Double gluon distribution from the single gluon distribution

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Work in collaboration with
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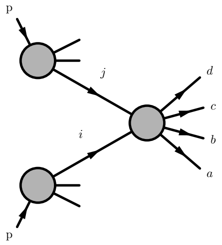
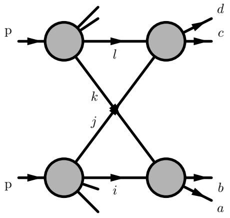
Single vs Double parton scattering





- Double Parton Scattering (DPS) seen for the first time by CDF and D0 in $\gamma +$ 3jets final state ($\gamma +$ jet $\times$ 2 jets)
- Smoking guns: absence of angular and momentum correlations in final states like 4 jets, $2\,l + 2\,\nu$, $W + 2$ jets enhanced $c\bar{c}c\bar{c}$ production, Mueller-Navelet jets ...
  CMS collaboration, JHEP 1403 (2014) 032, CMS preliminary, http://inspirehep.net/record/1405348, Maciula, Szczurek, van Hameren, Phys.Rev. D89 (2014) no.9, 094019 , Kutak, Maciula, M.S., Szczurek, van Hameren, arXiv:1602.06814 , Ducloué, Szymanowski,Wallon, Phys.Rev. D92 (2015) no.7, 076002
- The higher the energy, the smaller $x_{1,2}$ have to be to produce energetic enough final states $\Rightarrow$ DPS more significant at LHC
- Requires double parton distributions !

Evolution for sPDFS

Single parton scattering cross section:

$$\sigma^S = \sum_{i,j} \int dx_1\, dx_2\, D_i(x_1, \mu_F)\, D_j(x_2, \mu_F) \times \frac{1}{2\hat{s}} \prod_{l=i}^{4} \frac{d^3 k_l}{(2\pi)^3 2E_l} (2\pi)^4\, \delta\,(P_i - P_f)\, \overline{|\mathcal{M})|^2}$$

- Parton emission with $k_\perp \in [\Lambda_{QCD}, Q]$ makes single PDFs (sPDFs) scale-dependent
- Evolution is described by the well known DGLAP equations:

$$\frac{\partial}{\partial lnQ^2} D_f(x, Q) = \frac{\alpha_s(Q)}{2\pi} \sum_{f'} \int_x^1 \frac{du}{u}\, \mathcal{P}_{ff'}\left(\frac{x}{u}\right)\, D_{f'}(u, Q)$$

- Initial conditions at an initial scale Q_0 for DGLAP equations are known very well from several groups' fits. For this talk we stick to the Durham MSTW2008 parameterization, Martin, Stirling, Thorne, Watt, Eur.Phys.J. C63 (2009) 189-285 :

$$D_f(x, Q_0) = \sum_i A_f^i\, x^{\alpha_f^i}\, (1 - x)^{\beta_f^i} \quad \textbf{Dirichlet distributions}$$

Introducing dPDFs

Double parton scattering cross section (see Gaunt's talk here and MPI@LHC 2015):

$$\sigma^D = \mathcal{S} \sum_{i,j,k,l} \int \Gamma_{ij}(x_1, x_2, b; t_1, t_2)\, \Gamma_{kl}(x'_1, x'_2, b; t_1, t_2)\, \hat{\sigma}(x_1, x'_1)\, \hat{\sigma}(x_2, x'_2)\, dx_1 dx_2 dx'_1 dx'_2 d^2 b$$

Usual assumption: separation of longitudinal and transverse DOFs:

$$\Gamma_{ij}(x_1, x_2, b; t_1, t_2) = D_h^{ij}(x_1, x_2; t_1, t_2)\, F^{ij}(b) = D_h^{ij}(x_1, x_2; t_1, t_2)\, F(b)$$

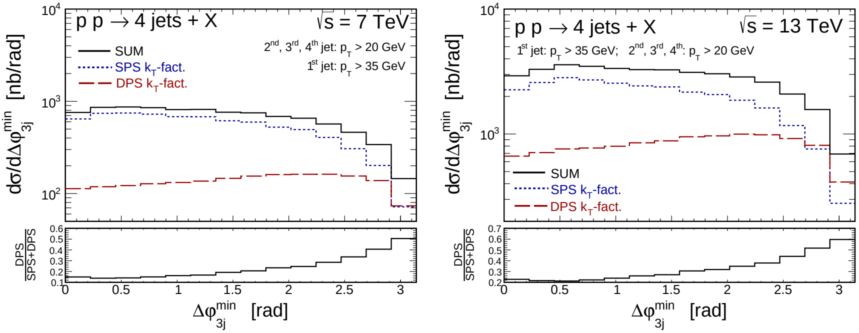
- Longitudinal correlations, morest often ignored or assumed to be negligible, especially at small x : $D_h^{ij}(x_1, x_2; t_1, t_2) = D^i(x_1; t_1)\, D^j(x_2; t_2)$
- Transverse correlation, assumed to be independent of the parton species, taken into account via $\sigma_{eff}^{-1} = \int d^2 b\, F(b)^2 \approx 15 mb$ (CDF and D0)

Usual final kind-of-crafty formula:

$$\sigma^D = \frac{\mathcal{S}}{\sigma_{eff}} \sum_{i_1, j_1, k_1, l_1; i_2, j_2, k_2, l_2} \sigma(i_1 j_1 \to k_1 l_1) \times \sigma(i_2 j_2 \to k_2 l_2)$$

DPS clear-cut discrimination for LHC

K. Kutak, R. Maciula, M. S., A. Szczurek, A. van Hameren, DPS in 4 jet production,
very preliminary



Differential cross sections for inclusive 4-jet production at 7 and 13 TeV
⇒ see Maciula's and M.S.'s talks, WG4 tomorrow

$$\Delta\phi_{3\,min} = \min\left(|\phi_i - \phi_j| + |\phi_k - \phi_j|\right)\ , \quad i,j,k \in [1,4]\ , \quad i \neq j \neq k$$

The appearance of such possibilities calls for more precise determination of the non-perturbative input.

Why factorised ansatz cannot hold: evolution

Factorisation of the dPDFs cannot hold for several reasons:

- For $t1 = t2 \equiv t$ an evolution equation at LLA similar to DGLAP exists:
  Kirschner, Phys. Lett. B84 (1979) 266
  Shelest, Snigiriev, Zinovjev, Phys. Lett. B113 (1982) 325

$$\frac{\partial}{\partial \ln Q^2} D_{f_1 f_2}(x_1, x_2, Q) = \frac{\alpha_s(Q)}{2\pi} \sum_{f'} \left\{ \int_{x_1}^{1-x_2} \frac{du}{u} \mathcal{P}_{f_1 f'}\left(\frac{x_1}{u}\right) D_{f' f_2}(u, x_2, Q) \right.$$

$$\left. + \int_{x_2}^{1-x_1} \frac{du}{u} \mathcal{P}_{f_2 f'}\left(\frac{x_2}{u}\right) D_{f_1 f'}(x_1, u, Q) + \frac{1}{x_1 + x_2} \mathcal{P}^R_{f' \to f_1 f_2}\left(\frac{x_1}{x_1 + x_2}\right) D_{f'}(x_1 + x_2, Q) \right\}$$

$\mathcal{P}_{f_1 f_2} =$ Altarelli-Parisi splitting function

$\mathcal{P}^R_{f \to f_1 f_2} =$ real emission part of the Altarelli-Parisi splitting function

Evolution predicts violation of factorised form of the ansatz

Why factorised ansatz cannot hold: sum rules

- $x_1 + x_2 \leq 1$ constraint not taken into account
- Sum rules are badly violated by a factorised ansatz: the probability of finding a second quark of flavour *a* **must** be correlated to the probability of finding a first one Gaunt, Stirling, JHEP 1003 (2010) 005

$$\sum_{f_1} \int_0^{1-x_2} dx_1\, x_1 D_{f_1 f_2}(x_1, x_2) = (1 - x_2) D_{f_2}(x_2)\,,$$

$$\int_0^{1-x_2} dx_1 \{D_{q f_2}(x_1, x_2) - D_{\bar{q} f_2}(x_1, x_2)\} = (N_q - \delta_{f_2 q} + \delta_{f_2 \bar{q}}) D_{f_2}(x_2)$$

where $q = u, d, s$ and $N_u = 2, N_d = 1, N_s = 0$.
Similar equation for $1 \leftrightarrow 2$; symmetry preserved by evolution.

Our approach for this work:
build initial conditions using the sum rules as constraints;
then solve evolution equarion...

Solving the constraints in the pure gluon case

- MSTW08 parameterisation at $Q_0 = 1GeV$: all parameters known

$$D_g(x) = \sum_{k=1}^{3} A_k\, x^{\alpha_k}\, (1-x)^{\beta}$$

- Hypothesis: Dirichlet distributions linear combinations for the dGDF:

$$D_{gg}(x_1, x_2) = \sum_{i=1}^{3} N_k\, (x_1 x_2)^{a_k}\, (1 - x_1 - x_2)^{b_k}$$

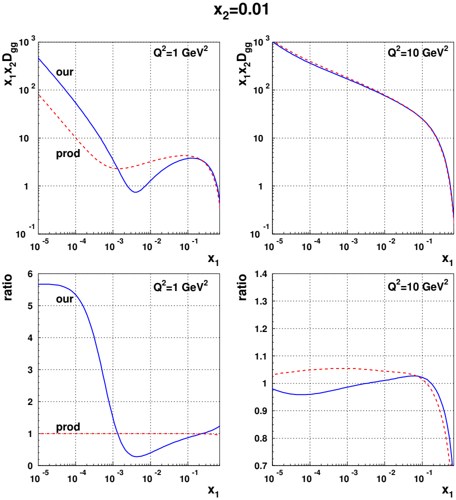
- Only sum rule for gluons is the momentum one

$$\sum_{f_1} \int_0^{1-x_2} dx_1\, x_1\, D_{f_1 f_2}(x_1, x_2) = (1-x_2)\, D_{f_2}(x_2)$$

- After solving, one ends up with the very simple constraints

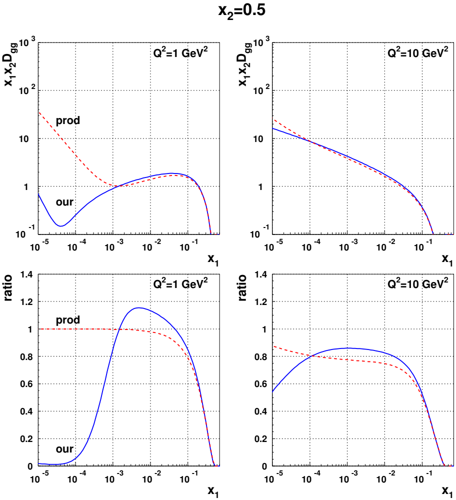
$$a_k = \alpha_k\,, \quad 2\,a_k + b_k + 3 = \alpha_k + \beta + 2\,, \quad N_k\, \Gamma(2 + a_k)\, \Gamma(1 + b_k) = A_k\, \Gamma(2 + \beta)$$

Evolution of the dGDF: $x_2 = 0.01$



- ratio $= \frac{D_{gg}(x_1, x_2)}{D_g(x_1)\, D_g(x_2)}$
- prod $=$ $D_g(x_1)\, D_g(x_2)\, \frac{(1-x_2-x_2)^2}{(1-x1)^2\,(1-x_2)^2}$ Gaunt,Stirling. Respects sum rules only approximately.
- Evolution washes out difference w.r.t. factorised case

Evolution of the dGDF: $x_2 = 0.5$



K. Golec-Biernat, E. Lewandowska, M.S., A. M. Stasto, Z. Snyder, Phys.Lett. B750 (2015) 559-564

Obstruction to including quarks

It is possible to include valence sum rules and extend the system to include quarks. Then the generalised expansion of a sPDF in terms of Dirichlet distributions is now

$$D_f(x) = \sum_k A_k \, x^{\alpha_k} \, (1-x)^{\beta_k}$$

- Reduces to a straightforward linear system in Mellin space
- Apparently $(2N_f+1)(N_f+1)$ equations for the same number of normalisation constants
- The system contains N_f redundant equations $\Rightarrow$ needs further assumptions
- Solving, for instance, k by k implies $\beta_k^{f_2} + \alpha_k^{f_1} = \beta_k^{f_1} + \alpha_k^{f_2}$, **manifestly violated by MSTW08** (does not work with other sets either)

Why: sPDFs simply do not contain enough information to fully determine dPDFs

Attempts based on a generalised valon model are at present underway:
W. Broniowski, K. Golec-Biernat, E. Ruiz Arriola, arXiv:1602.00254, C15-09-21.
In this approach, one first tries to reproduce known sPDFs from a light-cone Fock-space expansion at low energies. So far successful for the pion.
dPDFs in a few steps?

Summary and conclusions

- Factorised ansatz are not enough for Double Parton Scattering description
- A program to build explicitly dPDFs exploiting sum rules was successful for the pure gluon case
- For small longitudinal momentum fractions, the solution is never factorisable. Evolution washes this out significantly for high energies.
- Including quarks in this framework is still a challenge, because the resulting parameterisation of sPDFs does not quite fit the results in the literature. Attempts with light-front approaches are underway at present.

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Thank you for your attention !