
Top Quark Mass Calibration for Monte-Carlo Event Generators

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Outline

- Basic methods for top mass measurements
- Monte Carlo generators and the top quark mass
- Calibration of the Monte Carlo top mass parameter
- Preliminary results of first serious analysis

In collaboration with:

M. Butenschön

B. Dehnadi,

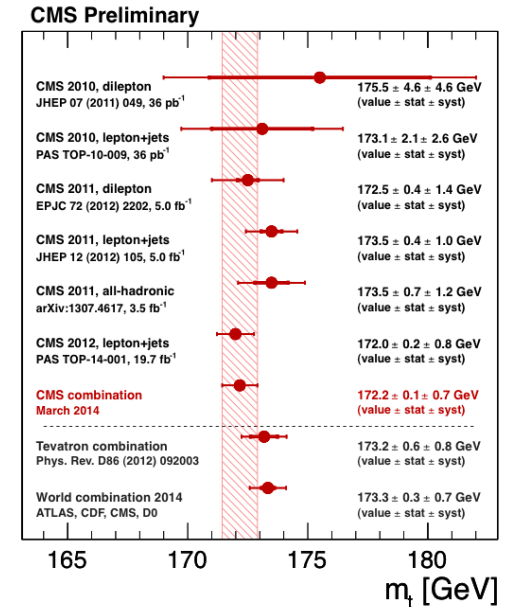
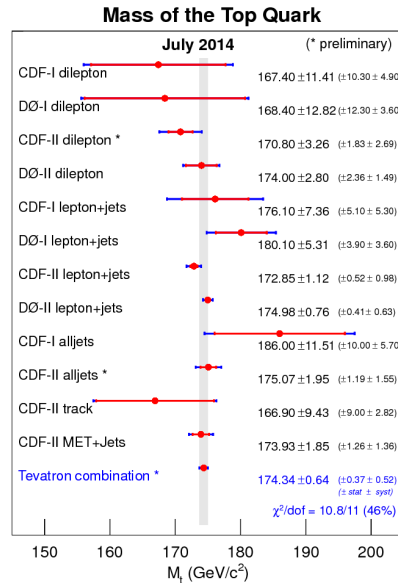
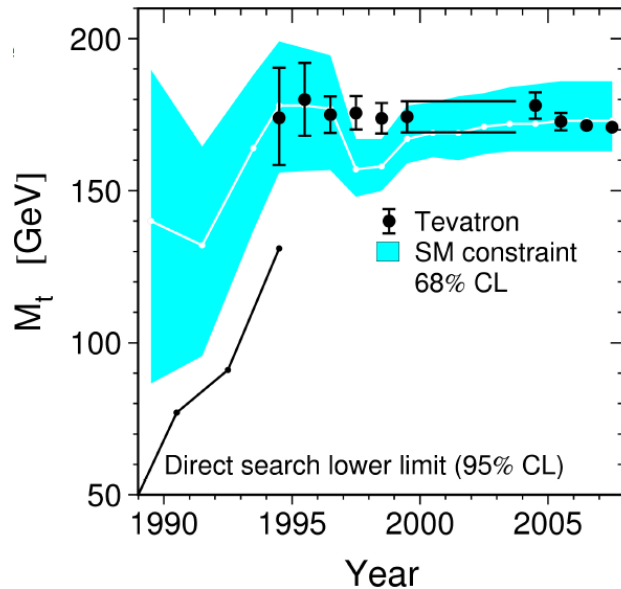
V. Mateu,

M. Preisser

I. Stewart



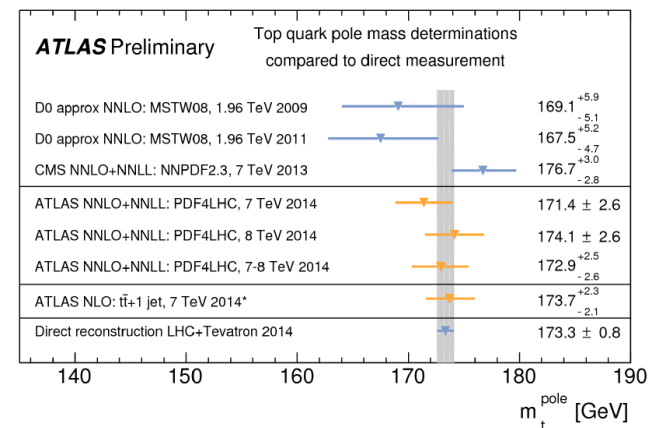
A small history on top mass reconstruction



- Many individual measurements with uncertainty below 1 GeV.
- Smallest errors from direct reconstruction/template fits
- World combination 2014:

$$m_t^{\text{MC}} = 173.34 \pm 0.76 \text{ GeV}$$

→ Talks by Shabalina, Bevilacqua, Thier



Main Top Mass Measurements Methods

LHC+Tevatron

Direct Reconstruction:

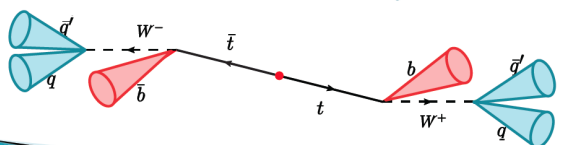
Kinematic Fit

Selected objects:

- 4 untagged jets
- 2 b-tagged jets

Constraints:

- $2 \times m_{jj} = m_W$
- $m_{\text{top}} = m_{jjb,1} = m_{jjb,2} = m_{\text{antitop}}$

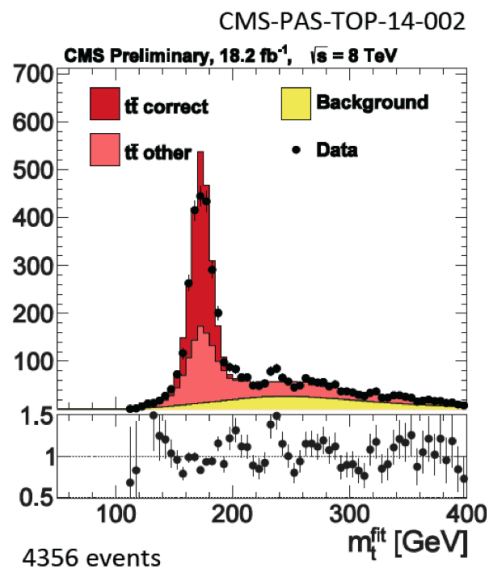


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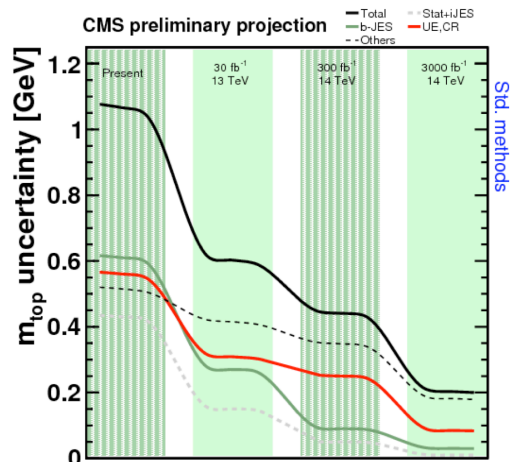
Eike Schlieckau - Universität Hamburg

September 30th 2014

kinematic mass determination



Determination of the best-fit value of the Monte-Carlo top quark mass parameter



⊕ High top mass sensitivity

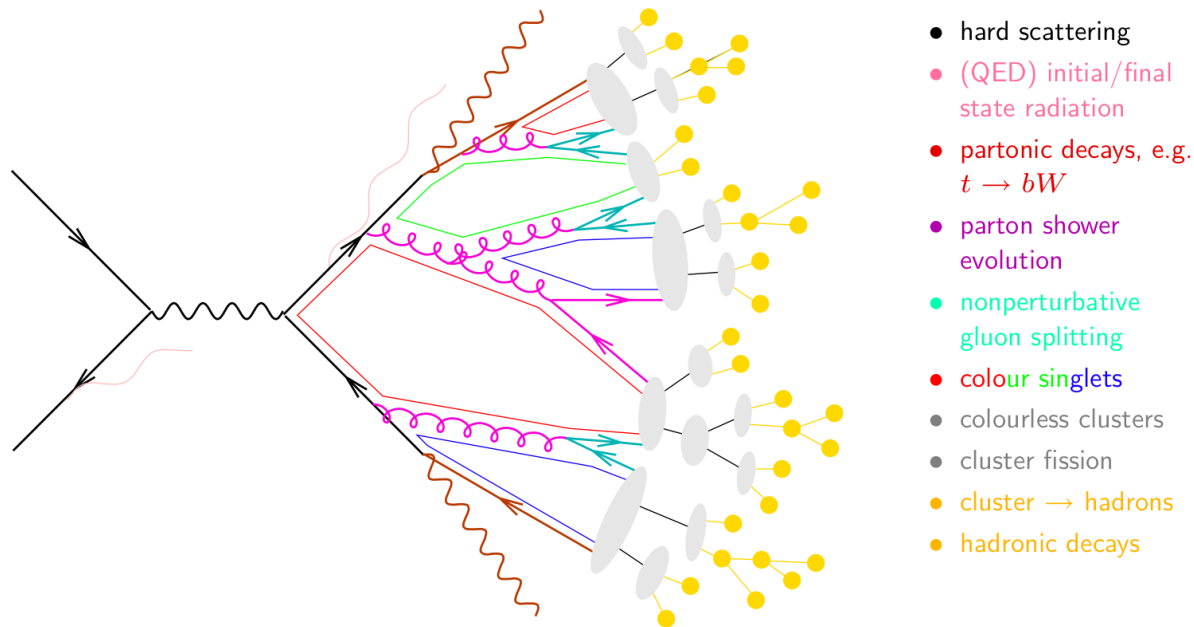
⊖ Precision of MC ?

⊖ Meaning of m_t^{MC} ?

$$\Delta m_t \gtrsim 0.5 \text{ GeV}$$

← $\Delta m_t \sim 200 \text{ MeV}$ (projection)

Monte-Carlo Event Generators



- Full simulation of all processes (all experimental aspects accessible)
- QCD-inspired: partly first principles QCD \Leftrightarrow partly model
- Description power of data better than intrinsic accuracy. (But how precise?)
- Top quark: treated like a real particle ($m_t^{\text{MC}} \approx m_t^{\text{pole}} + ?$).

But pole mass ambiguous by $O(1 \text{ GeV})$ due to confinement.

Better mass definition needed.

MC Top Quark Mass

AHH, Stewart 2008
AHH, 2014

$$m_t^{\text{MC}} = m_t^{\text{MSR}}(R = 1 \text{ GeV}) + \Delta_{t,\text{MC}}(R = 1 \text{ GeV})$$

$$\Delta_{t,\text{MC}}(1 \text{ GeV}) \sim \mathcal{O}(1 \text{ GeV})$$

- small size of $\Delta_{t,\text{MC}}$
- Renormalon-free
- little parametric dependence on other parameters

MSR Mass Definition

MS Scheme: $(\mu > \overline{m}(\overline{m}))$

$$\overline{m}(\overline{m}) - m^{\text{pole}} = -\overline{m}(\overline{m}) [0.42441 \alpha_s(\overline{m}) + 0.8345 \alpha_s^2(\overline{m}) + 2.368 \alpha_s^3(\overline{m}) + \dots]$$

MSR Scheme: $(R < \overline{m}(\overline{m}))$



$$m_{\text{MSR}}(R) - m^{\text{pole}} = -R [0.42441 \alpha_s(R) + 0.8345 \alpha_s^2(R) + 2.368 \alpha_s^3(R) + \dots]$$

$$m_{\text{MSR}}(m_{\text{MSR}}) = \overline{m}(\overline{m})$$

$\Rightarrow m_{\text{MSR}}(R)$ Short-distance mass that smoothly interpolates all R scales

Calibration of the MC Top Mass

Method:

- ✓ 1) **Strongly mass-sensitive observable** (closely related to reconstructed invariant mass distribution !)
- ✓ 2) Accurate **analytic hadron level QCD predictions at \geq NLL/NLO** with **full control over the quark mass scheme dependence**.
- ✓ 3) QCD masses as function of m_t^{MC} from **fits** of observable.
- 4) Cross check observable independence

$$m_t^{\text{MC}} = m_t^{\text{MSR}}(R = 1 \text{ GeV}) + \Delta_{t,\text{MC}}(R = 1 \text{ GeV})$$

$$\Delta_{t,\text{MC}}(1 \text{ GeV}) = \bar{\Delta} + \delta\Delta_{\text{MC}} + \delta\Delta_{\text{pQCD}} + \delta\Delta_{\text{param}}$$

Monte Carlo errors:

- different tunings
- parton showers
- color reconnection
- Intrinsic error, ...

QCD errors:

- perturbative error
- scale uncertainties
- electroweak effects

Parametric errors:

- strong coupling α_s
- Non-perturbative parameters

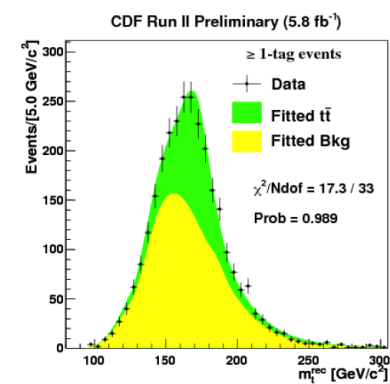
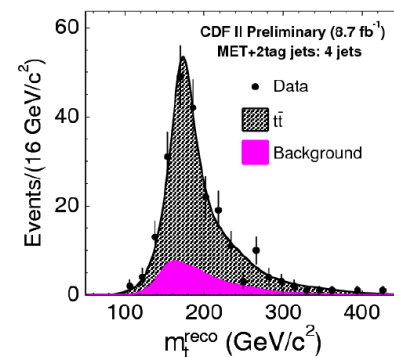
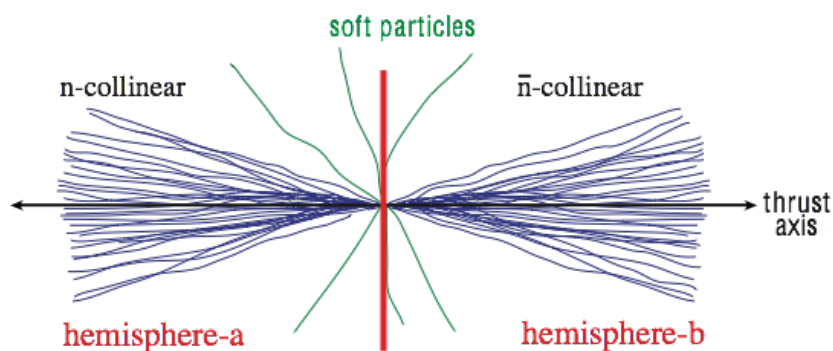
Thrust Distribution

Observable: 2-jettiness in $e+e^-$ for $Q \sim p_T \gg m_t$ (boosted tops)

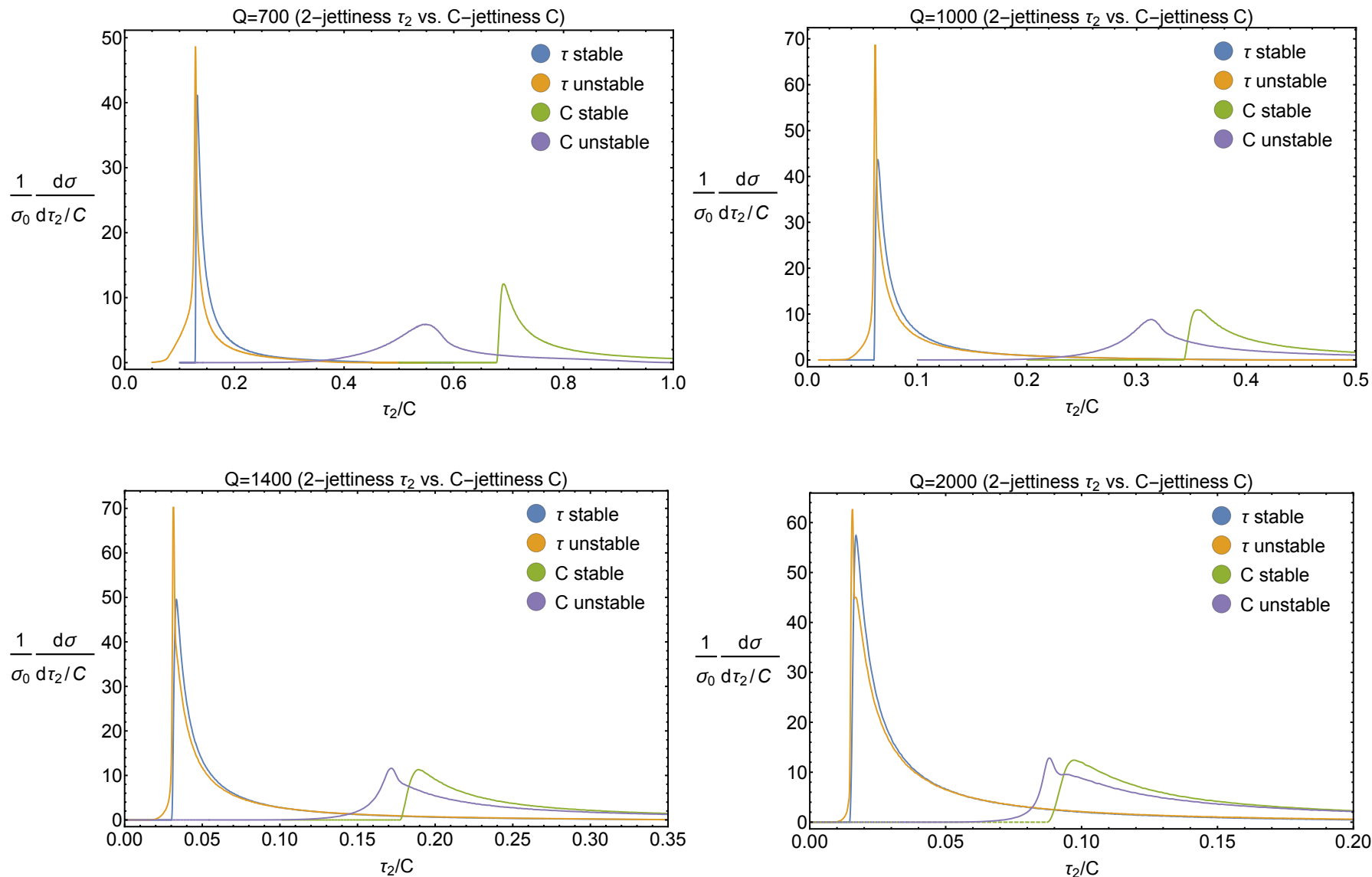
$$\tau = 1 - \max_{\vec{n}} \frac{\sum_i |\vec{n} \cdot \vec{p}_i|}{Q}$$

$$\tau \xrightarrow{\rightarrow 0} \frac{M_1^2 + M_2^2}{Q^2}$$

Invariant mass distribution in the resonance region of wide hemisphere jets !



Event Shape Distributions (Pythia 8.2)



Factorization for Event Shapes

$$\frac{d\sigma}{d\tau} = Q^2 \sigma_0 H_0(Q, \mu) \int d\ell J_0(Q\ell, \mu) S_0(Q\tau - \ell, \mu)$$

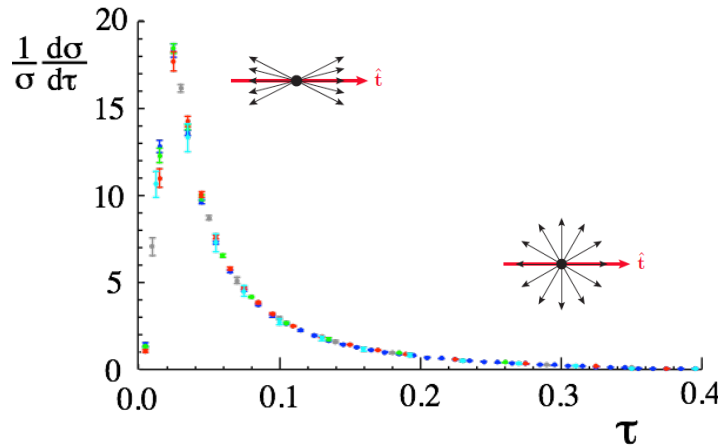
Massless quarks:

Korshemski, Sterman 1995-2000

Bauer, Fleming, Lee, Sterman (2008)

Becher, Schwartz (2008)

Abbate, AHH, Fickinger, Mateu, Stewart 2010



Extension to massive quarks:

- VFNS for final state jets (with massive quarks): log summation incl. mass
- Boosted fat top jets

Fleming, AHH, Mantry, Stewart 2007

Gritschacher, AHH, Jemos, Mateu Pietrulewicz 2013-2014

Butenschön, Dehnadi, AHH, Mateu 2016 (to appear soon)

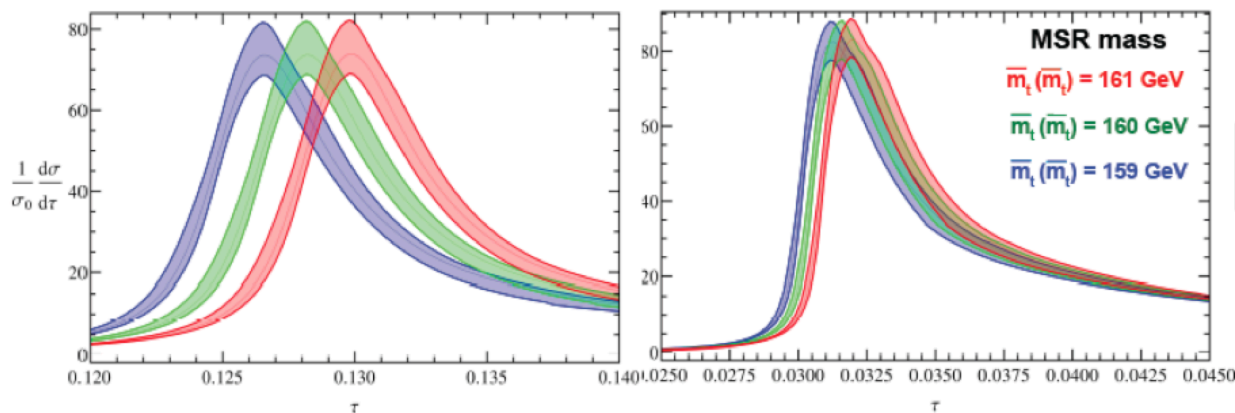
➡ NNLL + NLO + non-singular + hadronization + renormalon-subtraction

2-Jettiness for Top Production (QCD)

$$\frac{d\sigma}{d\tau_2} = f(\underbrace{m_t^{\text{MSR}}(R)}_{\text{any scheme possible}}, \underbrace{\alpha_s(M_Z), \Omega_1, \Omega_2, \dots}_{\text{Non-perturbative}}, \underbrace{\mu_h, \mu_j, \mu_s, \mu_m, R, \Gamma_t}_{\text{renorm. scales} \quad \text{finite lifetime}})$$

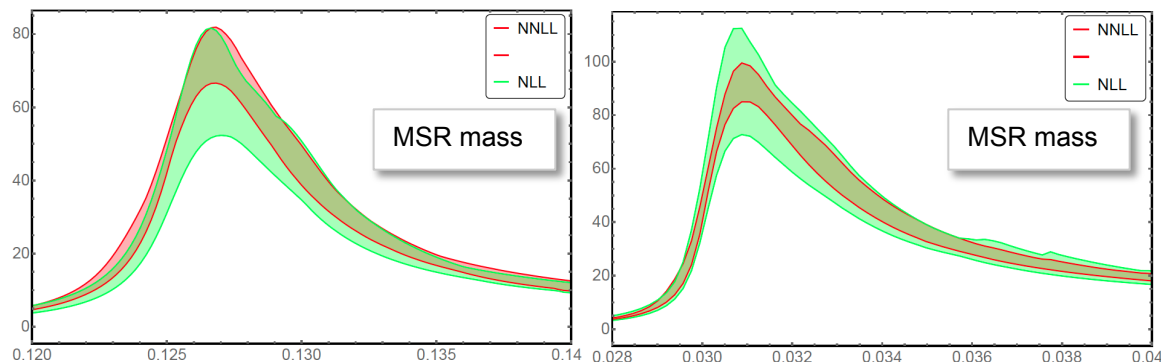
Q=700 GeV

Q=1400 GeV



Q=700 GeV

Q=1400 GeV



- Higher mass sensitivity for lower Q (p_T)
- Finite lifetime effects included
- Dependence on non-perturbative parameters
- Convergence: $\Omega_{1,2,\dots}$
- Good convergence
- Reduction of scale uncertainty (NLL to NNLL)
- Control over whole distribution

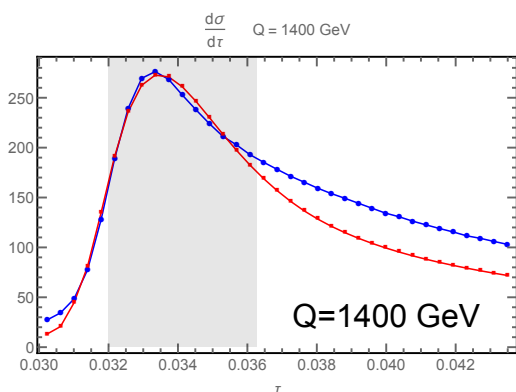
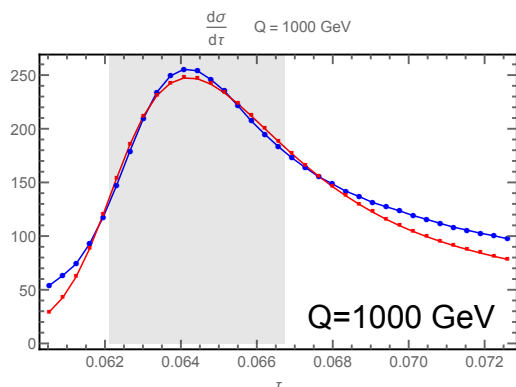
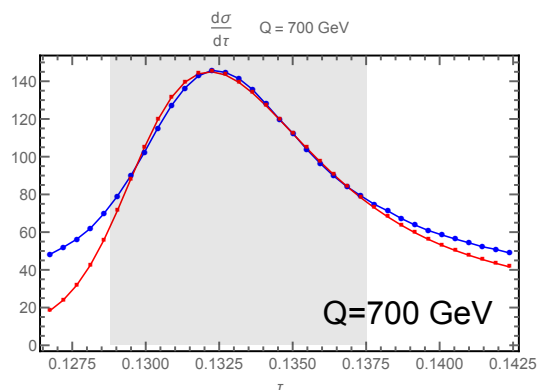
Fit Procedure Details

$$\frac{d\sigma}{d\tau_2} = f(\underbrace{m_t^{\text{MSR}}(R), \alpha_s(M_Z)}_{\text{any scheme possible}}, \underbrace{\Omega_1, \Omega_2, \dots}_{\text{Non-perturbative}}, \underbrace{\mu_h, \mu_j, \mu_s, \mu_m}_{\text{renorm. scales}}, \underbrace{R, \Gamma_t}_{\text{finite lifetime}})$$

QCD parameters measured from Pythia

- Fit parameters: $m_t^{\text{MSR}}(R), \alpha_s(M_Z), \Omega_1, \Omega_2, \dots,$
- Perturbative error: fits for 500 randomly picked sets of renorm. scales
- Tunings: 1, 3, 7 (default)
- Top quark width: $\Gamma_t = \text{dynamical (default), } 0.7, 1.4, 2.0 \text{ GeV}$
- External smearing (Detector effects): $\Omega_{1,\text{smear}} = 0, 0.5, \dots, 3.0, 3.5, \text{ GeV}$
- Pythia masses: $m_t^{\text{Pythia}} = 170, \dots, 175 \text{ GeV}$
- Fit possible for any mass scheme

Preliminary Peak Fits



Pythia
QCD

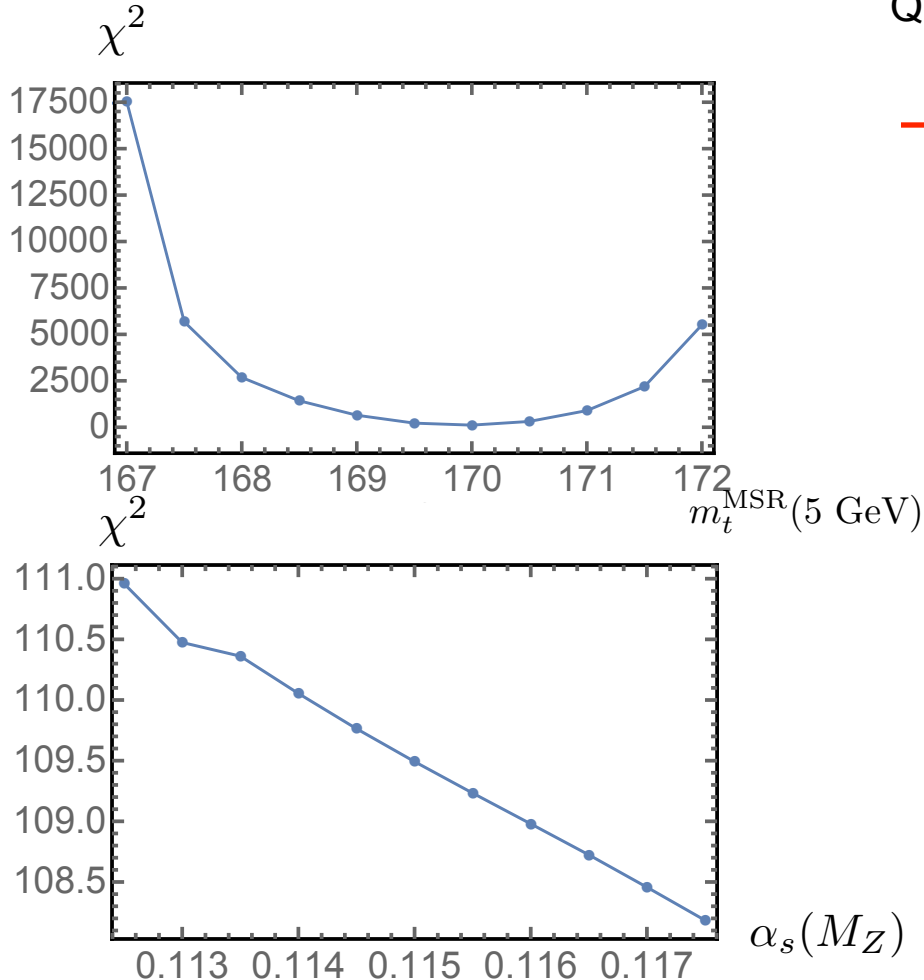
Default renormalization scales; $\Gamma_t = 1.4$ GeV, tune 7, $\Omega_{1,\text{smear}} = 2.5$ GeV, $m_t^{\text{Pythia}} = 171$ GeV, $Q = \{700, 1000, 1400\}$ GeV, peak fit (60/80)%

- Good agreement of Pythia 8.2 with NNLL+NLO QCD description
- Pythia statistics: 10^6 events
- Discrepancies in distribution tail and for higher energies (Pythia is less reliable where fixed-order results valid, well reliable in soft-collinear limit)
- Excellent sensitivity to the top quark mass.

Preliminary

Peak Fits

Default renormalization scales; $\Gamma_t = 1.4$ GeV,
tune 7, $\Omega_{1,\text{smear}} = 2.5$ GeV, $m_t^{\text{Pythia}} = 171$ GeV,
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→ $\chi^2_{\min} \sim \mathcal{O}(100)$

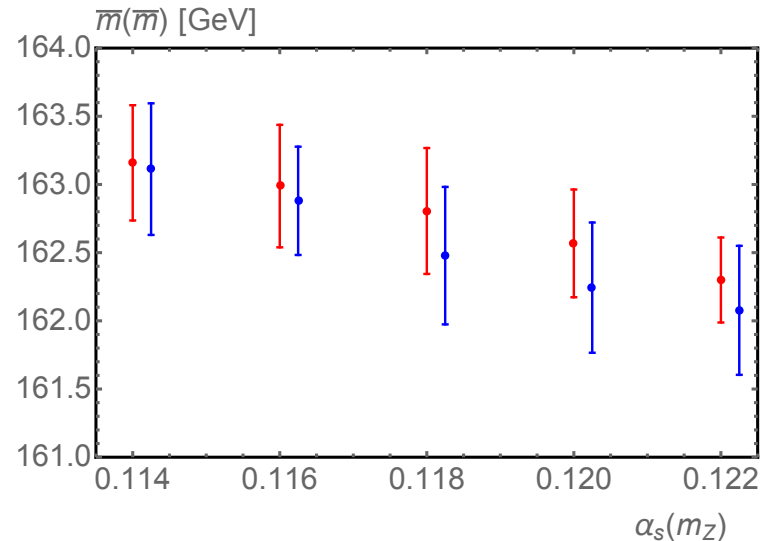
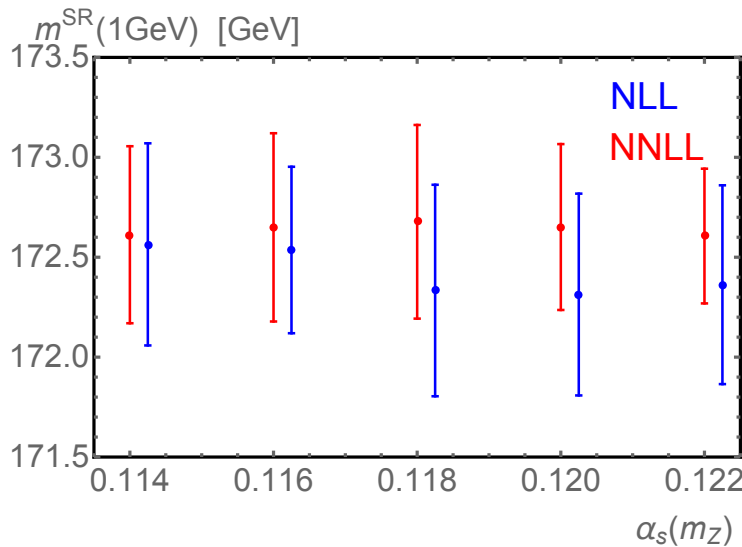
- Very strong sensitivity to m_t
- Low sensitivity to strong coupling
- Take strong coupling as input
- χ^2_{\min} and δm_t^{stat} do not have any physical meaning
- We use rescaled χ^2/dof (PDG prescription) to define “intrinsic MC compatibility uncertainty”

Preliminary

Peak Fits

First serious run: $\Gamma_t = 1.4$ GeV, tunes 1, 3, 7,
 $\Omega_{1,\text{smear}} = 1.5, 2.0, 2.5, 3.0, 3.5$ GeV,
 $Q = \{700, 1000, 1400\}$ GeV, peak fit (60/80)%
 $m_t^{\text{Pythia}} = 173$ GeV,
NLL: 177 scan survivors, NNLL: 254 scan survivors

Preliminary



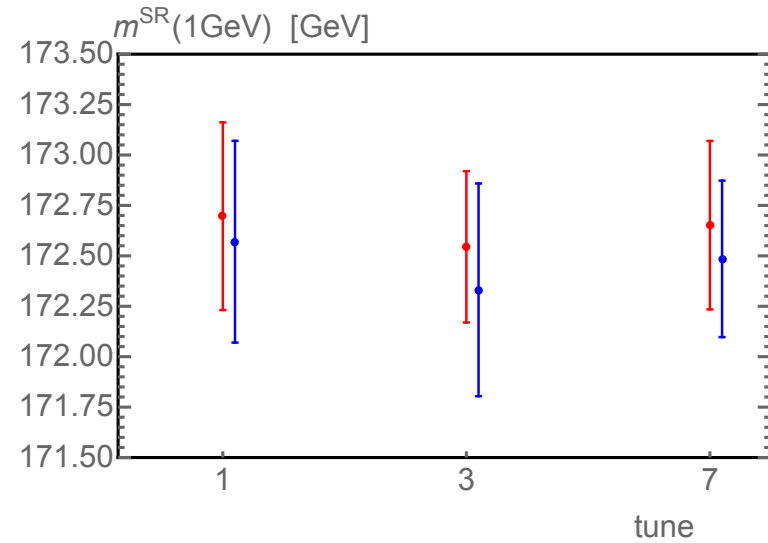
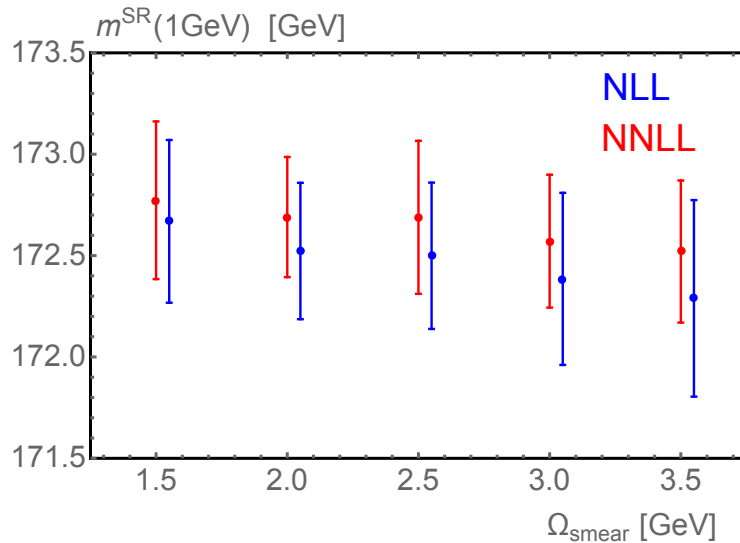
- Very low sensitivity of $m_t^{\text{MSR}}(5\text{GeV})$ on $\alpha_s(M_Z)$. ✓
- Large sensitivity of MSbar mass on $\alpha_s(M_Z)$. ✓

MC top mass indeed closely related to $m_t^{\text{MSR}}(R \sim 1 \text{ GeV})$!!

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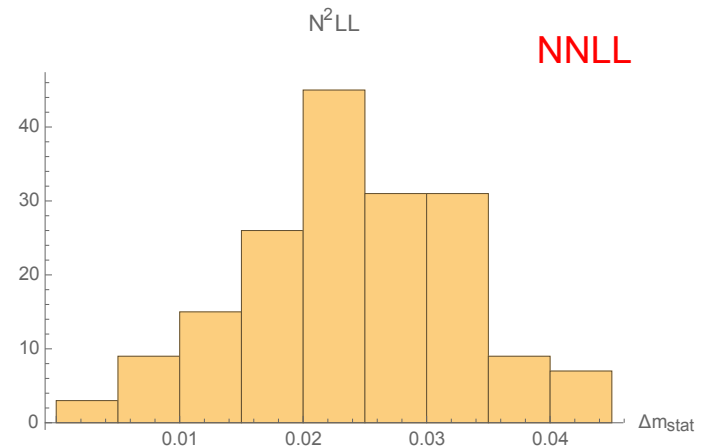
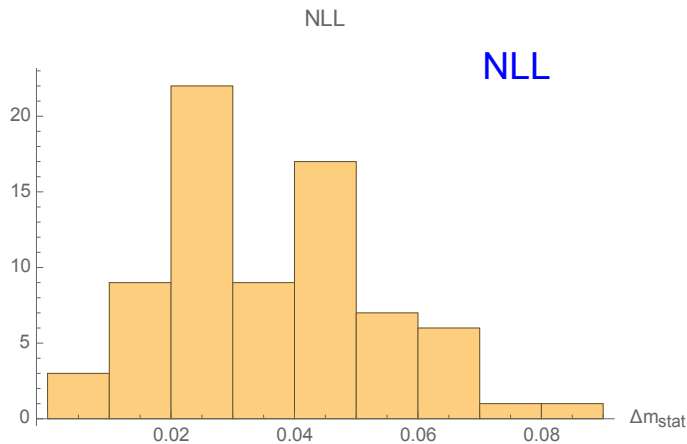
- “Detector effects” (~ 100 MeV) \ll perturbative uncertainty ($\lesssim 500$ MeV). ✓
- MC tune dependence ($\lesssim 100$ MeV) \ll perturbative uncertainty ($\lesssim 500$ MeV). ✓

MC top mass indeed closely related to $m_t^{\text{MSR}}(R \sim 1 \text{ GeV})$!!

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Preliminary



- “MC compatibility error” ~ tuning error ~ detector effect error ✓
- Effects are $O(100)$ MeV. (Maybe represents for ultimate precision)

Peak Fits

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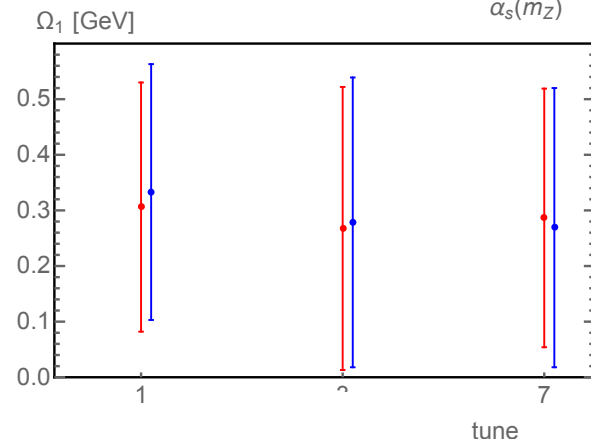
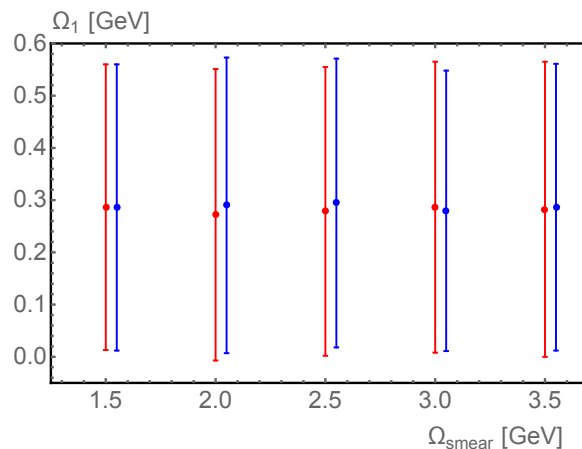
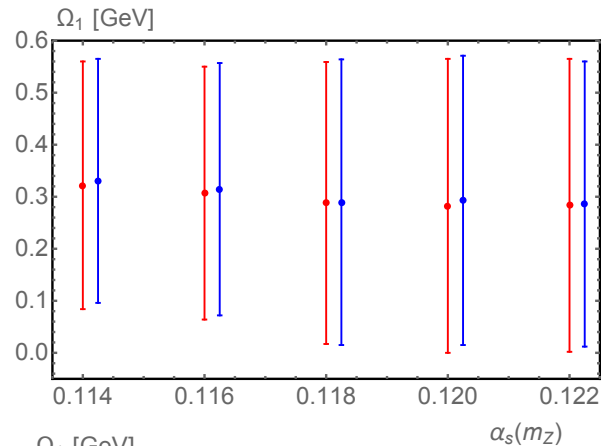
NLL

NNLL

$m_t^{\text{Pythia}} = 173$ GeV,

NLL: 177 scan survivors, NNLL: 254 scan survivors

Preliminary



- Reliable determination of non-perturbative matrix element Ω_1 (hadronization effects)
- Expected: $\delta m_t \sim \delta \Omega_1$ ✓
- Compatible with α_s -fits to e^+e^- data tail fits (Abbate et al, AHH et al.), larger err.

Peak Fits

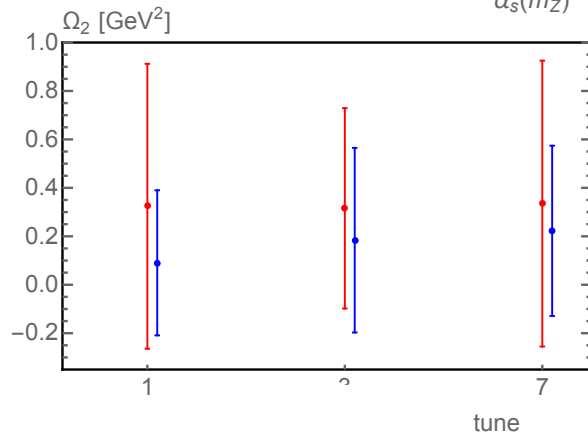
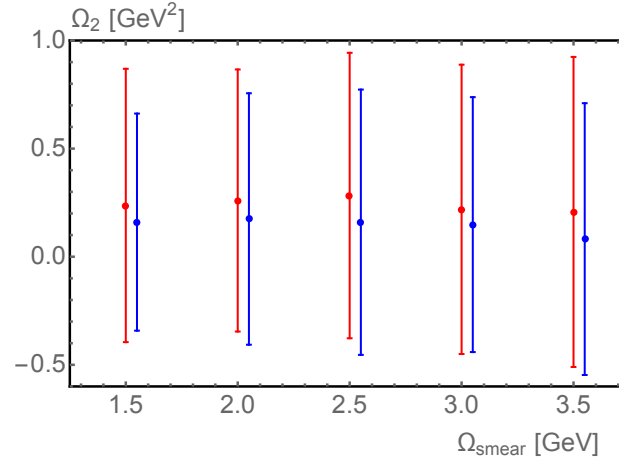
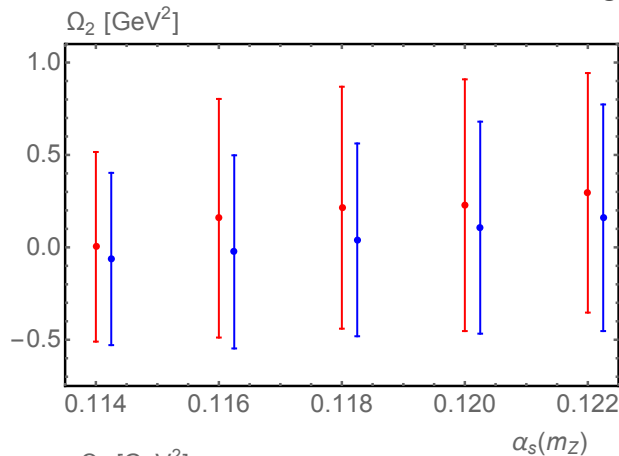
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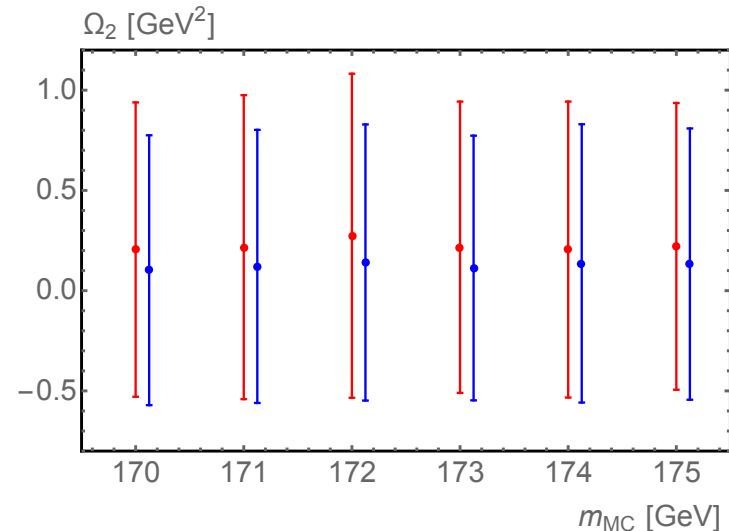
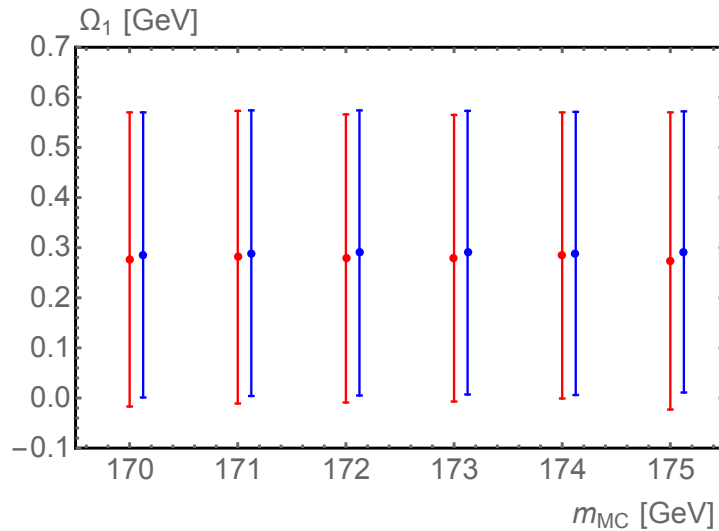


- Reliable determination of non-perturbative matrix element Ω_2 (hadronization effects)
- Found to be have huge error as expected due to little sensitivity ✓

Peak Fits

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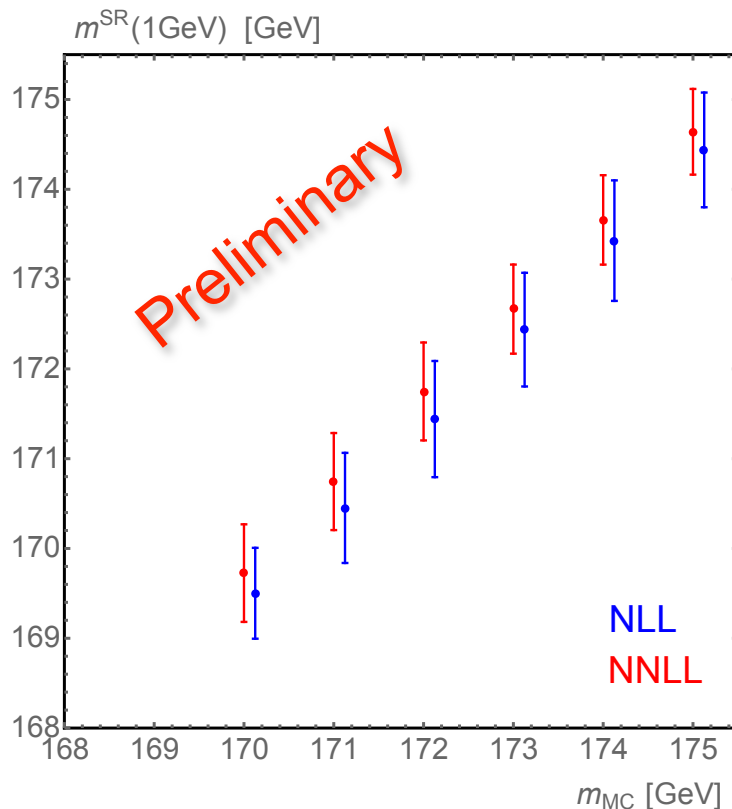
Preliminary



- Non-pert. matrix elements $\Omega_{1,2}$ independent of top mass. ✓

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- Many more cross checks to be done.
- Calibration error: 0.5 GeV seems feasible at NNLL !

Conclusions & Outlook

- First serious precise MC top quark mass calibration based on e^+e^- 2-jettiness (large p_T): preliminary results.
- NNLL+NLO QCD calculations based on an extension of the SCET approach concerning massive quark effects (all large logs incl. $\ln(m)$'s summed systematically).
- The Monte Carlo top mass calibration in terms of MSR mass with perturbative error $O(500 \text{ MeV})$ appears feasible at NNLL+NLO
- Intrinsic MC error seems $O(100 \text{ MeV})$.

Outlook:

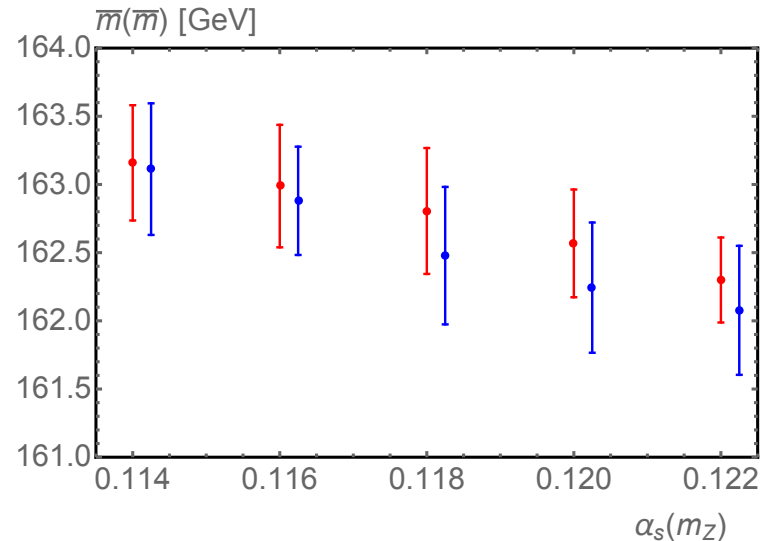
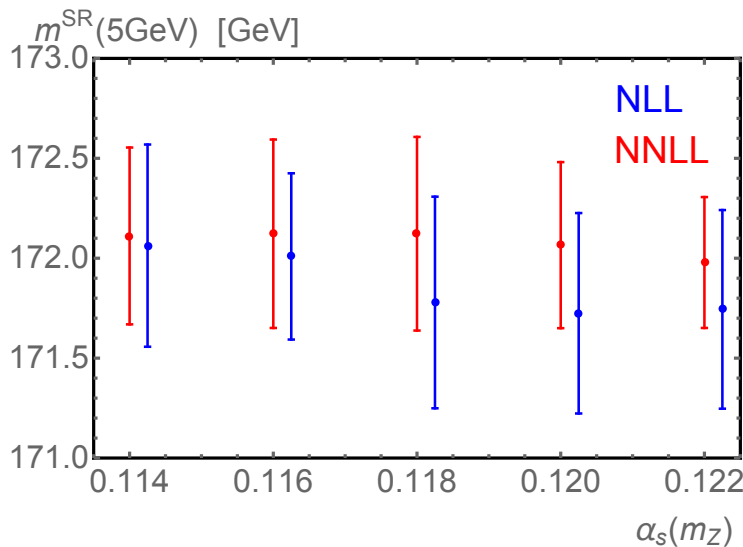
- Full verified error analysis @ NNLL+NLO on the way
- Calibration for other MC generators
- Heavy jet mass, C-parameter (NNLL), pp-2 jettiness analysis (NLL) w.i.p.
- NNNLL+NNLO (2jettiness for e^+e^-) w.i.p
- Mass (+ Yukawa coupling) conversions w. QCD + electroweak

Backup Slides

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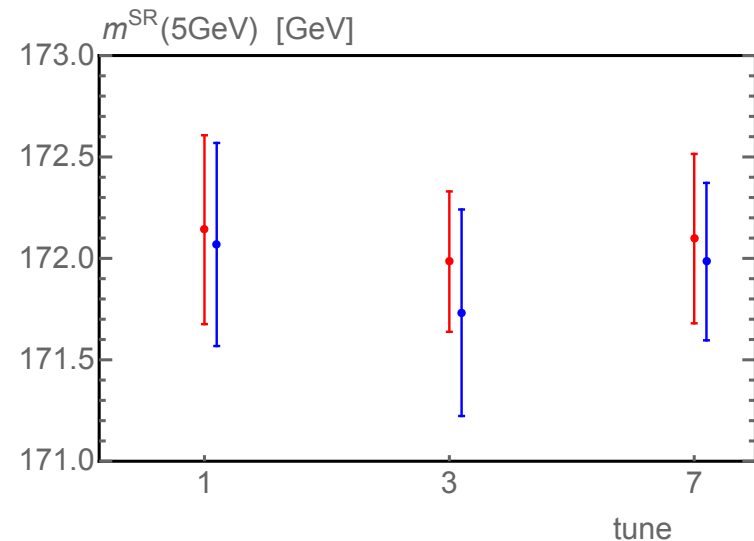
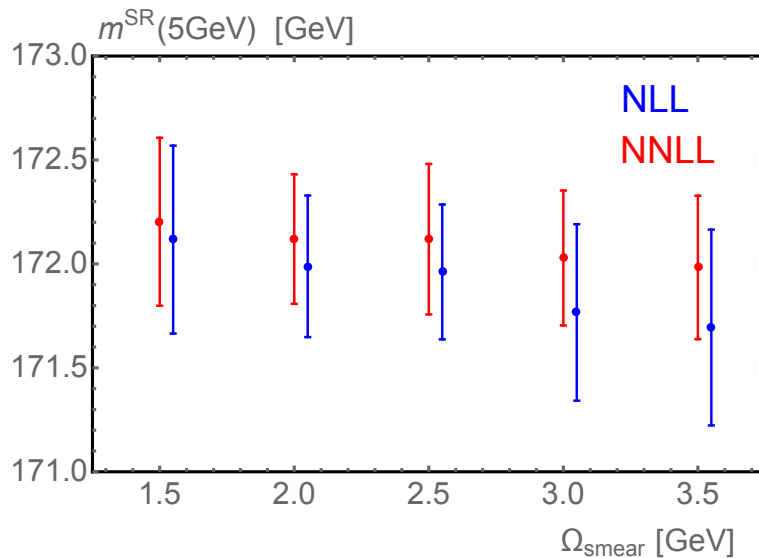
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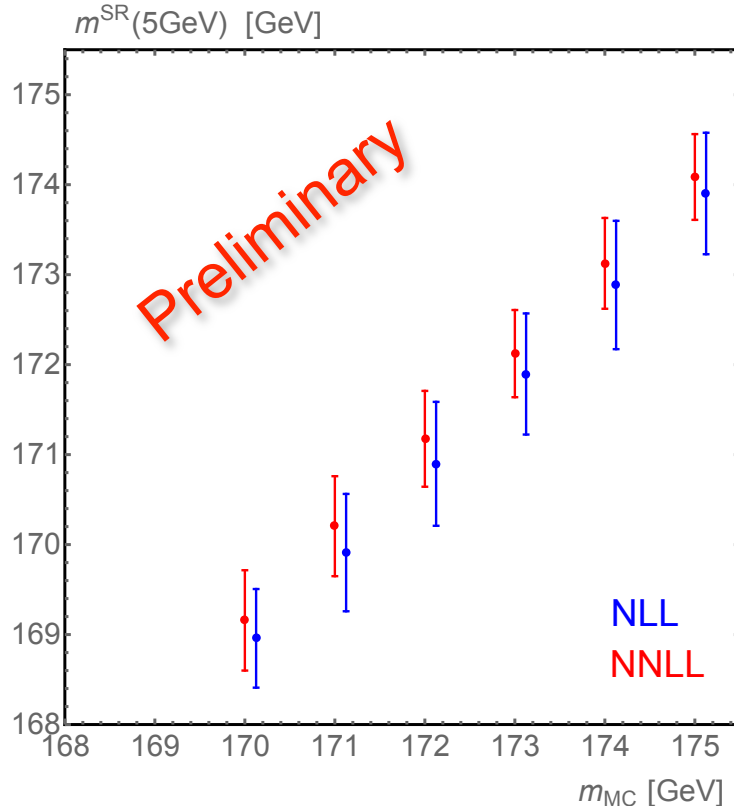


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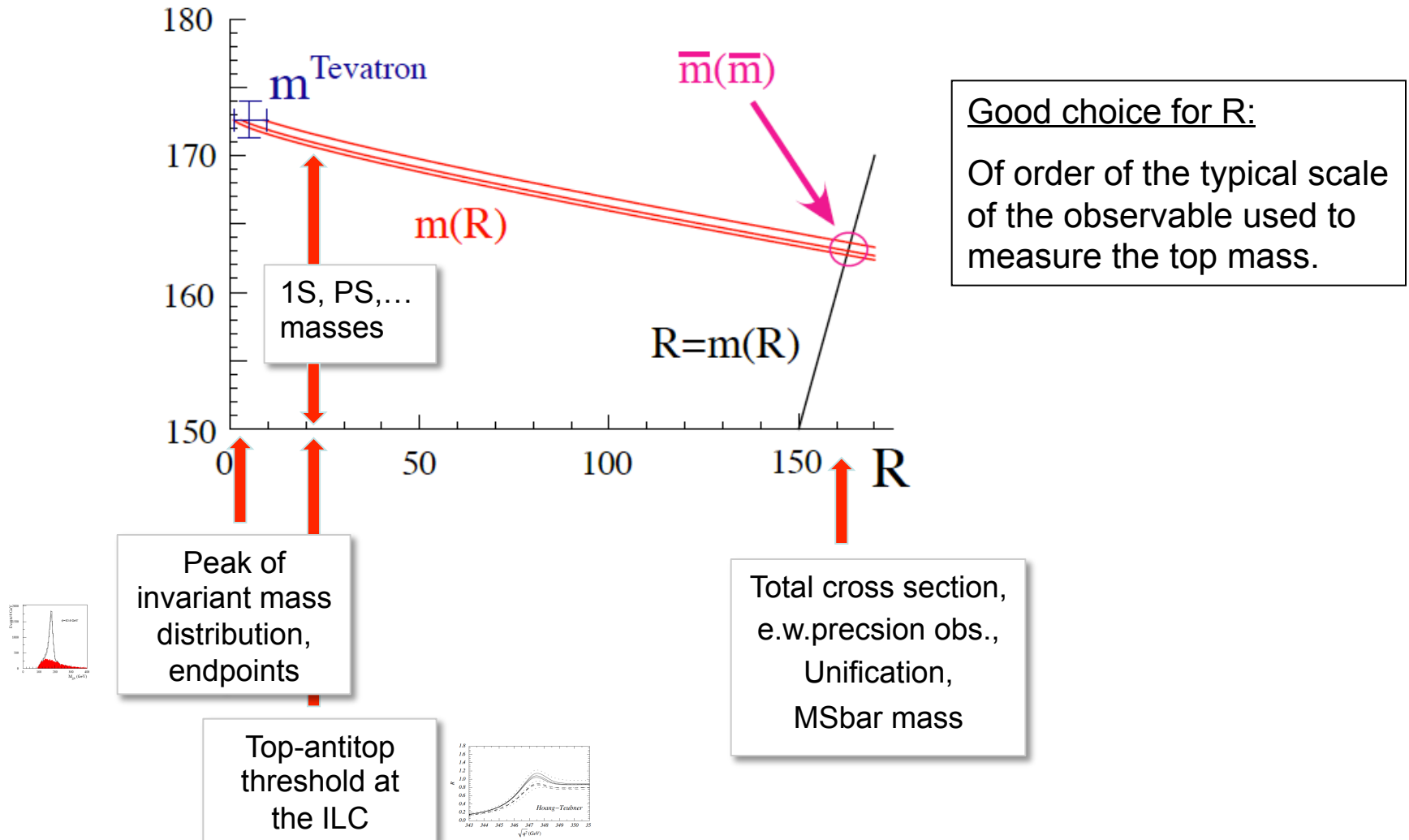


- Many more cross checks to be done.
- Calibration error: 0.5 GeV seems feasible at NNLL !

MSR Mass Definition

$$m_t^{\text{MC}} = m_t^{\text{MSR}}(3_{-2}^{+6} \text{ GeV}) = m_t^{\text{MSR}}(3 \text{ GeV})_{-0.3}^{+0.6}$$

AH, Stewart: arXiv:0808.0222



Masses Loop-Theorists Like to use

Total cross section (LHC/Tev):

$$m_t^{\text{MSR}}(R = m_t) = \bar{m}_t(\bar{m}_t)$$

$$M_t = M_t^{(O)} + M_t(0)\alpha_s + \dots$$

Threshold cross section (ILC):

$$m_t^{\text{MSR}}(R \sim 20 \text{ GeV}), m_t^{1S}, m_t^{\text{PS}}(R)$$

$$M_t = M_t^{(O)} + \langle p_{\text{Bohr}} \rangle \alpha_s + \dots$$

$$\langle p_{\text{Bohr}} \rangle = 20 \text{ GeV}$$

Inv. mass reconstruction (ILC/LHC):

$$m_t^{\text{MSR}}(R \sim \Gamma_t), m_t^{\text{jet}}(R)$$

$$M_t = M_t^{(O)} + \Gamma_t \alpha_s + \dots$$

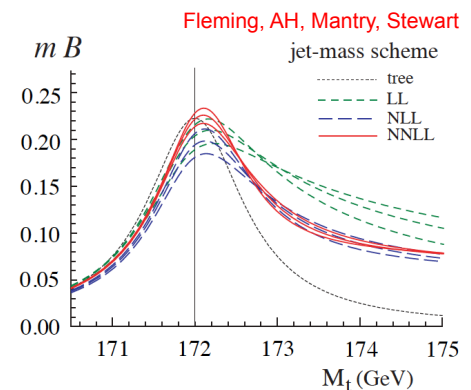
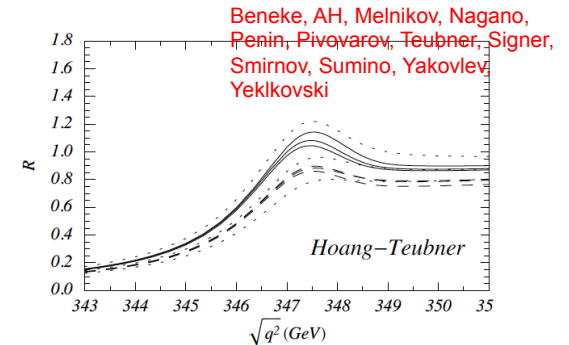
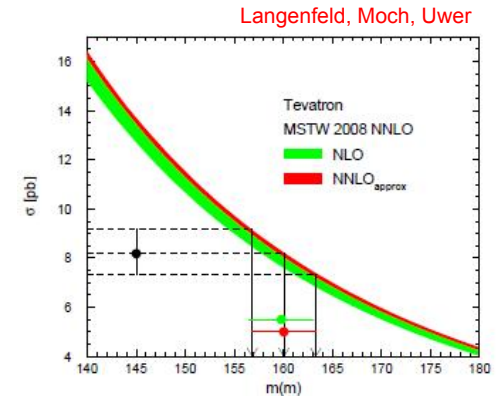
$$\Gamma_t = 1.3 \text{ GeV}$$

- more inclusive
- sensitive to top production mechanism (pdf, hard scale)
- indirect top mass sensitivity
- large scale radiative corrections

Mass schemes related to different computational methods

Relations computable in perturbation theory

- more exclusive
- sensitive to top final state interactions (low scale)
- direct top mass sensitivity
- small scale radiative corrections

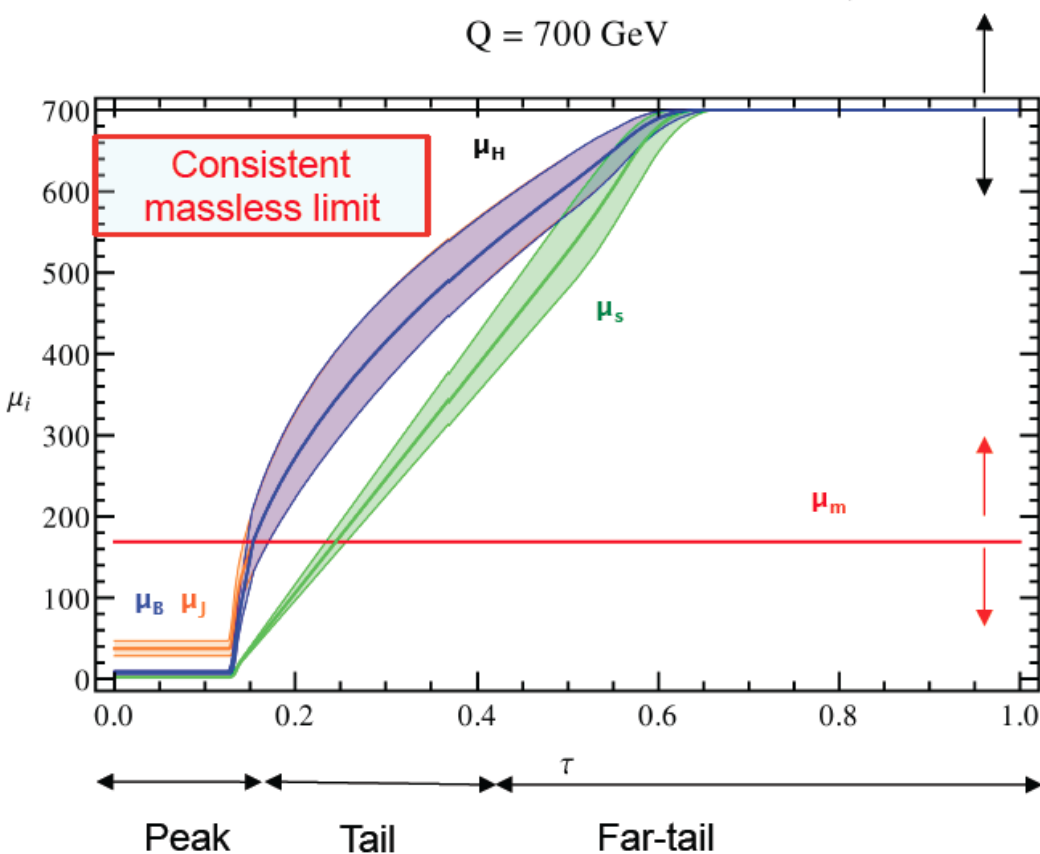


Profile Functions

Profile functions should sum up large logarithms and achieve smooth transition between the peak, tail and far-tail.

$$\log\left(\frac{Q}{\mu_H}\right) \quad \log\left(\frac{m_J}{\mu_m}\right) \quad \log\left(\frac{\mu_J^2}{Q\mu_s}\right) \quad \log\left(\frac{m_J\mu_B}{Q\mu_s}\right) \quad \log\left(\frac{Q(\tau - \tau_{\min}) + 2\Lambda_{\text{QCD}}}{\mu_s}\right)$$

$Q = 700 \text{ GeV}$



Scales Variation

- ✓ Generalized to arbitrary mass values
- ✓ Compatible with massless profiles

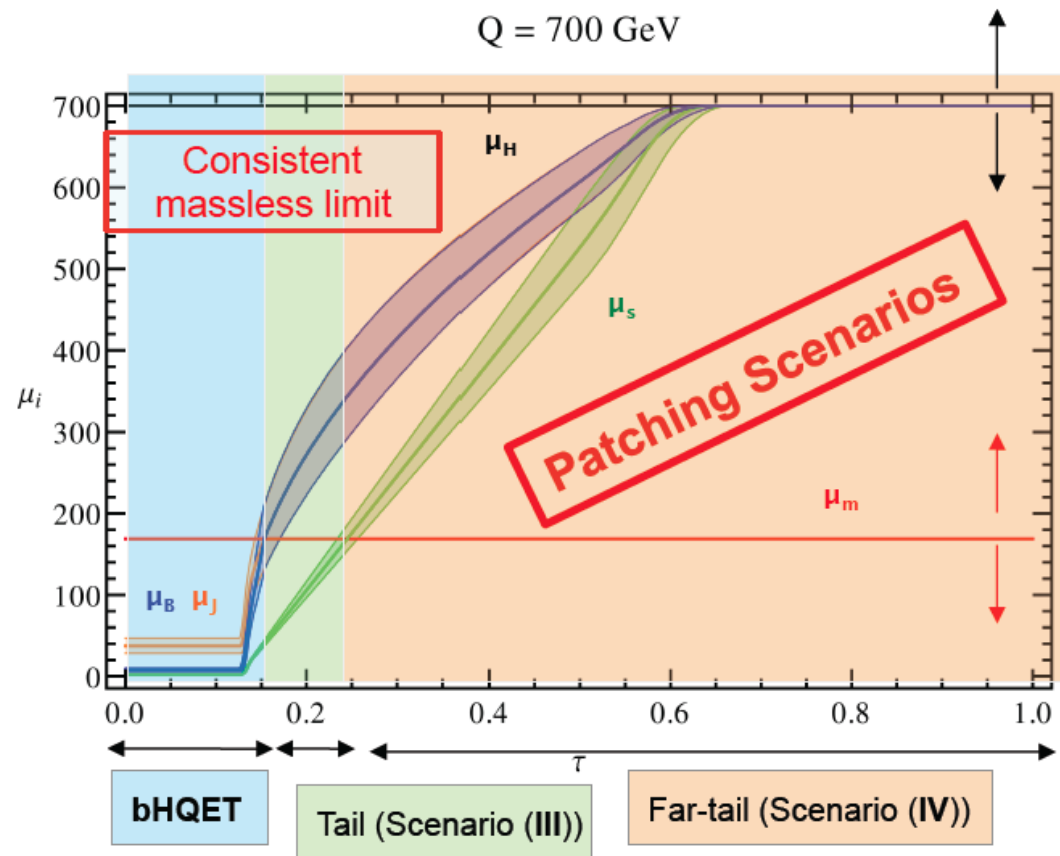
Proper scale variations are essential in reliable estimation of missing higher order terms.

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