Institute for Theoretical Physics

Eberhard Karls Universität Tübingen

Fragmentation Functions beyond Next-To-Leading Order

Hamburg, *12.04.2016*

Anderle, Kaufmann, Ringer, Stratmann

DIS 2016

Speaker:
Daniele Paolo ANDERLE

Outline

- Goal
- Time-Like Evolution
- Our e+e- NNLO Fit
- Improving: Small-z Resummation
- Conclusions & Outlook

TOWARDS A GLOBAL NNLO FF FIT

Anderle, Ringer, Stratmann

Ingredients needed to achieve the goal:

DATA SETS:

SI-e^+e^- → old: TPC(Phys. Rev. Lett 61, 1263 (1998)), SLD(Phys. Rev. D59,052001 (1999)), ALEPH(Phys. Lett. B357, 487 (1995)), DELPHI(Eur. Phys. J. C5, 585 (1998),Eur. Phys. J.C6, 19 (1999)) OPAL(Eur. Phys. J. C16, 407 (2000),Eur. Phys. J.C7, 369 (1999)), TASSO(Z. Phys.C42, 189 (1989))

SIDIS → old: EMC(Z. Phys. C52, 361 (1991)), JLAB(Phys. Rev. Lett. 98, 022001)

SI- p(anti-)p → old: CDF(Phys. Rev. Lett. 61,1819 (1988)), UA1(Nucl. Phys. B335,261 (1990)), UA2(Z. Phys. C27, 329 (1985))

Towards a Global NNLO FF Fit

Anderle, Ringer, Stratmann

Ingredients needed to achieve the goal:

DATA SETS:

SI-e^+e^- → new: BaBar(Phys. Rev. D 88, 032011 (2013)), Belle(Phys. Rev. Lett. 111, 062002 (2013))

SIDIS → new: HERMES(Ph.D. thesis, Erlangen Univ., Germany, September 2005), Compass(PoS DIS 2013, 202 (2013)), JLAB@12GeV

SI- p(anti-)p → new: Phenix(Phys. Rev. D 76,051106 (2007)), Alice(Phys. Lett. B 717, 162 (2012).), Brahms(Phys. Rev. Lett. 98, 252001 (2007)), Star(Phys. Rev. Lett. 97, 152302 (2006))

pp→ (Jet h)X → future: Star, CMS(JHEP 1210, 087 (2012)), Alice(arXiv:1408.5723), Atlas(Eur. Phys. J. C 71, 1795 (2011))

TOWARDS A GLOBAL NNLO FF FIT

Anderle, Ringer, Stratmann

Ingredients needed to achieve the goal:

NNLO EVOLUTION KERNELS:

Splitting functions →

NNLO-Non Singlet: Mitov, Moch, Vogt (Phys.Lett. B638 (2006) 61-67)

NNLO-Singlet: Moch, Vogt (Phys.Lett.B659 (2008) 290-296)

NNLO-Singlet: Almasy, Mitov, Moch, Vogt (Nucl.Phys. B854 (2012)) 133-152)

Both computed in x-Space and in Mellin Space

TOWARDS A GLOBAL NNLO FF FIT

Anderle, Ringer, Stratmann

Ingredients needed to achieve the goal:

NNLO COEFFICINT FUNCTIONS:

SI-e^+e^- → **x-Space**: Rijken, van Neerven (Phys.Lett.B386(1996)422, Nucl.Phys.B488(1997)233,Phys.Lett.B392(1997)207)

Mellin-Space: Mitov, Moch (Nucl.Phys.B751 (2006) 18-52)
Blümlein, Ravindran (Nucl.Phys.B749 (2006) 1-24)

SIDIS → NOT COMPUTED YET but work in progress

$$\gamma q' \to q\bar{q}q', \quad \gamma g \to q\bar{q}q'$$

Anderle, de Florian, Rotstein, Vogelsang

SI- p(anti-)p → NOT COMPUTED YET

pp→ (Jet h)X → NOT COMPUTED YET

TOWARDS A GLOBAL NNLO FF FIT

Anderle, Ringer, Stratmann

Ingredients needed to achieve the goal:

NNLO COEFFICINT FUNCTIONS:

SIDIS → Soft gluon Resummed results (can be expanded @ NNLO)

Anderle, Ringer, Vogelsang (Phys.Rev. D87 (2013) 094021,
Phys.Rev. D87 (2013) 3, 034014)

SI- p(anti-)p → Soft gluon Resummed results (can be expanded @ NNLO)

Work in progress for $\frac{d\sigma}{dp_T d\eta}^{(NNLL)}$ Hinderer, Ringer, Sterman, Vogelsang

pp→ (Jet h)X → Resummed results (can be expanded @ NNLO)

Work in progress from T. Kaufmann, Vogelsang

TOWARDS A GLOBAL NNLO FF FIT

Anderle, Ringer, Stratmann

Ingredients needed to achieve the goal:

NNLO COEFFICINT FUNCTIONS:

To include the last processes we need a

NNLO Mellin Space Fitting Program

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- Time-Like Evolution
- Our E+E- NNLO Fit
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The Time-Like Evolution

In the factorisation procedure, the absorption of *collinear singularities* by fragmentation functions (FF)(in case of massless partons) leads to scaling violation and the appearance of a factorisation scale μ_F

The scale dependance of FF is governed by the Time-Like DGLAP

$$\frac{\partial}{\partial \ln \mu_F^2} D_i^h(x, \mu_F^2) = \sum_j \int_x^1 \frac{dy}{y} P_{ji}\left(y, \alpha_s(\mu_F^2)\right) D_j^h\left(\frac{x}{y}, \mu_F^2\right)$$

Time-Like Splitting function perturbatively calculable $P_{ji}(y, \alpha_s) = \sum_{k=0} a_s^{k+1} P_{ji}^{(k)}(y)$

Usually rewritten into $2n_f - 1$ equations (charge conjugation and flavour symmetry)

NON-SINGLET

$$D^h_{\mathrm{NS};v} = \sum_{i=1}^{n_f} (D^h_{q_i} - D^h_{\bar{q}_i})$$

$$D^h_{\mathrm{NS};\pm} = (D^h_{q_i} \pm D^h_{\bar{q}_i}) - (D^h_{q_j} \pm D^h_{\bar{q}_j})$$

$$\frac{\partial}{\partial \ln \mu_F^2} D^h_{\mathrm{NS};\pm,v}(x,\mu_F^2) = P^{\pm,\mathrm{v}}(x,\mu_F^2) \otimes D^h_{\mathrm{NS};\pm,v}(x,\mu_F^2)$$

and two coupled

SINGLET

$$D^h_{\Sigma} = \sum_{i=1}^{n_f} (D^h_{q_i} + D^h_{\bar{q}_i})$$

$$D^h_g$$

$$\frac{\partial}{\partial \ln \mu_F^2} \begin{pmatrix} D^h_{\Sigma}(x,\mu_F^2) \\ D^h_g(x,\mu_F^2) \end{pmatrix} = \begin{pmatrix} P^{\mathrm{qq}} & 2n_f P^{\mathrm{gq}} \\ \frac{1}{2n_f} P^{\mathrm{qg}} & P^{\mathrm{gg}} \end{pmatrix} \otimes \begin{pmatrix} D^h_{\Sigma}(x,\mu_F^2) \\ D^h_g(x,\mu_F^2) \end{pmatrix}$$

The splitting functions are accordingly separated in the singlet and non-singlet sectors

NON-SINGLET

$$P_{\mathrm{ns}}^{\pm} = P_{\mathrm{qq}}^{\mathrm{v}} \pm P_{\mathrm{q\bar{q}}}^{\mathrm{v}}$$

$$P_{\mathrm{ns}}^{\mathrm{v}} = P_{\mathrm{qq}}^{\mathrm{v}} - P_{\mathrm{q\bar{q}}}^{\mathrm{v}} + n_f (P_{\mathrm{qq}}^{\mathrm{s}} - P_{\mathrm{q\bar{q}}}^{\mathrm{s}}) \equiv P_{\mathrm{ns}}^{-} + P_{\mathrm{ns}}^{\mathrm{s}}$$

SINGLET

$$P_{\mathrm{qq}} = P_{\mathrm{ns}}^{+} + n_f (P_{\mathrm{qq}}^{\mathrm{s}} + P_{\mathrm{\bar{q}q}}^{\mathrm{s}}) \equiv P_{\mathrm{ns}}^{+} + P_{\mathrm{ps}}$$

$$P_{\mathrm{gq}} \equiv P_{\mathrm{gq}_i} = P_{\mathrm{g\bar{q}}_i}$$

$$P_{\mathrm{qg}} \equiv n_f \, P_{\mathrm{q}_i\mathrm{g}} = n_f \, P_{\mathrm{\bar{q}}_i\mathrm{g}}$$

The splitting functions are accordingly separated in the singlet and non-singlet sectors

NON-SINGLET

$$P_{\mathrm{ns}}^{\pm} = P_{\mathrm{qq}}^{\mathrm{v}} \pm \cancel{P_{\mathrm{q\bar{q}}}^{\mathrm{v}}}$$

$$P_{\mathrm{ns}}^{\mathrm{v}} = P_{\mathrm{qq}}^{\mathrm{v}} - \cancel{P_{\mathrm{q\bar{q}}}^{\mathrm{v}}} + n_f\left(\cancel{P_{\mathrm{qq}}^{\mathrm{s}}} - \cancel{P_{\mathrm{q\bar{q}}}^{\mathrm{s}}}\right) \equiv P_{\mathrm{ns}}^{-} + \cancel{P_{\mathrm{ns}}^{\mathrm{s}}}$$

@LO

$$P_{\mathrm{ns}}^{\mathrm{v}} = P_{\mathrm{ns}}^{\pm}$$

SINGLET

$$P_{\mathrm{qq}} = P_{\mathrm{ns}}^{+} + n_f\left(\cancel{P_{\mathrm{qq}}^{\mathrm{s}}} + \cancel{P_{\mathrm{q\bar{q}}}^{\mathrm{s}}}\right) \equiv P_{\mathrm{ns}}^{+} + \cancel{P_{\mathrm{ps}}}$$

$$P_{\mathrm{gq}} \equiv P_{\mathrm{gq}_i} = P_{\mathrm{g\bar{q}}_i}$$

$$P_{\mathrm{qg}} \equiv n_f\, P_{\mathrm{q}_i\mathrm{g}} = n_f\, P_{\mathrm{\bar{q}}_i\mathrm{g}}$$

The splitting functions are accordingly separated in the singlet and non-singlet sectors

NON-SINGLET

$$P_{\mathrm{ns}}^{\pm} = P_{\mathrm{qq}}^{\mathrm{v}} \pm P_{\mathrm{q\bar{q}}}^{\mathrm{v}}$$

$$P_{\mathrm{ns}}^{\mathrm{v}} = P_{\mathrm{qq}}^{\mathrm{v}} - P_{\mathrm{q\bar{q}}}^{\mathrm{v}} + n_f (P_{\mathrm{qq}}^{\mathrm{s}} - P_{\mathrm{q\bar{q}}}^{\mathrm{s}}) \equiv P_{\mathrm{ns}}^{-} + P_{\mathrm{ns}}^{\mathrm{s}}$$

@NLO

$$P_{\mathrm{qq}}^{\mathrm{S}} = P_{\mathrm{q\bar{q}}}^{\mathrm{S}}$$

$$P_{\mathrm{ns}}^{\mathrm{v}} = P_{\mathrm{ns}}^{-}$$

SINGLET

$$P_{\mathrm{qq}} = P_{\mathrm{ns}}^{+} + n_f (P_{\mathrm{qq}}^{\mathrm{s}} + P_{\mathrm{\bar{q}q}}^{\mathrm{s}}) \equiv P_{\mathrm{ns}}^{+} + P_{\mathrm{ps}}$$

$$P_{\mathrm{gq}} \equiv P_{\mathrm{gq}_i} = P_{\mathrm{g\bar{q}}_i}$$

$$P_{\mathrm{qg}} \equiv n_f \, P_{\mathrm{q}_i\mathrm{g}} = n_f \, P_{\mathrm{\bar{q}}_i\mathrm{g}}$$

The splitting functions are accordingly separated in the singlet and non-singlet sectors

NON-SINGLET

$$P_{\mathrm{ns}}^{\pm} = P_{\mathrm{qq}}^{\mathrm{v}} \pm P_{\mathrm{q\bar{q}}}^{\mathrm{v}}$$

$$P_{\mathrm{ns}}^{\mathrm{v}} = P_{\mathrm{qq}}^{\mathrm{v}} - P_{\mathrm{q\bar{q}}}^{\mathrm{v}} + n_f (P_{\mathrm{qq}}^{\mathrm{s}} - P_{\mathrm{q\bar{q}}}^{\mathrm{s}}) \equiv P_{\mathrm{ns}}^{-} + P_{\mathrm{ns}}^{\mathrm{s}}$$

@NNLO

Responsable for $s, \ \bar{s}$ asymmetry

$$[s - \bar{s}](x, Q^2) \neq 0$$

German, Catani, de Florian, Vogelsang (arXiv:hep-ph/0406338)

SINGLET

$$P_{\mathrm{qq}} = P_{\mathrm{ns}}^{+} + n_f (P_{\mathrm{qq}}^{\mathrm{s}} + P_{\mathrm{\bar{q}q}}^{\mathrm{s}}) \equiv P_{\mathrm{ns}}^{+} + P_{\mathrm{ps}}$$

$$P_{\mathrm{gq}} \equiv P_{\mathrm{gq}_i} = P_{\mathrm{g\bar{q}}_i}$$

$$P_{\mathrm{qg}} \equiv n_f \, P_{\mathrm{q}_i\mathrm{g}} = n_f \, P_{\mathrm{\bar{q}}_i\mathrm{g}}$$

THE SOLUTION

We can solve the integro-differential DGLAP equation analytically in Mellin space at each fixed order since it becomes an Ordinary Differential Equation

$$\frac{\partial \boldsymbol{q}(N,a_s)}{\partial a_s} = \{\beta_{\mathrm{N^mLO}}(a_s)\}^{-1}\, \boldsymbol{P}_{\mathrm{N^mLO}}(N,a_s)\, \boldsymbol{q}(N,a_s)$$

$$= -\frac{1}{\beta_0 a_s}\Big[\boldsymbol{P}^{(0)}(N) + a_s\Big(\boldsymbol{P}^{(1)}(N) - b_1\boldsymbol{P}^{(0)}(N)\Big) + a_s^2\Big(\boldsymbol{P}^{(2)}(N) - b_1\boldsymbol{P}^{(1)}(N) + (b_1^2 - b_2)\boldsymbol{P}^{(0)}(N)\Big) + \dots\Big]\, \boldsymbol{q}(N,a_s)$$

$$f(N,\alpha_s) = \int_0^1 dy\, y^{N-1} f(y,\alpha_s) \qquad N \in \mathbb{C}$$

where here $\mathbf{P}(N,\alpha_S)$ and $\mathbf{q}(N,\alpha_S)$ are the Mellin-Transform of either singlet or non-singlet splitting function and FF respectively the general solution can be expressed in terms of the evolution matrices ***U*** (constructed from the splitting functions) as a simple multiplication

$$\begin{aligned} \boldsymbol{q}(N, a_s) &= \boldsymbol{U}(N, a_s)\,\boldsymbol{L}(N, a_s, a_0)\,\boldsymbol{U}^{-1}(N, a_0)\,\boldsymbol{q}(N, a_0) \\ &= \left[1 + \sum_{k=1}^{\infty} a_s^k\,\boldsymbol{U}_k(N)\right] \boldsymbol{L}(a_s, a_0, N) \left[1 + \sum_{k=1}^{\infty} a_0^k\,\boldsymbol{U}_k(N)\right]^{-1} \boldsymbol{q}(a_0, N) \end{aligned}$$

where ***L*** is defined by the LO solution

$$\boldsymbol{q}_{\,\mathrm{LO}}(N, a_s, N) = \left(\frac{a_s}{a_0}\right)^{-\boldsymbol{R}_0(N)} \boldsymbol{q}(N, a_0) \equiv \boldsymbol{L}(N, a_s, a_0)\,\boldsymbol{q}(N, a_0)$$

$$\boldsymbol{R}_0 \equiv \frac{1}{\beta_0}\boldsymbol{P}^{(0)}$$

TRUNCATED AND ITERATED SOLUTION

Since both $\beta_{\mathrm{N^mLO}}$ and $\boldsymbol{P}_{\mathrm{N^mLO}}$ have an expansion in powers of α_s

there are different ways of defining the N^{m}LO solution

$$\begin{aligned} \boldsymbol{q}_{\mathrm{N^3LO}}(a_s) \;=\; \Big[& \boldsymbol{L} + a_s \boldsymbol{U}_1 \boldsymbol{L} - a_0 \boldsymbol{L} \boldsymbol{U}_1 \\ & + a_s^2 \boldsymbol{U}_2 \boldsymbol{L} - a_s a_0 \boldsymbol{U}_1 \boldsymbol{L} \boldsymbol{U}_1 + a_0^2 \boldsymbol{L} \left(\boldsymbol{U}_1^2 - \boldsymbol{U}_2 \right) \\ & + a_s^3 \boldsymbol{U}_3 \boldsymbol{L} - a_s^2 a_0 \boldsymbol{U}_2 \boldsymbol{L} \boldsymbol{U}_1 + a_s a_0^2 \boldsymbol{U}_1 \boldsymbol{L} \left(\boldsymbol{U}_1^2 - \boldsymbol{U}_2 \right) \\ & - a_0^3 \boldsymbol{L} \left(\boldsymbol{U}_1^3 - \boldsymbol{U}_1 \boldsymbol{U}_2 - \boldsymbol{U}_1 \boldsymbol{U}_2 + \boldsymbol{U}_3 \right) \Big] \boldsymbol{q}(a_0) \end{aligned}$$

$$\boldsymbol{R}_0 \equiv \frac{1}{\beta_0} \boldsymbol{P}^{T,(0)} \,, \quad \boldsymbol{R}_k \equiv \frac{1}{\beta_0} \boldsymbol{P}^{T,(k)} - \sum_{i=1}^{k} b_i \boldsymbol{R}_{k-i} \,,$$

$$[\boldsymbol{U}_k, \boldsymbol{R}_0] = \boldsymbol{R}_k + \sum_{i=1}^{k-1} \boldsymbol{R}_{k-1} \boldsymbol{U}_i + k \boldsymbol{U}_k \,.$$

Truncated and Iterated Solution

TRUNCATED: Keep only terms up to $\alpha_s^{\,m}$ in the solution

$$\boldsymbol{q}_{\mathrm{N^3LO}}(a_\mathrm{s}) = \Big[\boldsymbol{L} + a_\mathrm{s}\boldsymbol{U}_1\boldsymbol{L} - a_0\boldsymbol{L}\boldsymbol{U}_1 + a_\mathrm{s}^2\boldsymbol{U}_2\boldsymbol{L} - a_\mathrm{s}a_0\boldsymbol{U}_1\boldsymbol{L}\boldsymbol{U}_1 + a_0^2\boldsymbol{L}\left(\boldsymbol{U}_1^2 - \boldsymbol{U}_2\right)$$
$$+ a_\mathrm{s}^3\boldsymbol{U}_3\boldsymbol{L} - a_\mathrm{s}^2a_0\boldsymbol{U}_2\boldsymbol{L}\boldsymbol{U}_1 + a_\mathrm{s}a_0^2\boldsymbol{U}_1\boldsymbol{L}\left(\boldsymbol{U}_1^2 - \boldsymbol{U}_2\right)$$
$$- a_0^3\boldsymbol{L}\left(\boldsymbol{U}_1^3 - \boldsymbol{U}_1\boldsymbol{U}_2 - \boldsymbol{U}_1\boldsymbol{U}_2 + \boldsymbol{U}_3\right)\Big]\boldsymbol{q}(a_0)$$

$$\boldsymbol{R}_0 \equiv \frac{1}{\beta_0}\boldsymbol{P}^{T,(0)}\,, \quad \boldsymbol{R}_k \equiv \frac{1}{\beta_0}\boldsymbol{P}^{T,(k)} - \sum_{i=1}^{k} b_i\boldsymbol{R}_{k-i}\,, \quad [\boldsymbol{U}_k, \boldsymbol{R}_0] = \boldsymbol{R}_k + \sum_{i=1}^{k-1}\boldsymbol{R}_{k-1}\boldsymbol{U}_i + k\boldsymbol{U}_k\,.$$

- It solves the equation exactly only up to terms of order n > m

TRUNCATED AND ITERATED SOLUTION

ITERATED: Keep the all the m-terms generated from $\beta_{\mathrm{N^mLO}}$ and $\boldsymbol{P}_{\mathrm{N^mLO}}$

$$\begin{aligned}\boldsymbol{q}_{\mathrm{N^3LO}}(a_s) = &\Big[\boldsymbol{L} + a_s \boldsymbol{U}_1 \boldsymbol{L} - a_0 \boldsymbol{L} \boldsymbol{U}_1 \\ &+ a_s^2 \boldsymbol{U}_2 \boldsymbol{L} - a_s a_0 \boldsymbol{U}_1 \boldsymbol{L} \boldsymbol{U}_1 + a_0^2 \boldsymbol{L}\left(\boldsymbol{U}_1^2 - \boldsymbol{U}_2\right) \\ &+ a_s^3 \boldsymbol{U}_3 \boldsymbol{L} - a_s^2 a_0 \boldsymbol{U}_2 \boldsymbol{L} \boldsymbol{U}_1 + a_s a_0^2 \boldsymbol{U}_1 \boldsymbol{L}\left(\boldsymbol{U}_1^2 - \boldsymbol{U}_2\right) \\ &- a_0^3 \boldsymbol{L}\left(\boldsymbol{U}_1^3 - \boldsymbol{U}_1 \boldsymbol{U}_2 - \boldsymbol{U}_1 \boldsymbol{U}_2 + \boldsymbol{U}_3\right)\Big] \boldsymbol{q}(a_0)\end{aligned}$$

$$\boldsymbol{R}_0 \equiv \frac{1}{\beta_0} \boldsymbol{P}^{T,(0)}\,, \quad \boldsymbol{R}_k \equiv \frac{1}{\beta_0} \boldsymbol{P}^{T,(k)} - \sum_{i=1}^{k} b_i \boldsymbol{R}_{k-i}\,, \qquad [\boldsymbol{U}_k, \boldsymbol{R}_0] = \boldsymbol{R}_k + \sum_{i=1}^{k-1} \boldsymbol{R}_{k-1} \boldsymbol{U}_i + k \boldsymbol{U}_k\,.$$

- It corresponds to the solution done in x-Space

- It introduces more higher order scheme-dependent terms

Truncated and Iterated Solution

ITERATED-TRUNCATED = theoretical uncertainty of order $\mathcal{O}({\alpha_s}^{m+1})$

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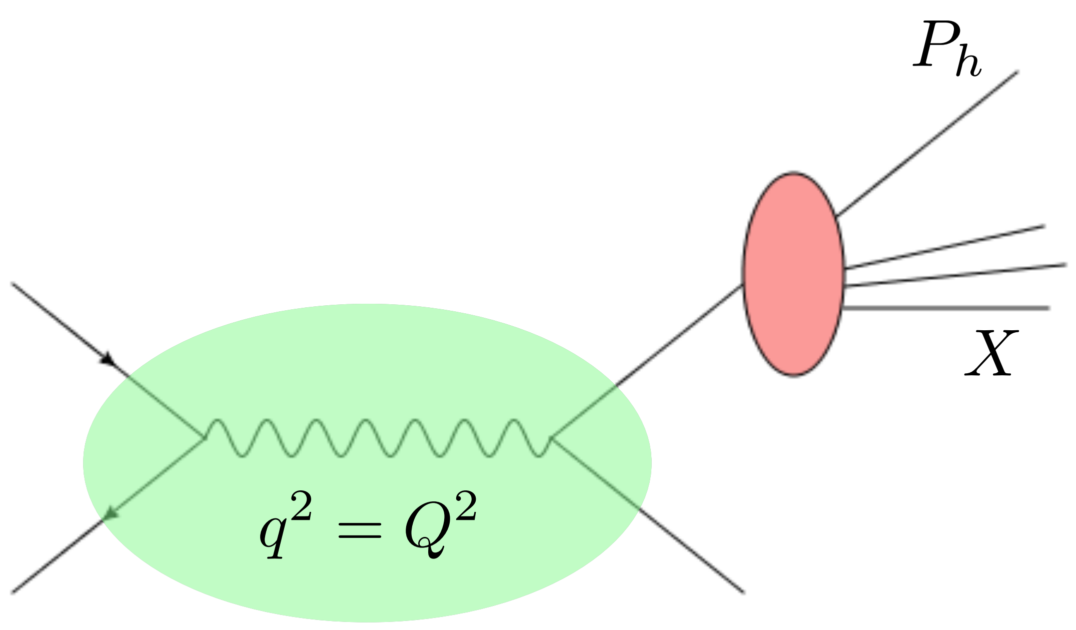
OUR SIA FIT



Hadron multiplicities

$$R^h_{e^+e^-} \equiv \frac{1}{\sigma^{\rm tot}} \frac{d^2\sigma^h}{dx_E\, d\cos\theta}$$

where $$x_E \equiv \frac{2P_h \cdot q}{Q^2}$$

$$\frac{d^2\sigma^h}{dx_E d\cos\theta} = \frac{\pi\alpha^2}{Q^2} N_C \left[\frac{1+\cos^2\theta}{2}\mathcal{F}^h_T(x_E,Q^2) + \sin^2\theta\,\mathcal{F}^h_L(x_E,Q^2)\right]$$

Nason, Webber; Furmanski, Petronzio

where
Structure Functions

$$\mathcal{F}^h_i(x_E,Q^2) = \sum_f \int_{x_E}^1 \frac{d\hat{z}}{\hat{z}} D^h_f\left(\frac{x_E}{\hat{z}},\mu^2\right) \mathcal{C}^i_f\left(\hat{z},\frac{Q^2}{\mu^2},\alpha_s(\mu^2)\right)$$

OUR SIA FIT

Parametrization of light patrons FF @ μ_0

$$D_i^h(z, Q_0) = \frac{N_i z^{\alpha_i}(1-z)^{\beta_i}[1+\gamma_i(1-z)^{\delta_i}]}{B[2+\alpha_i, \beta_i+1] + \gamma_i B[2+\alpha_i, \beta_i+\delta_i+1]}$$

So that $N_i = \int_0^1 z D_i^h dz$

Heavy Quark Treatment:

NON PERTURBATIVE INPUT: at $\mu > m_q$ the evolution is set to evolve with $n_f + 1$ for flavours and for the q-heavy quark FF the same functional form as for the light quark is set at $\mu = m_q$

Data sets:

15 Data Set: from Sld, Aleph, Delphi, Opal, Tpc, BaBar, Belle either inclusive, uds tagged, b tagged or c tagged. We use a GLOBAL CUT 0.075<z<0.95

PARAMETERS FOR PI+

With only SIA one needs some extra constrains:

parameter	LO	NLO	NNLO
$N_{u+\bar{u}}$	0.735	0.572	0.579
$\alpha_{u+\bar{u}}$	-0.371	-0.705	-0.913
$\beta_{u+\bar{u}}$	0.953	0.816	0.865
$\gamma_{u+\bar{u}}$	8.123	5.553	4.062
$\delta_{u+\bar{u}}$	3.854	1.968	1.775
$N_{s+\bar{s}}$	0.243	0.135	0.271
$\alpha_{s+\bar{s}}$	*-0.371*	*-0.705*	*-0.913*
$\beta_{s+\bar{s}}$	*4.807*	*2.784*	*2.640*
N_g	0.273	0.211	0.174
α_g	2.414	2.210	1.595
β_g	*8.000*	*8.000*	*8.000*
$N_{c+\bar{c}}$	0.405	0.302	0.338
$\alpha_{c+\bar{c}}$	-0.164	-0.026	-0.233
$\beta_{c+\bar{c}}$	5.114	6.862	6.564
$N_{b+\bar{b}}$	0.462	0.405	0.445
$\alpha_{b+\bar{b}}$	-0.090	-0.411	-0.695
$\beta_{b+\bar{b}}$	4.301	4.039	3.681
$\gamma_{b+\bar{b}}$	24.85	15.80	11.22
$\delta_{b+\bar{b}}$	12.25	11.27	9.908

5 free param needed

charge conjugation and isospin symmetry $D^{\pi^\pm}_{u+\bar{u}} = D^{\pi^\pm}_{d+\bar{d}}$,

1 free param, 2 fixed by

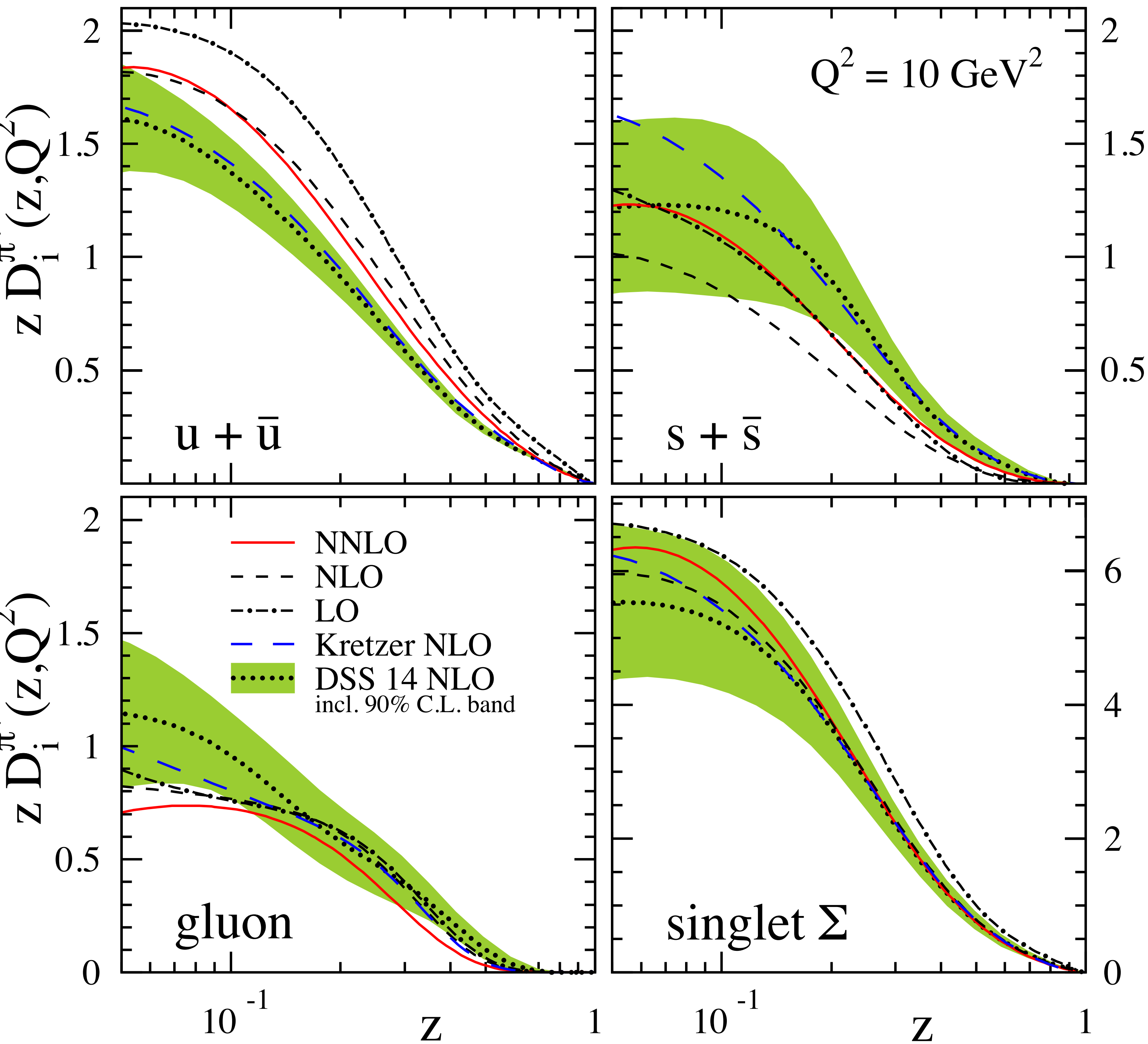
$\alpha_{s+\bar{s}} = \alpha_{u+\bar{u}}, \quad \beta_{s+\bar{s}} = \beta_{u+\bar{u}} + \delta_{u+\bar{u}}$

2 free param, 1 fixed

3 free param

5 free param

TOT = 16 free param

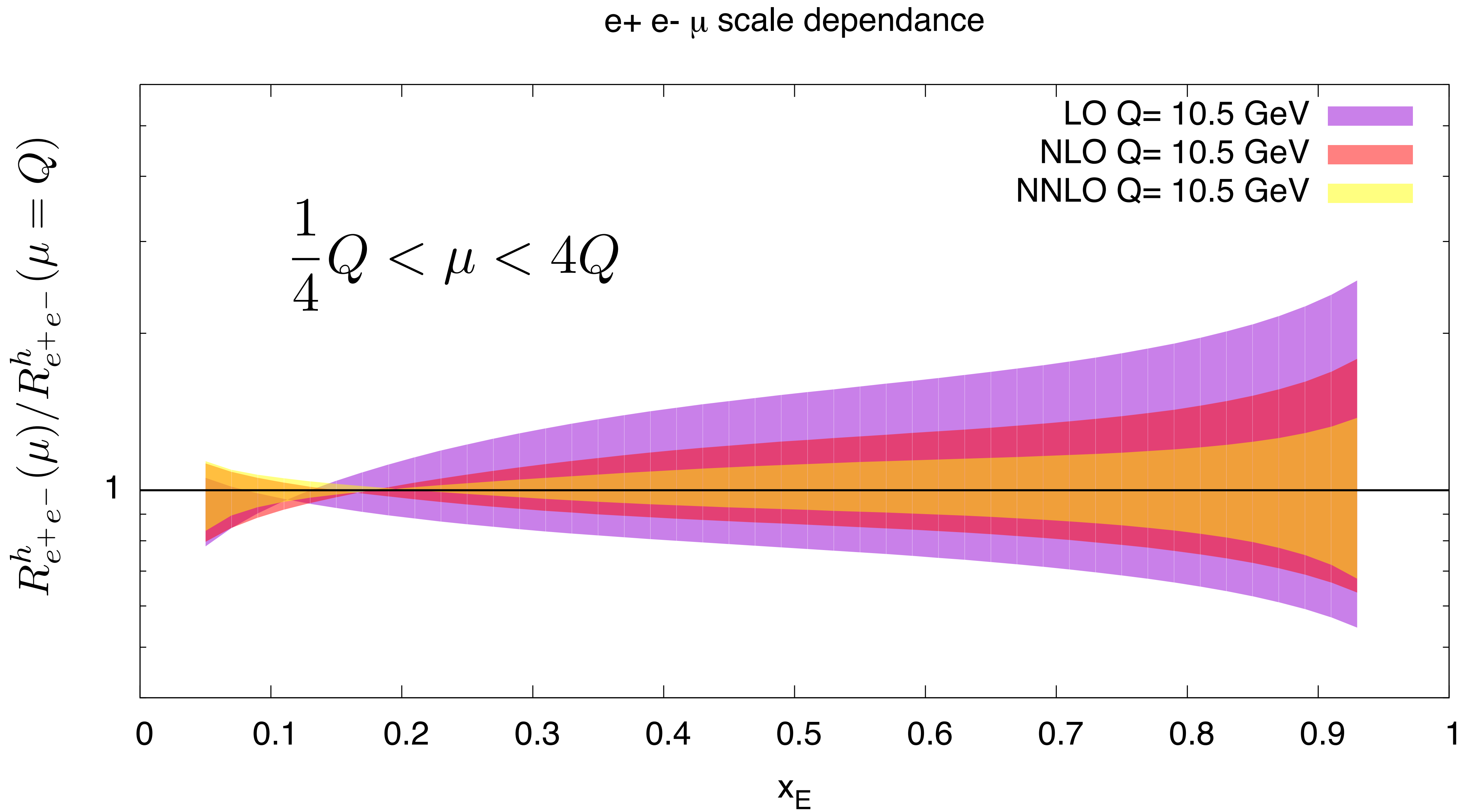


Kretzer FFS (Phys. Rev. D 62, 054001 (2000))

DSS FFS (Phys. Rev. D 91, 014035 (2015))

$$D^h_\Sigma = \sum_{i=1}^{n_f} \left(D^h_{q_i} + D^h_{\bar{q}_i} \right)$$

Scale Dependence



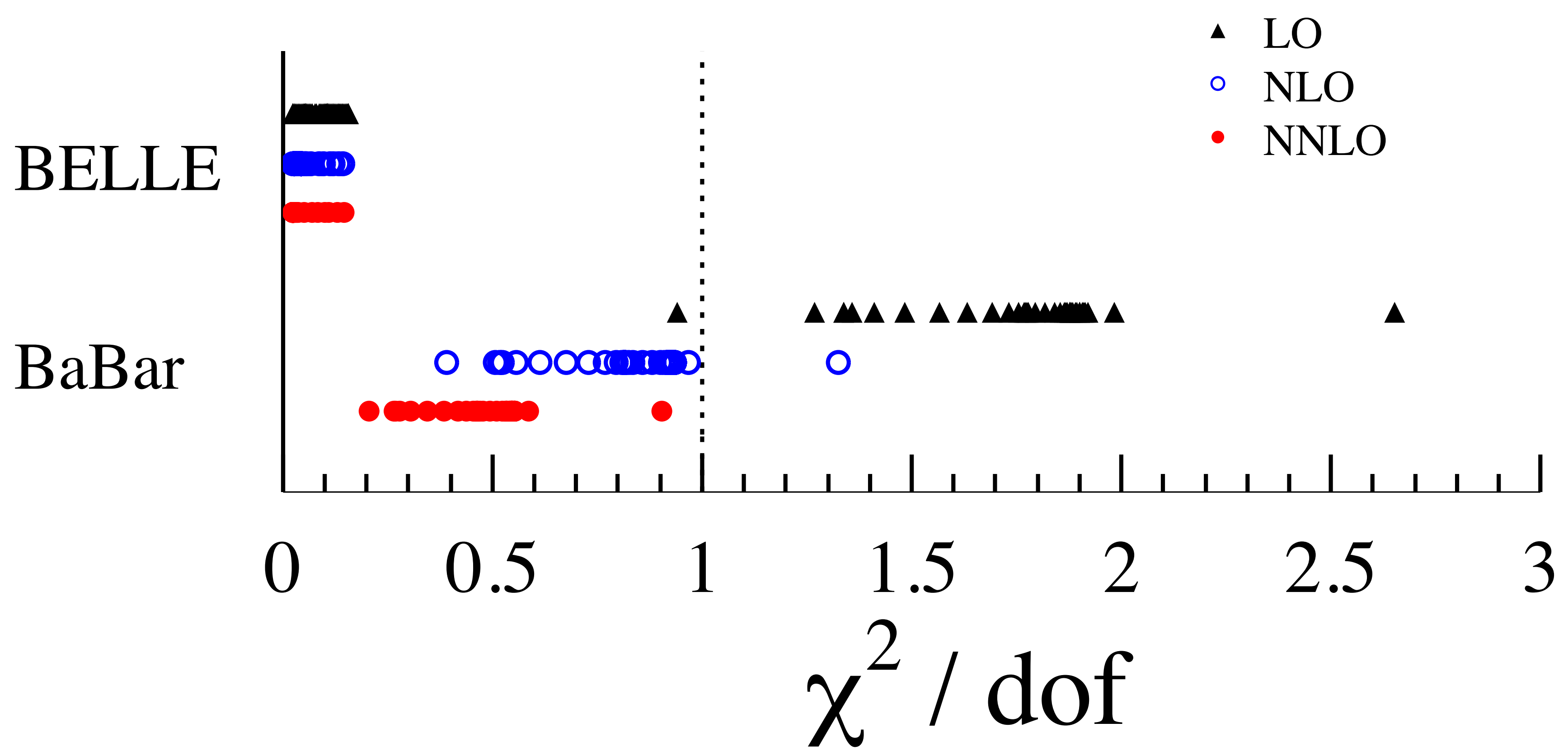
Multiplicity $R^h_{e^+e^-} \equiv \frac{1}{\sigma^{\text{tot}}} \frac{d^2\sigma^h}{dx_E \, d\cos\theta}$

using input parameter for FF of Kretzer (Phys.Rev. D62 (2000) 054001) and truncated-solution

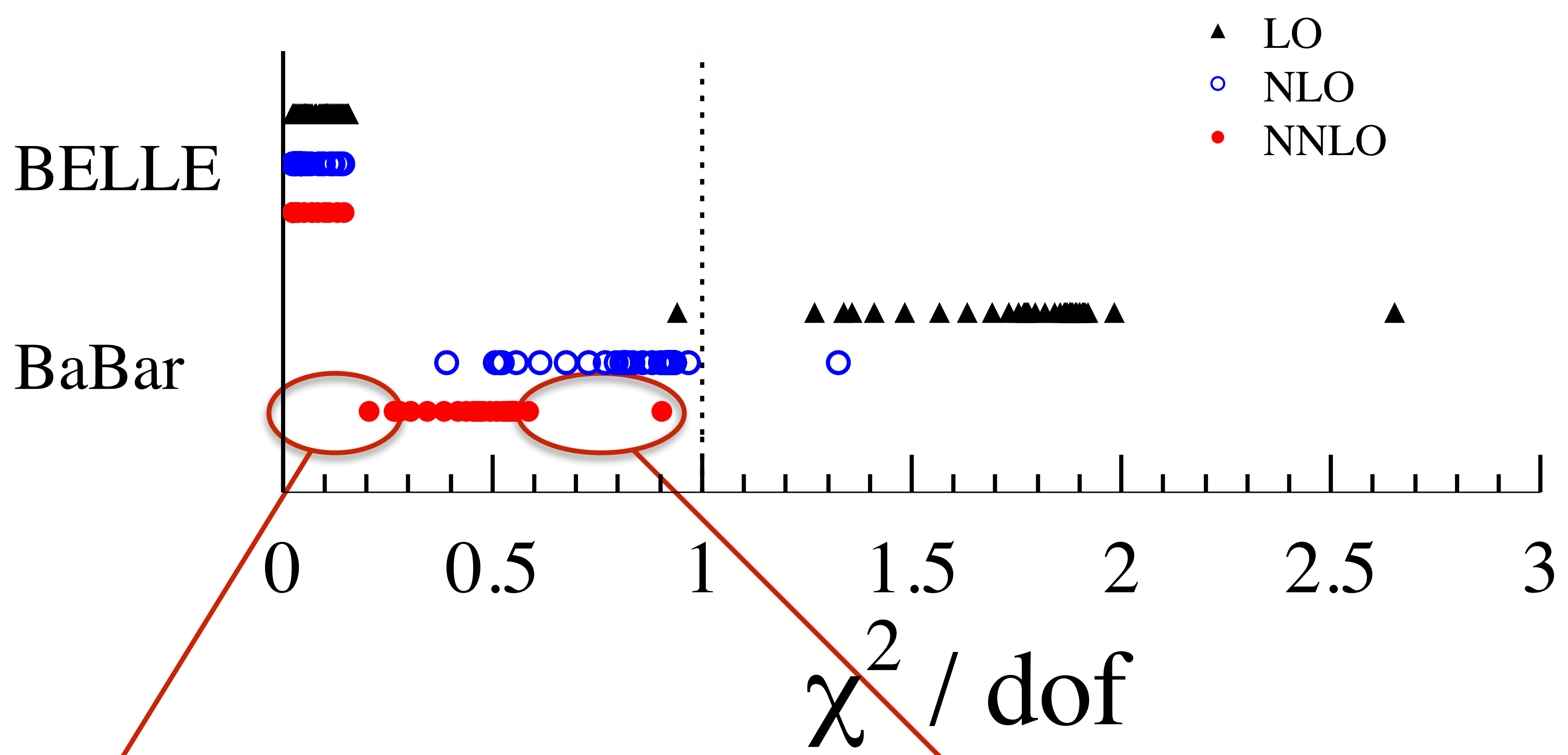
χ^2 COMPARISON

experiment	data type	# data in fit	χ^2 LO	NLO	NNLO
SLD [40]	incl.	23	15.0	14.8	15.5
	uds tag	14	9.7	18.7	18.8
	c tag	14	10.4	21.0	20.4
	b tag	14	5.9	7.1	8.4
ALEPH [41]	incl.	17	19.2	12.8	12.6
DELPHI [42]	incl.	15	7.4	9.0	9.9
	uds tag	15	8.3	3.8	4.3
	b tag	15	8.5	4.5	4.0
OPAL [43]	incl.	13	8.9	4.9	4.8
TPC [44]	incl.	13	5.3	6.0	6.9
	uds tag	6	1.9	2.1	1.7
	c tag	6	4.0	4.5	4.1
	b tag	6	8.6	8.8	8.6
BABAR [10]	incl.	41	108.7	54.3	37.1
BELLE [9]	incl.	76	11.8	10.9	11.0
TOTAL:		288	241.0	190.0	175.2

χ^2 Comparison



χ^2 Comparison



Small z Logs

$$\alpha_s^k \frac{\ln^{2k}(z)}{z}$$

+Hadron Mass Cor.

Threshold Logs

$$\alpha_s^k \left(\frac{\ln^{2k-1}(1-x)}{1-x}\right)_+$$

Anderle, Ringer, Vogelsang (Phys.Rev. D87 (2013) no.3, 034014)

Accardi, Anderle, Ringer (Phys.Rev. D91 (2015) 3, 034008)

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SMALL-Z LOGARITHMS (SIA)

N^{k}LO Small-z Logarithms in Splitting Functions and Singlet Coefficient Functions

$$P_{ij}^{(k-1)} \propto a_s^k \frac{\ln^{2k-1-a}(z)}{z}$$

$$C_{T,i}^{(k)} \propto a_s^k \frac{\ln^{2k-1-a}(z)}{z}$$

$$C_{L,i}^{(k)} \propto a_s^k \frac{\ln^{2k-2-a}(z)}{z}$$

$$a = 1, 2, 3 \qquad i, j \in \{q, g\}$$

Double Log Enhancement

spoils perturbative convergence even for $\alpha_s \ll 1$

In Mellin Space they correspond to N = 1 Poles

$$\mathcal{M}\left[\frac{\ln^{2k-1}(z)}{z}\right] \equiv \int_0^1 dx\, x^{N-1} \frac{\ln^{2k-1}(z)}{z} = \frac{(-1)^k (2k-1)!}{(N-1)^{2k}}$$

Resummation Accuracy

For example P_{gg} with $N-1=\bar{N}$

Resummation ↓ / Fixed Order →

LO	$\alpha_s/\bar{N}$	α_s				
NLO	$\alpha_s/\bar{N}^3$	$\alpha_s/\bar{N}^2$	$\alpha_s/\bar{N}$	α_s		
NNLO	$\alpha_s/\bar{N}^5$	$\alpha_s/\bar{N}^4$	$\alpha_s/\bar{N}^3$	$\alpha_s/\bar{N}^2$	$\alpha_s/\bar{N}$	α_s
...	...	...	...	...	...	
$N^{k-1}LO$	$\alpha_s/\bar{N}^{2k-1}$	$\alpha_s/\bar{N}^{2k-2}$	$\alpha_s/\bar{N}^{2k-3}$	$\alpha_s/\bar{N}^{2k-4}$	$\alpha_s/\bar{N}^{2k-5}$	...

NNLL : *Vogt (2011), Kom ,Vogt, Yeats (2012)*

NLL : *Mueller (83), Albino, Bolzoni, Kniehl, Kotikov (11)*

LL : *Mueller (81); Bassetto, Ciafaloni, Marchesini, Mueller (82).*

RESUMMATION VIA UNFACTORIZED SIA

van Neerven, Rijken (1996)
Vogt (2011), Kom,Vogt,Yeats(2012)

One can proceed by using "all-order" mass factorization: e.g.

A) starting from the *unfactorized gluon singlet transversal parton structure function in dimensional regularisation (IR-singularities not yet factorized out and "re-absorbed" in FF)*

$$\hat{\mathcal{F}}_g^T(N, a_s, \epsilon) = \sum_{i=q,g} \bar{C}_i^T(N, a_s, \epsilon)\, \Gamma_{ig}^N(N, a_s, \epsilon)$$

D-Dimensional coef. function:
only positive powers of ϵ

$$\bar{C}_i^T(N, a_s, \epsilon) = \delta_{iq} + \sum_{l=1}^{\infty} a_s^l \sum_{k=0}^{\infty} \epsilon^k c_{T,i}^{(l,k)}(N)$$

Transition function:
incorporates all IR $1/\epsilon$ poles, calculable order by order as a combination of splitting functions

$$\beta_D(a_s)\, \frac{\partial \Gamma_{ik}}{\partial a_s}\, \Gamma_{kj}^{-1} = P_{ij}$$

B) "Plug-in" the small $\bar{N} = N - 1$ limit for known fixed order coefficient functions and splitting functions (e.g. NNLO)

C) Impose equality order by order in a_s with the small $\bar{N} = N - 1$ limit for the unfactorized structure function which reads

A.Vogt JHEP10 (2011) 025

$$\hat{\mathcal{F}}_g^{T,(n)}(N,\epsilon) = a_s^n \frac{1}{\epsilon^{2n-1}} \sum_{l=0}^{n-1} \frac{1}{N-1-2(n-l)\epsilon} \left(A_{T,g}^{(l,n)} + \epsilon B_{T,g}^{(l,n)} + \epsilon^2 C_{T,g}^{(l,n)} + \dots\right)$$

LL NLL NNLL

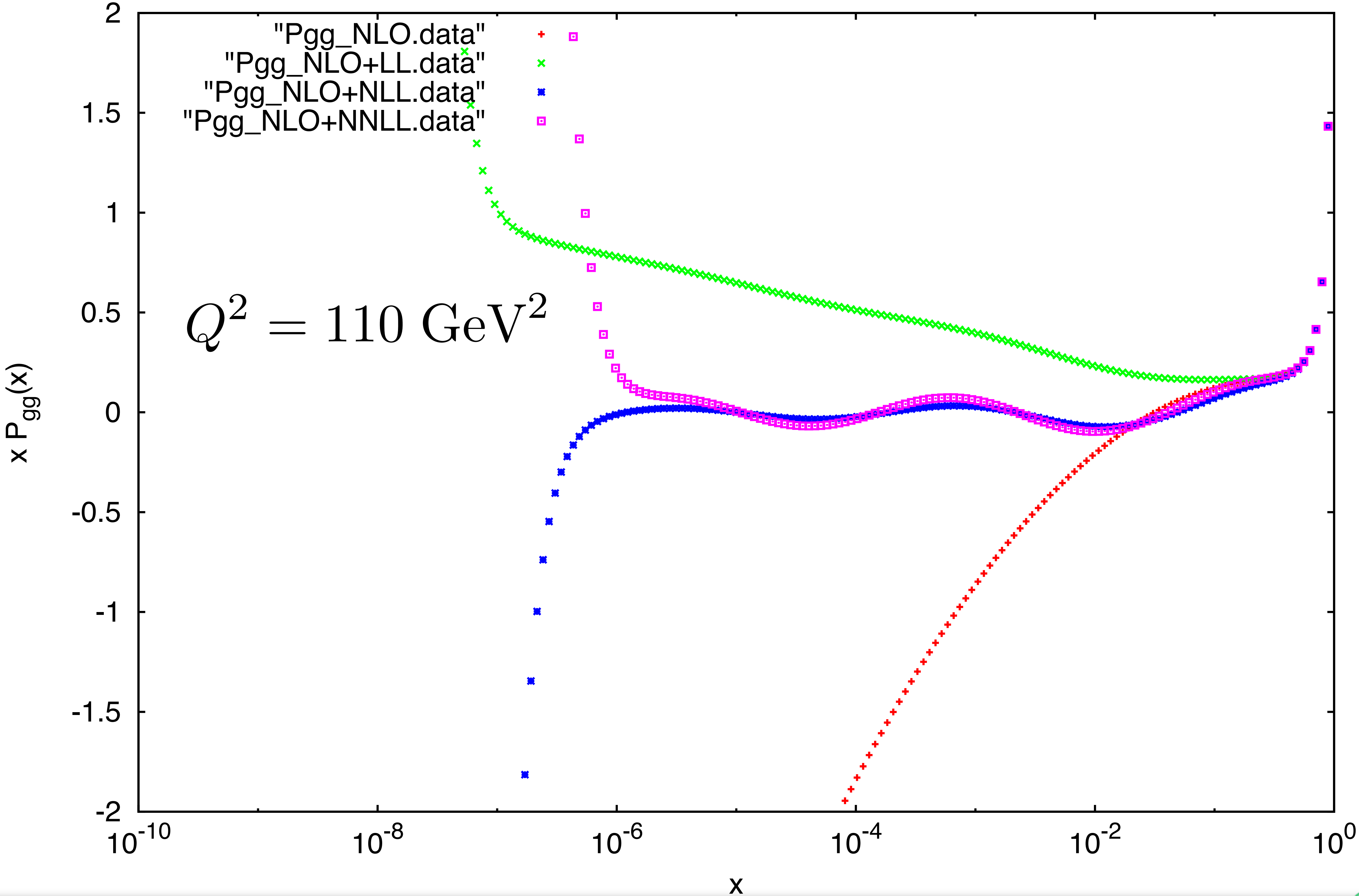
D) solve recursively order by order for $c_{T,i}^{(n,k)}$ $P_{ij}^{(n-1)}$ $A_{T,g}^{(l,n)}$ $B_{T,g}^{(l,n)}$ $C_{T,g}^{(l,n)}$:

- **KLN - Cancellations**
- **fixed order calculation constrains**

→ System of equation solvable

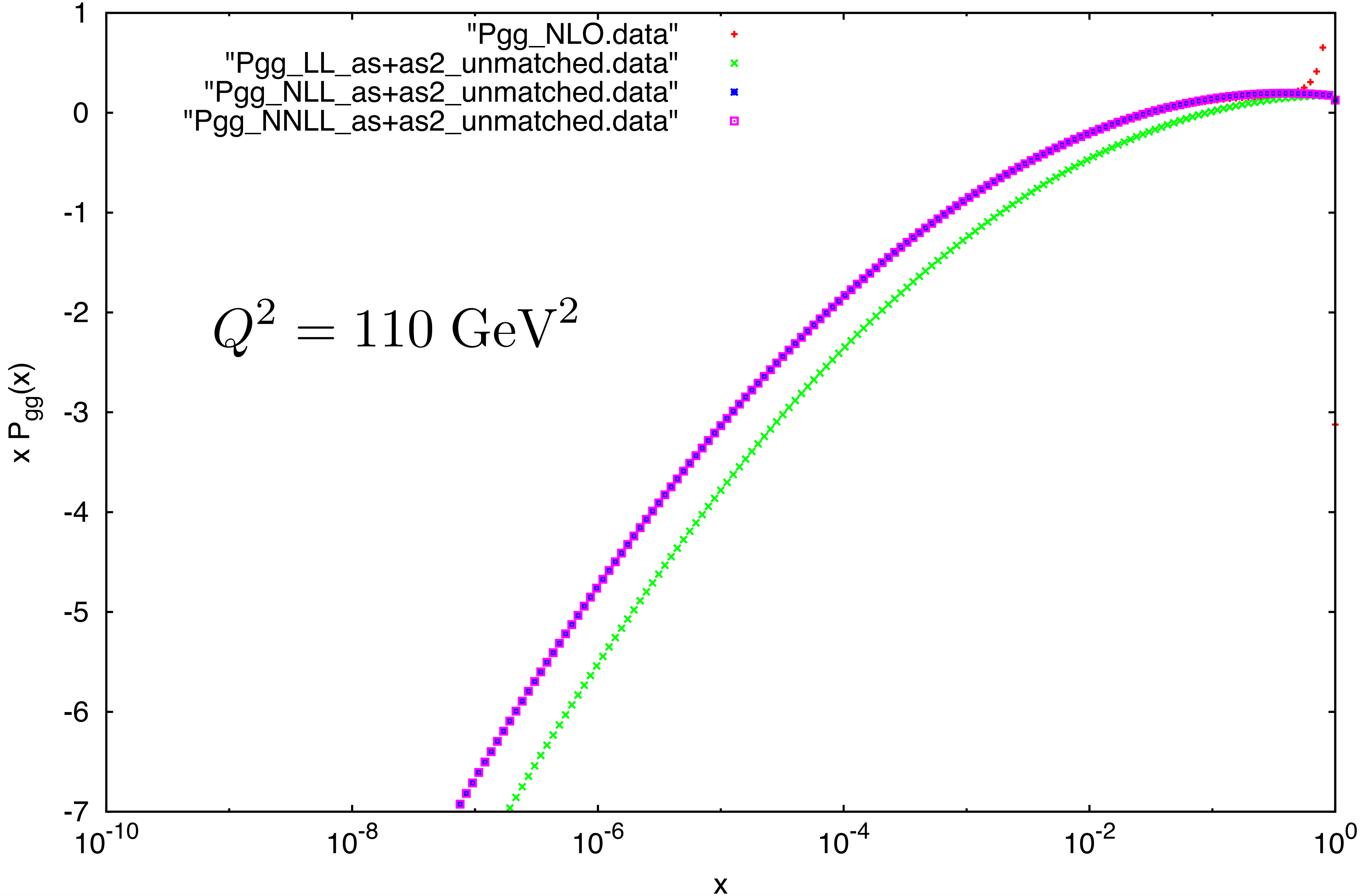
E) From the coefficient of the small N expansion deduct closed form

P_{gg}



"Pgg_NLO.data"
"Pgg_NLO+LL.data"
"Pgg_NLO+NLL.data"
"Pgg_NLO+NNLL.data"
$Q^2 = 110$ GeV^2
x P$_{gg}$(x)
x
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
10^{-10}
10^{-8}
10^{-6}
10^{-4}
10^{-2}
10^{0}

P_{gg}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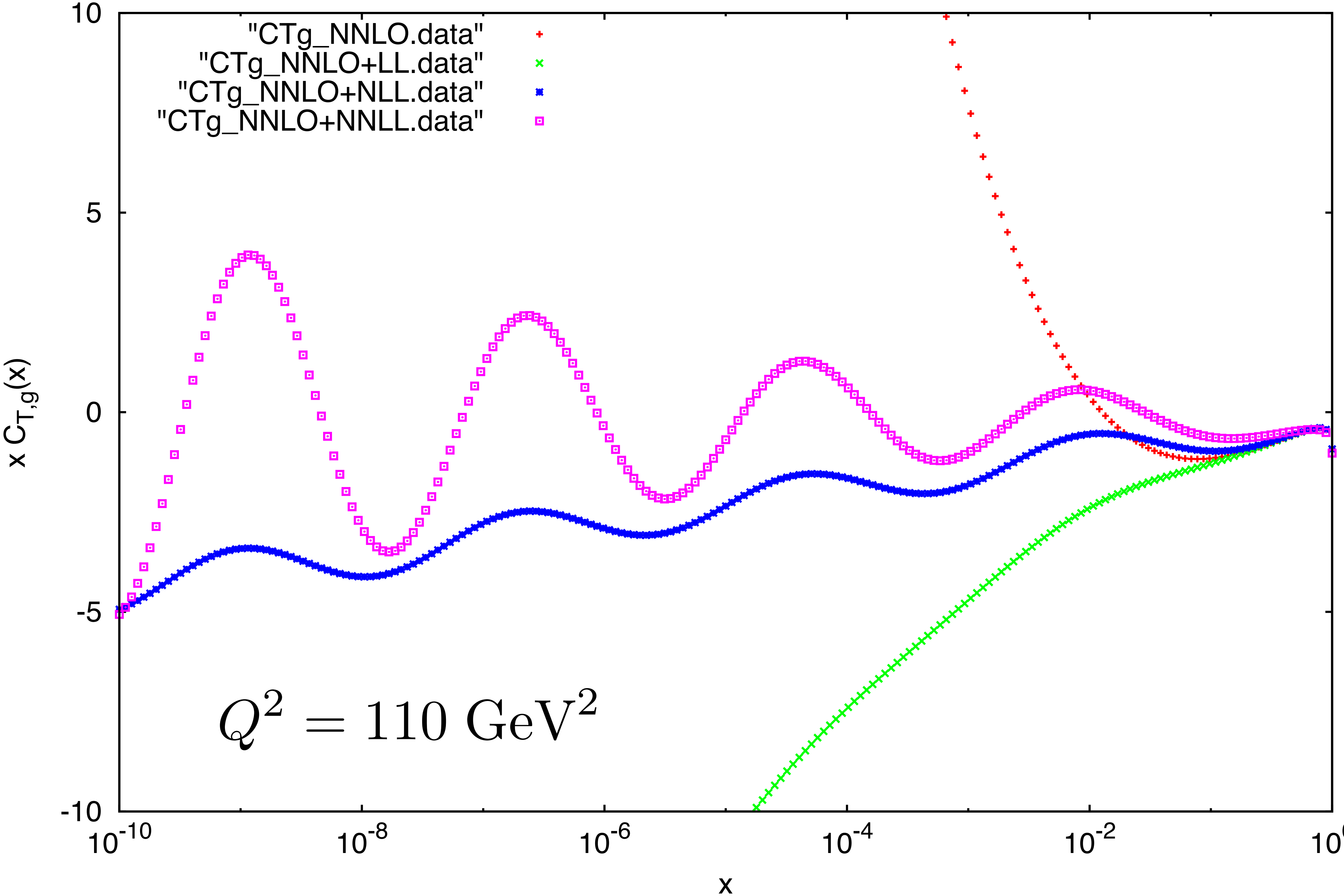
"Pgg_LL_as+as2_unmatched.data"
"Pgg_NLL_as+as2_unmatched.data"
"Pgg_NNLL_as+as2_unmatched.data"
$Q^2 = 110\ \mathrm{GeV}^2$
x P_gg(x)
x
1
0
-1
-2
-3
-4
-5
-6
-7
10^-10
10^-8
10^-6
10^-4
10^-2
10^0

C_g^T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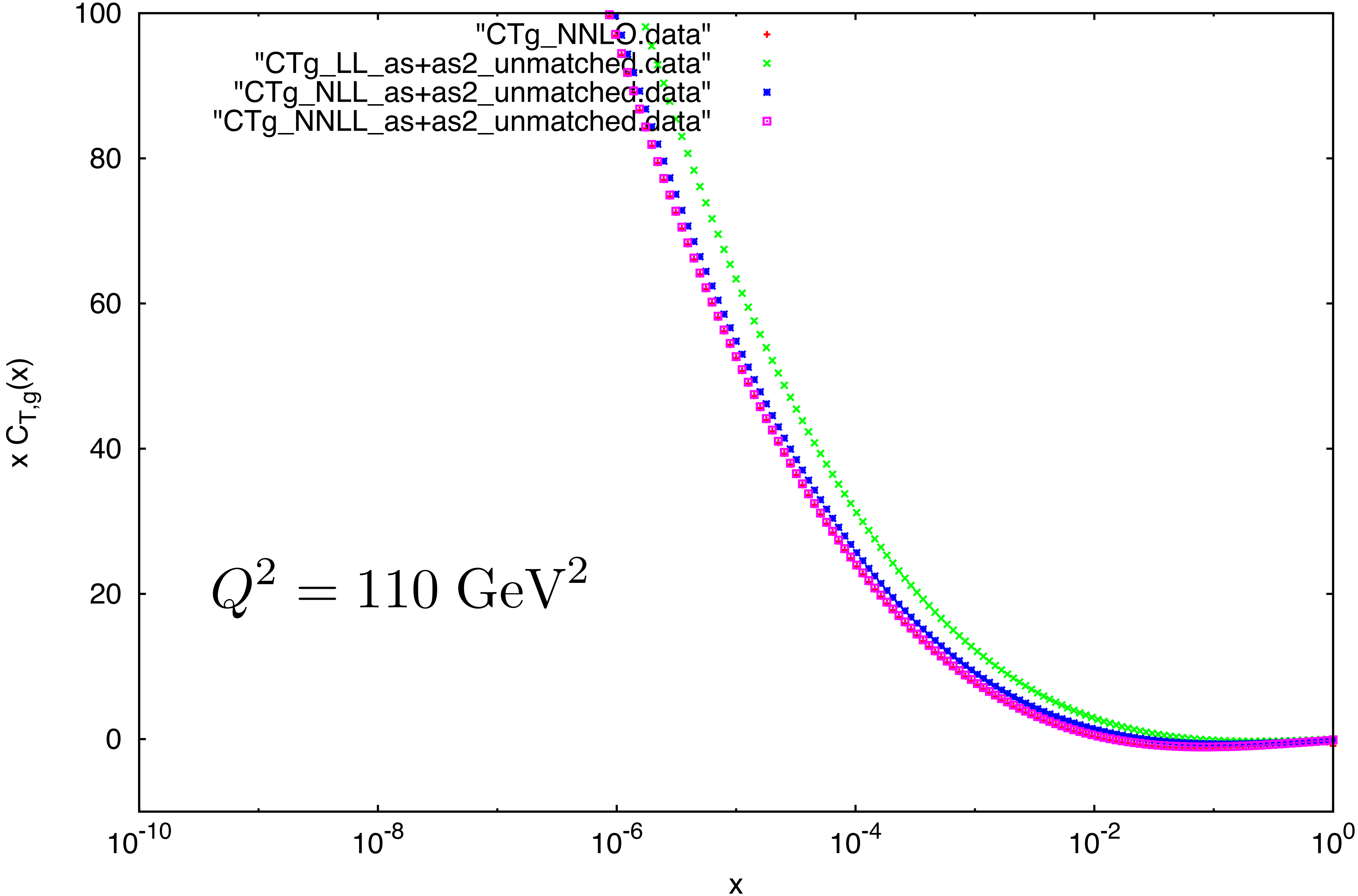
"CTg_NNLO+LL.data"
"CTg_NNLO+NLL.data"
"CTg_NNLO+NNLL.data"
$Q^2 = 110\ \mathrm{GeV}^2$
x $C_{T,g}(x)$
x
10
5
0
-5
-10
10^{-10}
10^{-8}
10^{-6}
10^{-4}
10^{-2}
10^{0}

C_g^T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"CTg_NNLL_as+as2_unmatched.data"
100
80
60
40
20
0
x C_{T,g}(x)
10^-10
10^-8
10^-6
10^-4
10^-2
10^0
x
Q^2 = 110 GeV^2

RESUMMED SOLUTION FOR EVOLUTION

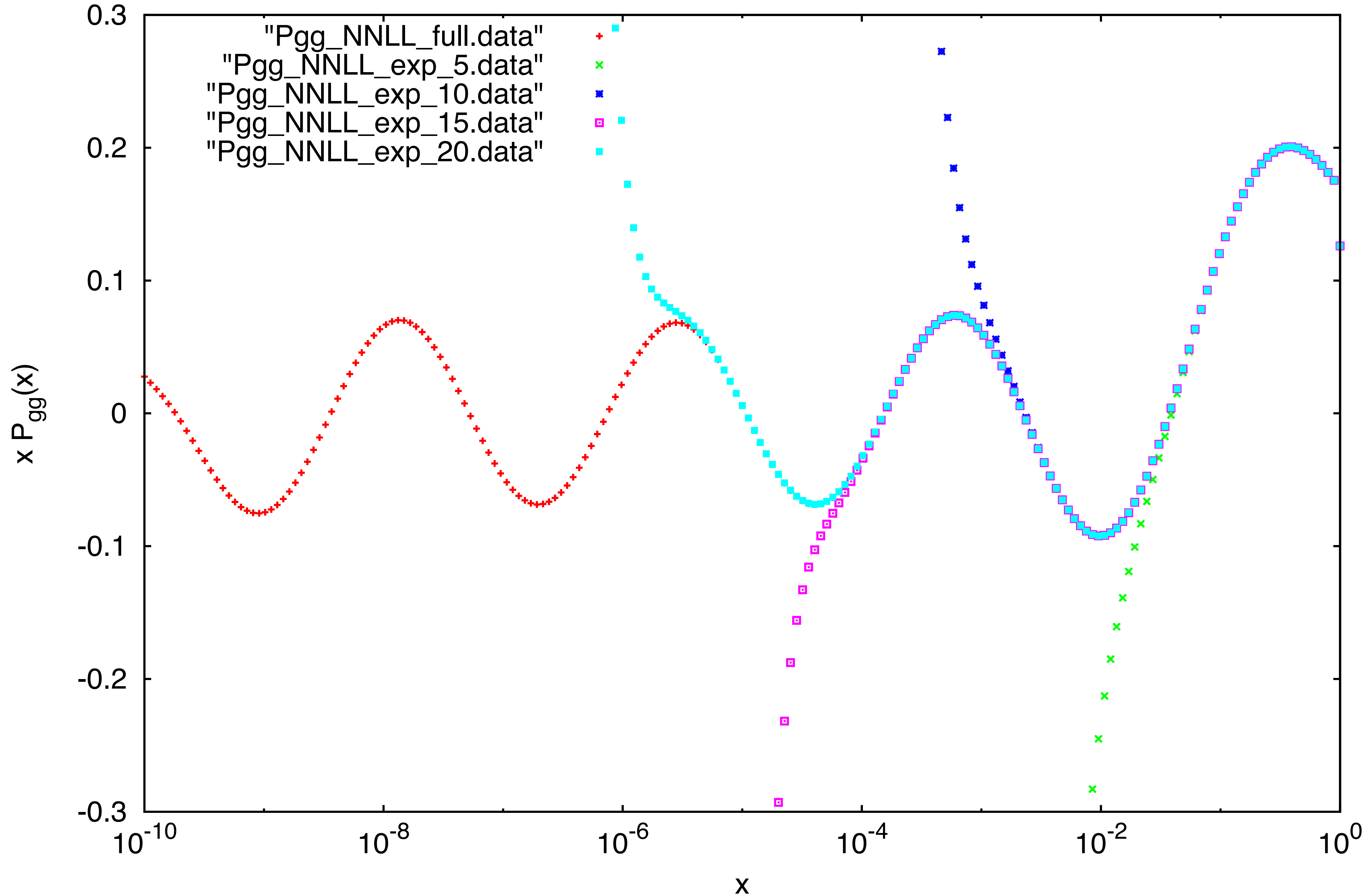
If NmLL accuracy then for k > m $\boldsymbol{P}^{T,(k)} = \boldsymbol{P}^{T,\mathrm{resum}}|_{a_s^k}$

$$\begin{aligned} \boldsymbol{q}(N,a_\mathrm{s}) &= \boldsymbol{U}(N,a_\mathrm{s})\,\boldsymbol{L}(N,a_\mathrm{s},a_0)\,\boldsymbol{U}^{-1}(N,a_0)\,\boldsymbol{q}(N,a_0) \\ &= \left[1+\sum_{k=1}^{\infty} a_\mathrm{s}^k\,\boldsymbol{U}_k(N)\right]\boldsymbol{L}(a_\mathrm{s},a_0,N)\left[1+\sum_{k=1}^{\infty} a_0^k\,\boldsymbol{U}_k(N)\right]^{-1}\boldsymbol{q}(a_0,N) \end{aligned}$$

$$\boldsymbol{R}_0 \equiv \frac{1}{\beta_0}\boldsymbol{P}^{T,(0)}\,, \quad \boldsymbol{R}_k \equiv \frac{1}{\beta_0}\boldsymbol{P}^{T,(k)} - \sum_{i=1}^{k} b_i \boldsymbol{R}_{k-i}\,,$$

$$[\boldsymbol{U}_k,\boldsymbol{R}_0] = \boldsymbol{R}_k + \sum_{i=1}^{k-1}\boldsymbol{R}_{k-1}\boldsymbol{U}_i + k\boldsymbol{U}_k\,.$$

How many terms?



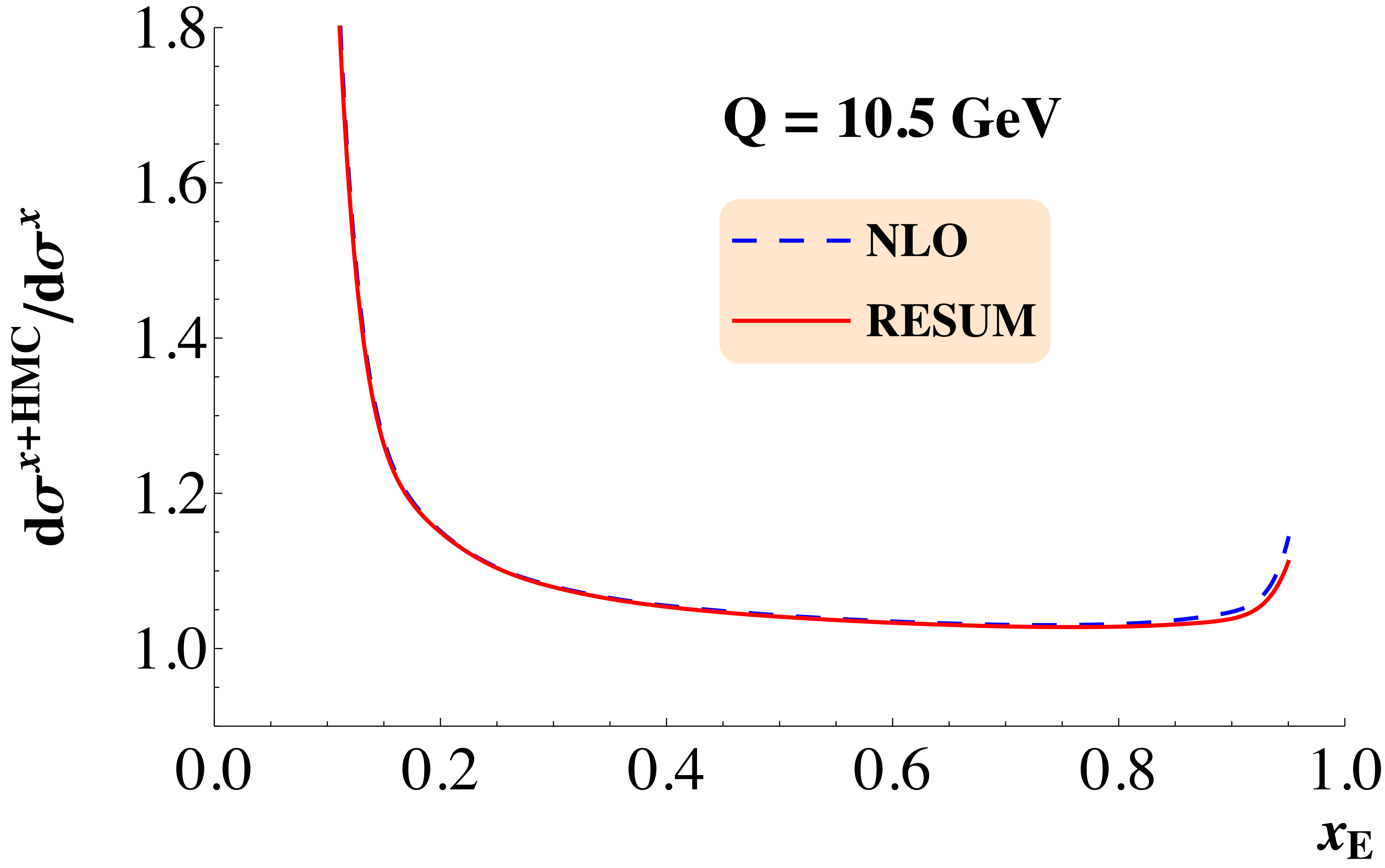
Conclusions & Outlook

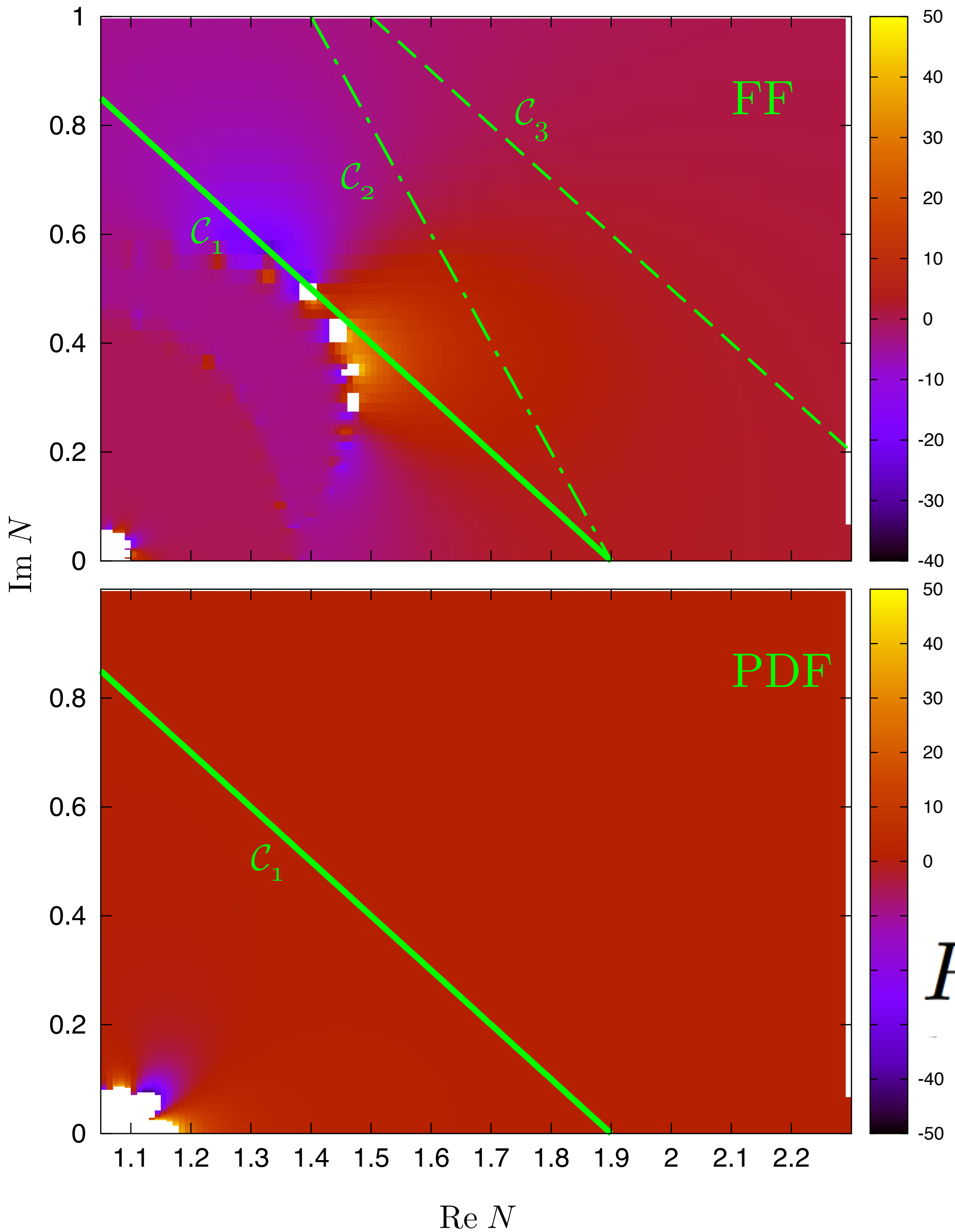
- We have performed a NNLO fit for SIA exploring the new features appearing @ NNLO.
- At NNLO it is clear that the extreme phase space regions are better described and that more of the small z and large z logs are taken into account
- We are working on a resummed version of our fit and looking to extend our analysis to an approximate NNLO global fit using expanded NLL results
- One other goal is to extend the small-z resummtion to the scale dependence in the coefficient functions

Thanks for Your Attention

Any questions?

No interplay between the two effects is found since they act independently on two different kinematical regions



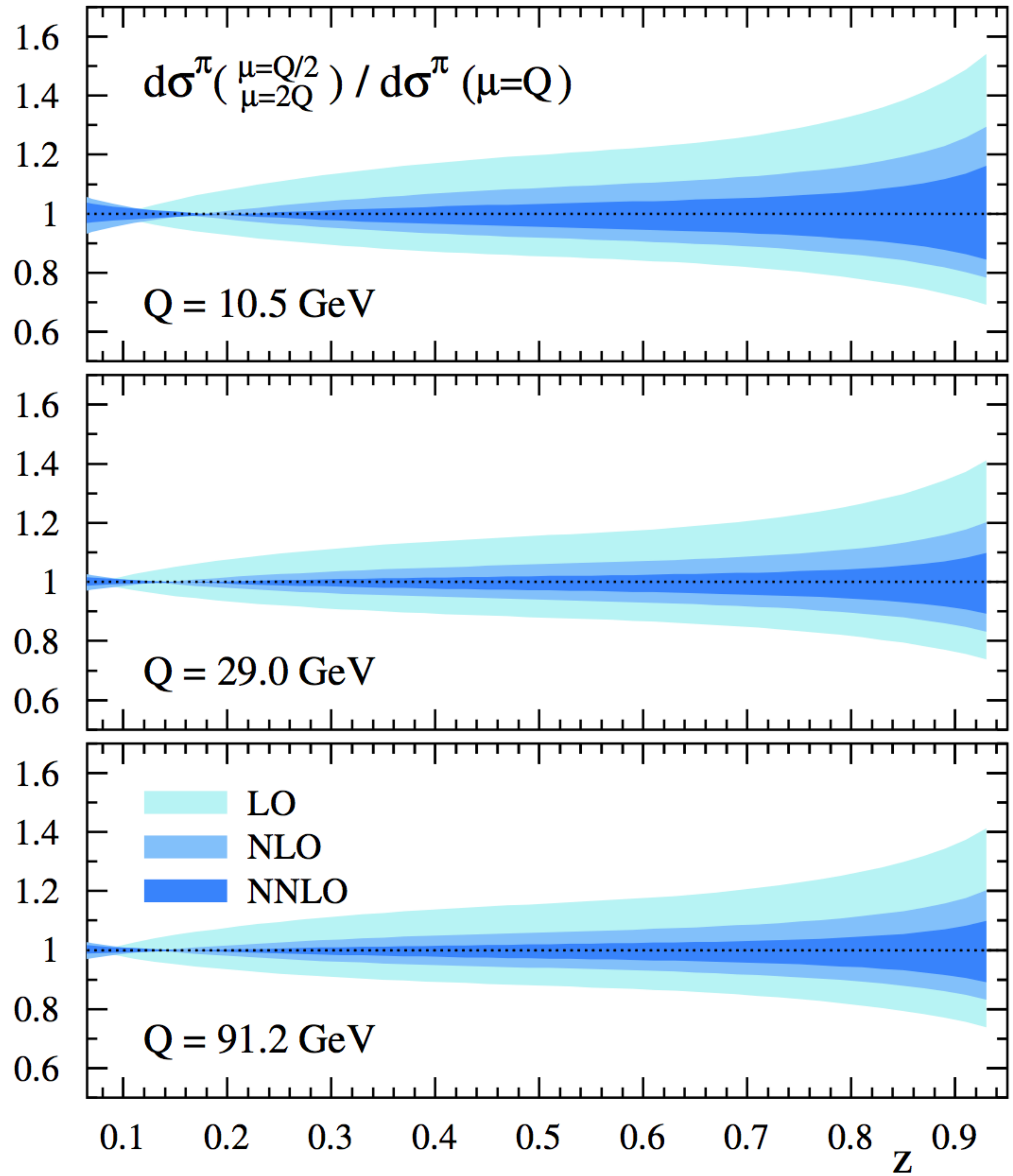


$$D(z) \otimes \mathbb{C}(z) = \frac{1}{2\pi i} \int_{\mathcal{C}_N} dN\, z^{-N}\, D(N)\, \mathbb{C}(N)$$

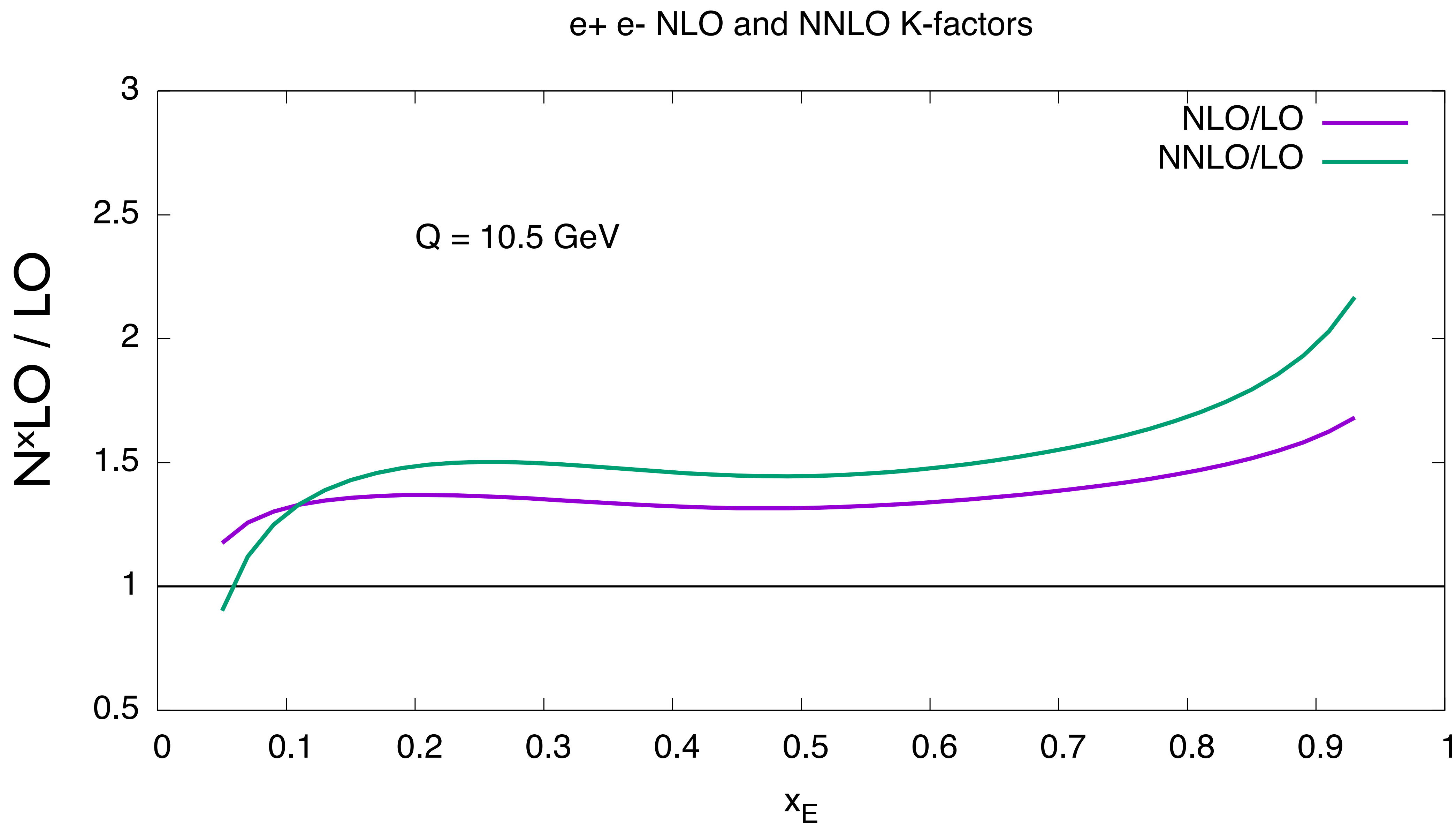
$$P_{gg}^{S,(1)}(x) \propto 1/z \longrightarrow \propto \ 1/(N-1)$$

$$P_{gg}^{T,(1)}(z) \propto \log^2(z)/z \longrightarrow \propto 1/(N-1)^3$$

$$P_{gg}^{T,(m)}(N) \propto 1/(N-1)^{(2m+1)}$$


dσ^π(μ=Q/2 μ=2Q) / dσ^π (μ=Q)
Q = 10.5 GeV
Q = 29.0 GeV
Q = 91.2 GeV
LO
NLO
NNLO
1.6
1.4
1.2
1
0.8
0.6
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
z

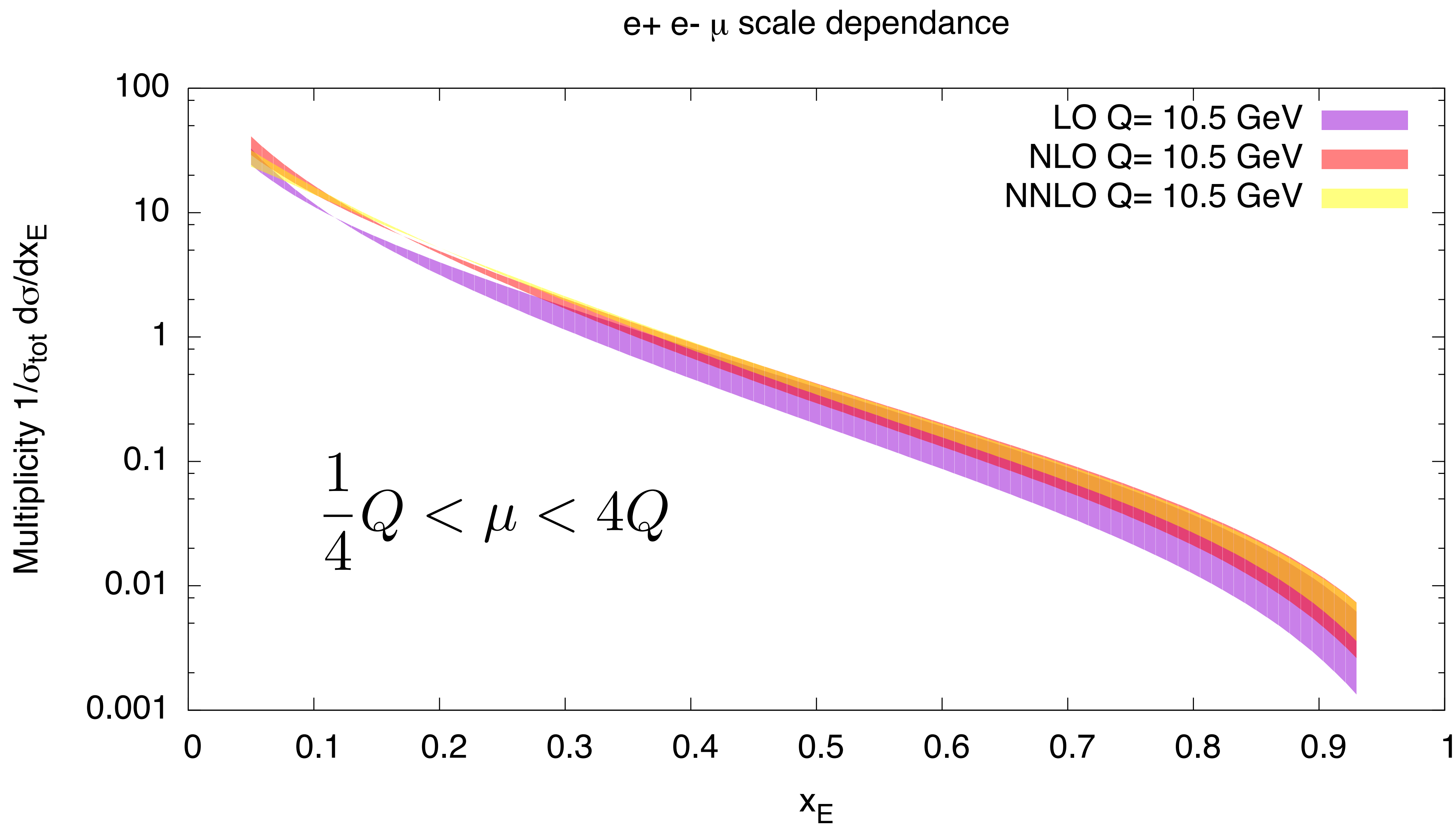
NNLO E+E- WITH "PEGASUS_FF"



Multiplicity $R^h_{e^+e^-} \equiv \frac{1}{\sigma^{tot}} \frac{d^2\sigma^h}{dx_E\, d\cos\theta}$

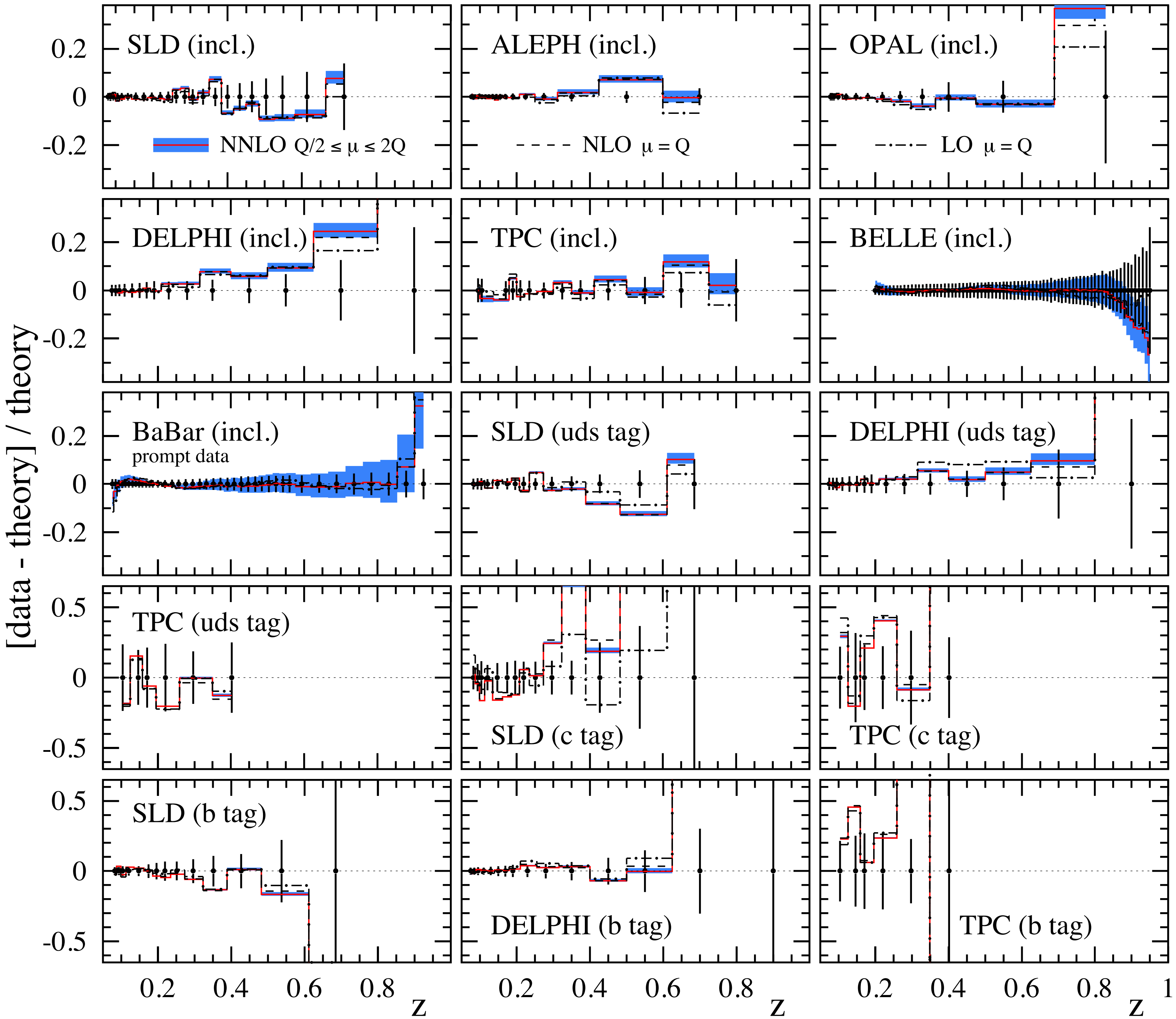
using input parameter for FF of Kretzer (Phys.Rev. D62 (2000) 054001) and truncated-solution

NNLO e+e- with "Pegasus_FF"



Multiplicity $R^h_{e^+e^-} \equiv \frac{1}{\sigma^{\rm tot}} \frac{d^2\sigma^h}{dx_E\, d\cos\theta}$

using input parameter for FF of Kretzer (Phys.Rev. D62 (2000) 054001) and truncated-solution



$$\frac{d\sigma_k^h}{dz} = \sigma_{\rm tot}^{(0)} \Big[D_{\rm S}^h(z,\mu^2) \otimes \mathbb{C}_{k,q}^{\rm S}\left(z, \frac{Q^2}{\mu^2}\right) + D_g^h\,(z,\mu^2) \otimes \mathbb{C}_{k,g}^{\rm S}\left(z, \frac{Q^2}{\mu^2}\right)\Big] + \sum_q \sigma_q^{(0)}\, D_{{\rm NS},q}^h(z,\mu^2) \otimes \mathbb{C}_{k,q}^{\rm NS}\left(z, \frac{Q^2}{\mu^2}\right)$$

THE NNLO EVOLUTION CODE "PEGASUS_FF"

Existing NNLO Evolution CODES:

X-SPACE APFEL(time-like version C/C++, Fortran77, Python)
Bertone1,Carrazza, Rojo (CERN-PH-TH/2013-209)

Mellin SPACE MELA(Fortran77)
Bertone1,Carrazza, Nocera (CERN-PH-TH-2014-265)

Newly born:

Mellin SPACE Pegasus_FF (Fortran77) → based on Pegasus(Fortran77)
Anderle, Ringer, Stratmann — Vogt (Comput.Phys.Commun.170:65-92,2005)

"PEGASUS_FF": HEAVY FLAVOURS

Parametrization of light patrons FF @ μ_0

$$D_i^h(z, Q_0) = \frac{N_i z^{\alpha_i}(1-z)^{\beta_i}[1+\gamma_i(1-z)^{\delta_i}]}{B[2+\alpha_i, \beta_i+1] + \gamma_i B[2+\alpha_i, \beta_i+\delta_i+1]}$$

So that $N_i = \int_0^1 z D_i^h dz$

"Pegasus_FF" OPTIONS

FIXED FLAVOUR SCHEME: the evolution is done for a fixed number of flavours for which the initial-scale functional form corresponds to the above one

NON PERTURBATIVE INPUT: at $\mu > m_q$ the evolution is set to evolve with $n_f + 1$ for flavours and for the q-heavy quark FF the same functional form as for the light quark is set at $\mu = m_q$

VARIABLE FLAVOUR SCHEME: at $\mu > m_q$ the evolution is set for $n_f + 1$ flavours and the q-heavy quark FF is fixed by matching-conditions at $\mu = m_q$

"PEGASUS_FF": HEAVY FLAVOURS

MATCHING CONDITION: computed by imposing the equality between the massive calculation and the massless (MS-bar) calculated cross section @ $\mu_f = m_q$

COMPUTED ONLY up to NLO: Cacciari, Nason, Oleari (JHEP 0510:034,2005)

$$D^{(n)}_{h/\bar{h}}(x,\mu) = \int_x^1 \frac{dy}{y} D_g(x/y,\mu) \times \frac{\alpha_S}{2\pi} C_F \frac{1+(1-y)^2}{y}\left[\log\frac{\mu^2}{m^2} - 1 - 2\log y\right]$$

$$D^{(n)}_{g}(x,\mu) = D^{(n_L)}_{g}(x,\mu)\left(1 - \frac{T_F\alpha_S}{3\pi}\log\frac{\mu^2}{m^2}\right)$$

$$D^{(n)}_{i/\bar{i}}(x,\mu) = D^{(n_L)}_{i/\bar{i}}(x,\mu) \qquad \text{for } i = q_1,\ldots,q_{n_L}$$

$$n_L = n_f + 1$$

TOWARDS A GLOBAL NNLO FF FIT

Ingredients needed to achieve the goal:

NNLO COEFFICINT FUNCTIONS:

SI-e^+e^- → x-Space: Rijken, van Neerven (Phys.Lett.B386(1996)422, Nucl.Phys.B488(1997)233,Phys.Lett.B392(1997)207)

Mellin-Space: Mitov, Moch (Nucl.Phys.B751 (2006) 18-52); Blümlein, Ravindran (Nucl.Phys.B749 (2006) 1-24)

SIDIS → NOT COMPUTED YET but work in progress

$$\gamma q' \to q\bar{q}q', \quad \gamma g \to q\bar{q}q'$$

Anderle, de Florian, Rotstein, Vogelsang

SI- p(anti-)p → NOT COMPUTED YET

TOWARDS A GLOBAL NNLO FF FIT

Ingredients needed to achieve the goal:

NNLO COEFFICINT FUNCTIONS:

SI-e^+e^- → **x-Space** Rijken, van Neerven (Phys.Lett.B386(1996)422, Nucl.Phys.B488(1997)233,Phys.Lett.B392(1997)207)

Mellin-Space Mitov, Moch (Nucl.Phys.B751 (2006) 18-52)
Blümlein, Ravindran (Nucl.Phys.B749 (2006) 1-24)

@ NNLO Harmonic PolyLogs(HPL) appear in the coefficient functions

↓

Calculation of Mellin moments non trivial

TOWARDS A GLOBAL NNLO FF FIT

@NLO the moments of the coefficient functions contain at worst SINGLE HARMONIC SUMS, which can be consistently continued in the complex plane

$$
\begin{aligned}
S_k(N) &= (-1)^{k-1}\frac{1}{(k-1)!}\psi^{(k-1)}(N+1)+c_k^+ \\
S_{-k}(N) &= (-1)^{k-1+N}\frac{1}{(k-1)!}\beta^{(k-1)}(N+1)-c_k^-
\end{aligned}
$$

$\psi(z)$ first derivative of Gamma Function

$$
\begin{aligned}
\beta(z) &= \frac{1}{2}\left[\psi\left(\frac{z+1}{2}\right)-\psi\left(\frac{z}{2}\right)\right] \\
c_1^+ &= \gamma_E \\
c_k^+ &= \zeta(k), \quad k \geq 2 \\
c_1^- &= \log(2) \\
c_k^+ &= \left(1-\frac{1}{2^{k-1}}\right)\zeta(k), \quad k \geq 2
\end{aligned}
$$

Towards a Global NNLO FF Fit

@NNLO ⟶ MULTIPLE HARMONIC SUMS from MT-HPLs

$$S_{k_1,\ldots,k_m}(N) = \sum_{n_1=1}^{N} \frac{[\mathrm{sign}(k_1)]^{n_1}}{n_1^{|k_1|}} \sum_{n_2=1}^{n_1} \frac{[\mathrm{sign}(k_2)]^{n_2}}{n_2^{|k_2|}} \cdots \sum_{n_m=1}^{n_{m-1}} \frac{[\mathrm{sign}(k_m)]^{n_m}}{n_m^{|k_m|}}$$

ANALITICAL CONTINUATIONS: provided by Blümlein,Kurth(Phys. Rev. D60 (1999) 014018)

also as FORTRAN77 routines Blümlein(Comput. Phys. Commun. 133 (2000) 76))

THRESHOLD RESUMMATION

For both DIS and SIA

in Mellin space: exponentiation of the one-loop results

$$\mathcal{C}_q^{T,res} \propto \exp\left[\int_0^1 d\xi \frac{\xi^N - 1}{1-\xi} \times \left\{ \int_{Q^2}^{(1-\xi)Q^2} \frac{dk_\perp^2}{k_\perp^2} A_q(\alpha_s(k_\perp^2)) + \frac{1}{2} B_q\left(\alpha_s((1-\xi)Q^2)\right) \right\}\right]$$

where
$$A^{(1)} = C_F \ , \quad A^{(2)} = \frac{1}{2} C_F\, K = \frac{1}{2}\, C_F \left[C_A \left(\frac{67}{18} - \frac{\pi^2}{6} \right) - \frac{5}{9} N_f \right]$$
$$B^{(1)} = -\frac{3}{2} C_F \ .$$

Catani, Trentadue; Stermann

Threshold Resummation acts for DIS and SIA in the same exact way and is relevant for the same Phase Space region:

DIS: $x \to 1$
SIA: $x_E \to 1$

TOWARDS A GLOBAL NNLO FF FIT

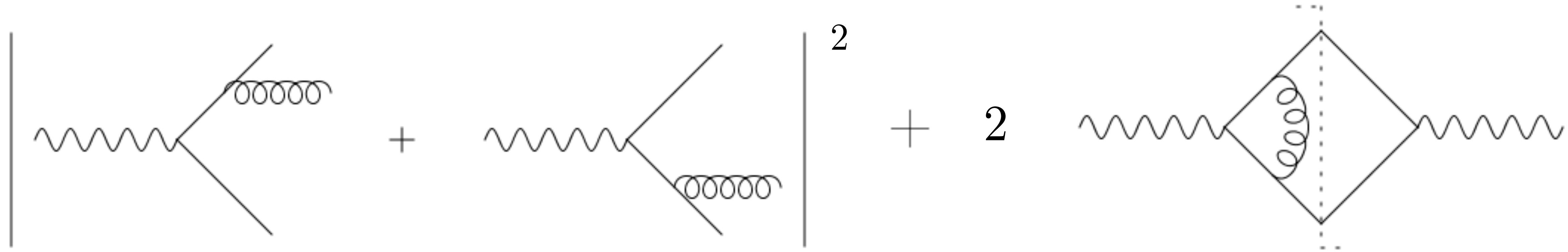
We have checked the Mellin moments calculation and the consistency between Mitov, Moch and Blümlein, Ravindran notation

NUMERICALLY and ANALITICALLY: making use of

- "HPL"-Mathematica package, D. Maître(Comput.Phys.Commun. 174 (2006) 222-240)

- "MT"-Mathematica package, Hoeschele,Hoff, Pak,Steinhauser, Ueda(arXiv:1307.6925)

NLO COEFFICIENT FUNCTION (SIA)



large corrections near threshold $\hat{z} \to 1$

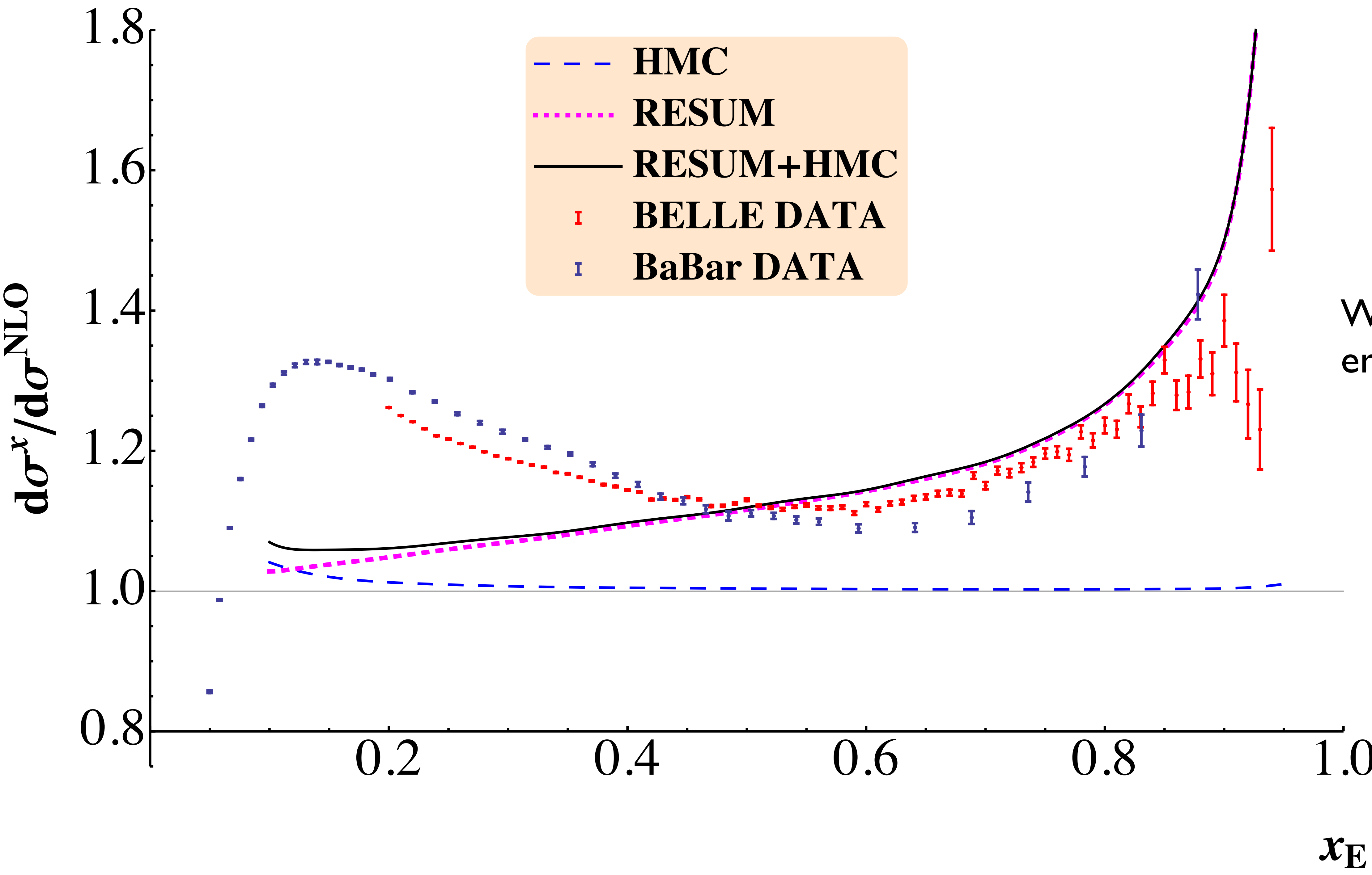
$$\hat{C}_q^{T,(1)} \sim e_q^2 C_F \left[2 \left(\frac{\log(1-\hat{z})}{1-\hat{z}} \right)_+ - \frac{3}{2} \frac{1}{(1-\hat{z})_+} + \left(\frac{2\pi^2}{3} - \frac{9}{2} \right) \delta(1-\hat{z}) \right]$$

$\overline{\text{MS}}$ scheme

Altarelli et al.; Furmanski, Petronzio; Nason, Webber...

$$\int_0^1 dz\, f(z) \left(\frac{\ln(1-z)}{1-z} \right)_+ \equiv \int_0^1 dz\, (f(z) - f(1)) \frac{\ln(1-z)}{1-z}$$

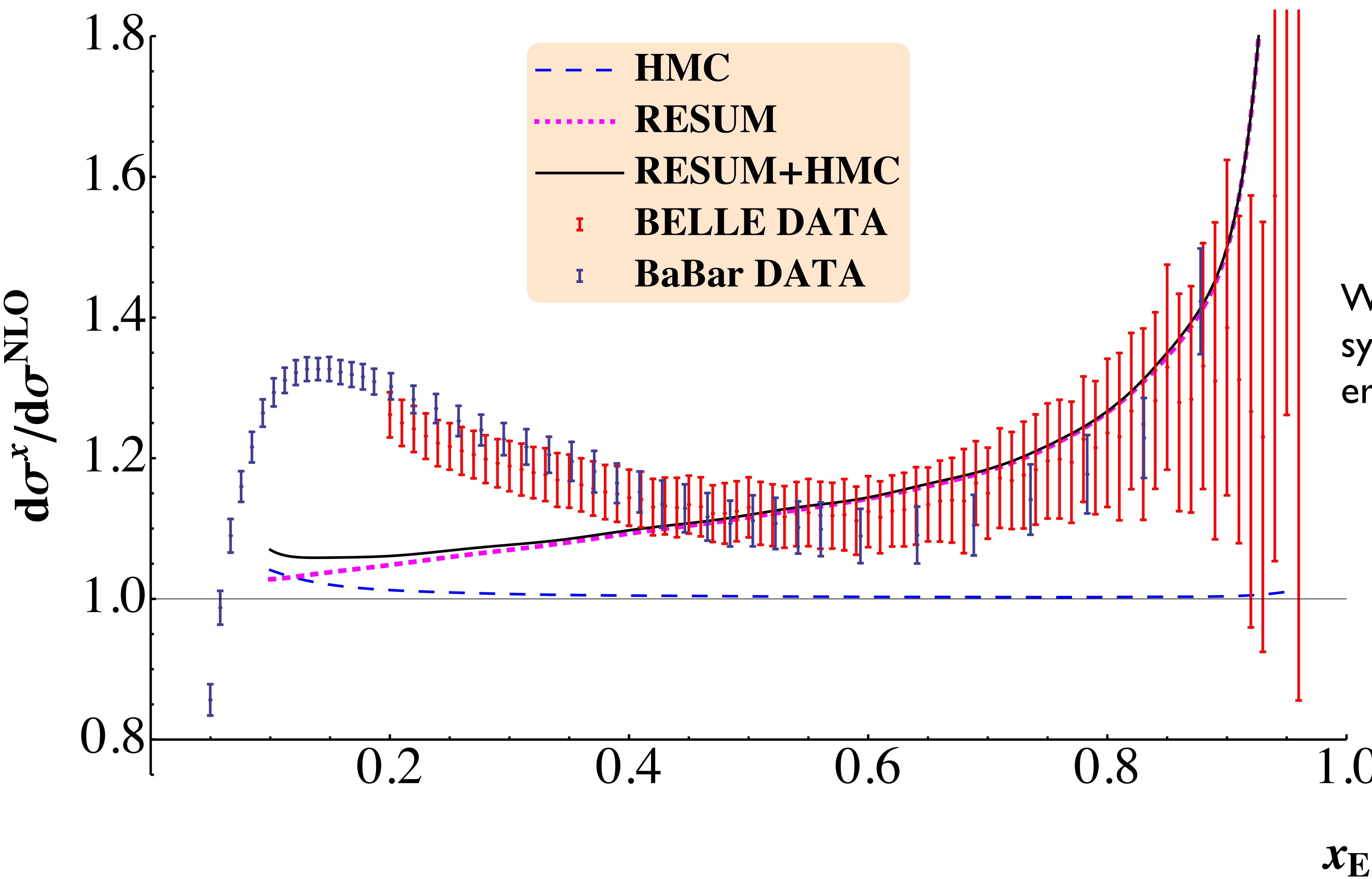
For Pions HMC is irrelevant



With statistical errors

Belle collaboration arXiv: 1301.6183; BaBar collaboration arXiv: 1306.2895

For Pions HMC is irrelevant

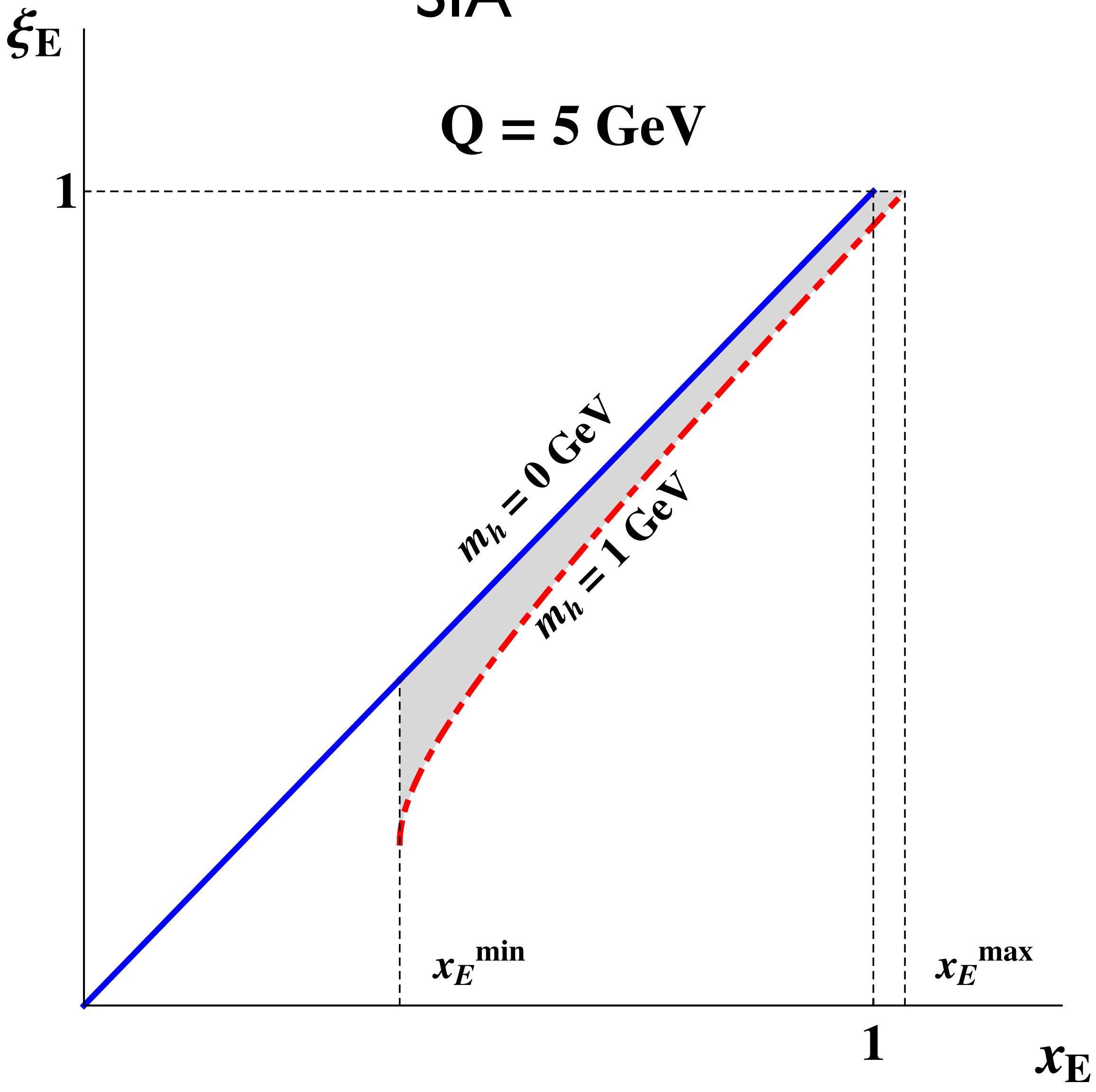


With systematical errors

Belle collaboration arXiv: 1301.6183; BaBar collaboration arXiv: 1306.2895

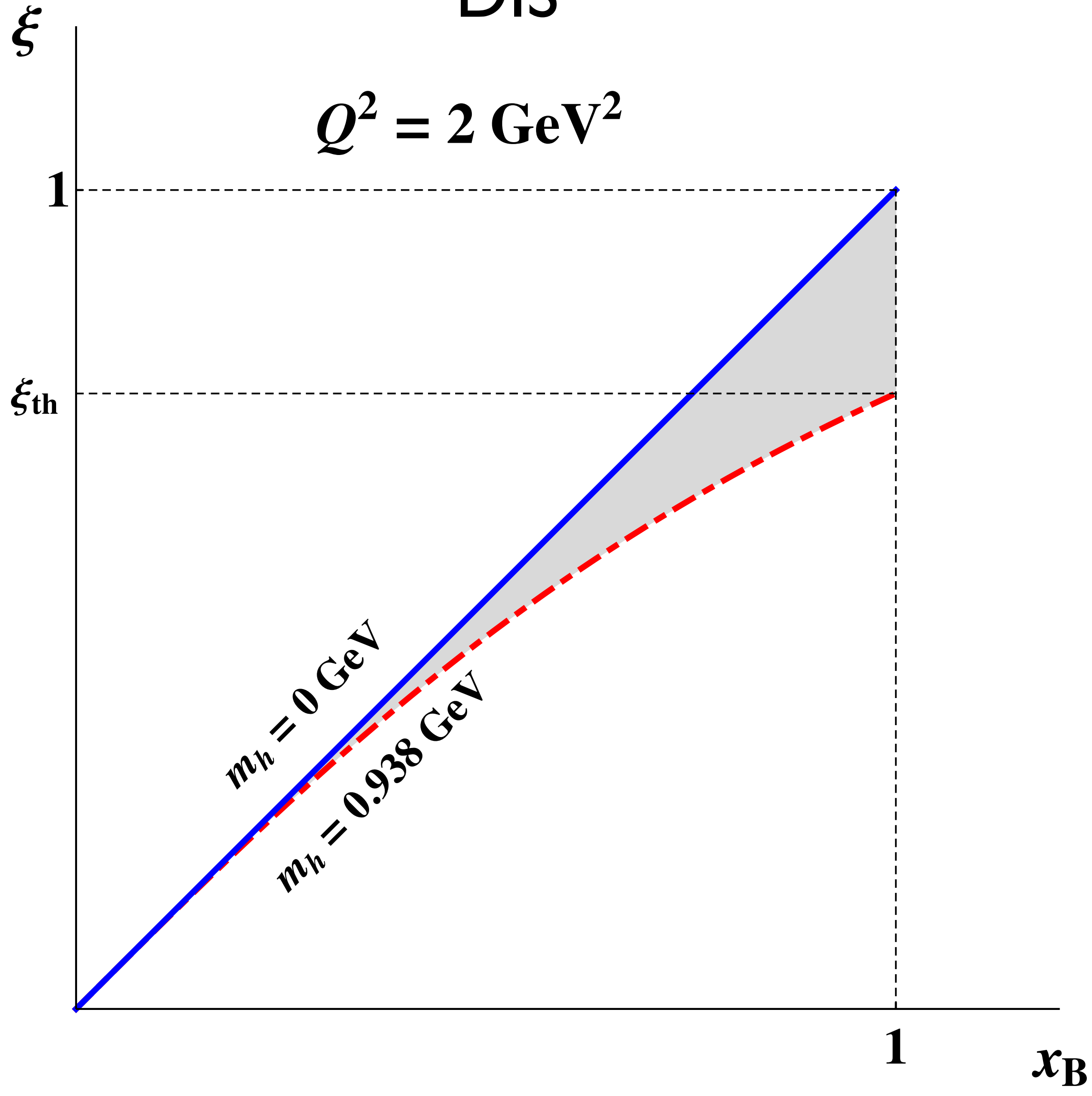
The hadron mass acts kinematically on the two processes in a very different way

SIA



Low x_E effect

DIS

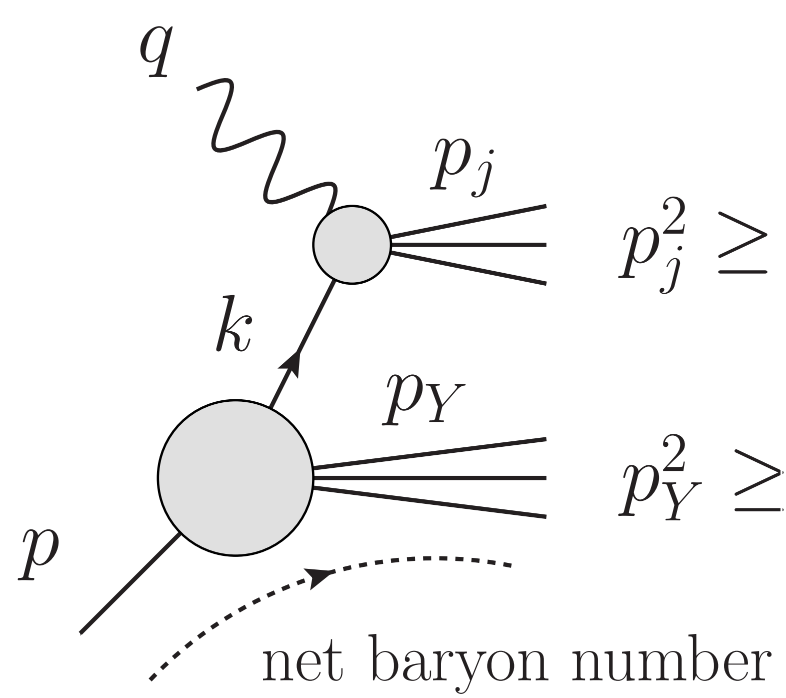


High x_B effect

AS RESUMMATION!!!

RESUMMATION AND HMC INTERPLAY (DIS)

Taking into account momentum conservation law and some simple algebra



$$p_j^2 \geq 0$$

$$p_Y^2 \geq m_N^2$$

$$0 \leq p_j^2 = (q+k)^2 = \left(1 - \frac{\xi}{\hat{x}}\right) \frac{Q^2 \hat{x}}{\xi}$$

$$(q+p)^2 = (p_j + p_Y)^2 \geq (q+k)^2 + m_N^2$$

$$1 - \frac{1}{x_B} \leq \left(1 - \frac{\hat{x}}{\xi}\right)$$

we find that the partonic momentum fraction $\hat{x}$ is limited as

$$\xi \leq \hat{x} = \frac{k^+}{P_h^+} \leq \xi / x_B$$

In the definition of the structure functions the integration limits need to be modified

$$\mathcal{F}_i(\xi, Q^2) = \sum_f \int_{\xi}^{\xi/x_B} \frac{d\hat{x}}{\hat{x}} f(\hat{x})\, \mathcal{C}_f^i\left(\frac{\xi}{\hat{x}}, Q^2\right)$$

Accardi and Qiu(JHEP 0807:090,2008)

This effects also Threshold Resummation correction

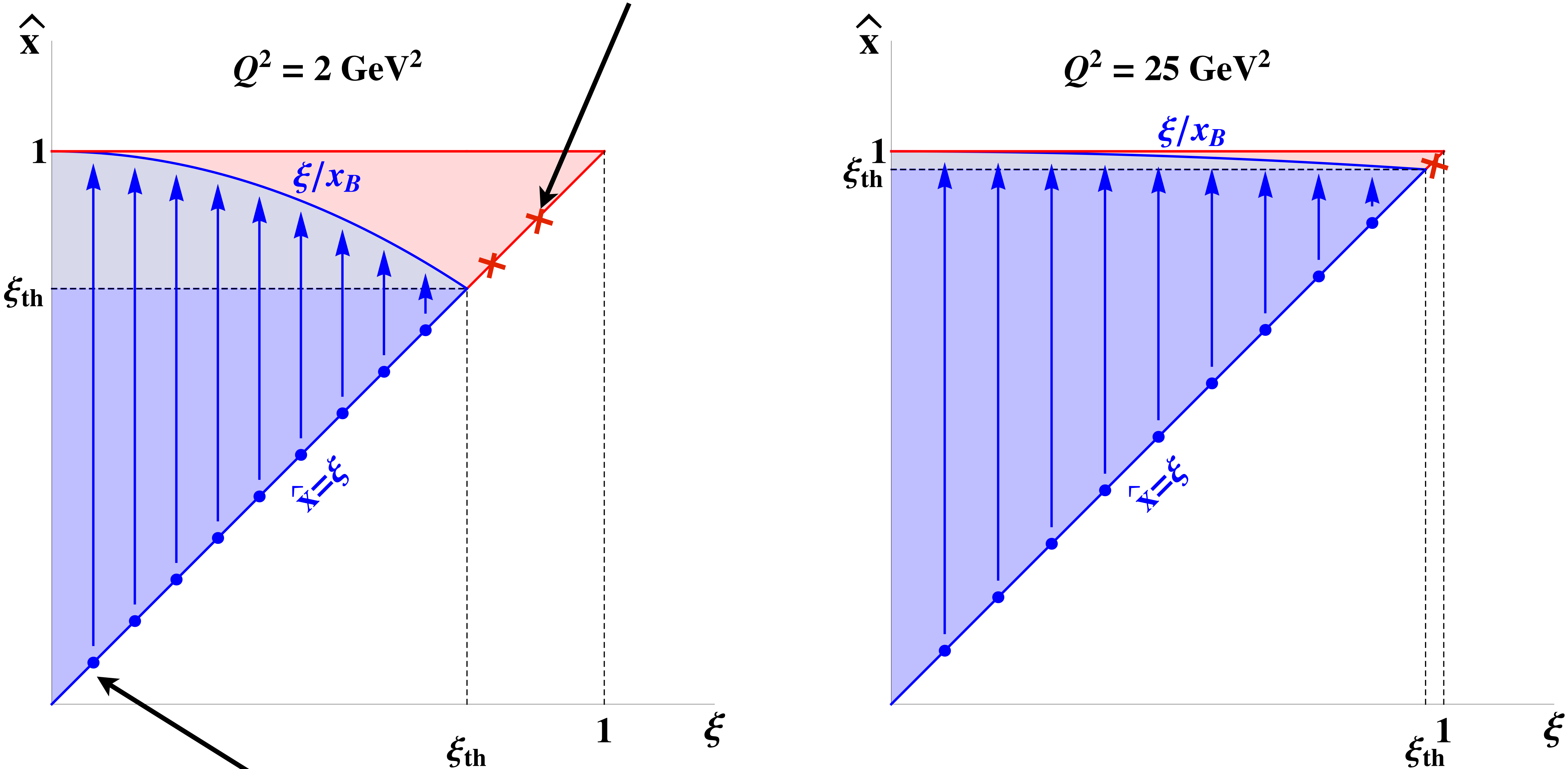
In order to be able to perform the Mellin Transform properly be able to use the resumption formula, we have to define

$$\begin{aligned}
\mathcal{F}_1^{\mathrm{TMC},N} &= \int_0^1 d\xi\, \xi^{N-1} \int_\xi^{\xi_{\mathrm{th}}} \frac{dx}{x} \mathcal{C}_f^1\left(\frac{\xi}{x}\right) f(x) \\
&= \int_0^1 d\xi\, \xi^{N-1} \int_0^1 dy \int_0^{\xi_{\mathrm{th}}} dx\, \mathcal{C}_f^1(y)\, f(x)\, \delta(xy-\xi) \\
&= \left(\int_0^1 dy\, y^{N-1} \mathcal{C}_f^1(y)\right) \left(\int_0^{\xi_{\mathrm{th}}} dx\, x^{N-1} f(x)\right) \\
&= \mathcal{C}_f^{1,N}\, f_{\xi_{\mathrm{th}}}^N
\end{aligned}$$

Truncated-Moments of PDF

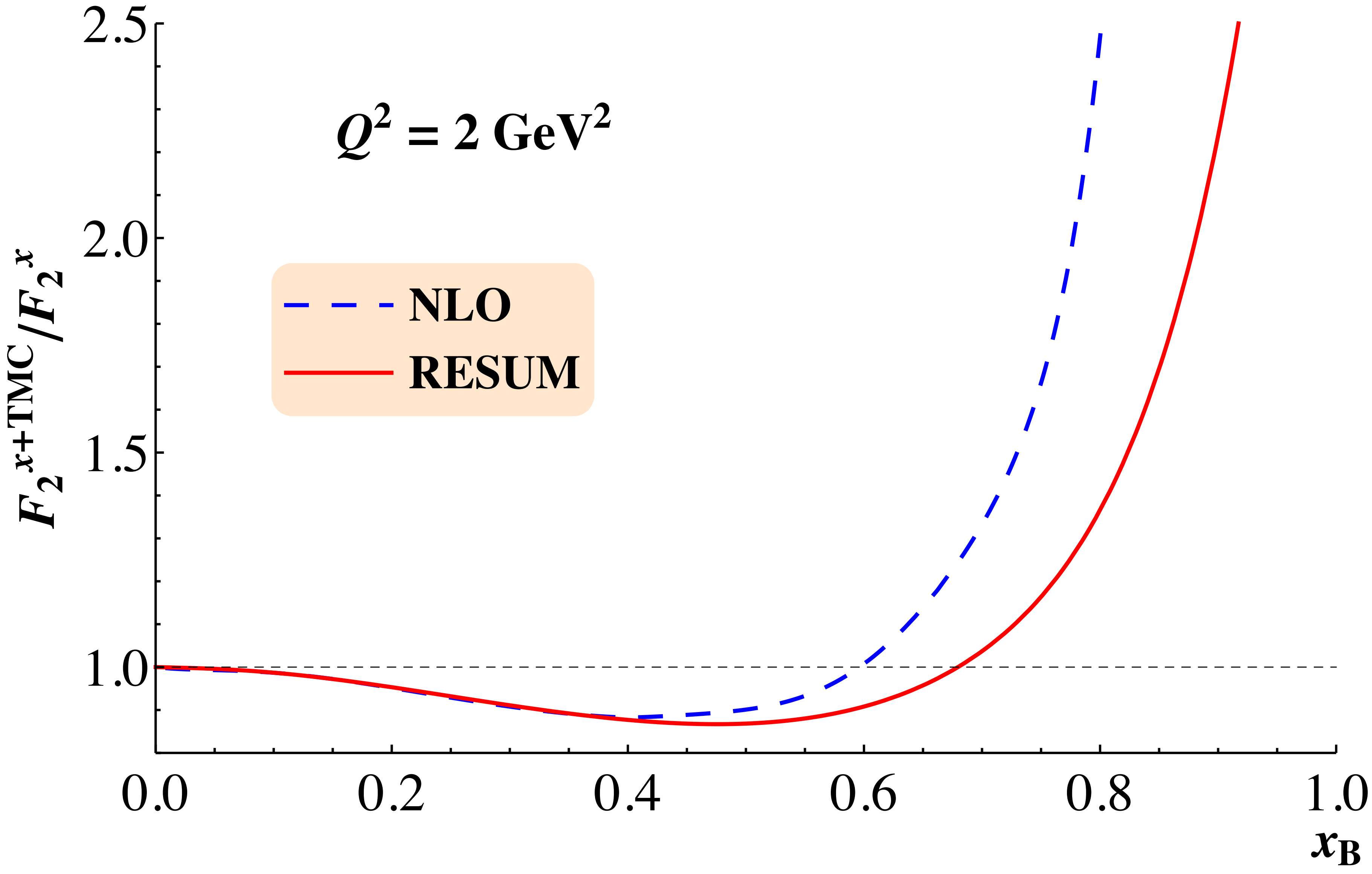
Integration support

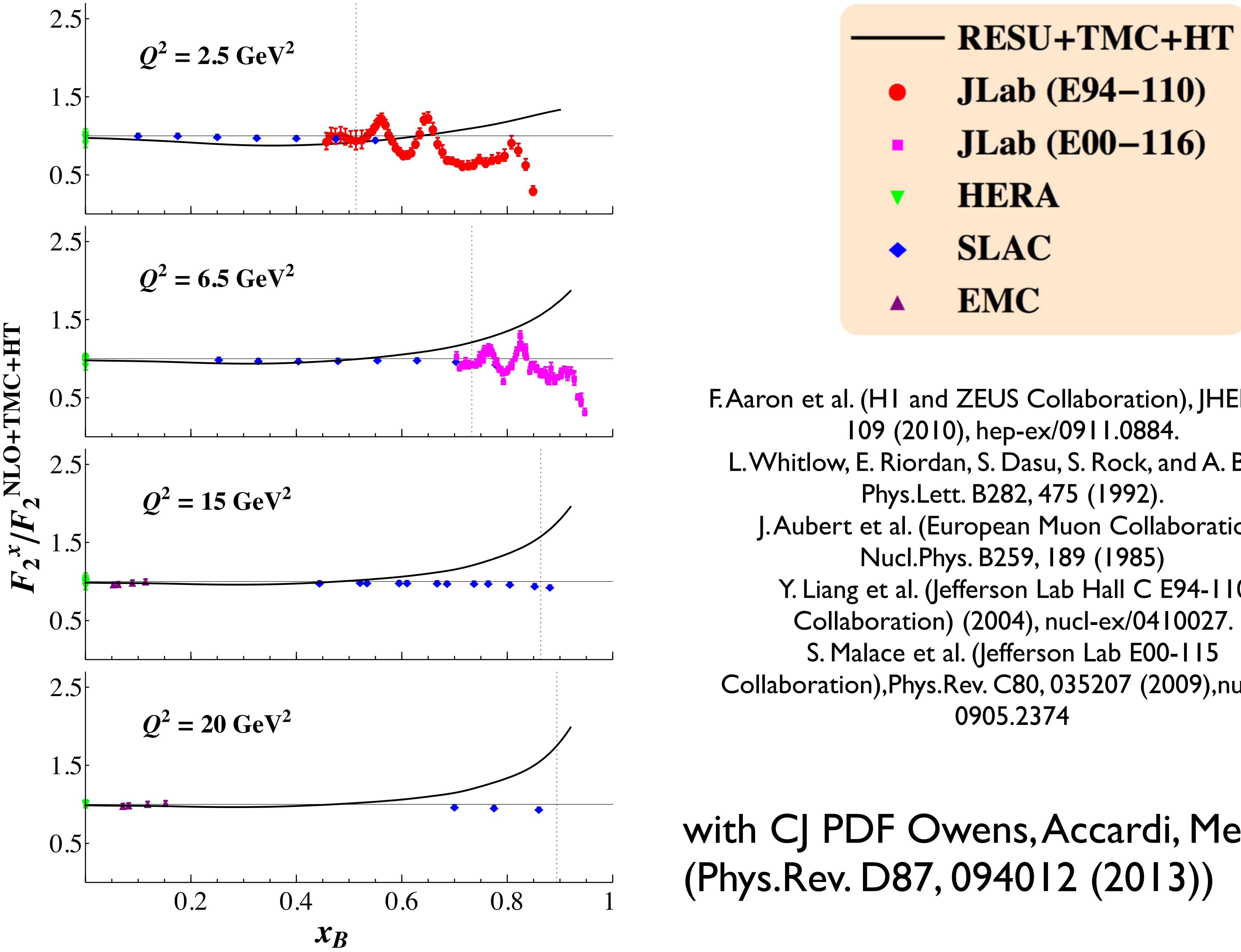
threshold logs excluded from integration



While integrating near the blue dots, the big threshold logs are encountered

For DIS the TMC and Threshold Resummation do not act independently





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