k_B = 50 GeV



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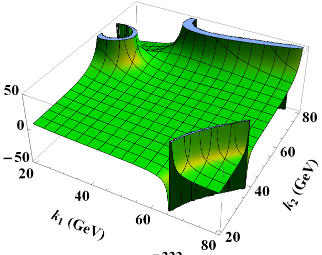
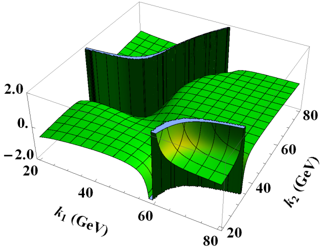


$\mathcal{R}^{222}_{211}$
k_A = 40 GeV; k_B = 50 GeV



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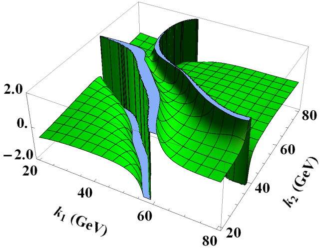

Conclusions...

Study of process with four tagged jets to propose new, more exclusive, BFKL observables

- LLA framework, taking ratios of correlation functions to minimize the influence of higher order corrections

  $\Rightarrow$ $\mathcal{R}^{MNL}_{PQR}$ exhibit interesting patterns:

  - similar to oscillation modes of a two-dimensional membrane
  - changes of sign in the denominator coefficient give ries to singularities

...Outlooks

- Give theoretical predictions on hadronic level by introducing PDFs in the jet vertices, using the strong coupling running and match the kinematical cuts of LHC [F. Caporale, F.G. C., G. Chachamis, D. Gordo Gómez, A. Sabio Vera (in progress)]
- Perform a study in the NLA accuracy
- Comparison with BFKLex results and other Monte Carlo codes
- Comparison with analyses where the four-jet predictions stem from two independent gluon ladders [R. Maciula, A. Szczurek (2014, 2015)] [K. Kutak, R. Maciula, M. Serino, A. Szczurek, A. van Hameren (2016)]

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Thanks for your attention!!

BACKUP slides

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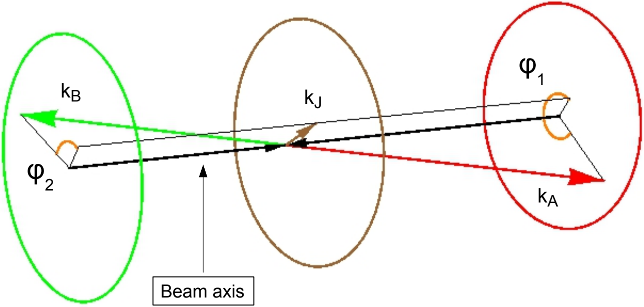
An event with three tagged jets



$Y_B < y_J < Y_A$