Parton Shower Development

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Cross Section

The main goal is to calculate the cross section as precise as possible and gain access to all properties of the partonic final state.

Observable
- Probes the partonic final state
- IR safe

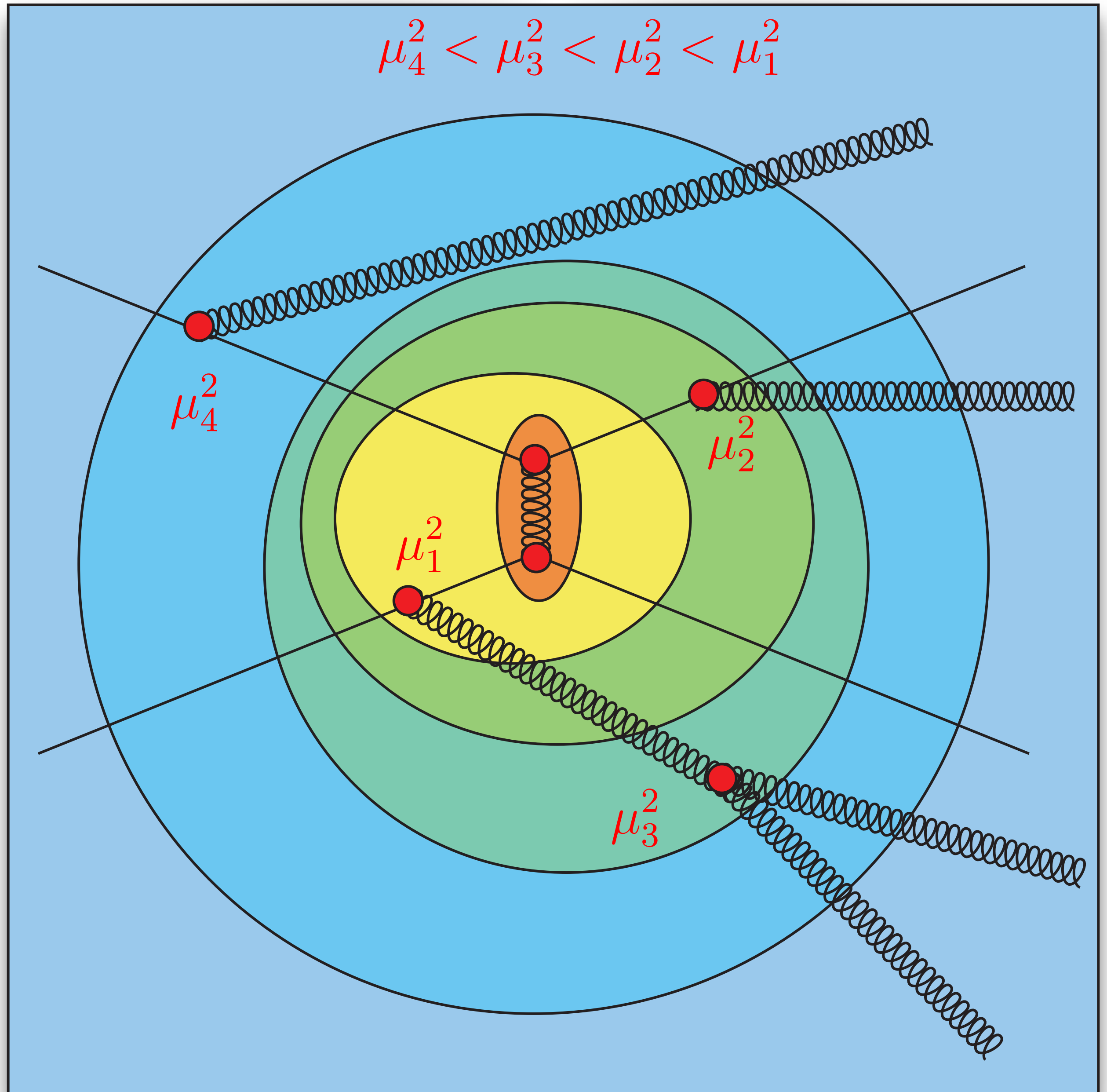
$$\sigma[O_J] = \sum_m \int d\{p,f\}_m \, \mathrm{Tr}\{\underbrace{\rho(\{p,f\}_m)}_{\text{QCD quantum density matrix}} \overbrace{O_J(\{p,f\}_m)}^{\text{Observable}}\} = \sum_{m=2}^{\infty} \big(1\big|\mathcal{O}_J\big|\rho_m\big) = \big(1\big|\mathcal{O}_J\big|\rho\big)$$

QCD quantum density matrix
- Summed over loops
- A matrix in the color and spin space

This is meaningful only in d = 4-2ϵ dimensions.

IMPORTANT: The basic object is the QCD density matrix and not the matrix element square.

IR Safe Observable



- Let us consider an infrared safe observable and it has a typical resolution scale μ_J^2 .
- This means every radiation under this scale is unresolvable and not visible by the observable.
- The observable is IR safe at μ_J^2, so the soft and collinear radiation are integrated out.
- But integrating over the unresolved phase space leads to singularities.
- The big challenge in the N^{k}LO calculation to arrange the cancellation of the IR singularities and do the integrals in *d=4* dimensions.
- Fortunately these singularities has universal structure that allows us to do NLO, NNLO,… calculation in process independent way.

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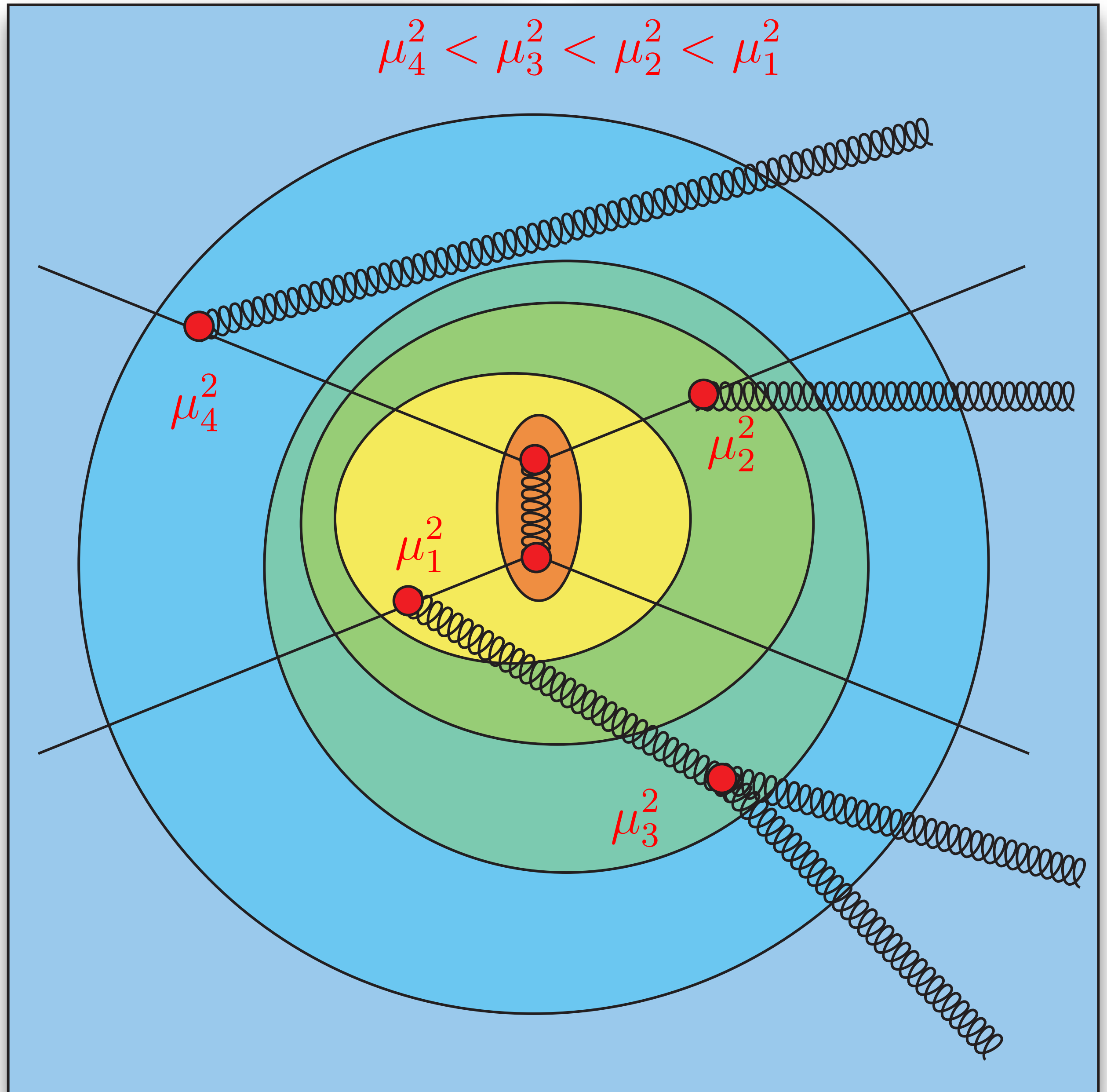

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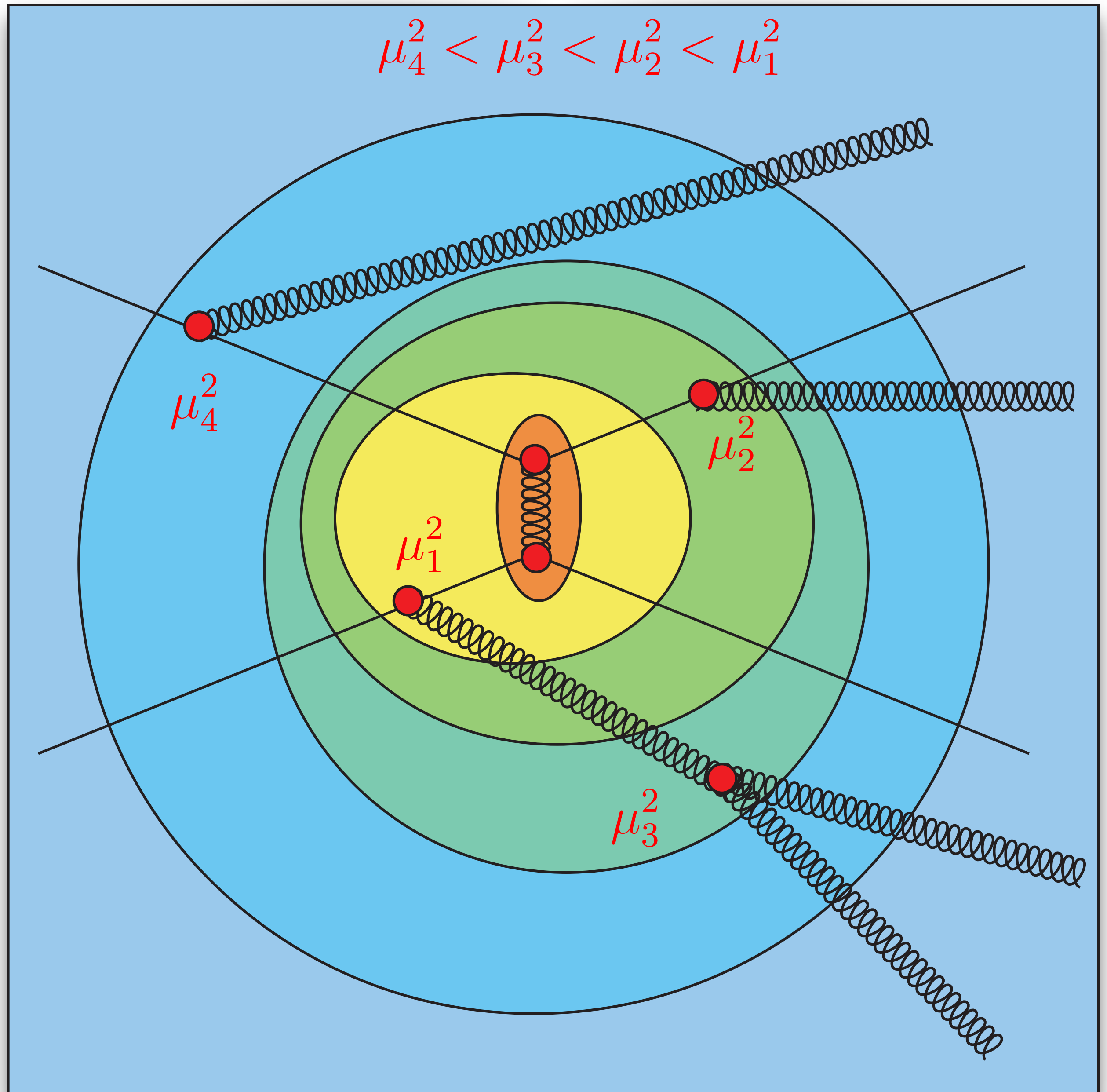

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To understand parton showers, first we should do N$^\infty$LO calculation…

…at least formally, of course.

Factorization

We know that the cross section is factorizes to a product of the PDFs and the "hard cross section". Thus, our all order expression can be written as

Collins, Soper, Sterman

$$\sigma[O_J] = \big(1\big|\big[\underbrace{\mathcal{F}(\mu^2)}_{\text{PDFs of the incoming hadrons}} \circ \overbrace{\mathcal{Z}_F(\mu^2)}^{\text{Removes the initial state collinear } 1/\epsilon \text{ singularities.}}\big]\mathcal{O}_J(\mu_J^2)\,\underbrace{\big|\hat{\rho}(\mu^2)\big)}_{\text{Still sum of zillions of Feynman graphs, and each graph comes with very reach and colorful singularity structure.}} + \mathcal{O}\big(1\mathrm{GeV}^2/\mu_J^2\big)$$

This is still meaningful only in d = 4-2ϵ dimensions.

$$\mu_R^2 = \mu_F^2 = \mu^2$$

Note, that the product

$$\big[\mathcal{F}(\mu_1^2) \circ \mathcal{Z}_F(\mu_1^2)\big] = \big[\mathcal{F}(\mu_2^2) \circ \mathcal{Z}_F(\mu_2^2)\big]$$

is independent of the scale choice and this leads to the well known DGLAP equation.

Singular Operator

Since the QCD amplitudes has universal factorization properties both for real and virtual emission in the IR sensitive regions, thus we can define a singular operator

$$\big|\hat{\rho}(\mu^2)\big) = \overbrace{\mathcal{D}(\mu^2)}^{\text{All the IR singularity resides to here}} \underbrace{\left[\mathcal{D}^{-1}(\mu^2)\big|\hat{\rho}(\mu^2)\big)\right]}_{\substack{\text{Finite and well defined} \\ \text{in } d = 4 \text{ dimension.} \\ \text{This is the hard state.}}} = \mathcal{D}(\mu^2)\,\big|\hat{\rho}_H(\mu^2)\big)$$

$\mathcal{D}(\mu^2)$ describes all the radiation that happens under the scale μ^2.

Cross Section

We would like to commute the observable and the singular operator to be able the perform the integrals over the potentially unresolvable phase space region.

$$\sigma[O_J] = \big(1\big|\big[\mathcal{F}(\mu^2) \circ \mathcal{Z}_F(\mu^2)\big]\mathcal{O}_J(\mu_J^2)\mathcal{D}(\mu^2)\big|\hat{\rho}_H(\mu^2)\big)$$

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$$+\mathcal{O}\Big(\alpha_s^n \log^{2n}\big(\min\{1,\, \mu_J^2/\mu^2\}\big)\Big)$$

Now we can perform the integral over the unresolved phase space region and the singularities get cancelled between the real and virtual contributions. So,

$$\big(1\big|\big[\mathcal{F}(\mu^2)\circ \mathcal{Z}_F(\mu^2)\big]\mathcal{D}(\mu^2) = \big(1\big|\big[\mathcal{F}(\mu^2)\circ \underbrace{\mathcal{I}(\mu^2)}\big] = \text{finite}$$

Finite in d= 4 dimension. In the Catania-Seymour subtraction scheme this is called insertion operator.

Shower Cross Section

To get the shower cross section we have to insert a unit operator several times.

$$\sigma[O_J] = \big(1\big|\big[\mathcal{F}(\mu^2) \circ \mathcal{Z}_F(\mu^2)\big]\,\mathcal{O}_J(\mu_J^2)\mathcal{D}(\mu^2)\big|\hat{\rho}_H(\mu^2)\big)$$

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$$\mathcal{D}(\mu^2)$$

$$\big|\hat{\rho}_H(\mu^2)\big)$$

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$$\mathcal{D}(\mu^2)\,\mathcal{F}^{-1}(\mu^2)$$
$$\mathcal{F}(\mu^2)\big|\hat{\rho}_H(\mu^2)\big)$$

The state $\mathcal{F}(\mu^2)\big|\hat{\rho}_H(\mu^2)\big)$ is like a “hard cross section”, PFDs times the parton level cross section. This is finite in $d = 4$ dimension and the “matching” to $N^k LO$ is done.

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$$\mathcal{D}^{-1}(\mu_\mathrm{f}^2)\mathcal{D}(\mu^2)\,\mathcal{F}^{-1}(\mu^2)$$
$$\mathcal{F}(\mu^2)\big|\hat{\rho}_H(\mu^2)\big)$$

The operator $\mathcal{D}(\mu^2)$ has all the singularities, but this singularities can be factorized out at a lower scale than μ^2. We choose

$$\mu_J^2 \gg \mu_\mathrm{f}^2 \approx 1\mathrm{GeV}^2 \quad .$$

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$$\mathcal{F}(\mu_{\rm f}^2)\,\mathcal{D}^{-1}(\mu_{\rm f}^2)\,\mathcal{D}(\mu^2)\,\mathcal{F}^{-1}(\mu^2)$$
$$\mathcal{F}(\mu^2)\big|\hat\rho_H(\mu^2)\big)$$

The operator $\mathcal{D}(\mu_f^2)$ describes the radiations under the scale $\mu_f^2 \ll \mu_J^2$. The observable $\mathcal{O}_J(\mu_J^2)$ has no chance to resolve anything. Thus, we have

$$\mathcal{O}_J(\mu_J^2)\mathcal{D}(\mu_f^2) = \mathcal{D}(\mu_f^2)\mathcal{O}_J(\mu_J^2) + O(1GeV^2/\mu_J^2) \quad .$$

(That's why we need infrared safety.)

Shower Cross Section

To get the shower cross section we have to insert a unit operator several times.

$$\sigma[O_J] = \big(1\big|\big[\mathcal{F}(\mu_{\rm f}^2) \circ \mathcal{Z}_F(\mu_{\rm f}^2)\big]\mathcal{D}(\mu_{\rm f}^2)\,\mathcal{O}_J(\mu_J^2)\,\mathcal{F}^{-1}(\mu_{\rm f}^2)$$
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$$\mathcal{F}(\mu^2)\big|\hat{\rho}_H(\mu^2)\big)$$

Since $[\mathcal{F}(\mu^2) \circ \mathcal{Z}_F(\mu^2)]$ is independent of the scale we can replace it by

$$[\mathcal{F}(\mu_f^2) \circ \mathcal{Z}_F(\mu_f^2)]\ .$$

Shower Cross Section

To get the shower cross section we have to insert a unit operator several times.

$$\begin{aligned}\sigma[O_J] = \big(1\big|\big[\mathcal{F}(\mu_{\mathrm{f}}^2)\circ\mathcal{I}(\mu_{\mathrm{f}}^2)\big]\mathcal{F}^{-1}(\mu_{\mathrm{f}}^2)\mathcal{O}(\mu_J^2)\\ \mathcal{F}(\mu_{\mathrm{f}}^2)\,\mathcal{D}^{-1}(\mu_{\mathrm{f}}^2)\,\mathcal{D}(\mu^2)\,\mathcal{F}^{-1}(\mu^2)\\ \mathcal{F}(\mu^2)\big|\hat{\rho}_H(\mu^2)\big)\end{aligned}$$

Now we can perform the integral over the unresolved phase space regions in

$$\big(1\big|\big[\mathcal{F}(\mu^2)\circ\mathcal{Z}_F(\mu^2)\big]\mathcal{D}(\mu^2)$$

and the sigularities cancel each other. The result is well defined in $d = 4$ dimensions.

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$$\mathcal{U}(\mu_\mathrm{f}^2,\mu^2)$$
$$\mathcal{F}(\mu^2)\big|\hat{\rho}_H(\mu^2)\big)$$

The shower evolution operator is defined by

$$\mathcal{U}(\mu_f^2,\mu^2) = \mathcal{F}(\mu_f^2)\,\mathcal{D}^{-1}(\mu_f^2)\,\mathcal{D}(\mu^2)\,\mathcal{F}^{-1}(\mu^2) \quad .$$

This is well defined in $d = 4$ dimensions.

Singular Operator

- If we can define operator $D(\mu^2)$ order by order then we have fixed order subtractions scheme and parton shower.
- Does it exist? — I think YES.
  - KLN theorem
  - PDF factorization theorem
  - QCD is a renormalizable theory
  - The proof of this is not easy at all and it is not the purpose of this talk.
- Is it easy to define order-by-order? — Of course NOT.
- Can we do it at NLO level? — YES, we can!
  - Still better be “Careful with that Axe, Eugene”!

Shower Evolution Operator

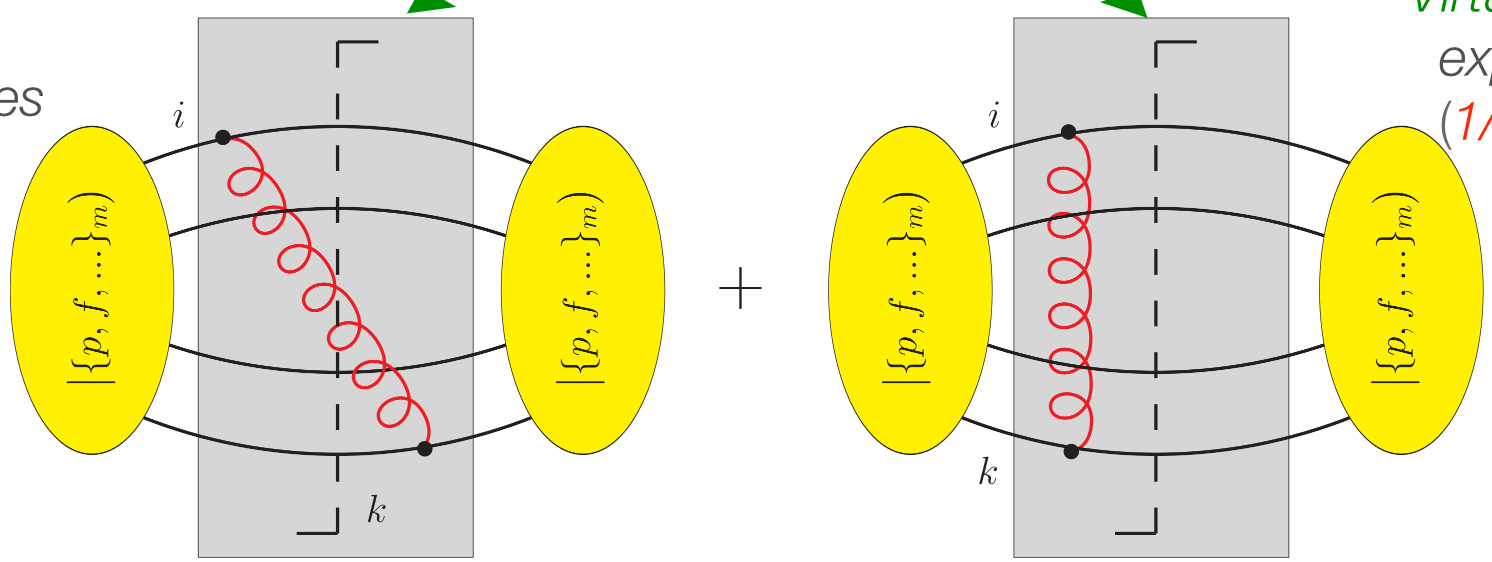
We might want to write the shower operator in a more familiar form. Well, that is

$$\mathcal{U}(\mu_2^2,\mu_1^2) = \mathcal{F}(\mu_2^2)\,\mathcal{D}^{-1}(\mu_2^2)\,\mathcal{D}(\mu_1^2)\,\mathcal{F}^{-1}(\mu_1^2)$$
$$= \mathbb{T}\exp\left\{\int_{\mu_2^2}^{\mu_1^2} d\mu^2 \left[\mathcal{D}(\mu^2)\,\mathcal{F}^{-1}(\mu^2)\right]^{-1}\frac{d}{d\mu^2}\left[\mathcal{D}(\mu^2)\,\mathcal{F}^{-1}(\mu^2)\right]\right\}$$
$$= \mathbb{T}\exp\left\{\int_{\mu_2^2}^{\mu_1^2} \frac{d\mu^2}{\mu^2}\frac{\alpha_s(\mu^2)}{2\pi}\left(\mathcal{S}^{(1)}(\mu^2) + \frac{\alpha_s(\mu^2)}{2\pi}\mathcal{S}^{(2)}(\mu^2) + \cdots\right)\right\}$$

At first oder level

$$\mathcal{D}(\mu^2) = 1 + \frac{\alpha_s(\mu^2)}{2\pi}\left[\mathcal{H}_R(\mu^2) + \mathcal{H}_V(\mu^2)\right] + \cdots$$

Real contributions
implicit singularities
($1/\epsilon$ and $1/\epsilon^2$)



Virtual contributions
explicit singularities
($1/\epsilon$ and $1/\epsilon^2$)

Evolution Kernel

Skipping a rather long derivation, the first order evolution kernel is

Phase part

Breaks color coherence.
Leads to “super-leading-logs”

Unitary part of the shower

This term alone preserves normalization.

$$
\frac{1}{\mu^2}\mathcal{S}^{(1)}(\mu^2)\mathcal{F}(\mu^2) = \overbrace{\mathcal{F}(\mu^2)\frac{d\mathcal{H}_R(\mu^2)}{d\mu^2} - \Big[\mathcal{F}(\mu^2)\circ\frac{d\mathcal{V}_R(\mu^2)}{d\mu^2}\Big]} + \overbrace{\mathrm{i}\,\mathrm{Im}\frac{d\mathcal{H}_V(\mu^2)}{d\mu^2}\mathcal{F}(\mu^2)}
$$
$$
+ \Big[\mathcal{F}(\mu^2)\circ\frac{d}{d\mu^2}\Big(\underbrace{\mathrm{Re}\big[\mathcal{H}_V(\mu^2)\big] + \mathcal{V}_R(\mu^2) + \mathcal{Z}_F^{(1)}(\mu^2)}_{\mathcal{I}^{(1)}(\mu^2)}\Big)\Big]
$$

“Threshold” part

All the threshold logs sit here.
Breaks probability conservation even in LC approximation.

The *inclusive splitting* operator is defined by the unitary condition

$$
\big(1\big|\big[\mathcal{F}(\mu^2)\circ\mathcal{V}_R(\mu^2)\big] = \big(1\big|\mathcal{F}(\mu^2)\mathcal{H}_R(\mu^2)
$$

Available Parton Showers

Most widely used

- Herwig
  - New name (Herwig++ ➪ Herwig7), new logo (***H7***)
  - Default shower is still the angular ordered one, Herwig/Angular
  - Dipole shower based on Catan-Seymour splitting functions, Herwig/Dipole
- Pythia
  - Standard Pythia dipole shower (dipole picture is fully implemented only in the final state)
  - Pythia/Dire another implementation of the CS based dipole shower
- Sherpa
  - Default shower is the CS based dipole shower
  - Sherpa/Dire same as in Pythia

Other parton showers: Ariadne, Deductor, Vincia,…

Unitary Parton Shower

The unitary parton showers are motivated by the classical picture that a parton either splits or not. Such a model relies on a *rather crude approximation*, dropping the phase and threshold terms from the shower evolution kernel.

$$\frac{1}{\mu^2}\mathcal{S}^{(1)}(\mu^2)\mathcal{F}(\mu^2) = \mathcal{F}(\mu^2)\frac{d\mathcal{H}_R(\mu^2)}{d\mu^2} - \left[\mathcal{F}(\mu^2) \circ \frac{d\mathcal{V}_R(\mu^2)}{d\mu^2}\right]$$

$$+\mathrm{i}\,\mathrm{Im}\frac{d\mathcal{H}_V(\mu^2)}{d\mu^2}\mathcal{F}(\mu^2) + \left[\mathcal{F}(\mu^2) \circ \frac{d\mathcal{I}^{(1)}(\mu^2)}{d\mu^2}\right]$$

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Dropping the threshold part we lose shower corrections to the absolute normalization.

The claim is that we are still able to describe accurately enough the shape of the distributions for sufficiently inclusive observables.

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J. Gaunt's talk on Glauber gluon tomorrow

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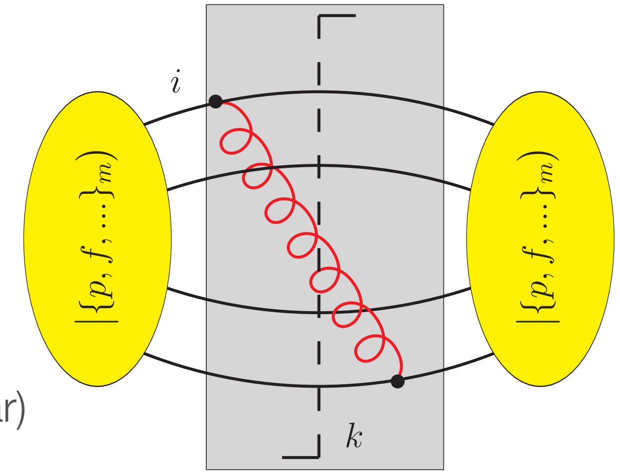
Gain from dropping these terms: It is much much EASIER to implement!

Splitting Operator

We have to define only the real splitting operator then everything is done. The inclusive splitting operator is defined from unitary condition.



$$\left(\{\hat{p}, \hat{f}, \hat{c}', \hat{c}\}_{m+1}\middle|\mathcal{H}_R(\mu^2)\middle|\{p, f, c', c\}_m\right) =$$

Evolution scale,
Allows the phase space region only around the singularities.

Momentum and flavor mappings
- 1→ 2 global recoil (Deductor, Herwig/Angular)
- 2→ 3 local recoil (Ariadne, Vincia)
- 1→ 2 local recoil (everybody else)

$$\sum_{l,k} \overbrace{\Theta\left(\{\hat{p}, \hat{f}\}_{m+1}, \mu^2\right)} \overbrace{\left(\{\hat{p}, \hat{f}\}_{m+1}\middle|\mathcal{P}_l\middle|\{p, f\}_m\right)}$$

$$\times \underbrace{\Psi_{l,k}(\{\hat{p}, \hat{f}\}_{m+1})} \underbrace{\left(\{\hat{c}', \hat{c}\}_{m+1}\middle|t_k^\dagger \otimes t_l + t_l^\dagger \otimes t_k\middle|\{c', c\}_m\right)}$$

Splitting function

Color operator
Takes care about color correlations
Usually we approximate it, because it is impossible to exponentiate it in practice.

Splitting Function

The splitting function in different implementation differs only in power suppressed terms.

Collinear term

This is the contribution of the pure collinear radiation

$$\Psi_{lk} = \frac{1}{\hat{p}_l \cdot \hat{p}_{m+1}} \left[\underbrace{A_{lk} \frac{2\hat{p}_l \cdot \hat{p}_k}{\hat{p}_k \cdot \hat{p}_{m+1}}} + \overbrace{H_{ll}^{\text{coll}}(\{\hat{f}, \hat{p}\}_{m+1})} \right]$$

Soft, soft-collinear term

A_{lk} function distributes the soft gluon along the "jet" directions

Important:

$$A_{lk} + A_{kl} = 1$$

In the collinear limit it goes to the usual Altarelli-Parisi splitting function

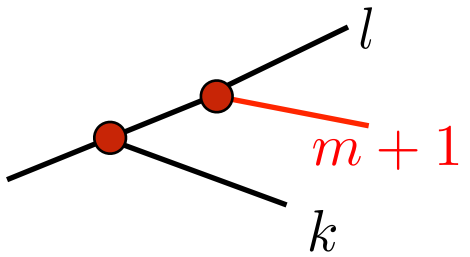
Angular Ordered Shower

Everybody consider it as special because of the angular ordering but it is partitioned dipole shower with some special choice of ordering variable and with an extra approximation. A special chose of the A_{lk} function is equivalent to the azimuthal angle averaging.

$$A_{lk} = \theta(\vartheta_{l,m+1} < \vartheta_{l,k}) \frac{1 - \cos\vartheta_{m+1,k}}{1 - \cos\vartheta_{l,k}}$$

$$A_{lk} + A_{kl} \approx 1$$

If we do angular ordering this condition is trivial and the splitting function becomes independent of parton k.



$$\Psi_{l,k} = \frac{\overbrace{\theta(\vartheta_{l,m+1} < \vartheta_{l,k})}}{\hat{p}_l \cdot \hat{p}_{m+1}} \left[\frac{2\hat{p}_l \cdot \hat{Q}}{\hat{p}_{m+1} \cdot \hat{Q}} + H_{ll}^{\mathrm{coll}}\left(\{\hat{f}, \hat{p}\}_{m+1}\right) \right]$$

The color structure simplifies a lot and we can exponentiate the color operators without approximation

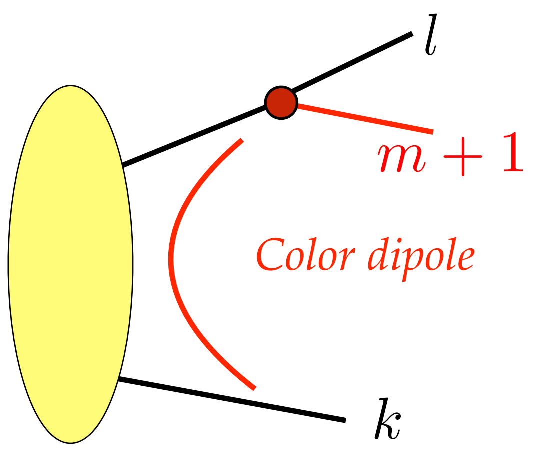
$$\sum_k \left[t_k^\dagger \otimes t_l + t_l^\dagger \otimes t_k \right] = -2t_l^\dagger \otimes t_l$$

Implementations: HERWIG/ANGULAR

Dipole Showers

There are Pythia dipole shower and a bunch of implementation of the parton shower based on the Catani-Seymour factorization formulae. In fact Pythia is organized in the same way.

Ordering variable: is the transverse momentum of the emitted parton respect to partons l and k



Soft partitioning function:

$$A_{lk} = \frac{\hat{p}_k \cdot \hat{p}_{m+1}}{\hat{p}_k \cdot \hat{p}_{m+1} + \hat{p}_l \cdot \hat{p}_{m+1}}$$

Momentum mapping: Local, always the color connected parton absorbs the recoil. Usually in initial-initial state dipoles the whole event absorbs the recoil. Herwig/Dipole uses global mapping also for the initial-final state dipoles.

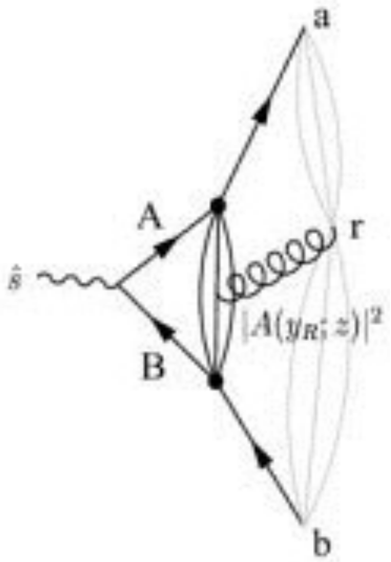
In leading color approximation the soft gluon color structure becomes simple and the splitting kernel can be exponentiated.

Implementations: HERWIG/DIPOLE, PYTHIA/DIRE, PYTHIA/DEFAULT, SHERPA/DEFAULT, SHERPA/DIRE

Antenna Dipole Showers

The antenna dipole shower is rather a *reorganization of the leading color* partitioned dipole *shower*.

$$\mathcal{H}_{lk}^{\text{part}}(t) \propto [\mathcal{P}_l\, A_{lk} + \mathcal{P}_k\, A_{kl}] \frac{\hat{p}_l \cdot \hat{p}_k}{\hat{p}_{m+1} \cdot \hat{p}_l\ \hat{p}_{m+1} \cdot \hat{p}_k}$$



The antenna shower tries to remove the ambiguity of the soft partitioning function A_{lk} by using a new momentum mapping

$$\mathcal{H}_{lk}^{\text{ant}}(t) \propto \mathcal{P}_{lk} \frac{\hat{p}_l \cdot \hat{p}_k}{\hat{p}_{m+1} \cdot \hat{p}_l\ \hat{p}_{m+1} \cdot \hat{p}_k}$$

Now the freedom to choose A_{lk} function resides in the freedom to choose P_{lk}.

Momentum mapping: Local 2→3 mapping. The two partons in the dipole emit an extra gluon, momentum conversation in the antenna.

Implementations: ARIADNE, VINCIA

Deductor

Ordering variable: The default ordering is Lambda (virtuality divided by the mother parton energy). Also kT (with respect to the emitter and rest of the event), angular.

Soft partitioning function:

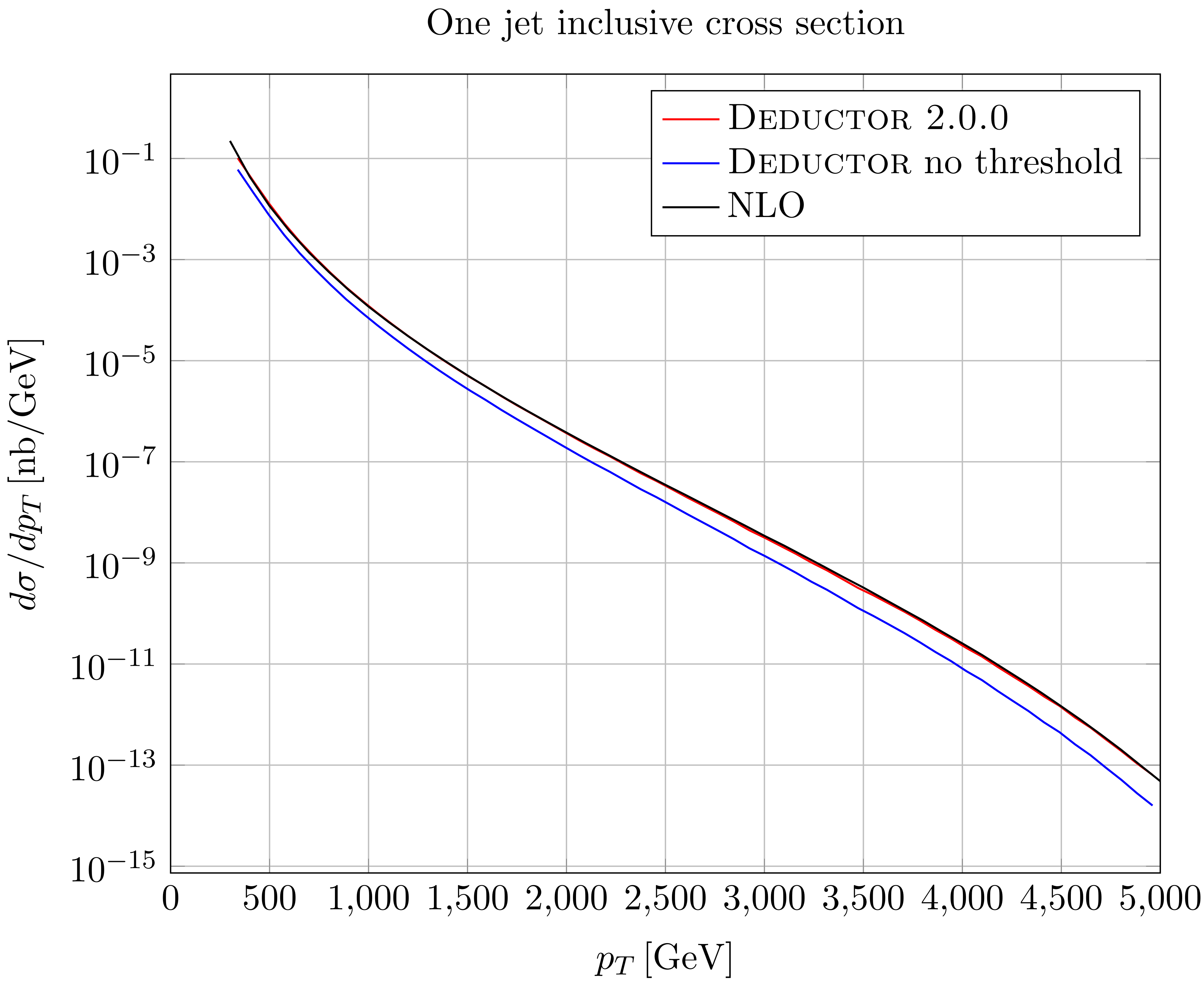
$$A_{lk} = \frac{\hat{p}_k \cdot \hat{p}_{m+1}\, \hat{p}_l \cdot \hat{Q}}{\hat{p}_k \cdot \hat{p}_{m+1}\, \hat{p}_l \cdot \hat{Q} + \hat{p}_l \cdot \hat{p}_{m+1}\, \hat{p}_k \cdot \hat{Q}}$$

Momentum mapping: Global mapping, the whole event absorbs the recoil.

Color treatment: Leading Color+ (LC+) It can evolve color interference contributions. Essentially it is LC at amplitude level.

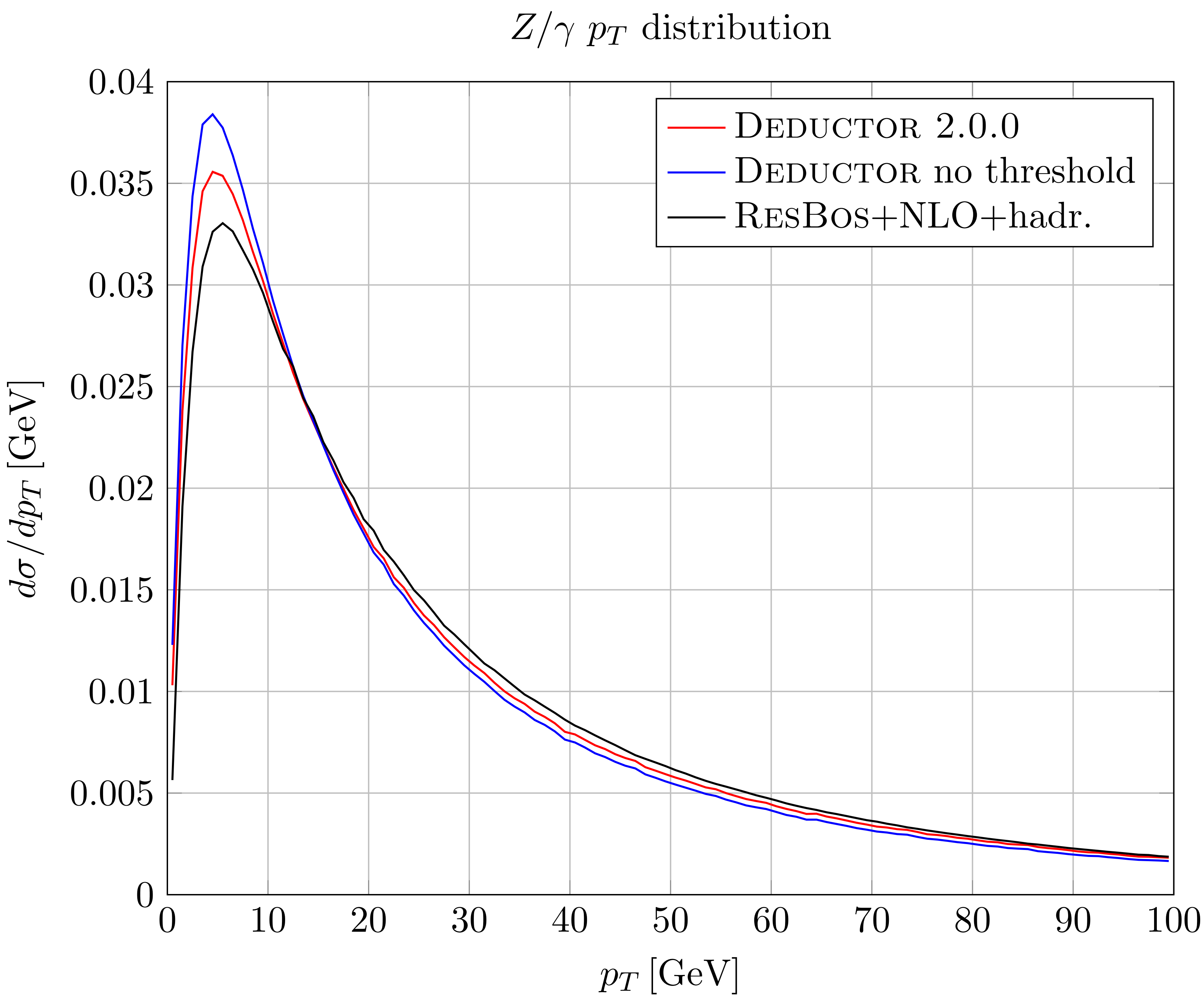
$$\frac{1}{\mu^2}\mathcal{S}^{(1)}(\mu^2)\mathcal{F}(\mu^2) = \mathcal{F}(\mu^2)\frac{d\mathcal{H}_R(\mu^2)}{d\mu^2} - \Big[\mathcal{F}(\mu^2) \circ \frac{d\mathcal{V}_R(\mu^2)}{d\mu^2}\Big] + \underbrace{\Big[\mathcal{F}(\mu^2) \circ \frac{d\mathcal{I}^{(1)}(\mu^2)}{d\mu^2}\Big]}_{\text{We include the threshold part!}}$$

Inclusive Jet Production



Ecm = 13TeV
Anti-kT R = 0.4
|y| < 2
CT14

Drell-Yan ZpT



Ecm = 8TeV
Q > 400GeV
CT14

Conclusion

Conclusion

➡ I think parton showers has a bright future and become important tools to make real prediction.