Soft gluon resummation for $pp \to t\bar{t}H$

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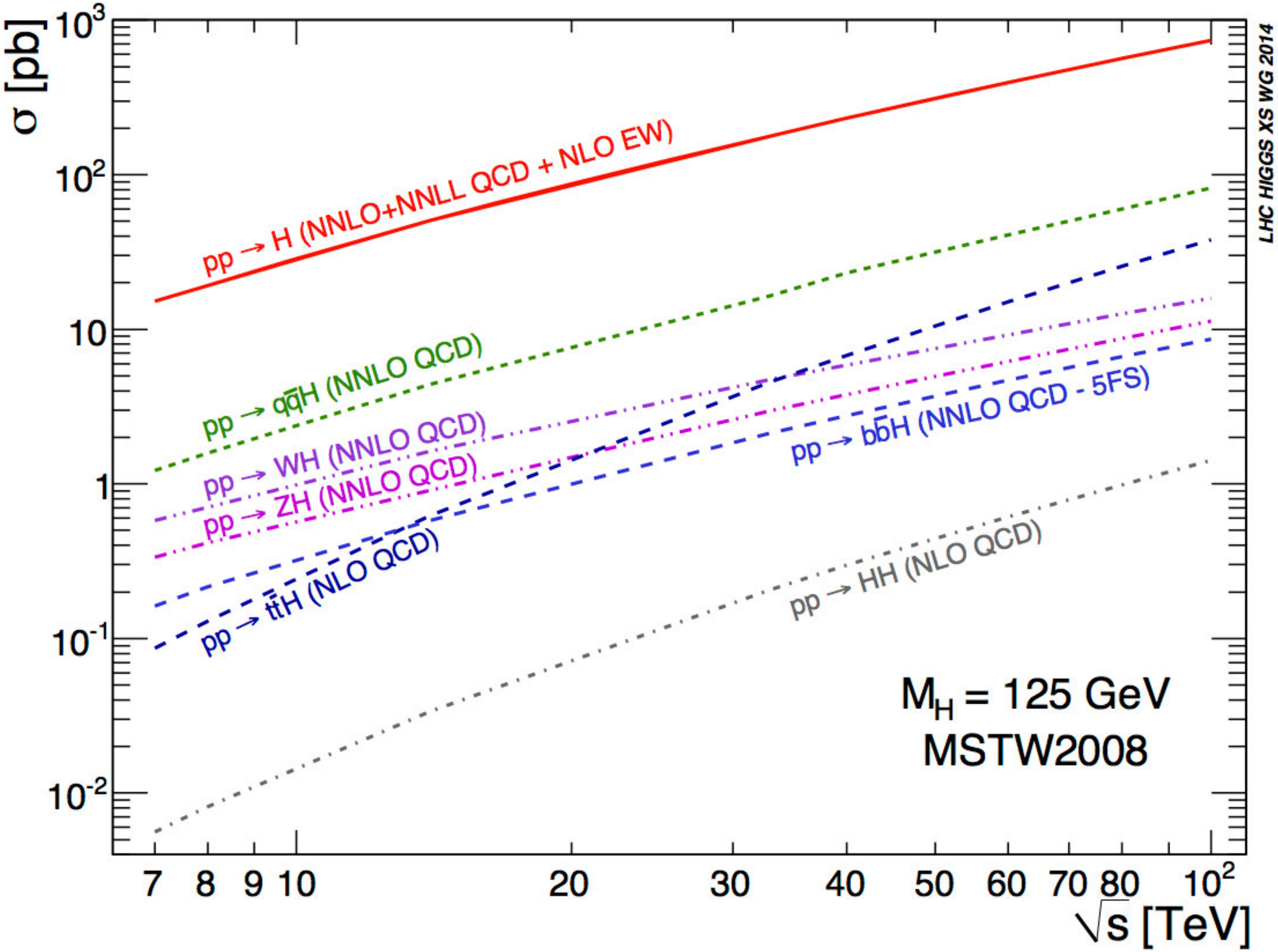
based on JHEP 03(2016)065, *arXiv:1509.02780*

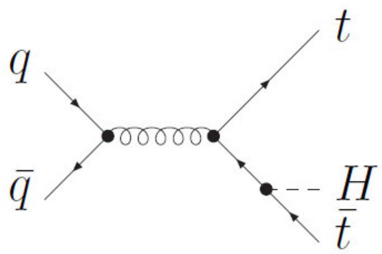
Why resummation for $pp \to t\bar{t}H$?

- Measure of top-Higgs Yukawa coupling -> new physics.
- It is $2 \to 3$ process -> something new in soft gluon resummation.
- It is simple (Higgs is colorless).

Higgs production at the LHC

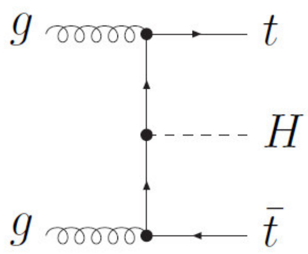


$pp \to t\bar{t}H$ at the leading order (LO)



Color channels:

Octet $T^{j}_{a_q a_{\bar{q}}}\, T^{j}_{a_t a_{\bar{t}}}$



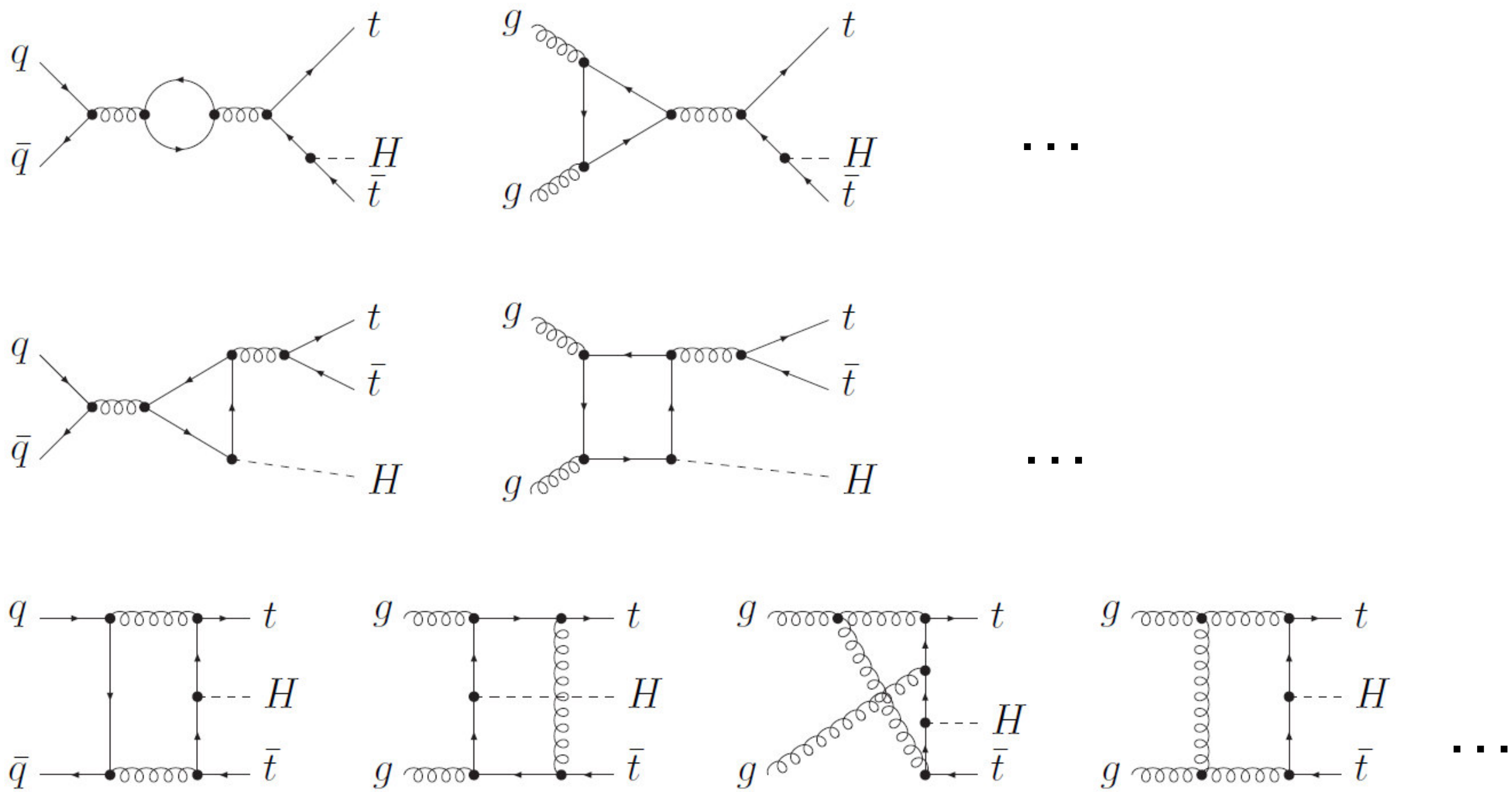
Singlet $\delta^{A_{g1} A_{g2}} \delta_{a_t a_{\bar{t}}}$

Sym. octet $d^{A_{g1} A_{g2}\, j}\, T^{j}_{a_t a_{\bar{t}}}$

Antisym. octet $i f^{A_{g1} A_{g2}\, j}\, T^{j}_{a_t a_{\bar{t}}}$

- 3 particles in the final state ⇨ 5 independent Mandelstam variables

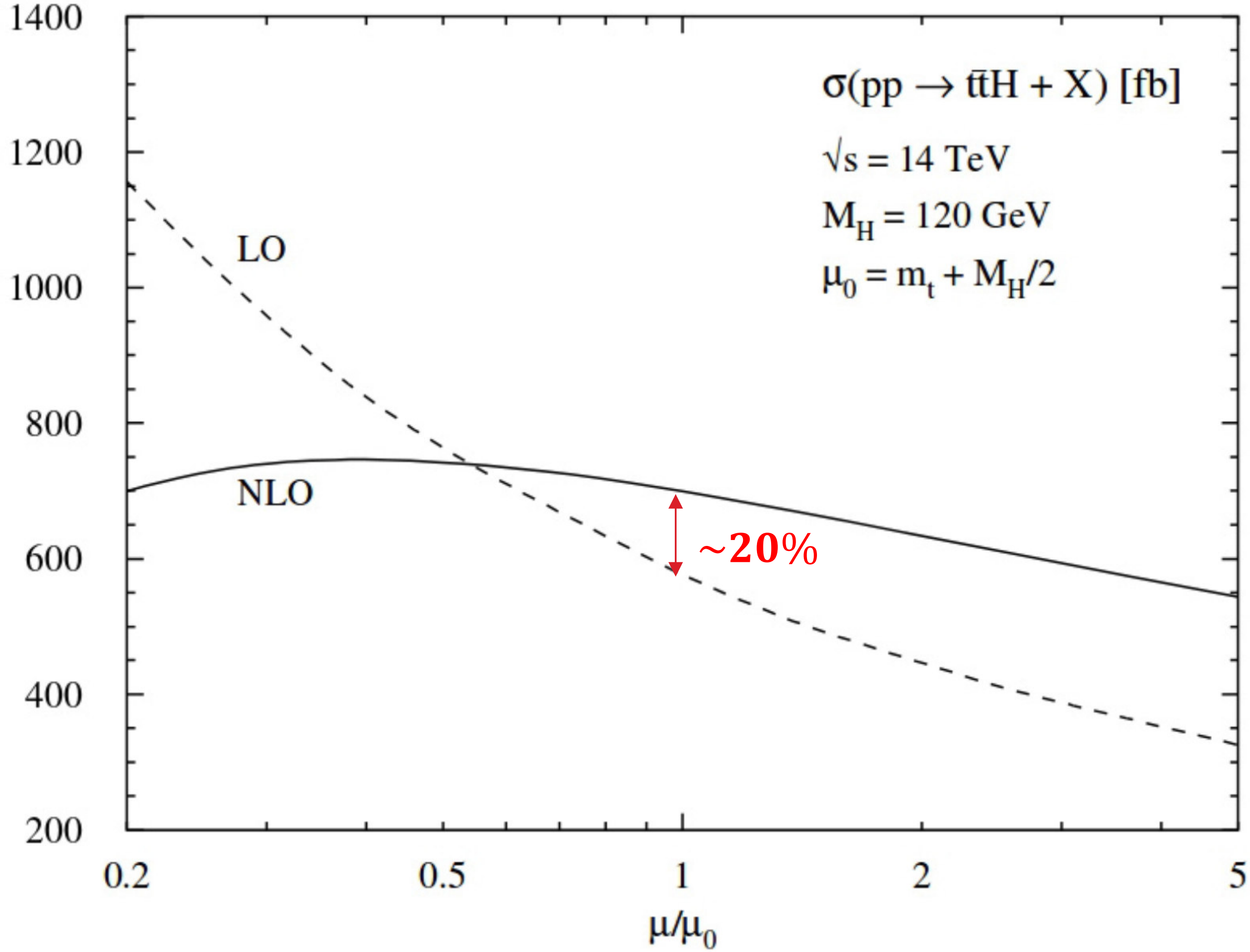
Next-to-leading order (NLO)



Beenakker et. al (2001,03): hep-ph/0107081, hep-ph/0211352,
Reina et. al (2001-03): hep-ph/0107101, hep-ph/0109066, hep-ph/0211438, hep-ph/0305087

LO vs. NLO

Beenakker et. al, *hep-ph/0211352*

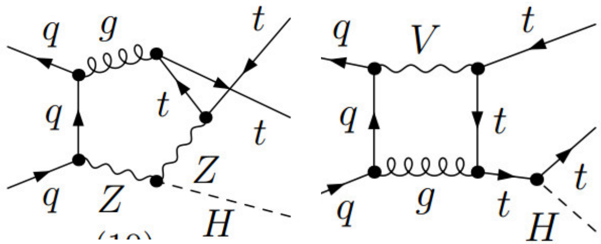


$\mu = \mu_F = \mu_R$

What next?

- NNLO QCD is out of reach…
- QCD-Electroweak corrections (NLO) -> 1-2%

Frixione et. al (2014,15), arXiv:1407.0823, arXiv:1504.03446
Zhang et. al (2014), arXiv:1407.1110

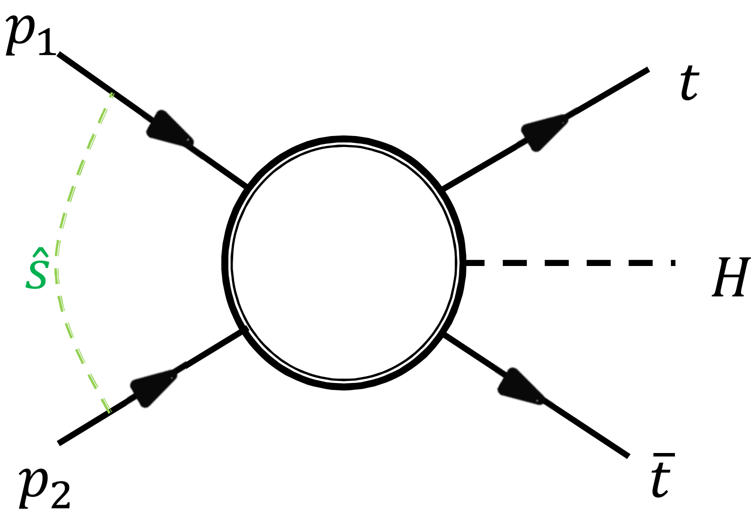


- This talk: Soft gluon NLL resummation at the threshold.

- NNLL at NNLO soft gluon resummation using SCET *Broggio, Ferroglia, Pecjak, Signer and Yang, arXiv:1510.01914*

Notation

Partonic amplitude:



$$\hat{s} = (p_1 + p_2)^2$$

$$s_0 = (2m_t + m_H)^2$$

$$\beta = \sqrt{1 - \frac{s_0}{\hat{s}}}$$

Absolute threshold at $\beta = 0$.

Total cross-section (LO):

$$\sigma^{LO} = \sum_{i,j} \int dx_1 dx_2 \int d\Pi_3 \, f_i(x_1, \mu_F) f_j(x_1, \mu_F) \;\; \hat{\sigma}^{LO}_{ij \to t\bar{t}H}(\mu_F, \Pi_3, \beta)$$

PDFs — Partonic cross-section

Soft gluon resummation

- At threshold $\beta \to 0$ cross-sections is divergent because of terms:

$$\underbrace{\alpha_s^n \log^{2n} \beta}_{\text{Leading logarithms (LL)}}, \quad \underbrace{\alpha_s^n \log^{2n-1} \beta}_{\text{Next-to-leading logarithms (NLL)}}, \quad \underbrace{\alpha_s^n \log^{2n-2} \beta}_{\text{Next-to-Next-to leading logarithms (NNLL)}}, \ \ldots$$

- Our goal: resum all logarithms to NLL precision.

  To do this we use Mellin space technique *(Sterman 1987)*.

- Large logarithms:

„β space”		„N space”
$\log \beta \, , \beta \to 0$	$\longrightarrow$	$\log N \, , N \to \infty \; (complex)$

Soft gluon resummation

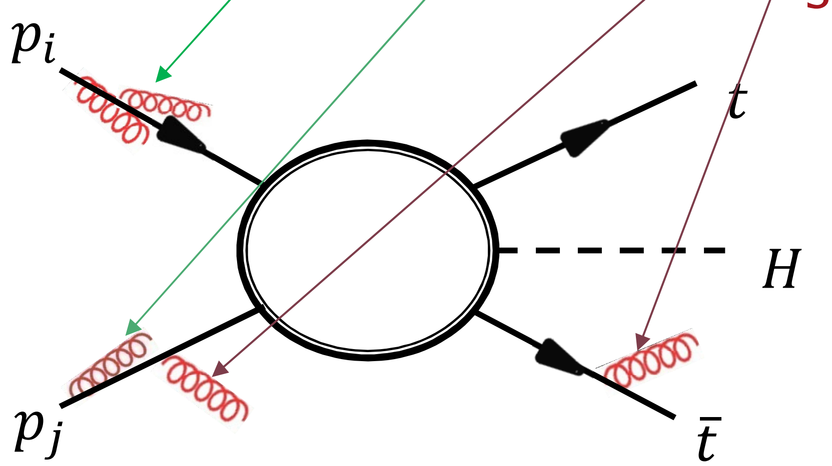
- Factorization in the Mellin space:

Hard matching coefficients

$$\tilde{\tilde{\sigma}}^{(res)}_{ij\to t\bar{t}H,I}(\mathrm{N}) = \sum_{I,J} C^{I}_{ij\to t\bar{t}H}\,\Delta_i(N+1)\,\Delta_j(N+1)\,S_{ij\to t\bar{t}H,IJ}(N+1)\;\tilde{\tilde{\sigma}}^{(LO)}_{ij\to t\bar{t}H,J}(\mathrm{N})$$

Soft collinear, collinear

Soft wide angle



Soft anomalous dimension Γ

- To find matrix S one needs to calculate soft anomalous dimension

Botts, Sterman (1989), Nucl. Phys. B325, 62
Kidonakis, Oderda, Sterman (1998), hep-ph/9801268

$$\left(\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}\right)S^{(N)}_{JI}=-\Gamma^{\dagger}_{JK}S(N)_{KI}-S(N)_{JL}\Gamma_{LI}$$

$$\Gamma^{gg\to klB}=\frac{\alpha_s}{\pi}\begin{bmatrix}\Gamma^{gg}_{11} & 0 & \Omega_3\\ 0 & \Gamma^{gg}_{22} & \frac{N_c}{2}\Omega_3\\ 2\Omega_3 & \frac{N_c^2-4}{2N_c}\Omega_3 & \Gamma^{gg}_{33}\end{bmatrix},$$

$$\Gamma^{gg}_{11}=-C_{\rm F}(L_{\beta,kl}+1),$$
$$\Gamma^{gg}_{22}=\Gamma^{gg}_{33}=\frac{1}{2}((C_{\rm A}-2C_{\rm F})(L_{\beta,kl}+1)+C_{\rm A}\Lambda_3),$$

$$\Lambda_3=(T_1(m_k)+T_2(m_l)+U_1(m_l)+U_2(m_k))/2,$$
$$\Omega_3=(T_1(m_k)+T_2(m_l)-U_1(m_l)-U_2(m_k))/2,$$

$$L_{\beta,kl}=\frac{\kappa^2+\beta^2_{kl}}{2\kappa\beta_{kl}}\left(\log\left(\frac{\kappa-\beta_{kl}}{\kappa+\beta_{kl}}\right)+i\pi\right),$$
$$T_i(m)=\frac{1}{2}\left(\ln((m^2-t_i)^2/(m^2\hat{s}))-1+i\pi\right),$$
$$U_i(m)=\frac{1}{2}\left(\ln((m^2-u_i)^2/(m^2\hat{s}))-1+i\pi\right),$$
$$\kappa=\sqrt{1-(m_k-m_l)^2/s_{kl}},\qquad s_{kl}=(p_k+p_l)^2,$$
$$t_1=(p_i-p_k)^2,\qquad t_2=(p_j-p_l)^2,\quad u_1=(p_i-p_l)^2,\quad u_2=(p_j-p_k)^2.$$

Soft gluon resummation at the threshold

Soft anomalous dimension is diagonal at the threshold
so soft matrix doesn't mix color channels:

$$S_{ij\to t\bar{t}H,IJ} = \delta_{IJ}\, \Delta_{ij\to t\bar{t}H,I}$$

We can resum cross-section in each color channel separately:

$$\tilde{\hat{\sigma}}^{(res)}_{ij\to t\bar{t}H,I}(\mathrm{N}) = C^{I}_{ij\to t\bar{t}H} \Delta_i(N+1)\Delta_j(N+1)\Delta_{ij\to t\bar{t}H,I}(N+1)\, \tilde{\hat{\sigma}}^{(LO)}_{ij\to t\bar{t}H,I}(\mathrm{N})$$

Resummation factors:

$$\Delta_i(N)\Delta_j(N)\Delta_{ij\to t\bar{t}H,I}(N) = \exp\{\log N * g_{LL}(\alpha_s \log N) + g_{NLL}(\alpha_s \log N) + \cdots (NNLL)\}$$

Hard matching coefficients $C^{I}_{ij\to t\bar{t}H}$

- They are coming from other contributions constant at threshold (<u>don't contain</u> $\log N$).

$$C_{ij\to t\bar{t}H,I} = 1 + \frac{\alpha_s}{\pi} C^{(1)}_{ij\to t\bar{t}H,I} + \cdots$$

NNLL correction

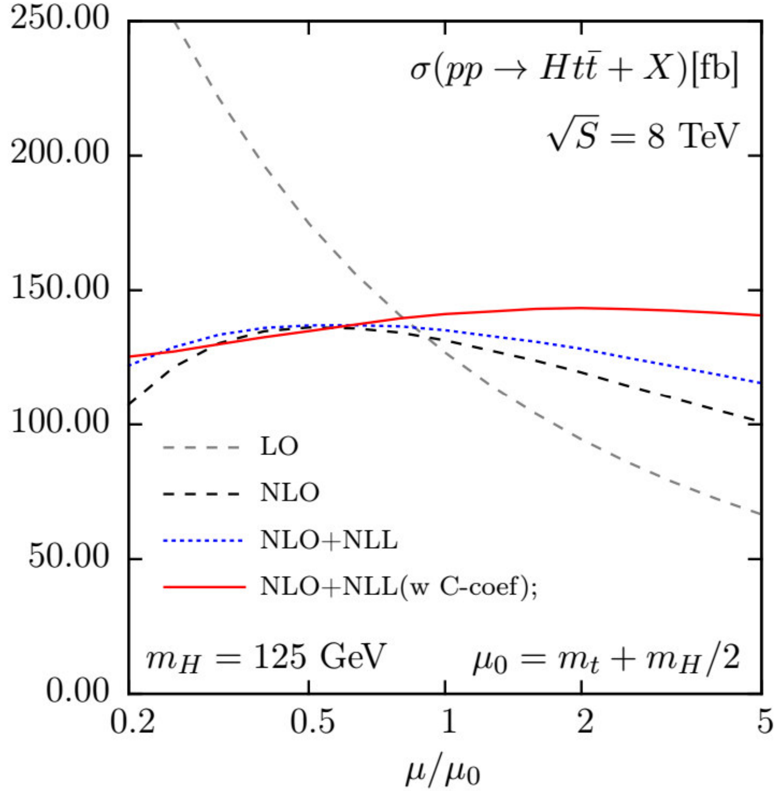
- But NNLL correction can be calculated using NLO virtual corrections + dipole subtraction formalism.

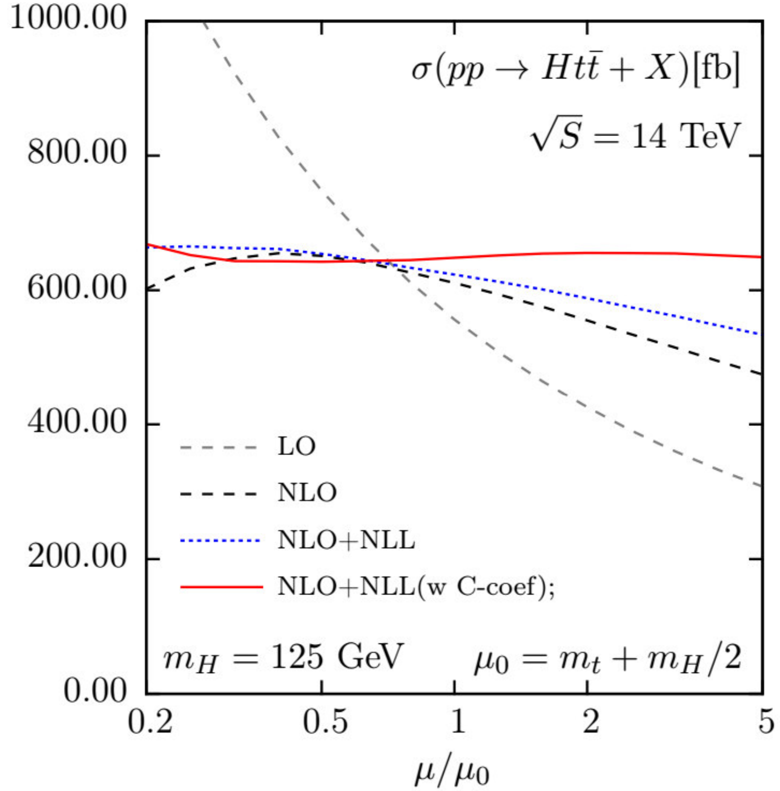
Hartanto et. al, arXiv:1501.04498 *Catani, Seymour, Nucl.Phys. B485 (1997) 291-419*

Results (matched to NLO from aMC@NLO)

J. Alwall et. al, arXiv:1405.0301

$\mu = \mu_F = \mu_R$





PDF: MMHT 2014

7-point method for error estimation

- Idea: change μ_F and μ_R independently:

μ_F \ μ_R	$\mu_0/2$	μ_0	$2\mu_0$
$\mu_0/2$	✖	✖	
μ_0	✖	✖	✖
$2\mu_0$		✖	✖

$\mu_0 = m_t + m_H/2$

Estimate error using min(...) and max(...)

$\sqrt{S}$ [TeV]	NLO [fb]	NLO+NLL		NLO+NLL with C		pdf error
		Value [fb]	K-factor	Value [fb]	K-factor	
8	$132^{+3.9\%}_{-9.3\%}$	$135^{+3.0\%}_{-5.9\%}$	1.03	$141^{+7.7\%}_{-4.6\%}$	1.07	$^{+3.0\%}_{-2.7\%}$
13	$506^{+5.9\%}_{-9.4\%}$	$516^{+4.6\%}_{-6.5\%}$	1.02	$537^{+8.2\%}_{-5.5\%}$	1.06	$^{+2.3\%}_{-2.3\%}$
14	$613^{+6.2\%}_{-9.4\%}$	$625^{+4.6\%}_{-6.7\%}$	1.02	$650^{+7.9\%}_{-5.7\%}$	1.06	$^{+2.3\%}_{-2.2\%}$

K-factor

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- K-factor is quite low. The reason is $2 \to 3$ phase space in which cross-section is strongly suppressed at the threshold:

$$\sigma^{LO} \sim \beta^4 \,, \quad \text{for } \beta \to 0$$

(we are resumming large $\log \beta$).

Thank You