Factorization and Resummation for Massive Quarks in Drell-Yan at low q_T

Piotr Pietrulewicz

based on work with
Daniel Samitz, Anne Spiering, and Frank Tackmann

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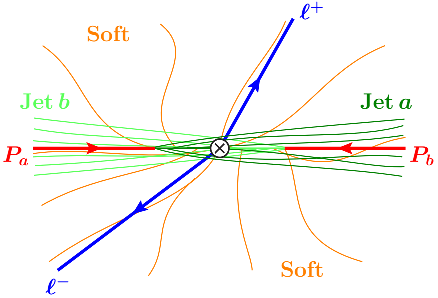
Motivation

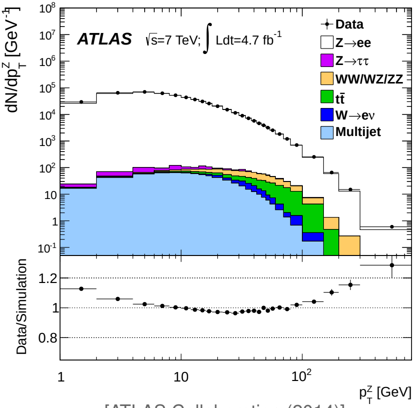
- m_b-effects frequently relevant at the LHC (e.g. in Higgs production)
- systematic treatment for many inclusive processes available
  e.g. for $pp \to b\bar{b}H$: [Bonvini, Papanastasiou, Tackmann (2015)] $\to$ Andrew's talk on Tuesday (WG4)
- aim: systematic approach for mass effects in exclusive processes (with jet veto)
  for e^+e^- event shapes, DIS for $x \to 1$: [Gritschacher et al. (2014); Hoang, P.P., Samitz (2016)]
- here: q_T-distribution in Drell-Yan, $pp \to Z/\gamma^* \to \ell\bar{\ell}$
  effective veto on additional jets: $q_T \equiv |\vec{p}_T^{\,\ell\bar{\ell}}| \ll \hat{s} \equiv Q$



taken from: [Stewart, Tackmann, Waalewijn (2010)]

Drell-Yan at small q_T

- spectrum measured with high precision up to low q_T
  $\rightarrow$ see talks in the morning
- analytic high precision calculations, up to NNLL$'$+NNLO (soon N^{3}LL)
- no systematic description of b-mass effects at low q_T
- in MC's: initial state splitting with massive quarks not well understood



[ATLAS Collaboration (2014)]

$\Rightarrow$ Goal: Factorization framework for massive quark effects (using EFTs), explicit results at NNLL$'$ (i.e. for NNLL resummation with FO ingredients at $\mathcal{O}(\alpha_s^2)$)

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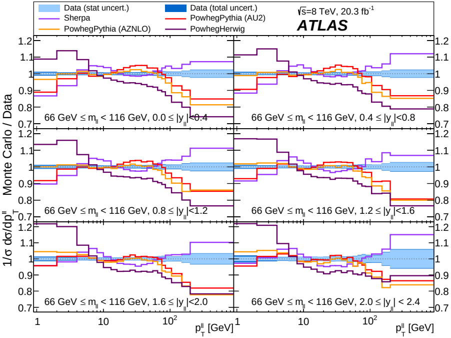

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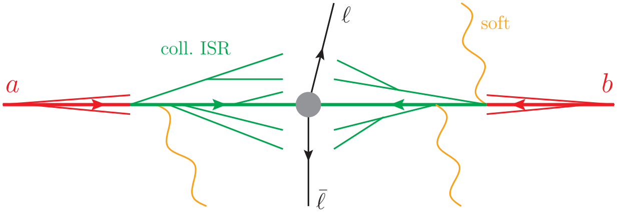
Outline

1. Factorization with massless quarks
2. Factorization with massive quarks
3. Resummation with massive quarks
4. Conclusions & Outlook

Outline

Scales in DY for small q_T



DY with $q_T \ll Q$ is a multiscale process:

- hard process: $q\bar{q} \to Z/\gamma^* \to \ell\bar{\ell}$ at scale $\mu \sim Q$
- n_a-/n_b-collinear ISR at scale $\mu \sim q_T$
- wide-angle soft ISR at scale $\mu \sim q_T$
- nonperturbative collinear proton at scale $\mu \sim \Lambda_{\mathrm{QCD}}$

large scale hierarchies ⇒ large logs $\ln(q_T/Q)$, $\ln(\Lambda_{\mathrm{QCD}}/q_T)$ in perturbation theory

Factorization for small q_T

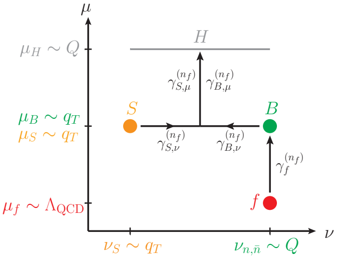
[Collins, Soper, Sterman (1985); Catani, de Florian, Grazzini (2001), Becher, Neubert (2011); Echevarria, Idilbi, Scimemi (2011); Chiu, Jain, Neill, Rothstein (2012); ...]

Factorization theorem for n_f massless quarks: $(i, j, k \in \{q, \bar{q}, g\})$

$$\frac{d\sigma}{dq_T^2} \sim \sum_{i,j} H_{ij}^{(n_f)} \times \left[\sum_k \mathcal{I}_{ik}^{(n_f)} \otimes f_k^{(n_f)} \right]^2 \otimes S^{(n_f)} + \mathcal{O}\left(\frac{q_T}{Q}, \frac{\Lambda_{\rm QCD}}{q_T} \right)$$

- hard function $H_{ij}(Q)$: process dependence
- beam function/TMD $B_i = \sum_k \mathcal{I}_{ik} \otimes_x f_k$
  - $\rightarrow$ collinear ISR matching $\mathcal{I}_{ik}(\vec{q}_T, x)$
  - $\rightarrow$ nonperturbative PDF $f_k(\Lambda_{\rm QCD}, x)$
- soft function $S(\vec{q}_T)$: wide-angle soft radiation
- resummation of logs via evolution factors (implicit)
- rapidity divergences in S and B_i
  - $\rightarrow$ associated rapidity logarithms
  - $\rightarrow$ resummed via rapidity RGE

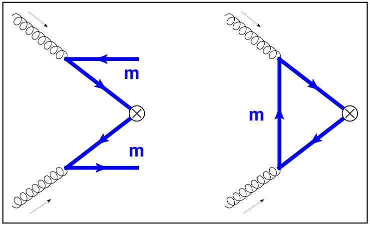
  [Chiu, Jain, Neill, Rothstein (2012)]



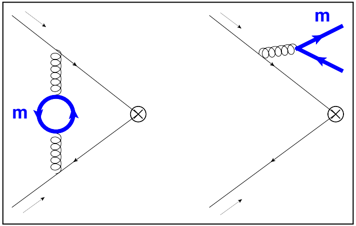
Outline

Massive quark effects

primary massive quark corrections:

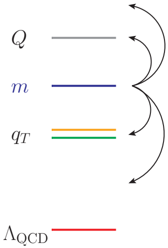


secondary massive quark corrections:

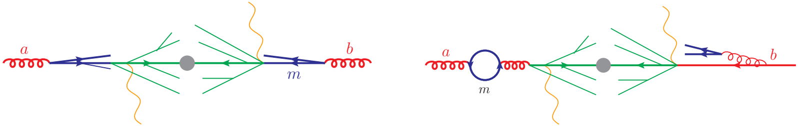


- $\mathcal{O}(\alpha_s^2)$ contributions relevant for NNLL′ resummation
- additional scale m: different hierarchies possible
- different features for primary and secondary effects
- in the following: focus on $q_T \gtrsim m \gg \Lambda_{\rm QCD}$ (mainly relevant for m_b effects)

  $m \gg q_T$ analogous to previous work
  [Gritschacher et al. (2014); Hoang, P.P., Samitz (2016)]



$\Lambda_{\rm QCD} \ll m \ll q_T \ll Q$



Factorization theorem: $(i, j, m \in \{q, \bar{q}, Q, \bar{Q}, g\}, k \in \{q, \bar{q}, g\})$

$$\frac{d\sigma}{dq_T^2} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times \Big[\sum_{m,k} \mathcal{I}_{im}^{(n_l+1)} \otimes \mathcal{M}_{mk}(m) \otimes f_k^{(n_l)} \Big]^2 \otimes S^{(n_l+1)} + \mathcal{O}\left(\frac{m^2}{q_T^2}, \frac{\Lambda_{\rm QCD}^2}{m^2}\right)$$

- hard function, collinear ISR matching and soft function with $n_l + 1$ massless flavors
- primary and secondary massive quark corrections in PDF matching

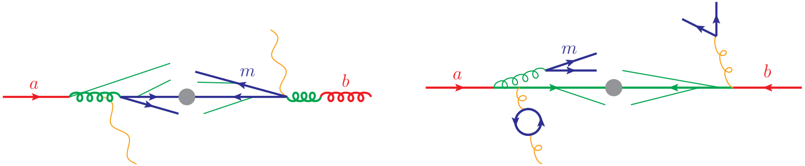
$$f_i^{(n_l+1)}(x,m) = \sum_{k\in\{q,\bar{q},g\}} \mathcal{M}_{ik}(x,m) \otimes f_k^{(n_l)}(x)$$

$\rightarrow$ one-loop primary massive: $\mathcal{M}_{Qg}(x,m)$ [$\rightarrow$ massive quark PDF]

$\rightarrow$ two-loop secondary massive: $\mathcal{M}_{qq}(x,m)$

[Buza, Matiounine, Smith, van Neerven (1998)]

$\Lambda_{\mathrm{QCD}} \ll m \sim q_T \ll Q$



Factorization theorem: $(i,j \in \{q,\bar{q},Q,\bar{Q},g\}, k \in \{q,\bar{q},g\})$

$$\frac{d\sigma}{dq_T^2} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times \Big[\sum_k \mathcal{I}_{ik}(m) \otimes f_k^{(n_l)}\Big]^2 \otimes S^{(n_l+1)}(m) + \mathcal{O}\left(\frac{m^2}{Q^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{m^2}\right)$$

- hard function with $n_l + 1$ massless flavors
- primary and secondary massive quark corrections to beam functions/TMDs:

$$B_i^{(n_l+1)}(\vec{q}_T, x, m) = \sum_{k\in\{q,\bar{q},g\}} \mathcal{I}_{ik}(\vec{q}_T, x, m) \otimes f_k^{(n_l)}(x)$$

→ one-loop primary massive: $\mathcal{I}_{Qg}(\vec{q}_T, x, m)$ ✓ [→ massive quark beam fct.]

→ two-loop secondary massive: $\mathcal{I}_{qq}(\vec{q}_T, x, m)$ ✓

- two-loop secondary massive quark corrections to soft function: $S^{(n_l+1)}(\vec{q}_T, m)$ ✓

Relations between factorization theorems

Factorization theorem for $m \sim q_T$:

$$\frac{d\sigma}{dq_T^2} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times \Big[\sum_k \mathcal{I}_{ik}(m) \otimes f_k^{(n_l)}\Big]^2 \otimes S^{(n_l+1)}(m)$$

Factorization theorem for $m \ll q_T$:

$$\frac{d\sigma}{dq_T^2} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times \Big[\sum_{m,k} \mathcal{I}_{im}^{(n_l+1)} \otimes \mathcal{M}_{mk}(m) \otimes f_k^{(n_l)}\Big]^2 \otimes S^{(n_l+1)} + \mathcal{O}\left(\frac{m^2}{q_T^2}\right)$$

$\Rightarrow$ Relations between ingredients:

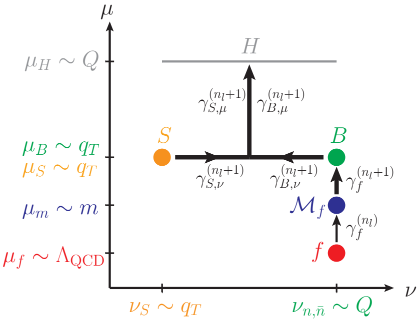
$$\mathcal{I}_{ik}(m) = \sum_j \mathcal{I}_{ij}^{(n_l+1)} \otimes \mathcal{M}_{jk}(m) \left[1 + \mathcal{O}\left(\frac{m^2}{q_T^2}\right)\right], \quad S^{(n_l+1)}(m) = S^{(n_l+1)} \left[1 + \mathcal{O}\left(\frac{m^2}{q_T^2}\right)\right]$$

- checked explicitly at $\mathcal{O}(\alpha_s^2)$ using known massless results for $\mathcal{I}_{ij}^{(n_f)}$ and $S^{(n_f)}$ ✓
  [Gehrmann, Luebbert, Yang (2012); Luebbert, Oredsson, Stahlhofen (2016)]
- resummation of $\ln(m^2/q_T^2)$ can be combined with power corrections of $\mathcal{O}(m^2/q_T^2)$
- similar relations between other hierarchies
  $\Rightarrow$ continuous description over complete spectrum = (GM-)VFNS

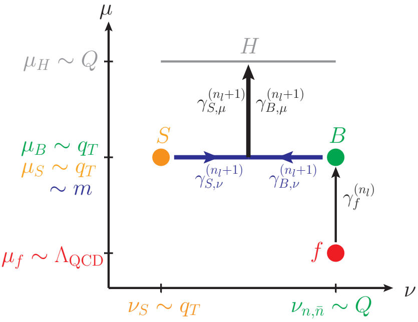
Outline

Resummation of logs

$\Lambda_{\mathrm{QCD}} \ll m \ll q_T \ll Q$:



$\Lambda_{\mathrm{QCD}} \ll m \sim q_T \ll Q$:



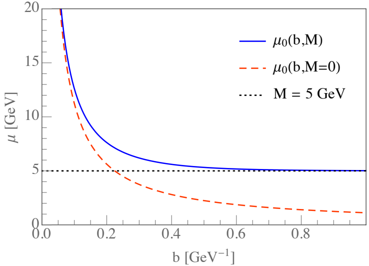
- μ-evolution with $n_l = 4$ quark flavors below the mass scale
- μ-evolution with $n_l + 1 = 5$ quark flavors above the mass scale
- ν-evolution affected by quark mass for $q_T \sim m$ (due to secondary effects)

Resummation of rapidity logs

- convenient in impact parameter (= Fourier) space ($\vec{p}_T \leftrightarrow \vec{b}$)
- avoid (or otherwise resum) large logarithms in anomalous dimension $\gamma_\nu(b,\mu)$
  $\rightarrow$ e.g. via scale setting of μ in b space
- illustration at one-loop for $\gamma_\nu \equiv \gamma_{S,\nu}$ with massless/massive gluon:

$$M=0:\ \gamma_\nu(b,\mu) = -\frac{\alpha_s(\mu)C_F}{2\pi^3}\ln\Big(\frac{b^2\mu^2 e^{2\gamma_E}}{4}\Big) \qquad \Rightarrow \mu \sim \mu_0(b) \equiv \frac{2e^{-\gamma_E}}{b}$$

$$M\neq 0:\ \gamma_\nu(b,M,\mu) = \frac{\alpha_s(\mu)C_F}{2\pi^3}\Big(\ln\frac{M^2}{\mu^2} + 2K_0(b\,M)\Big) \quad \Rightarrow \mu \sim \mu_0(b,M) \equiv M e^{K_0(b\,M)}$$



$$\mu_0(b,M) \xrightarrow{b\to 0} \frac{2e^{-\gamma_E}}{b}$$

$$\mu_0(b,M) \xrightarrow{b\to\infty} M$$

$\Rightarrow$ mass introduces IR cutoff

$\Rightarrow$ no Landau pole for $b\to\infty$

- similar for secondary massive quarks at $\mathcal{O}(\alpha_s^2)$ (with correct schemes for α_s)

Outline

Summary

Conclusions:

- quark mass effects important in several exclusive processes at hadron colliders
- factorization with massive quarks for Drell-Yan + 0 jets at low q_T $\surd$
- computation of required ingredients for resummation of mass related logarithms at NNLL$'$ accuracy $\surd$
- setup & ingredients also for beam thrust $\surd$

Outlook:

- phenomenological analysis of m_b effects for q_T-spectrum
- application to other processes (e.g. $pp \to Hb\bar{b}$)

Outline

Massless factorization theorem (explicit)

Factorization for $q_T \ll Q$: (quark initiated channels)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}q_T^2\,\mathrm{d}Q^2\,\mathrm{d}Y} = \sum_{i,j\in\{q,\bar{q}\}} H_{ij}^{(n_f)}(Q,\mu) \int \mathrm{d}^2p_{T,a}\,\mathrm{d}^2p_{T,b}\,\mathrm{d}^2p_{T,s}\,\delta(q_T^2 - |\vec{p}_{T,a}+\vec{p}_{T,b}+\vec{p}_{T,s}|^2)$$
$$\times B_i^{(n_f)}\Big(\vec{p}_{T,a}, x_a, \mu, \frac{\nu}{\omega_a}\Big)\, B_j^{(n_f)}\Big(\vec{p}_{T,b}, x_b, \mu, \frac{\nu}{\omega_b}\Big)\, S^{(n_f)}(\vec{p}_{T,s},\mu,\nu)$$
$$\times \Big[1+\mathcal{O}\Big(\frac{q_T}{Q}, \frac{\Lambda_{\mathrm{QCD}}^2}{q_T^2}\Big)\Big]$$

with

$$\omega_a = Qe^{Y}\,,\quad \omega_b = Qe^{-Y}\,,\quad x_{a,b} = \frac{\omega_{a,b}}{E_{\mathrm{cm}}}\,.$$

$$B_i^{(n_f)}\Big(\vec{p}_T, x, \mu, \frac{\nu}{\omega}\Big) = \sum_k \underbrace{\int_x^1 \frac{\mathrm{d}z}{z}\,\mathcal{I}_{ik}^{(n_f)}\Big(\vec{p}_T, \frac{x}{z}, \mu, \frac{\nu}{\omega}\Big)\, f_k^{(n_f)}(z,\mu)}_{\equiv \mathcal{I}_{ik}^{(n_f)}(\vec{p}_T,x,\mu,\frac{\nu}{\omega})\otimes f_k^{(n_f)}(x,\mu)} \Big[1+\mathcal{O}\Big(\frac{\Lambda_{\mathrm{QCD}}^2}{|\vec{p}_T|^2}\Big)\Big],$$

$\Lambda_{\mathrm{QCD}} \ll q_T \ll m \sim Q$

Factorization theorem: $(i, j, k \in \{q, \bar{q}, g\})$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}q_T^2} \sim \sum_{i,j} H_{ij}(m) \times \Big[\sum_k \mathcal{I}_{ik}^{(n_l)} \otimes f_k^{(n_l)}\Big]^2 \otimes S^{(n_l)} + \mathcal{O}\left(\frac{q_T^2}{m^2}\right)$$

- hard function with contributions from primary and secondary massive quarks
- beam and soft function with n_l massless flavors

$\Lambda_{\mathrm{QCD}} \ll q_T \ll m \ll Q$

Factorization theorem: $(i, j, k \in \{q, \bar{q}, g\})$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}q_T^2} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times |H_n(m)|^2 \times H_s(m) \times \Big[\sum_k \mathcal{I}_{ik}^{(n_l)} \otimes f_k^{(n_l)}\Big]^2 \otimes S^{(n_l)} + \mathcal{O}\left(\frac{m^2}{Q^2}, \frac{q_T^2}{m^2}\right)$$

- two step matching: (1) QCD $\to$ SCET$^{(n_l+1)}$, (2) SCET$^{(n_l+1)}$ $\to$ SCET$^{(n_l)}$
- hard function with $n_l + 1$ massless flavors (at $\mu_H \sim Q$)
- collinear & soft massive matching factors H_n, H_s (at $\mu_m \sim m$)
  $\to$ only due to secondary massive quarks
  $\to$ independent of hard process (for 0 jets) and measurement
  see also [Gritschacher, Hoang, Jemos, Mateu, P.P. (2014); Hoang, P.P., Samitz (2016)]
  $\to$ rapidity logarithm $\ln(Q/m)$ can be resummed using rapidity RGE
  [Hoang, Pathak, P.P., Stewart (2015)]
- beam and soft functions with n_l massless flavors

GMVFNS for $m \lesssim q_T$:

Relations between ingredients:

$$\mathcal{I}_{ik}(m) = \sum_j \mathcal{I}_{ij}^{(n_l+1)} \otimes \mathcal{M}_{jk}(m) \left[1 + \mathcal{O}\Big(\frac{m^2}{q_T^2}\Big)\right], \quad S^{(n_l+1)}(m) = S^{(n_l+1)} \left[1 + \mathcal{O}\Big(\frac{m^2}{q_T^2}\Big)\right]$$

define

$$\mathcal{I}_{ik}^{(n_l+1)}(m) = \mathcal{I}_{ik}^{(n_l+1)} + \Big(\mathcal{I}_{ik}(m) - \sum_j \mathcal{I}_{ij}^{(n_l+1)} \otimes \mathcal{M}_{jk}(m)\Big)$$

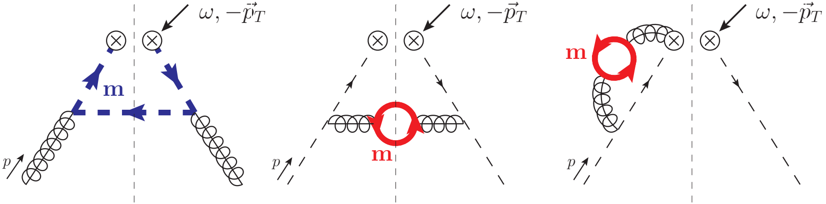
Factorization theorem for $m \lesssim q_T$:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}q_T^2} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times \Big[\sum_k \mathcal{I}_{ik}^{(n_l+1)} \otimes \mathcal{M}_{mk}(m) \otimes f_k^{(n_l)}\Big]^2 \otimes S^{(n_l+1)}(m)$$

$\Rightarrow$ resums $\ln(m^2/q_T^2)$ & includes all power corrections of $\mathcal{O}(m^2/q_T^2)$

Diagrams

beam function diagrams: primary and secondary



soft function diagrams: only secondary