Statistical Fragmentation in pp & ep Collisions

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Motivation

- ***Goal***

  Hadronisation of off-shell partons

- ***Proposed model***

  Statistical Model

- ***Suggestion***

  It might be more suitable to

  characterise JETs with their MASS

  instead of thier P or E

Outline

- ***3D Statistical Jet fragmentation model***

  hadron distributions in jets in e^+e^-, ep, pp collisions

- ***Applications***

  - Transverse momentum spectra in pp collisions

  from a pQCD parton model calculation

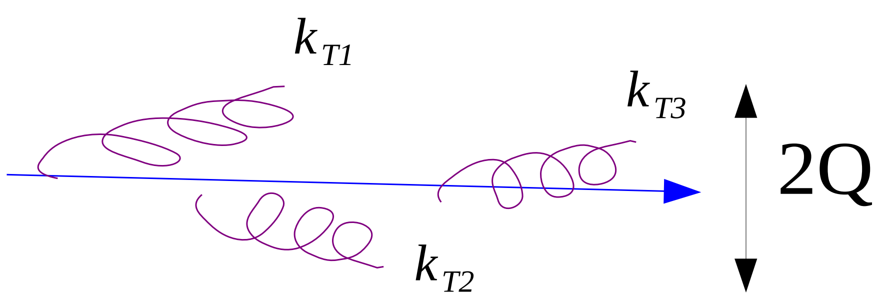
  - Spectra & anisotropy of hadrons in heavy-ion collisions

Q^2 Scale of the jet

- **parton branching**

DGLAP for Fragmentation Functions goes with $\boldsymbol{Q^2}$

$$k_{Ti}^2 = k_{i\,\mu}\, k_i^{\mu} \le Q^2$$

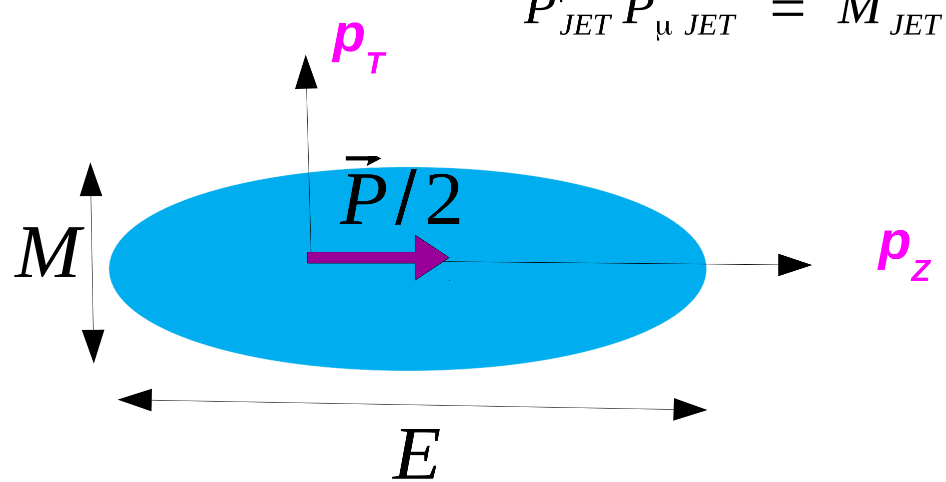


- ***Hadrons in the jet***

Energy-momentum conservation

$$\sum_h p_h^{\mu} = P_{JET}^{\mu}$$

$$P_{JET}^{\mu} P_{\mu\, JET} = M_{JET}^2$$



$$Q \sim M_{JET}/2$$

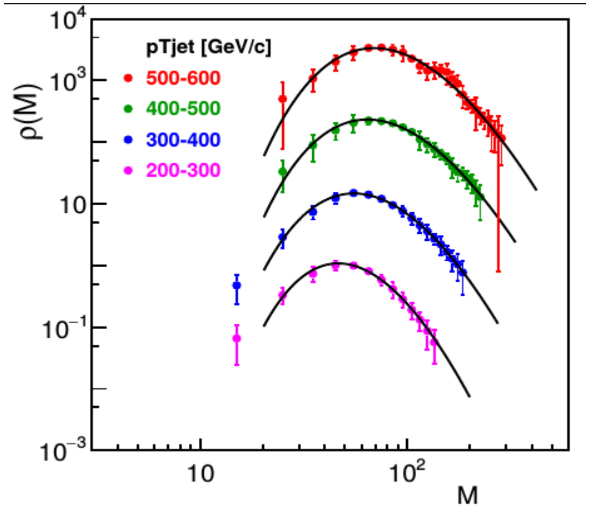
- ***Suggestion***

It might be more suitable to

characterise JETs with their MASS

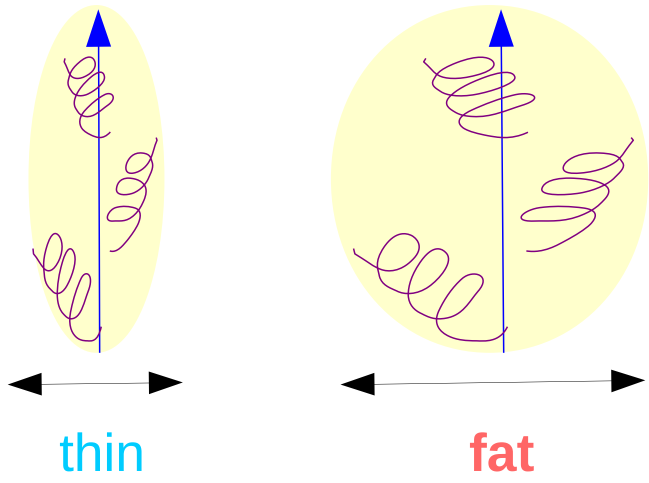
instead of thier P or E

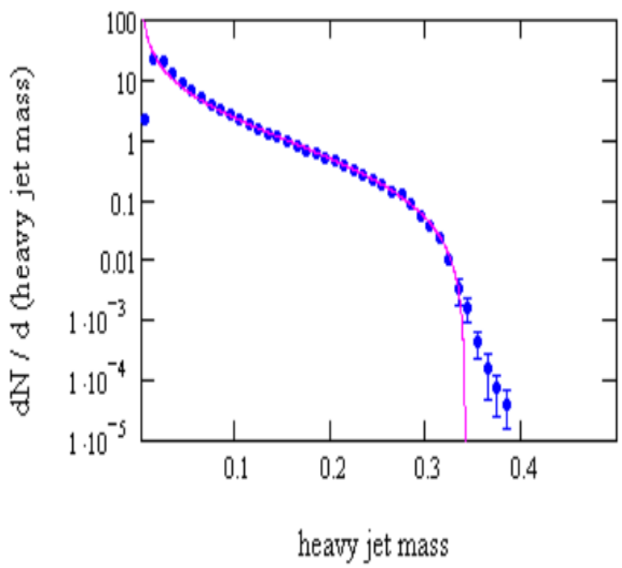
Problems



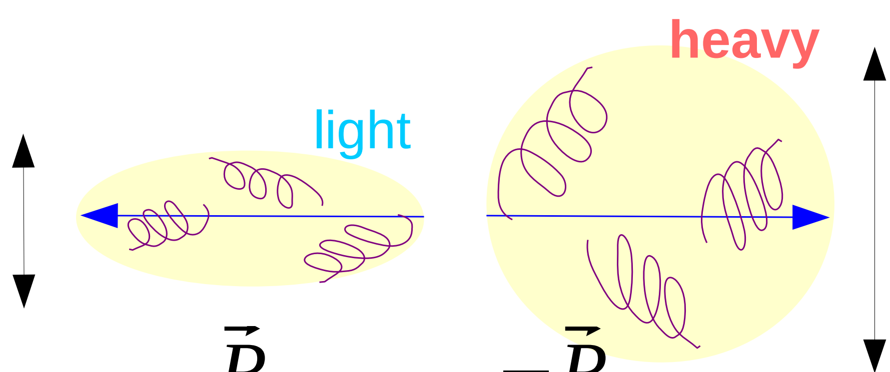
Data: ATLAS, *JHEP* 1205 (2012) 128

pp collisions: ***jet*** $\vec{P}$ **is measured, *E, M* fluctuates**



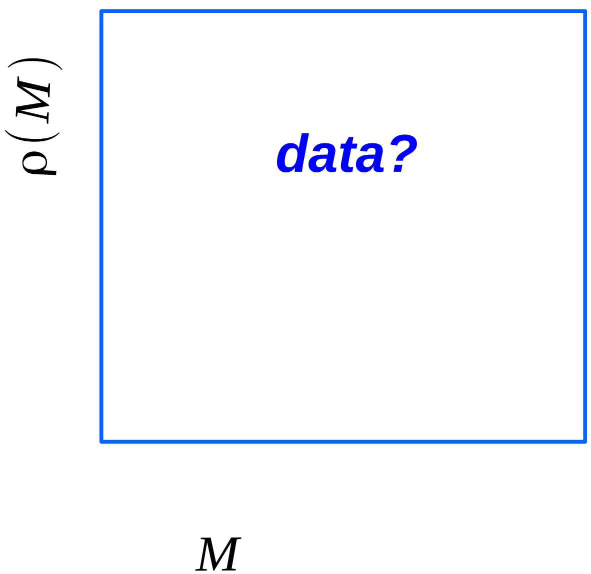


e+e- → 2 jet: **both *E* and $\vec{P}$ of the jets fluctuate**



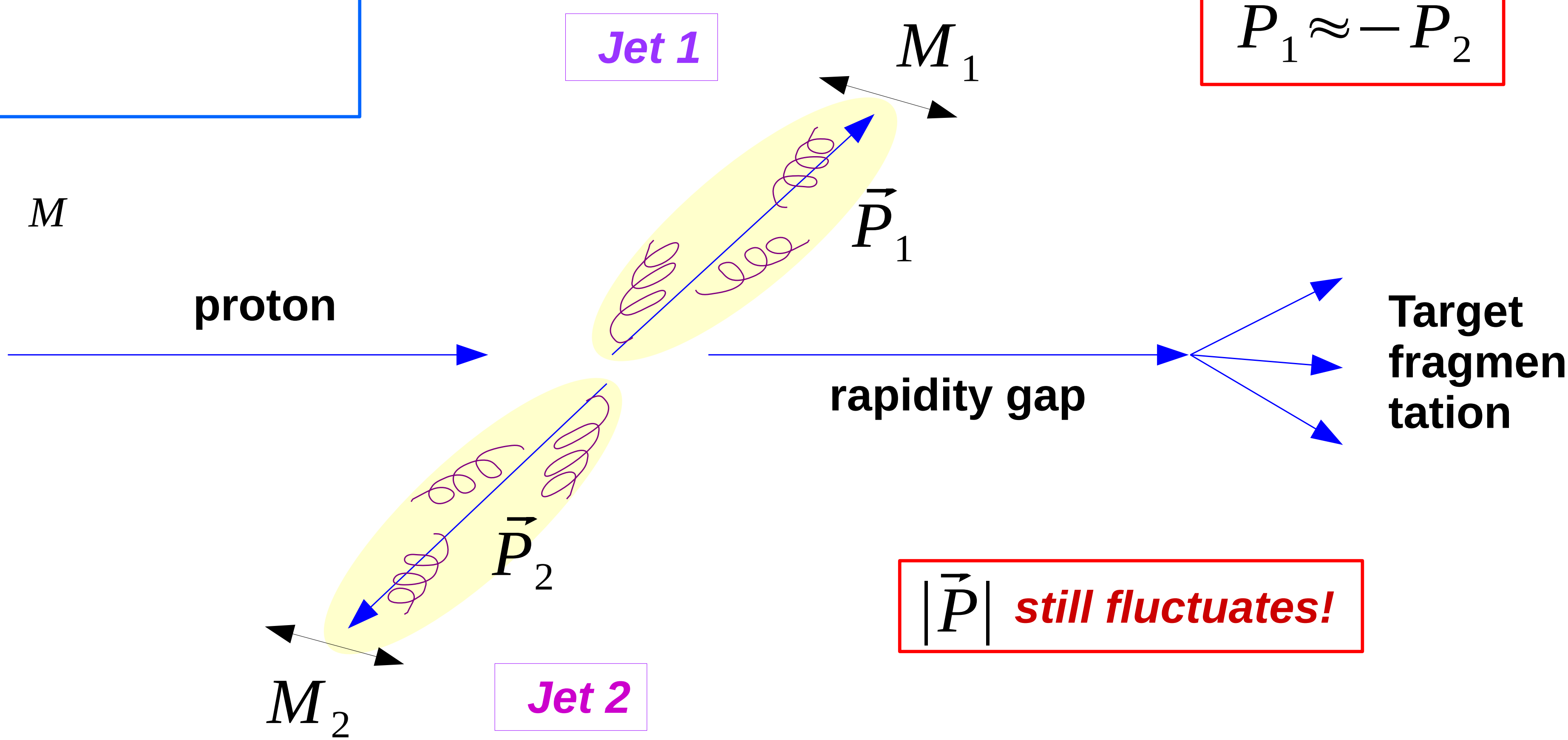
M_1 $\vec{P}$ $-\vec{P}$ M_2

Problems



ep → 2 jets of approximately *same E*nergy and large rapidity gap:

$$E_1 \approx E_2$$
$$\vec{P}_1 \approx -\vec{P}_2$$



$|\vec{P}|$ ***still fluctuates!***

Statistical Fragmentation

Statistical jet-fragmentation

The cross-section of the creation of hadrons $h_1, \ldots, h_N$ in a jet of N hadrons

$$d\sigma^{h_1,\ldots,h_N} = |M|^2 \delta^{(4)}\left(\sum_i p^{\mu}_{h_i} - P^{\mu}_{tot}\right) d\Omega_{h_1,\ldots,h_N}$$

If *|M| ≈ constans*, *we arrive at a microcanonical ensemble*:

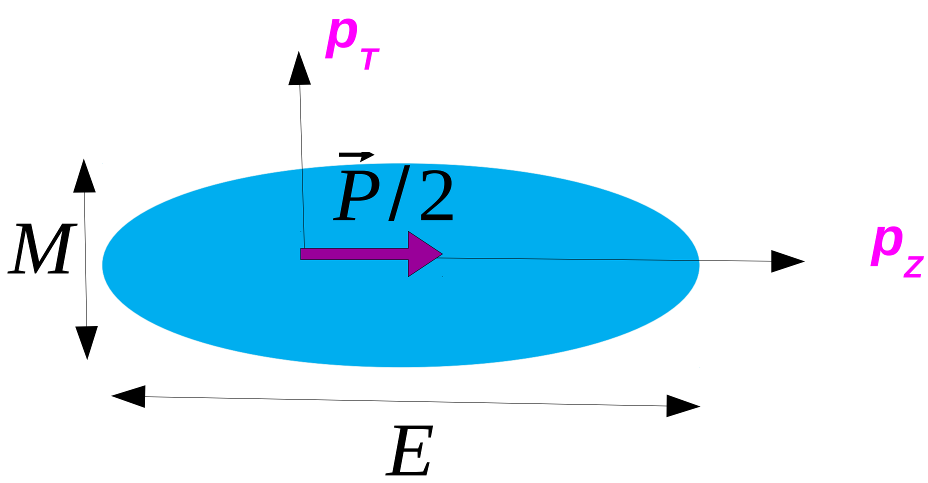
$$d\sigma^{h_1,\ldots,h_n} \sim \delta\left(\sum_i p^{\mu}_{h_i} - P^{\mu}_{tot}\right) d\Omega_{h_1,\ldots,h_n} \propto \left(P_{\mu} P^{\mu}\right)^{n-2} = M^{2n-4}$$

Thus, the haron distribution in a jet of *n* hadron is

$$p^0 \frac{d\sigma}{d^3 p}^{n=fix} \propto \frac{\Omega_{n-1}\left(P_{\mu} - p_{\mu}\right)}{\Omega_n\left(P_{\mu}\right)} \propto \left(1-x\right)^{n-3}, \qquad x = \frac{P_{\mu} p^{\mu}}{M^2/2}$$

Energy of the hadron in the co-moving frame

The haron distribution in a jet of *n* hadron with total momentum *P*



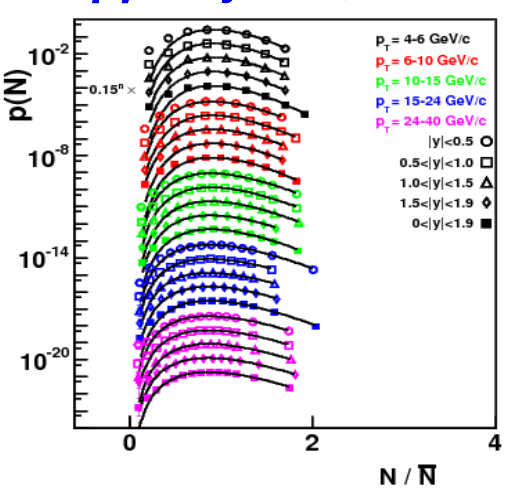
$$p^0 \frac{d\sigma}{d^3 p}^{n=fix} \propto (1-x)^{n-3}, \qquad x = \frac{P_\mu p^\mu}{M^2/2}$$

$$P(n) = \binom{n+r-1}{r-1} \tilde{p}^n (1-\tilde{p})^r$$

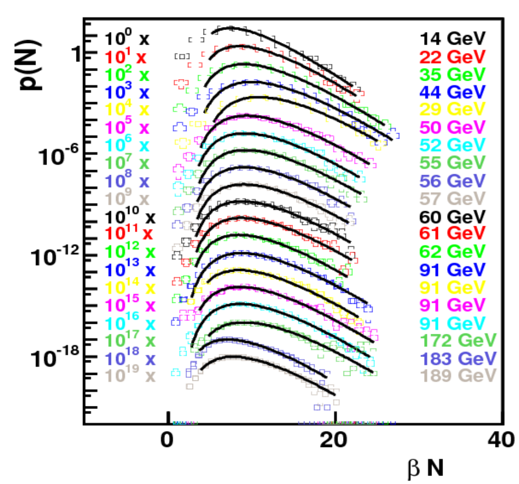
Problems

- The ***hadron multiplicity*** in a jet ***fluctuates***

pp → jets @ 7 TeV



e^-e^+ → $h^\pm$



Refs.:

Urmossy et.al.,*PLB*, **701**: 111-116 (2011)

Urmossy et. al., *PLB*, **718**, 125-129, (2012)

Averaging over n fluctuations

The distribution in a jet with *fix n*

$$p^0 \frac{d\sigma}{d^3p}^{n=fix} \propto (1-x)^{n-3}, \qquad x = \frac{P_\mu p^\mu}{M^2/2}$$

The multiplicity distribution

$$P(n) = \binom{n+r-1}{r-1} \tilde{p}^n (1-\tilde{p})^r$$

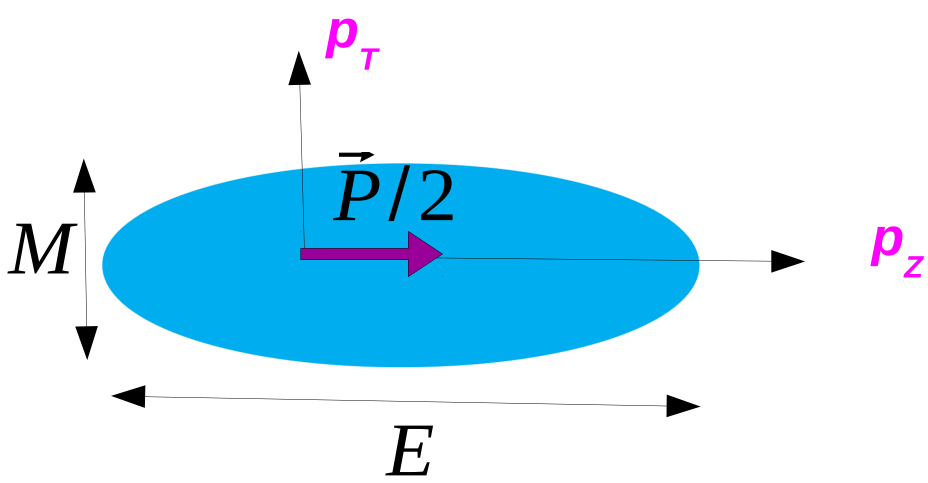
The *n-averaged* distribution

$$p^0 \frac{d\sigma}{d^3p} = A\left[1 + \frac{q-1}{\tau} x\right]^{-1/(q-1)}$$

$$\tau = \frac{1-\tilde{p}}{\tilde{p}(r+3)}$$

$$q = 1 + \frac{1}{r+3}$$

The haron distribution in a jet of *n* hadron with total momentum *P*



$$p^0 \left.\frac{d\sigma}{d^3p}\right|^{n=fix} \propto (1-x)^{n-3}, \qquad x = \frac{P_\mu p^\mu}{M^2/2}$$

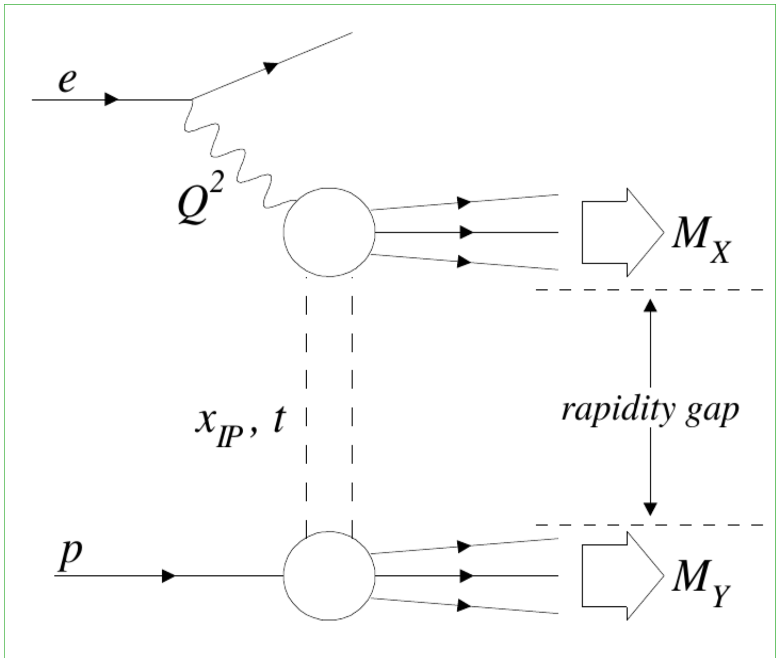
Problems

The *jet E, P fluctuate*

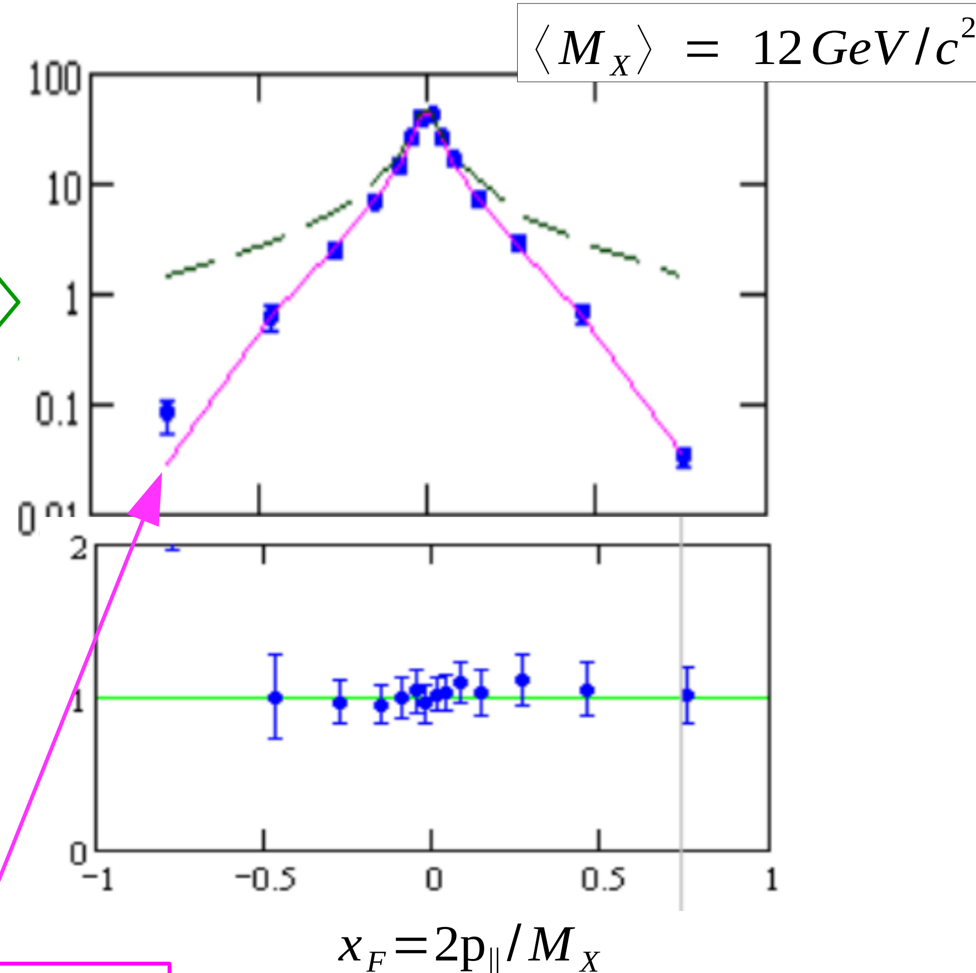
- ***pp*** **collisions:** $\vec{P}$ **is measured, *E* fluctuates**
- ***e+e- → 2 jet:*** **both *E* and** $\vec{P}$ **of the jets fluctuate**

Results

Charged hadrons from *diffractive* eP collisions in a frame *co-moving with X*



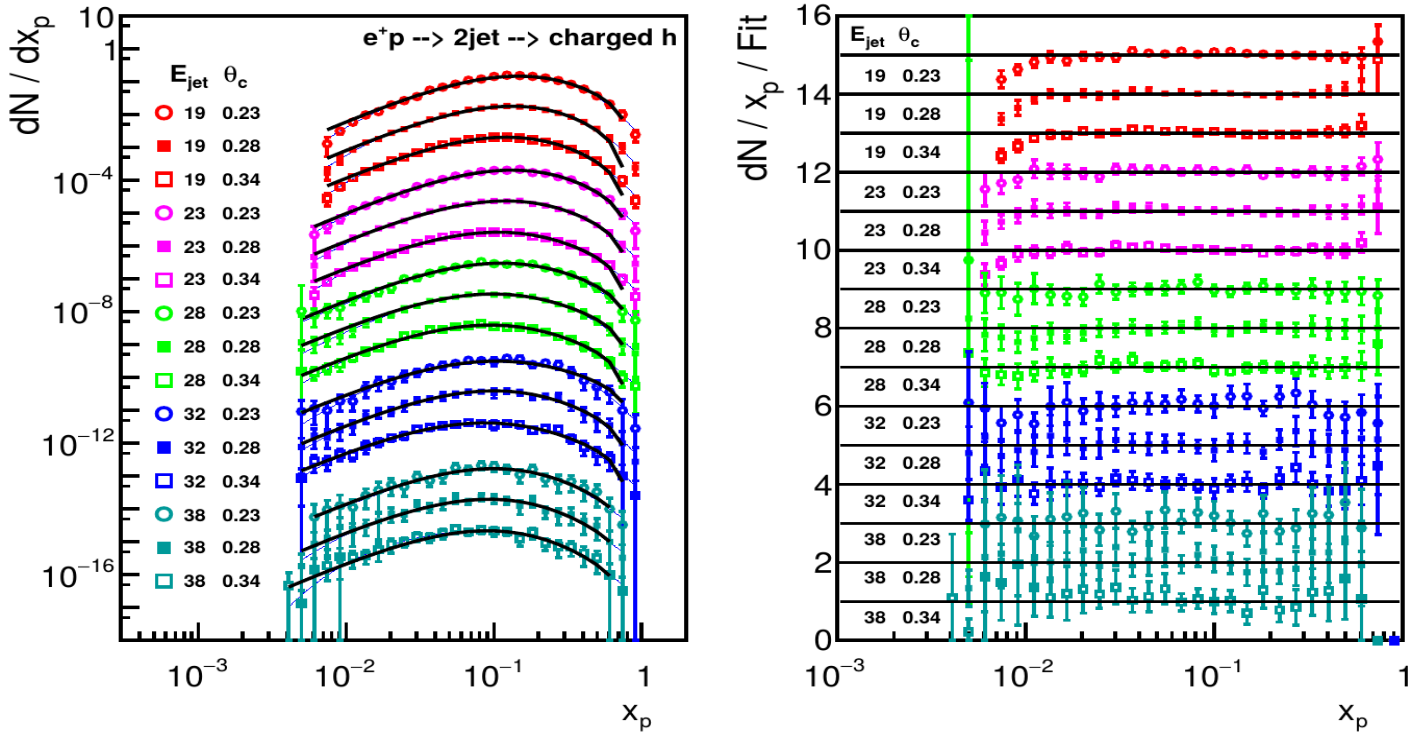
Phys.Lett.B **428**: 206-220 (1998)



$$\frac{d\sigma}{x_F\, dx_F} \sim \left[1 + \frac{q-1}{\tau} x_F\right]^{-1/(q-1)}$$

$e^+P \rightarrow$ 2 jets $\rightarrow$ charged hadrons

with large rapidity gap



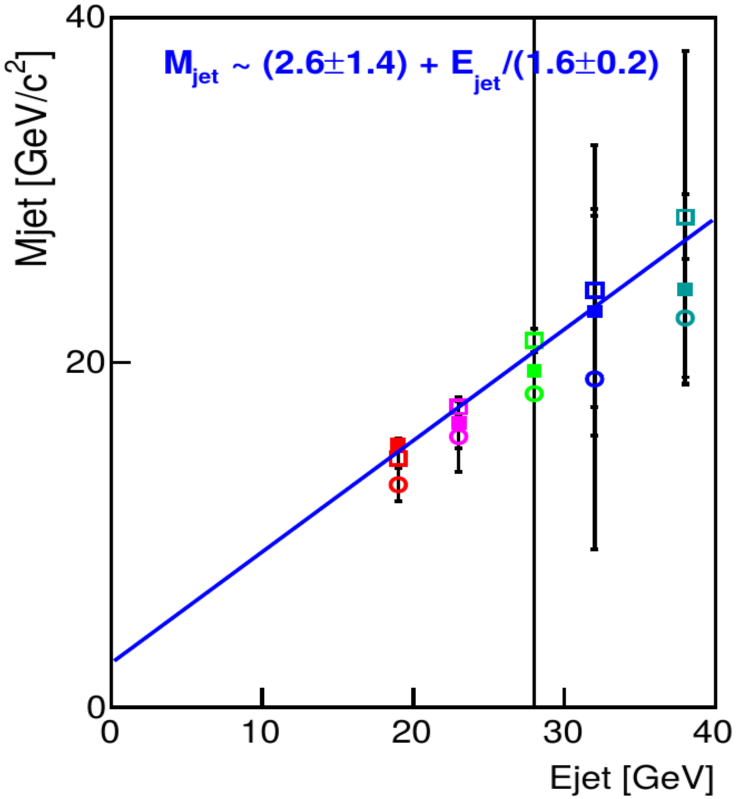
$$\frac{d\sigma}{dx_p} \sim x_p \left[1 + \frac{q-1}{\tau} x_p\right]^{-1/(q-1)}$$

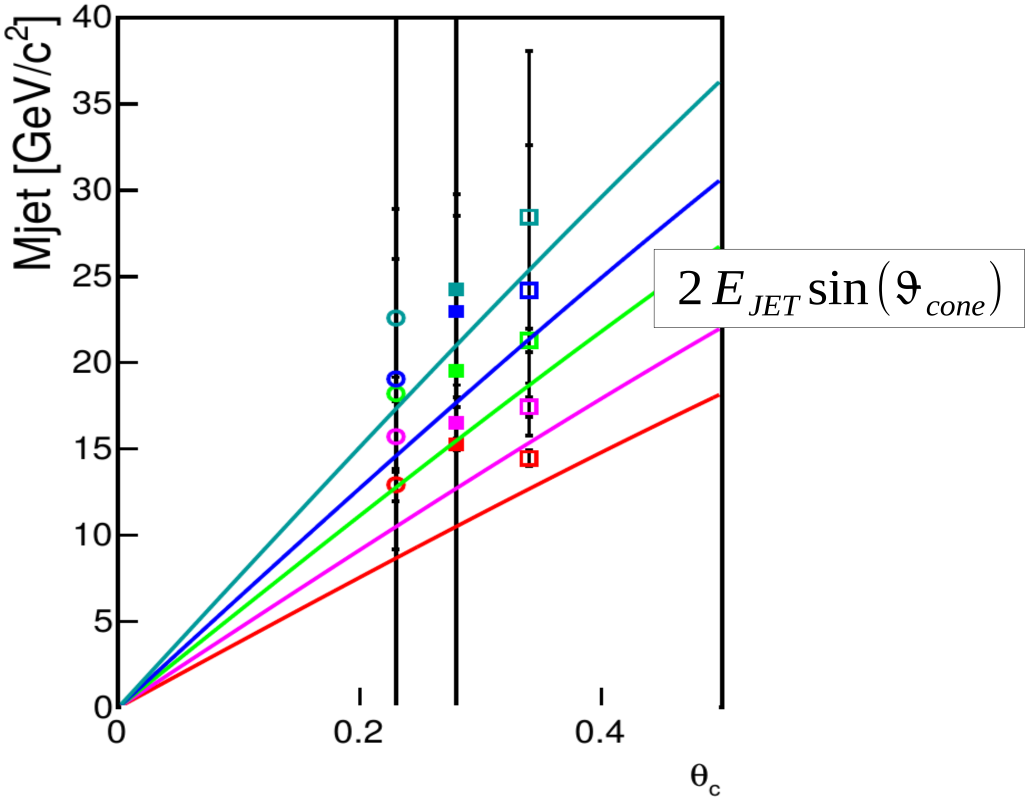
$$M_{2JET} = \frac{E_1 + E_2}{2}$$

$$x_p = 2p / M_{2JET}$$

$$\frac{E_1}{E_2} = 1 \pm 0.2$$

Fitted average characteristic *jet mass*



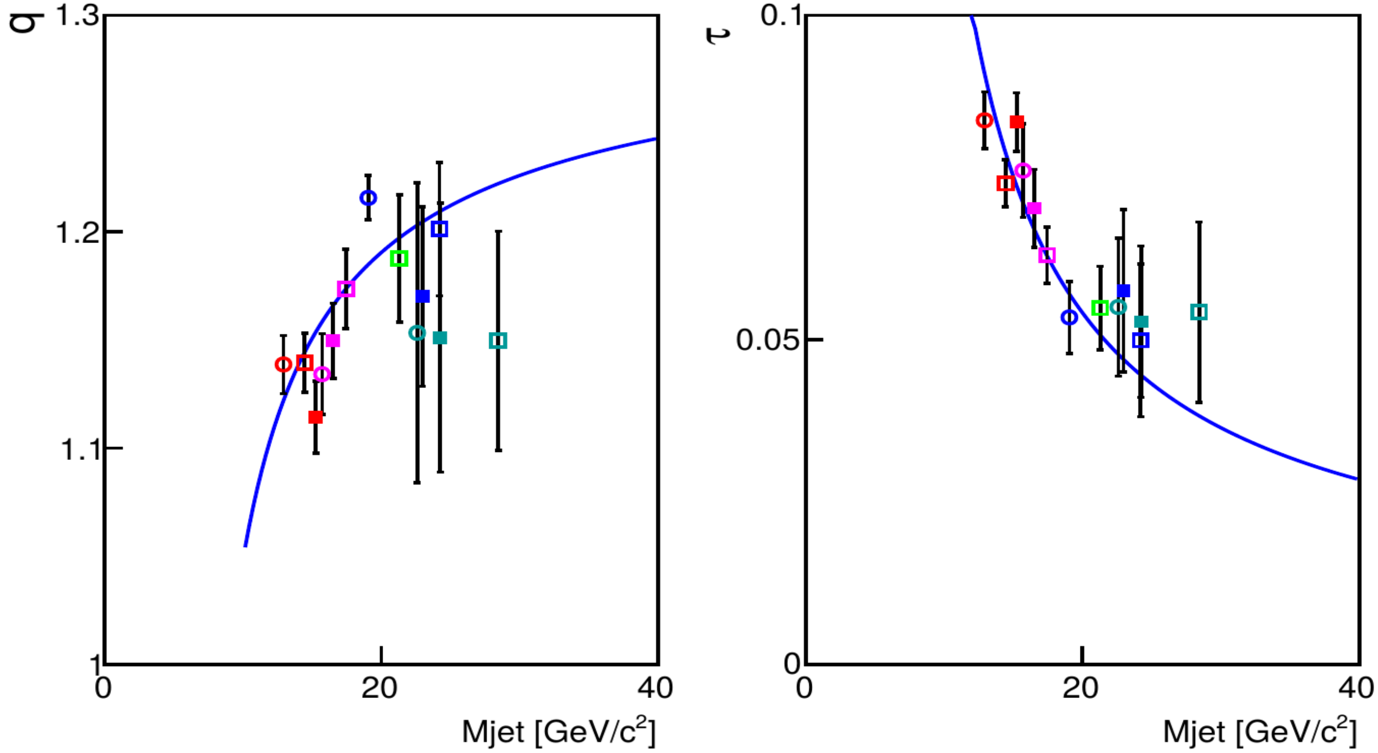


fitted $\langle M_{JET}\rangle = M_0 + E_{JET}/E_0$

Fitted average jet mass is of the order of that used in DGLAP calcs.

$$\langle M_{JET}\rangle \sim 2\,E_{JET}\sin(\vartheta_{cone})$$

Scale evolution of the fit parameters



What we have:

- **an *approximate* formula for the *fragmentation function* which *does not solve DGLAP***

$$D(x) \sim \left[1 + \frac{q-1}{\tau} x\right]^{-1/(q-1)}$$

- **Let us use *this ansatz* with *scale dependent parameters***

$$q, T \sim q(t), T(t)$$

- **along with some other conjectures**

First step: in the Φ^3 theory

The Φ^3 theory case

Resummation of branchings with DGLAP

$$\frac{d}{dt} D(x,t) = g^2(t) \int_x^1 \frac{dz}{z} P(z) D(x/z,t), \qquad t = \ln(Q^2/Q_0^2)$$

with LO splitting function:

$$P(z) = z(1-z) - \frac{1}{12}\delta(1-z)$$

M. Grazzini,
Nucl. Phys. Proc. Suppl.
64: 147-151, 1998

Let the non-perturbative input at starting scale Q_0 be:

$$D_0(x) = \left(1 + \frac{q_0 - 1}{\tau_0} x\right)^{-1/(q_0-1)}$$

The full solution is

$$\frac{dN}{dx} = \int_x^1 \frac{dz}{z} f(z,t) d(x/z, t')$$

with

$$f(x) \sim \delta(1-x) + \sum_{k=1}^{\infty} \frac{b^k}{k!(k-1)!} \sum_{j=0}^{k-1} \frac{(k-1+j)!}{j!(k-1-j)!} x \ln^{k-1-j}\left[\frac{1}{x}\right] \left[(-1)^j + (-1)^k x\right]$$

$$b = \beta_0^{-1} \ln\left(\frac{\ln(Q^2/\Lambda^2)}{\ln(Q_0^2/\Lambda^2)}\right)$$

K. Urmossy, J. Rak, *Proc. Of Conf.* Gribov 85 (2015)

Approximations

Let the FF preserve its form:

$$D_{apx}(x,t) = \left(1+\frac{q(t)-1}{\tau(t)}x\right)^{-1/(q(t)-1)} \quad \textbf{with} \quad D(x,0) = \left(1+\frac{q_0-1}{\tau_0}x\right)^{-1/(q_0-1)}$$

From DGLAP:

$$\tilde{D}(s,t) = \tilde{D}(s,0)\exp\{b(t)\tilde{P}(s)\} \quad \textbf{with} \quad b(t) = \beta_0^{-1}\ln(t), \quad t=\frac{\ln(Q^2/\Lambda^2)}{\ln(Q_0^2/\Lambda^2)}$$

Let us prescribe the approximations:

$$\int D_{apx}(x,t) = \int D(x,t)$$

$$\int x\,D_{apx}(x,t) = \int x\,D(x,t) = 1$$

(by definition)

$$\int x^2 D_{apx}(x,t) = \int x^2 D(x,t)$$

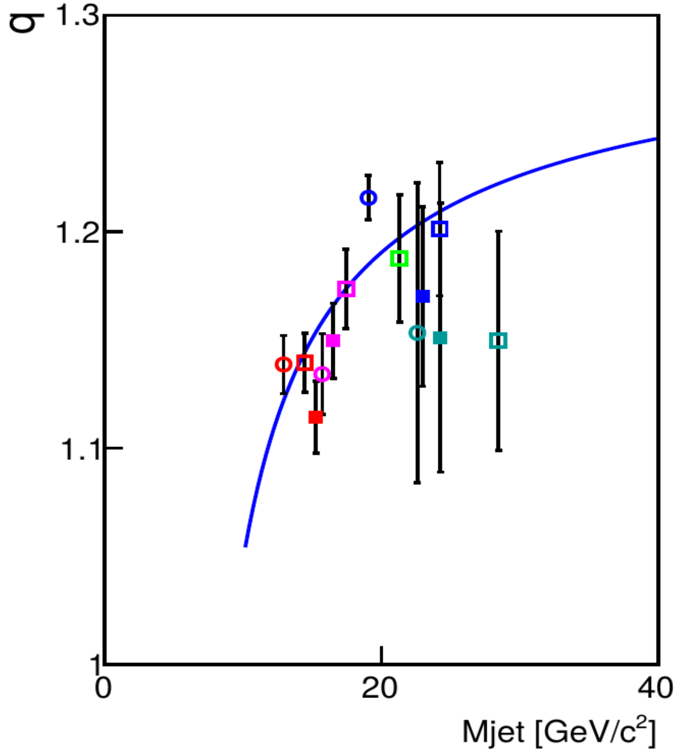
$$q(t)=\frac{(8q_0-12)t^{a1}-(9q_0-12)t^{-a2}}{(6q_0-9)t^{a1}-(6q_0-8)t^{-a2}}$$

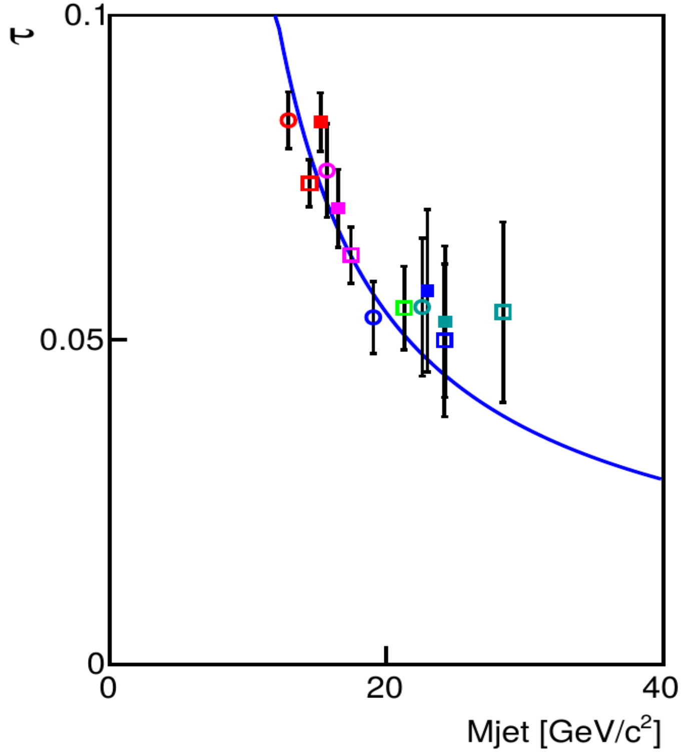
$$\tau(t)=\frac{\tau_0}{(6q_0-8)t^{-a2}-(6q_0-9)t^{a1}}$$

$$a_1=\tilde{P}(1)/\beta_0, \quad a_2=\tilde{P}(3)/\beta_0$$

U.K. J. Rak, *Proc. Of Conf.* Gribov 85 (2015)

Scale evolution of the fit parameters





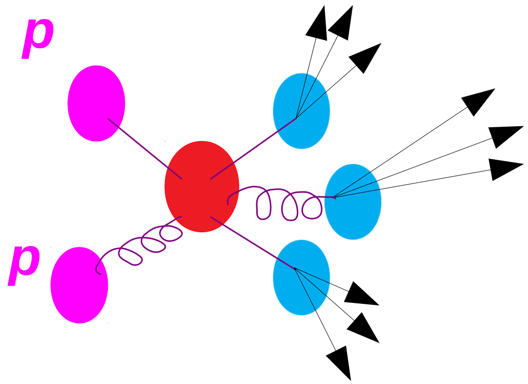
$$q(t)=\frac{(8q_0-12)t^{a1}-(9q_0-12)t^{-a2}}{(6q_0-9)t^{a1}-(6q_0-8)t^{-a2}}$$

$$\tau(t)=\frac{\tau_0}{(6q_0-8)t^{-a2}-(6q_0-9)t^{a1}}$$

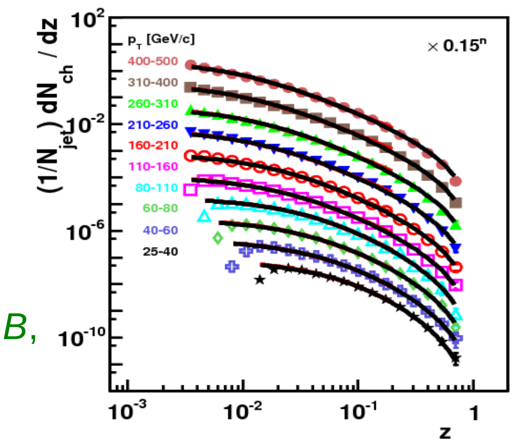
$$t=\frac{\ln(M^2/\Lambda^2)}{\ln(Q_0^2/\Lambda^2)}$$

pp & ee collisions

pp → jets @LHC (pT = 25–500 GeV/c)

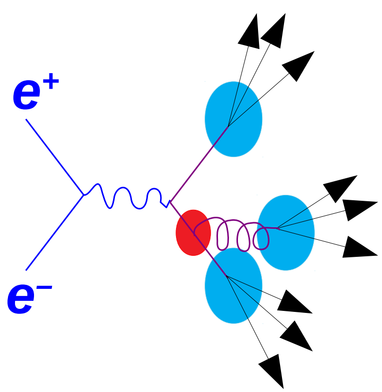


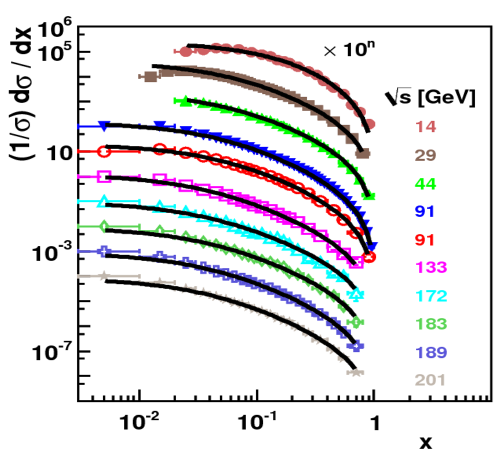
$$\frac{dN}{dz} \propto \left[1 - a\ln(1-z)\right]^{-b}$$



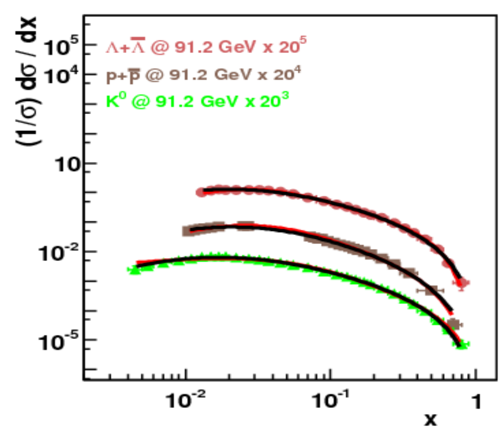
Urmossy et.al. *Phys. Lett. B*, **718**, 125-129, (2012)

e+e- annihilation @LEP (√s = 14–200 GeV)

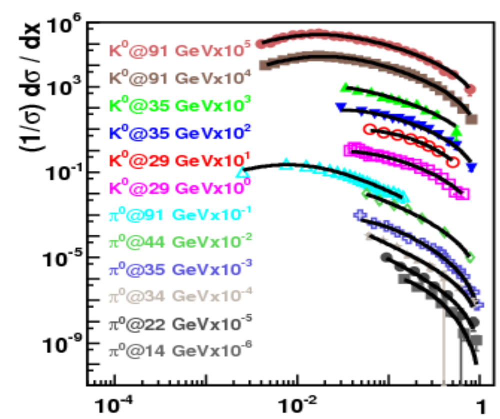




Urmossy et. al., *Phys. Lett. B*, **701**, 111-116 (2011)



Urmossy et.al., *Acta Phys. Polon. Supp.* **5** (2012) 363-368

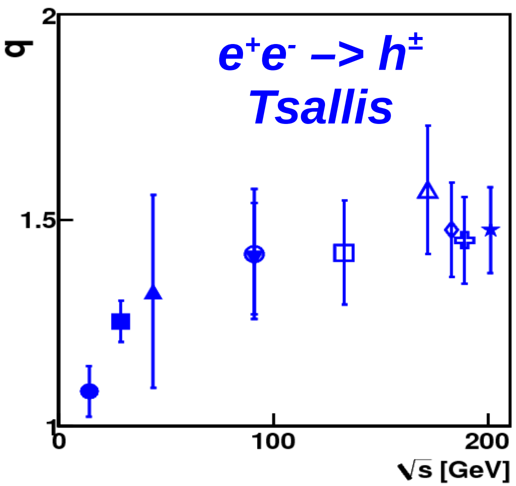


T. S. Biró et.al., *Acta Phys. Polon. B*, **43** (2012) 811-820

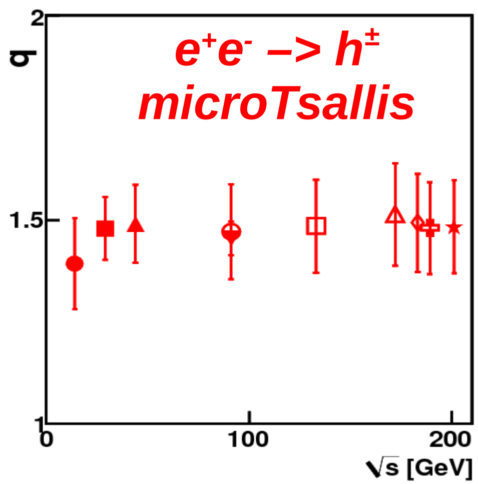
Scale Evolution

Fitts:

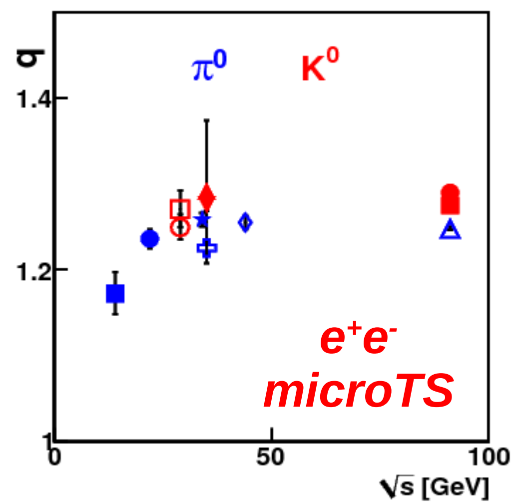
1)



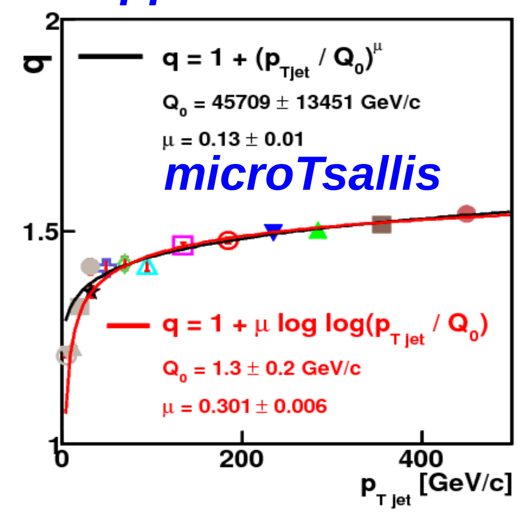
2)



3)

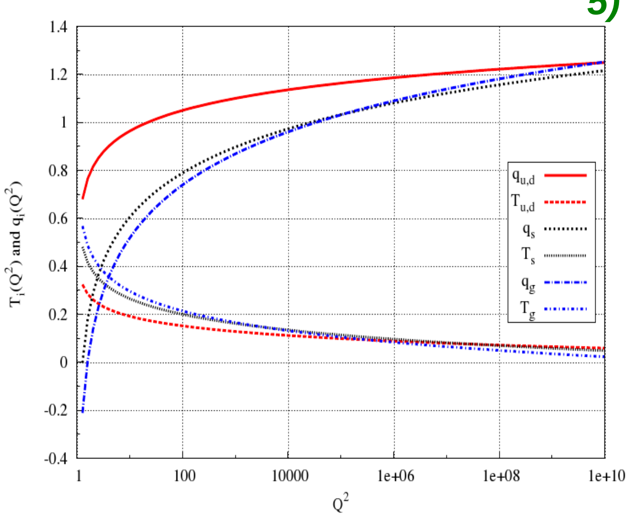


pp –> Jet –> $h^\pm$ 4)



Theory: Scale evolution of *q*, *T* from fits to AKK Frag. Funcs:

5)



$$D^{\pi^+}_{p_i}(z) \sim \left(1+(q_i-1)z/T_i\right)^{-1/(q_i-1)}$$

$$q_i = q_{0i} + q_{1i}\ln\left(\ln\left(Q^2\right)\right)$$

1-2) U.K. etal., *Phys.Lett. B*, **701** (2011) 111-116

3) T. S. Biró etal., *Acta Phys.Polon. B*, **43** (2012) 811-820

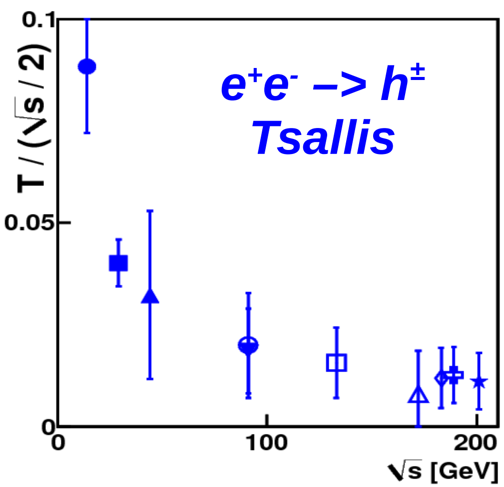
4) U.K. etal., *Phys.Lett. B*, **718** (2012) 125-129

5) Barnaföldi etal., *Gribov-80 Conf*: C10-05-26.1, p.357-363

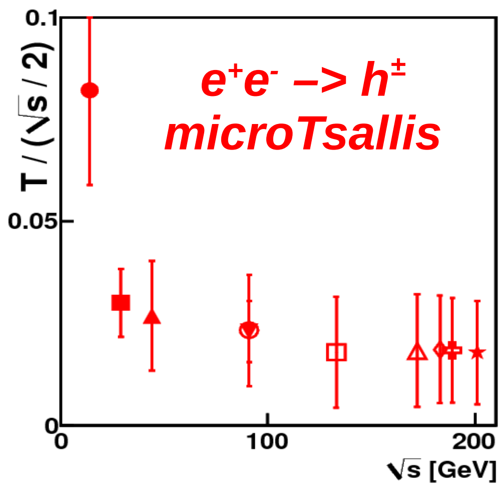
Scale Evolution

Fitts:

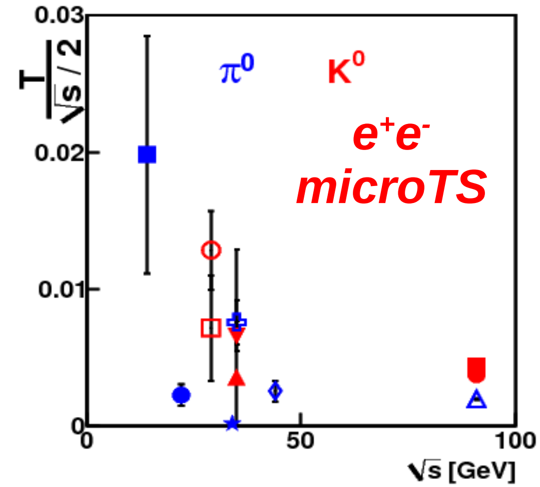
1)



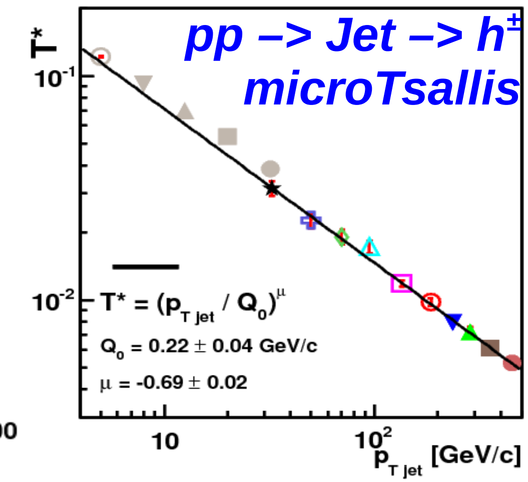
2)



3)



4)



Theory: Scale evolution of *q*, *T* from fits to AKK Frag. Funcs:

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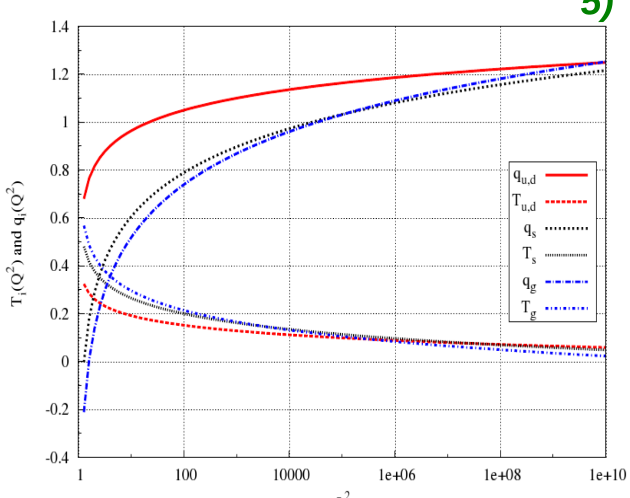


$$D_{p_i}^{\pi^+}(z) \sim \left(1+(q_i-1)z/T_i\right)^{-1/(q_i-1)}$$

$$T_i = T_{0i}+T_{1i}\ln(\ln(Q^2))$$

1-2) U.K. etal., *Phys.Lett. B*, **701** (2011) 111-116

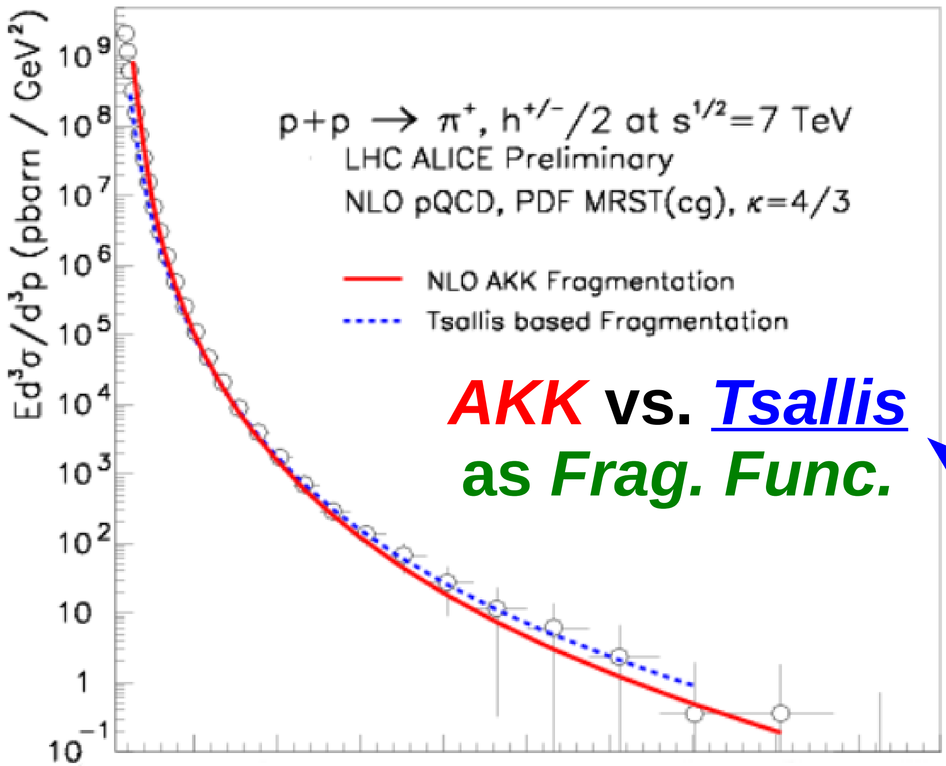
3) T. S. Biró etal., *Acta Phys.Polon. B*, **43** (2012) 811-820

4) U.K. etal., *Phys.Lett. B*, **718** (2012) 125-129

5) Barnaföldi etal., *Gribov-80 Conf*: C10-05-26.1, p.357-363

Application in a pQCD calculation

π^+ spectrum in pp --> $\pi^\pm X$ @ √s=7 TeV (NLO pQCD)

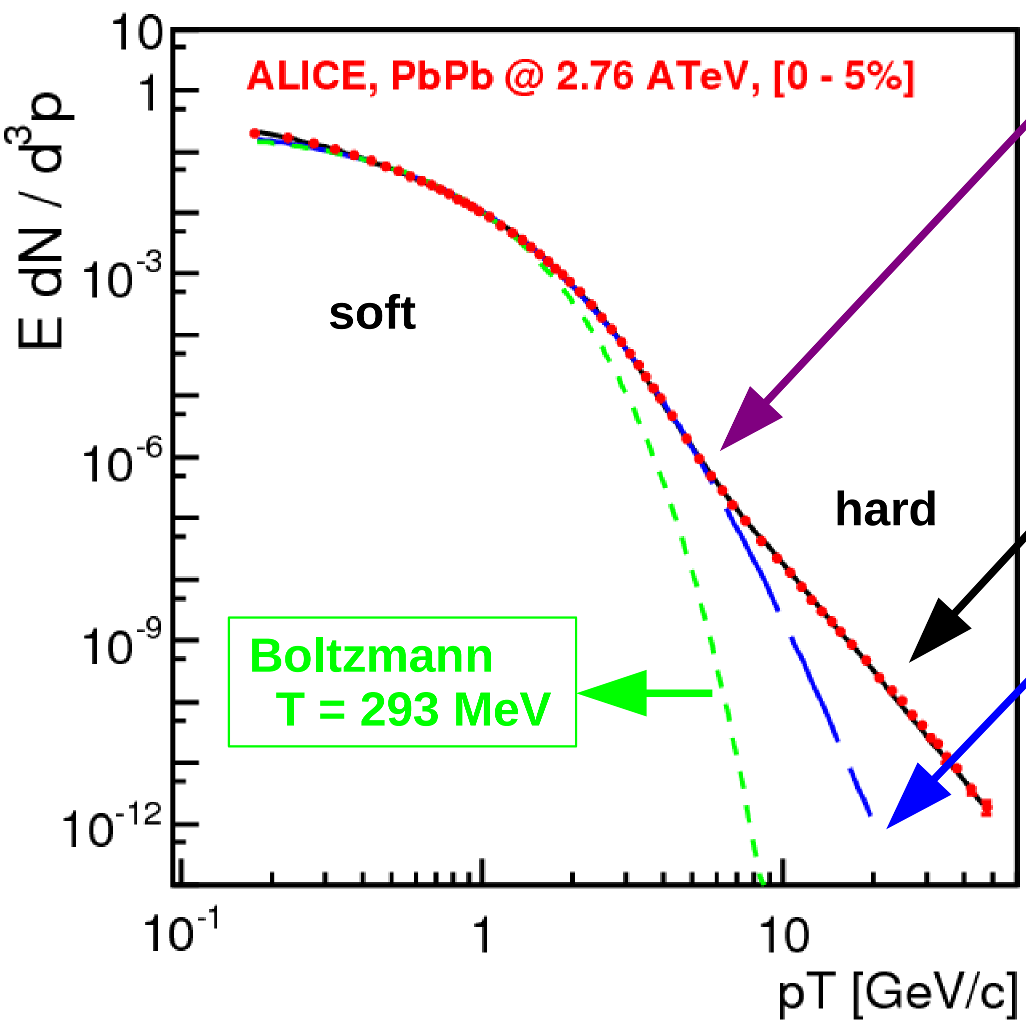


AKK vs. Tsallis as Frag. Func.

$$D_{p_i}^{\pi^+}(z) \sim \left(1+(q_i-1)\,z/T_i\right)^{-1/(q_i-1)}$$

Barnaföldi et. al., *Proceedings of the Workshop Gribov '80* (2010)

How about the *soft* part?



The power of the spectrum changes drastically at pT ~ 6 GeV/c.

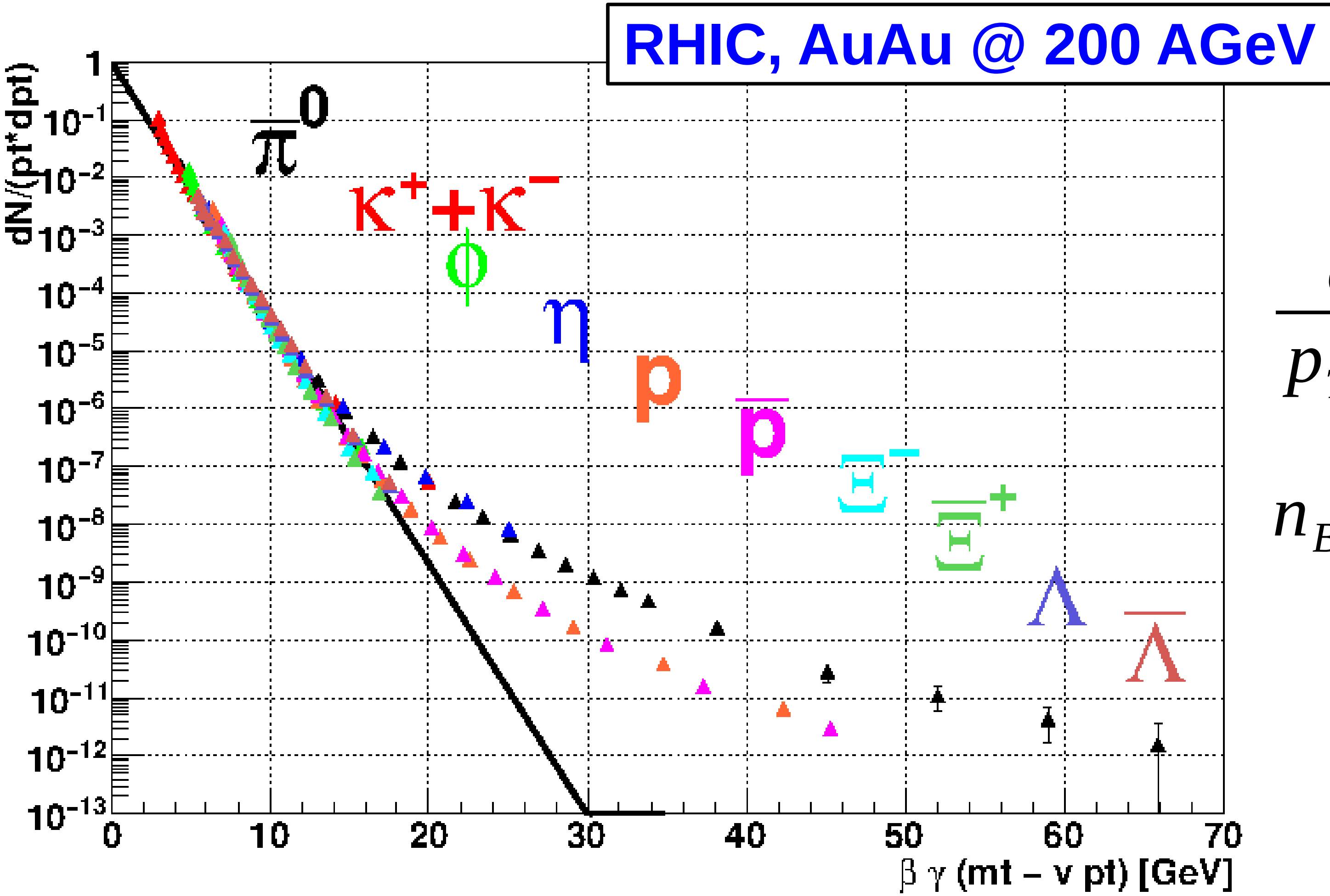
$\sim \; p_T^{-6.08}$

Tsallis $\sim \; p_T^{-13.7}$

A *hard + soft* model:

$$E\frac{dN}{d^3\mathbf{p}} = E\frac{dN}{d^3\mathbf{p}}^{hard} + E\frac{dN}{d^3\mathbf{p}}^{soft}$$

Different q for baryons and mesons



$$\frac{dN}{p_T\, dp_T} \sim p_T^{-n}$$

$$n_{Baryon} \neq n_{Meson}$$

T S Biró etal, J. Phys. G-Nucl. Part. Phys., 37, 9, (2010)

- **Hadronisation:** ***rapid coalescence*** **of** ***thermal quarks*** **and** ***gluon fibres*** **:**

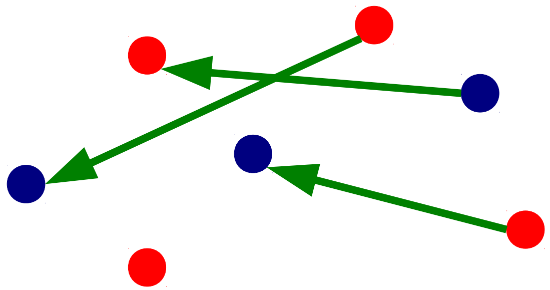
$$F_h(P_h, x) = f_q(x, p_{q1}) * \dots * f_q(x, p_{qn}) \; G(m) \; C(p_{qi}, m)$$

quarks: $$f_q(x, \vec{p}_q) = \left(1 + \frac{q-1}{T}\epsilon_q\right)^{-1/(q-1)}$$

gluon fibres: $$G(m) = \exp\left(-\left[\Gamma(1+1/d)\frac{m}{\langle m \rangle}\right]^d\right)$$

kernel: $$C(x, p_{qi}) = \delta^3\left(\sum \vec{p}_i - \vec{P}_h\right) \prod_{i,j} \delta^3(\vec{p}_i - \vec{p}_j) \; \delta\left(\sum \epsilon_i + m - E_h\right)$$

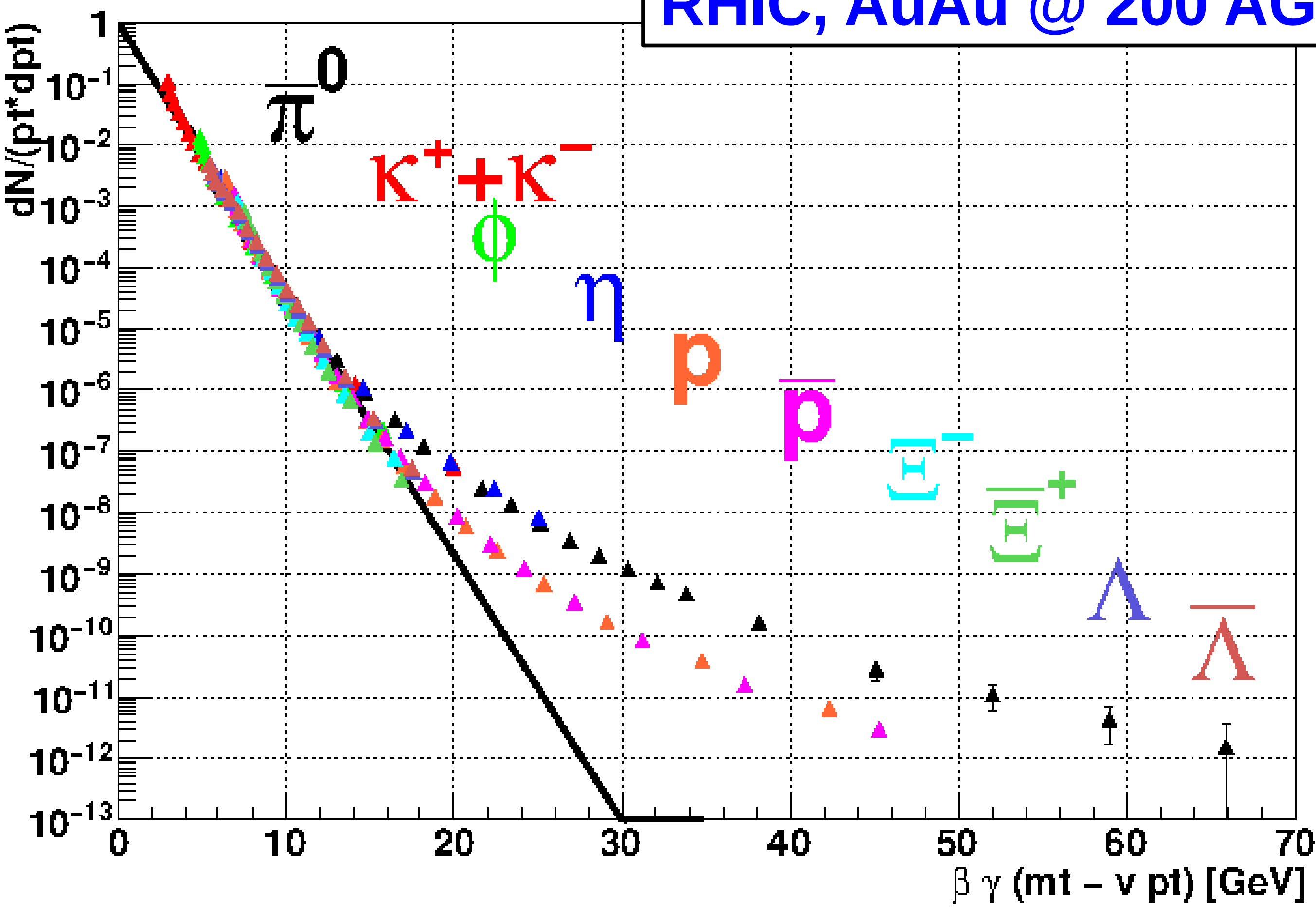
- **The** ***distribution of the length of gluon fibres:*** **is the probability of finding two quarks at distance** $l = \sigma/m$ **in a hogenous quark sea with fractal dimension** ***d*****.**



T S Biró etal,
J. Phys. G-Nucl. Part. Phys., 37, 9, (2010)
J. Phys. G., G36, 064044, (2009)
Eur. Phys. J. A, 40, 325-340, (2009)

Different q for baryons and mesons

RHIC, AuAu @ 200 AGeV



$$\frac{dN}{p_T\, dp_T} \sim p_T^{-n}$$

$$\frac{n_B}{2} \approx \frac{n_M}{3}$$

T S Biró etal, J. Phys. G-Nucl. Part. Phys., 37, 9, (2010)

(1) Statistical description of hadron spectra:

$$E\frac{dN}{d^3\mathbf{p}} = \sum_{sources} f\left[u_\mu p^\mu\right]$$

(2) Space-time dependence only through $u_\mu(x)$ *Bjorken + Blast Wave*

$$u_\mu = \left(\gamma\cosh\zeta, \gamma\sinh\zeta, \gamma v\cos\alpha, \gamma v\sin\alpha\right), \qquad \zeta = \frac{1}{2}\ln\left(\frac{t+z}{t-z}\right)$$

$$v(\alpha) = v_0 + \sum_1^N \delta v_m \cos(m\alpha)$$

Then, the spectrum and the v2 are

$$\left.\frac{dN}{p_T\, dp_T\, dy}\right|_{y=0} \propto f\left[E(v_0)\right] + O(\delta v^2)$$

$$v_2 \propto \delta v_2 (v_0 m_T - p_T)\frac{f'\left[E(v_0)\right]}{f\left[E(v_0)\right]} + O(\delta v^2) \qquad E(v_0) = \gamma_0(m_T - v_0 p_T)$$

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$$v_2 \propto \delta v_2 (v_0 m_T - p_T) \frac{f'\left[E(v_0)\right]}{f\left[E(v_0)\right]} + O(\delta v^2)$$

Boltzmann-distribution:

$$f\left[E(v_0)\right] \propto e^{-E(v_0)/T}$$

$$v_2 \propto p_T - v_0 m_T$$

Tsallis-distribution:

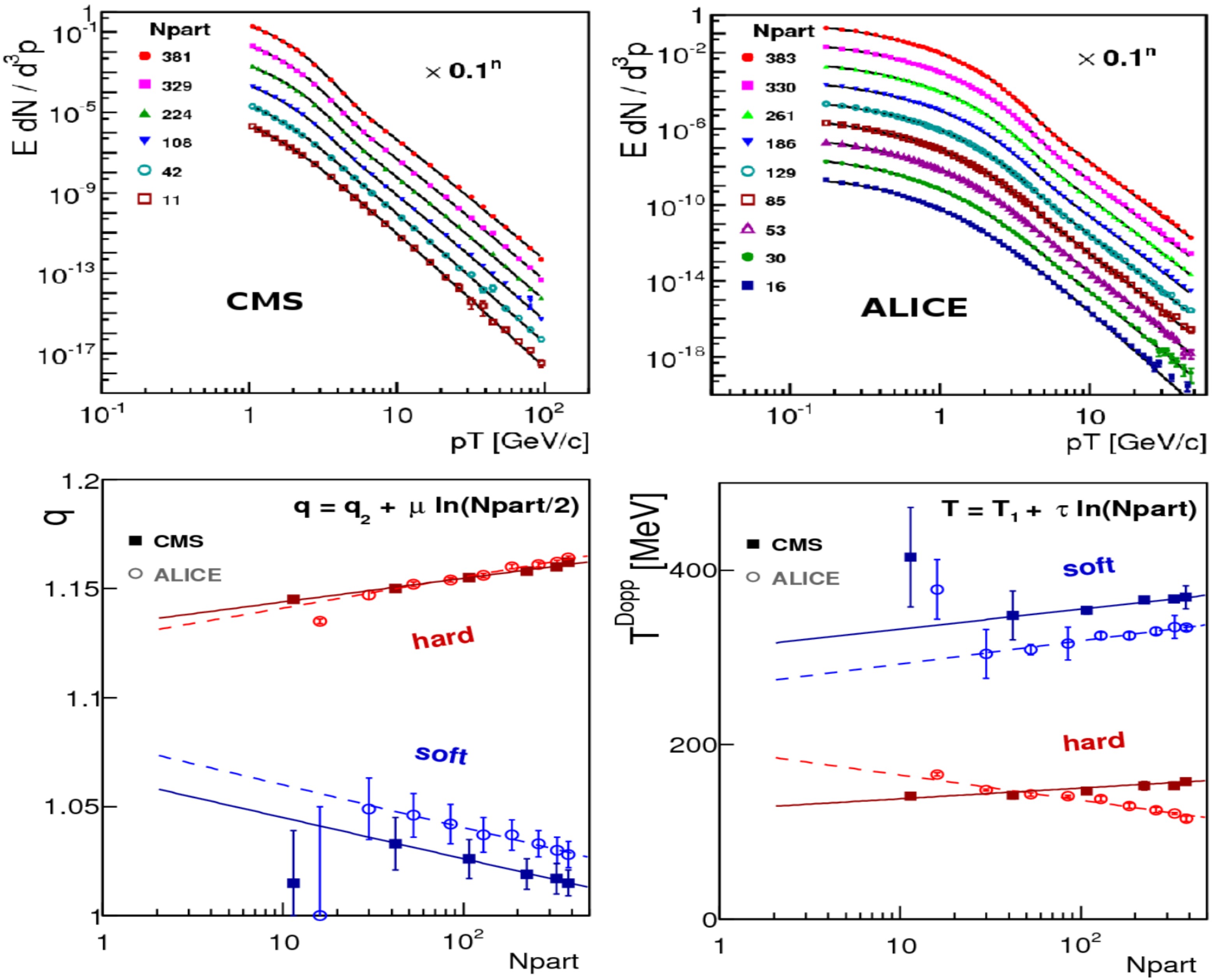
$$f\left[E(v_0)\right] \propto \left[1 + (q-1)\frac{E(v_0) - m}{T}\right]^{-1/(q-1)}$$

$$v_2 \propto \frac{p_T - v_0 m_T}{1 + \frac{q-1}{T}\left[\gamma_0(m_T - v_0 p_T) - m\right]}$$

Barnaföldi etal, (Hot Quarks 2014) J. Phys. Conf. Ser. 612 (2015) 1, 012048

Urmossy etal, (WPCF 2014) arXiv:1501.05959, Conference: C14-08-25.8

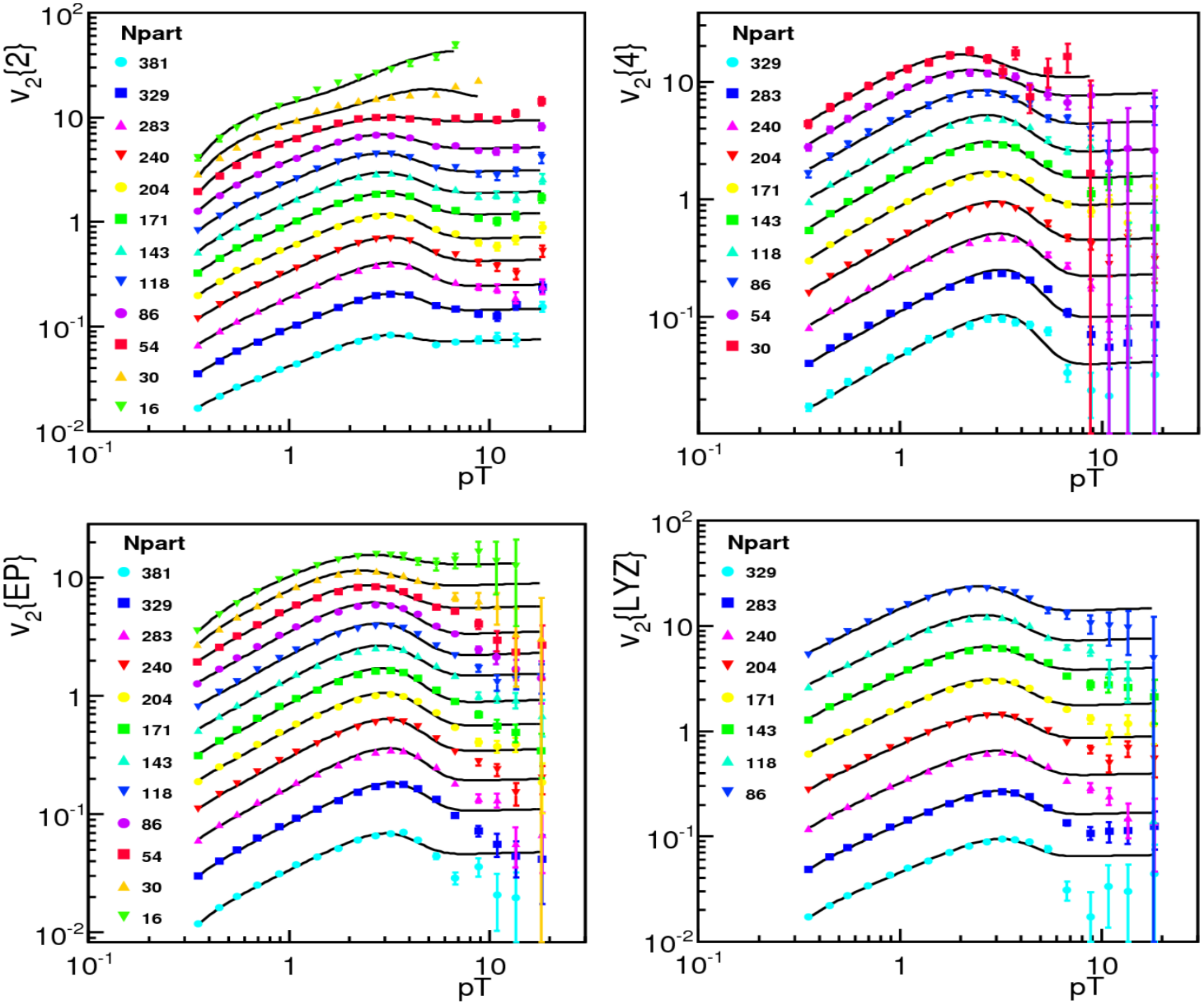
Urmossy etal, (High-pT 2014), arXiv:1501.02352, arXiv:1405.3963



PbPb → $h^{\pm}$
CMS

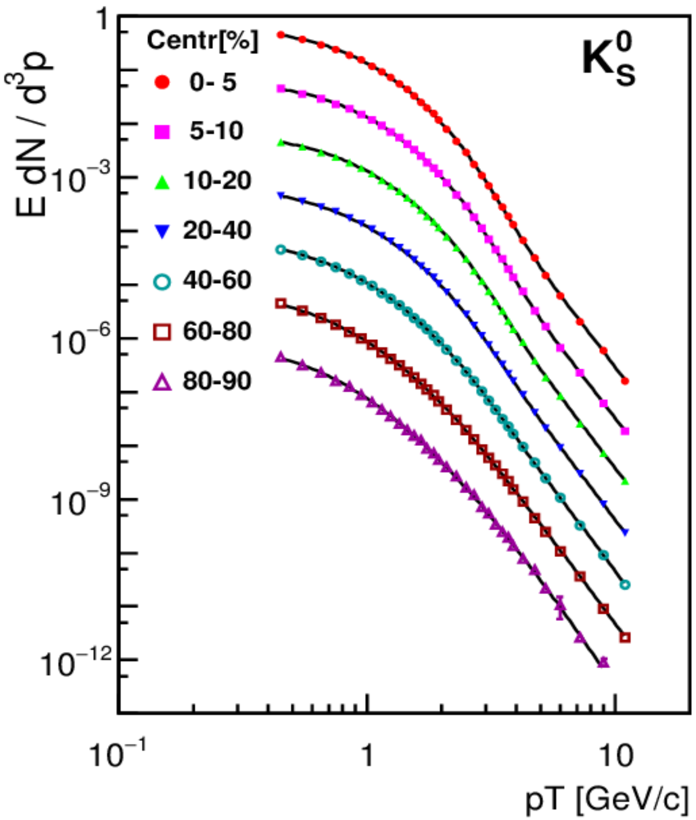
arXiv:1405.3963

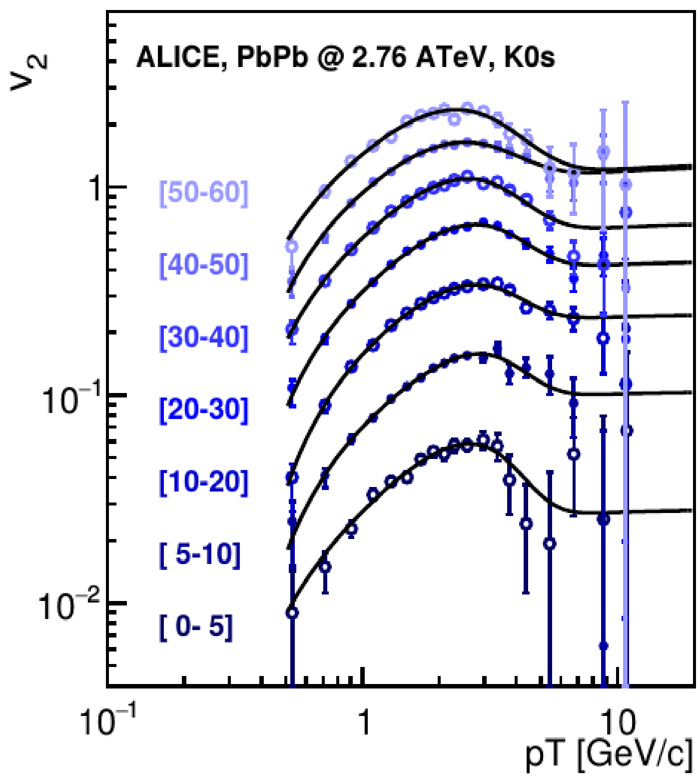
v2 of $h^{\pm}$

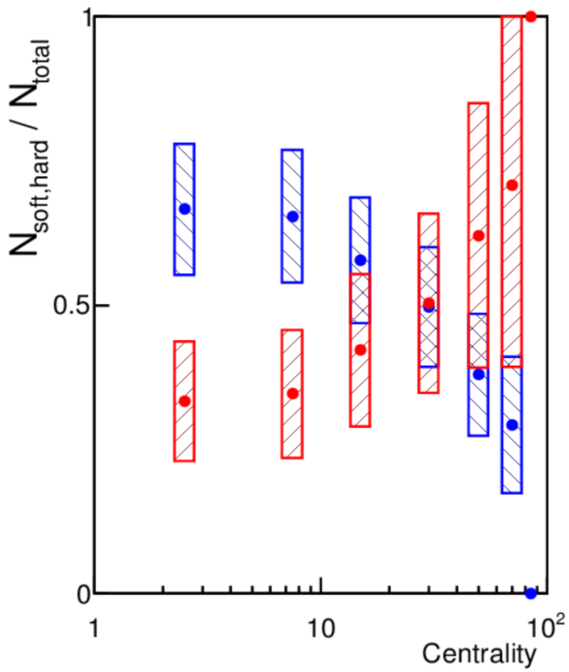


arXiv:1405.3963

PbPb → K^0s *ALICE*

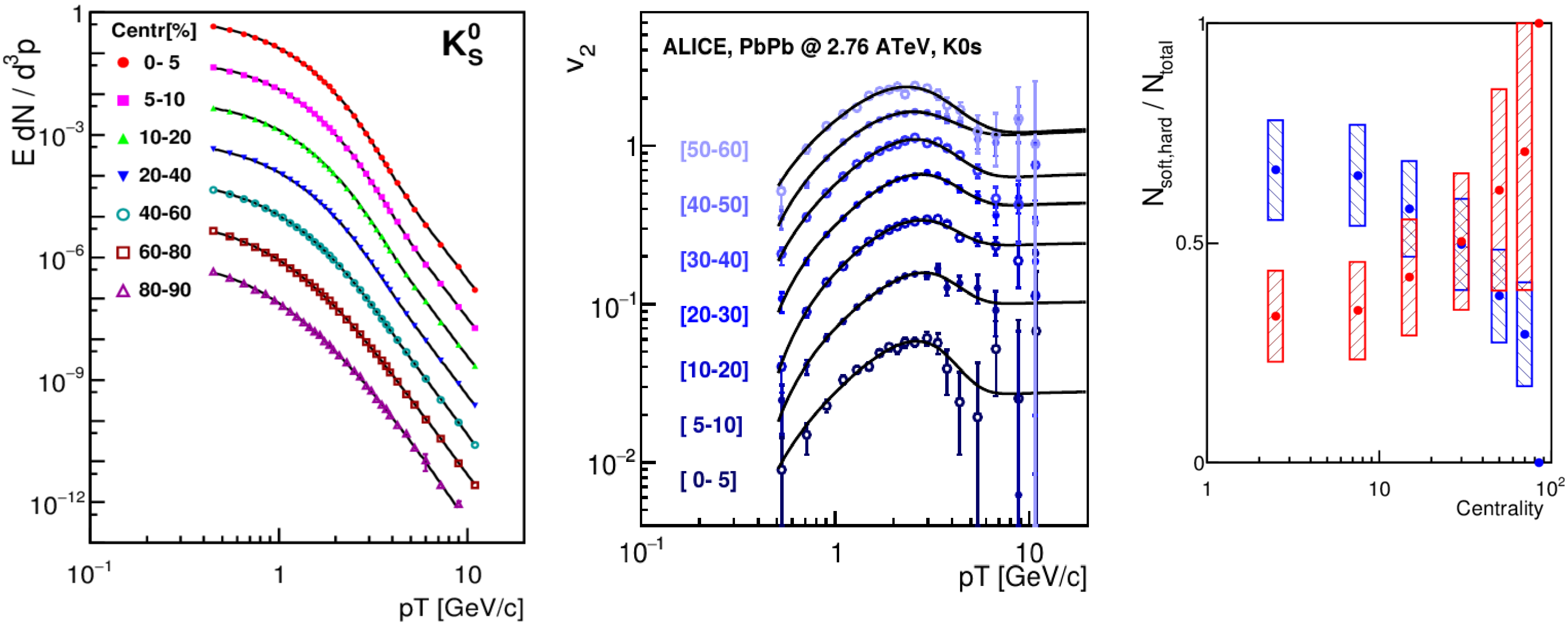






Preliminary

Application in heavy-ion collisions



Preliminary

Conclusion

Jet-fragmentation might be statistical?

- ***Suggestion***

  It might be more suitable to

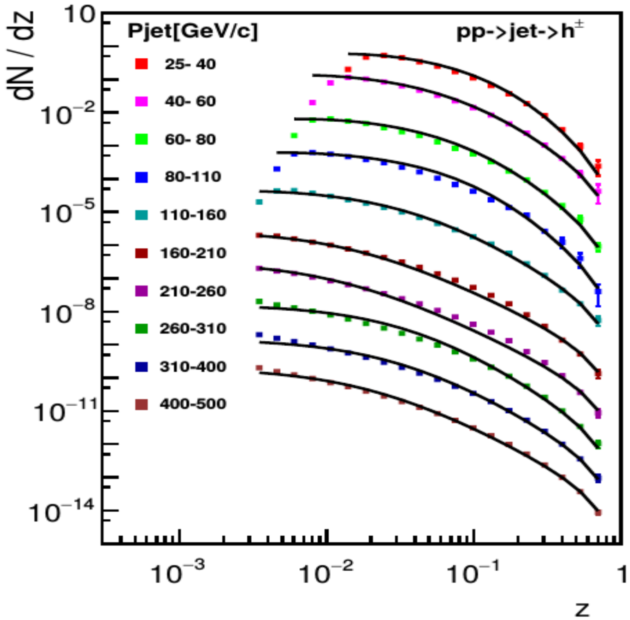
  characterise JETs with their MASS

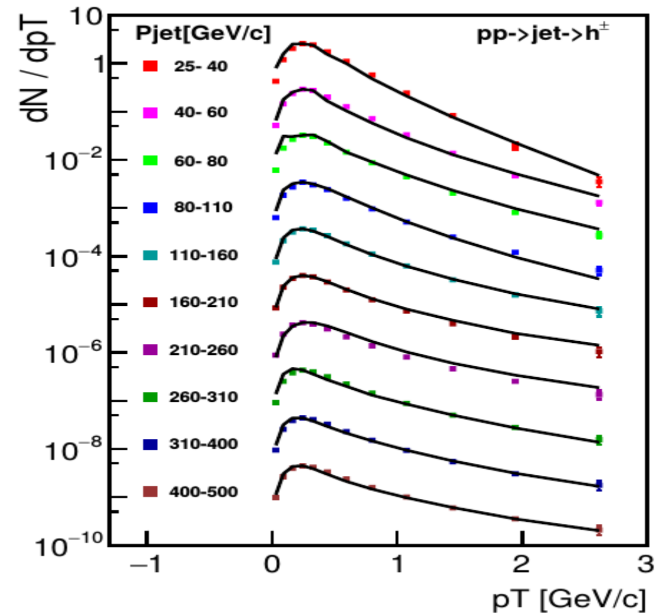
  instead of thier P or E

Thanks for the attention

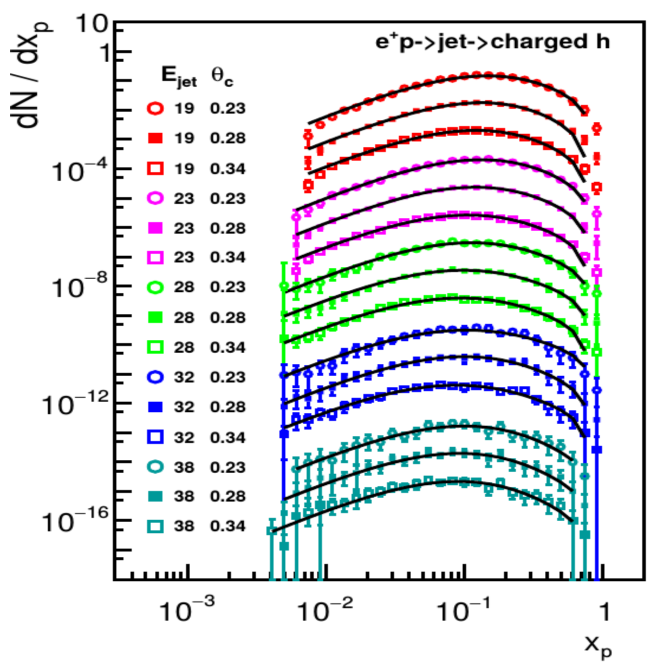
Results

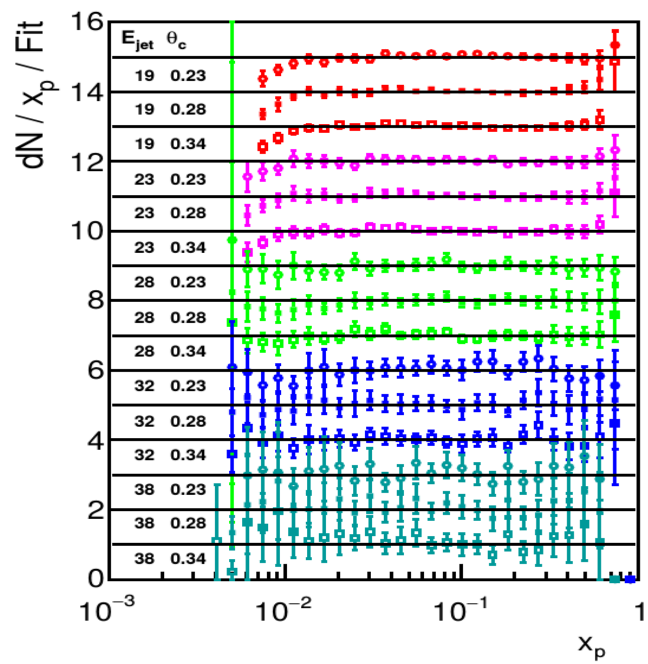
PP
collisions





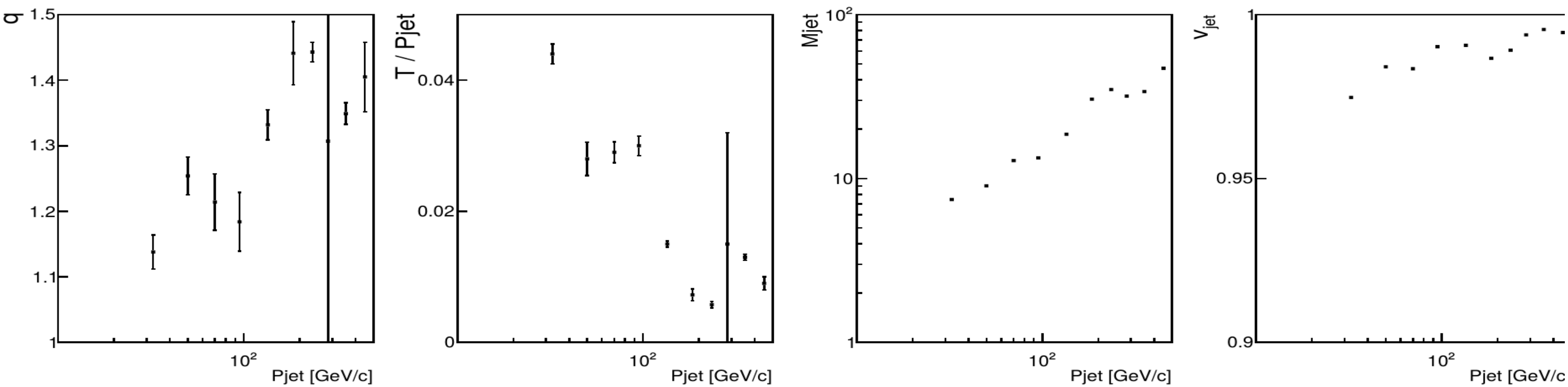
eP
collisions



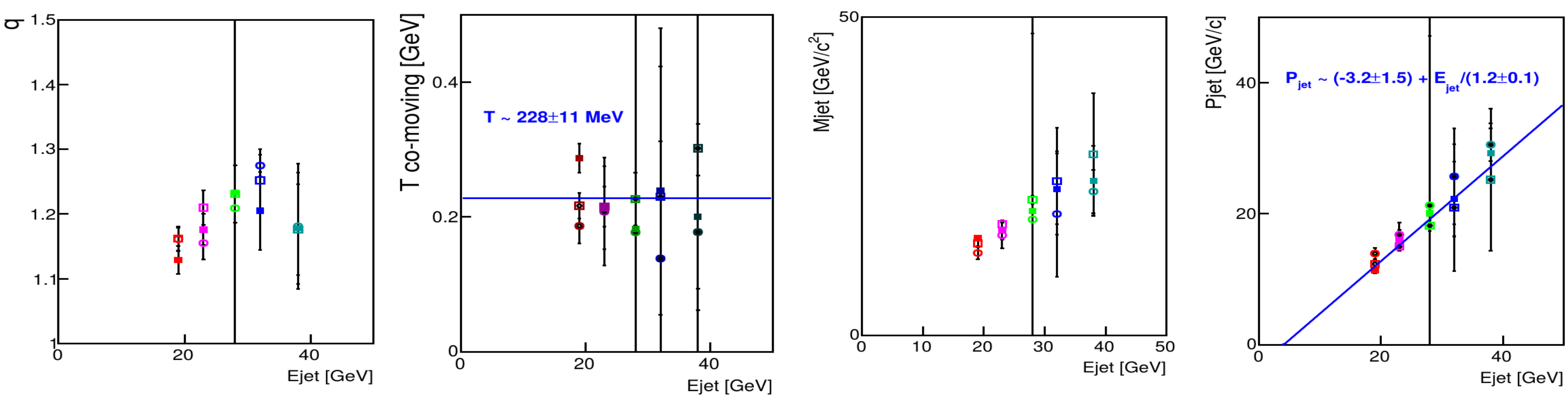


Results

PP



eP



Averaging over n fluctuations

The distribution in a jet with *fix n*

$$p^0 \frac{d\sigma}{d^3p}^{n=fix} \propto (1-x)^{n-3}, \qquad x = \frac{P_\mu p^\mu}{M^2/2}$$

The multiplicity distribution

$$P(n) = \binom{n+r-1}{r-1} \tilde{p}^n (1-\tilde{p})^r$$

The *n-averaged* distribution

$$p^0 \frac{d\sigma}{d^3p} = A\left\{\left(1+\frac{\tilde{p}}{1-\tilde{p}}x\right)^{-r-3} - \sum_3^{n_0-1} P(n)\, n f_n(x)\right\}$$

What is T?

If in a single event / jet, we have equipartition:

$$1\,event: \quad \frac{E_{event}}{N_{event}} = D\,T_{event}$$

On the average, we have:

$$\frac{E}{N} = \frac{\int \epsilon f_{TS}(\epsilon)}{\int f_{TS}(\epsilon)} = \frac{D\,T}{1-(q-1)(D+1)}$$

(m ≈ 0 particles)