

Collins functions from recent e⁺e⁻ data

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- Umberto D'Alesio (Univ. of Cagliari)

Outline

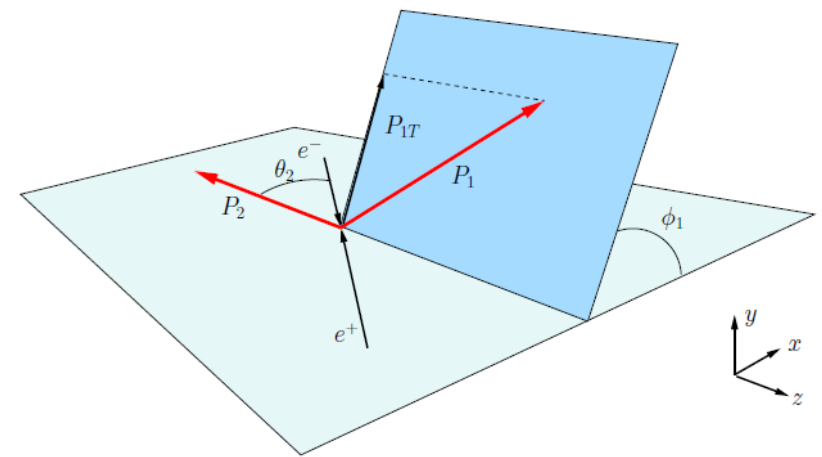
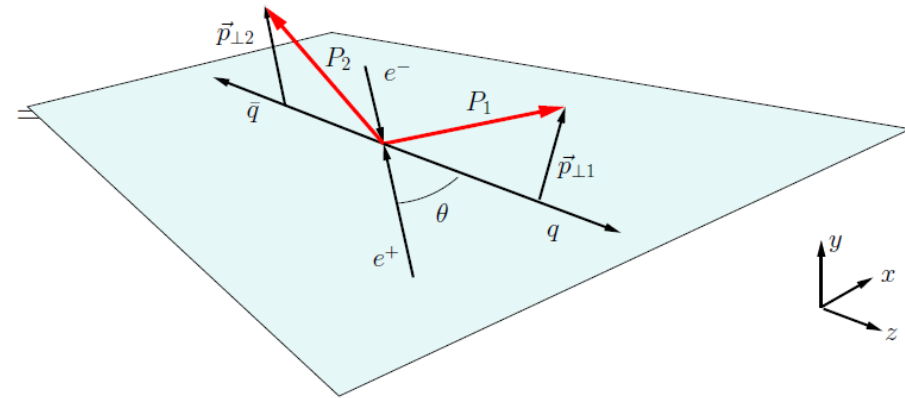
- **Description of available data.**
- **Ingredients to extract Collins function.**
- **Results on pion Collins function and its transverse momentum dependence.**
- **Results on kaon Collins function.**

Transversity and Collins function

$$e^+e^- \rightarrow \pi\pi X$$

Collins TMDFF

$$\frac{d\sigma^{e^+e^- \rightarrow h_1 h_2 X}}{dz_1 dz_2 d^2\mathbf{P}_{1T} d\cos\theta_2} = \frac{3\pi\alpha^2}{2s} \left\{ D_{h_1 h_2} + \boxed{N_{h_1 h_2}} \cos 2\phi_1 \right\}$$



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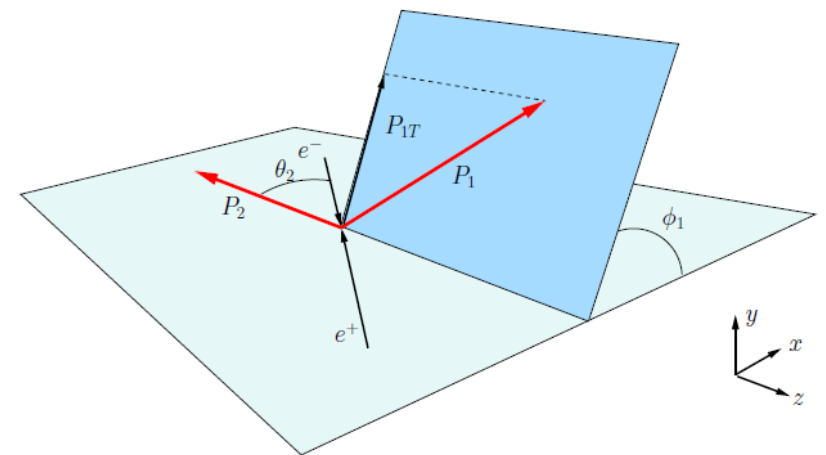
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$$P_0^{U,L,C} = \frac{N^{U,L,C}}{D^{U,L,C}}$$

Ratio

$$\begin{aligned} D^U &= D_{\pi^+\pi^-} + D_{\pi^-\pi^+} & N^U &= N_{\pi^+\pi^-} + N_{\pi^-\pi^+} \\ D^L &= D_{\pi^+\pi^+} + D_{\pi^-\pi^-} & N^L &= N_{\pi^+\pi^+} + N_{\pi^-\pi^-} \\ D^C &= D^U + D^L & N^C &= N^U + N^L, \end{aligned}$$

Data from Belle, Babar at
 $Q^2 = 112 \text{ GeV}^2$



Transversity and Collins function

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Collins TMDFF

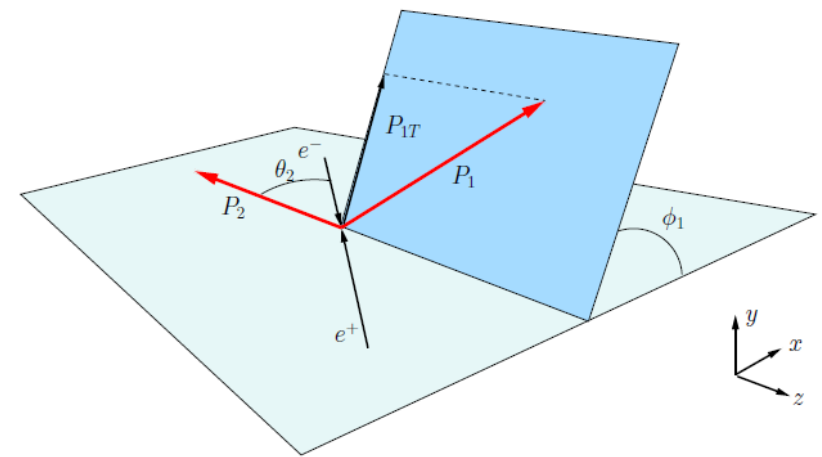
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$$\frac{A_0^U}{A_0^{L(C)}} \equiv 1 + \cos(2\phi_1) \boxed{A_0^{UL(C)}} \quad \text{Double Ratio}$$

Different charge combinations are considered: Like (L), Unlike (U), Charged (C)



Transversity and Collins function

$$e^+e^- \rightarrow \pi\pi X$$

Unpolarized TMDFF

Collins TMDFF

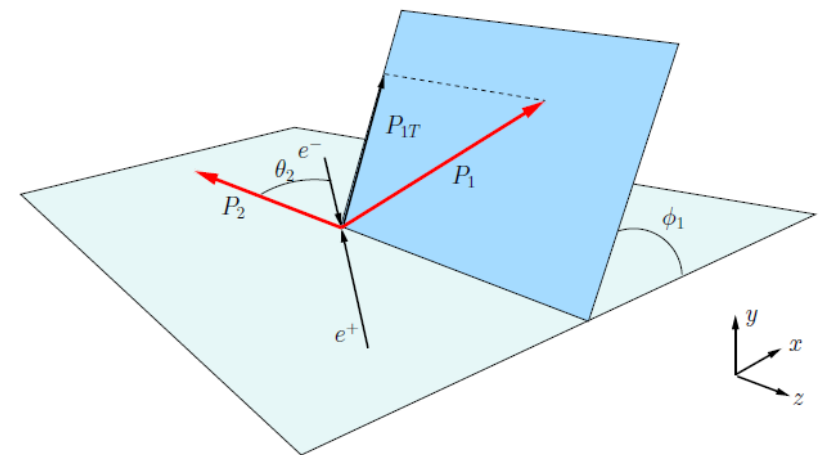
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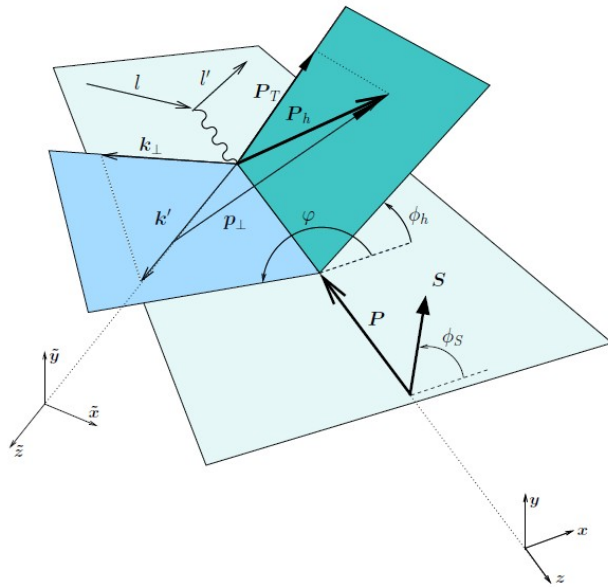
$$\begin{aligned} D^U &= D_{\pi^+\pi^-} + D_{\pi^-\pi^+} & N^U &= N_{\pi^+\pi^-} + N_{\pi^-\pi^+} \\ D^L &= D_{\pi^+\pi^+} + D_{\pi^-\pi^-} & N^L &= N_{\pi^+\pi^+} + N_{\pi^-\pi^-} \\ D^C &= D^U + D^L & N^C &= N^U + N^L, \end{aligned}$$

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Transversity and Collins function



HERMES and COMPASS data

$Q^2 = 2-3 \text{ GeV}^2$

SIDIS

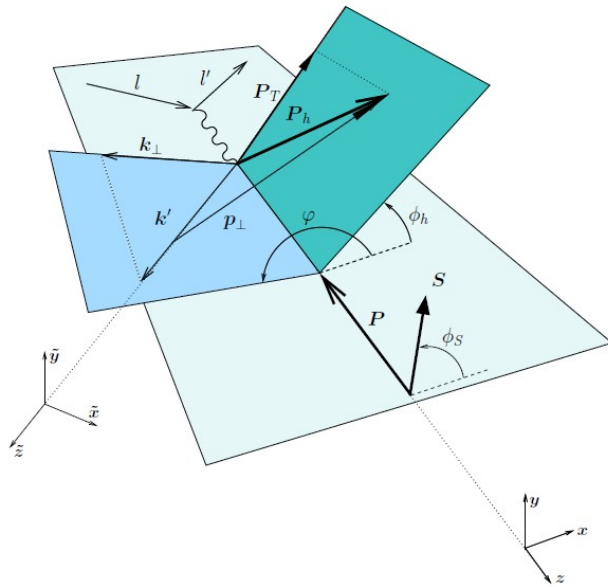
$$\frac{d\sigma^{\ell(S_e)+p(S)\rightarrow\ell' hX}}{dx_B dQ^2 dz_h d^2\mathbf{P}_T d\phi_S} =$$

$$\frac{2\alpha^2}{Q^4} \left\{ \frac{1 + (1-y)^2}{2} F_{UU} + \dots \right.$$

$$\left. + S_T(1-y)(\sin(\phi_h + \phi_S) F_{UT}^{\sin(\phi_h + \phi_S)}) \right\}.$$

$$A_{UT}^{\sin(\phi_h + \phi_S)} \sim \frac{F_{UT}^{\sin(\phi_h + \phi_S)}}{F_{UU}}$$

Transversity and Collins function



Simultaneous extraction

SIDIS

$$\frac{d\sigma^{\ell(S_e)+p(S)\rightarrow\ell'hX}}{dx_B dQ^2 dz_h d^2\mathbf{P}_T d\phi_S} =$$

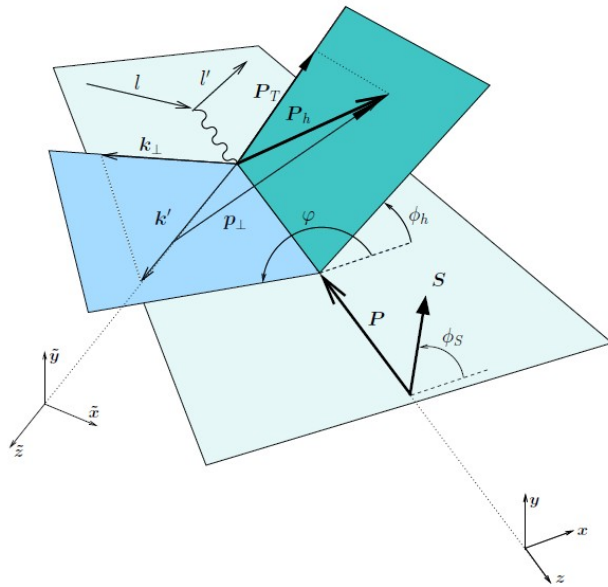
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$$A_{UT}^{\sin(\phi_h + \phi_S)} \sim \frac{F_{UT}^{\sin(\phi_h + \phi_S)}}{F_{UU}}$$

TMD Transversity
& Collins function

Transversity and Collins function



Extraction of the Collins and Transversity functions is contingent on the knowledge of Unpolarized functions.

SIDIS

$$\frac{d\sigma^{\ell(S_e)+p(S)\rightarrow\ell'hX}}{dx_B dQ^2 dz_h d^2\mathbf{P}_T d\phi_S} =$$

$$\frac{2\alpha^2}{Q^4} \left\{ \frac{1 + (1-y)^2}{2} F_{UU} + \dots \right.$$

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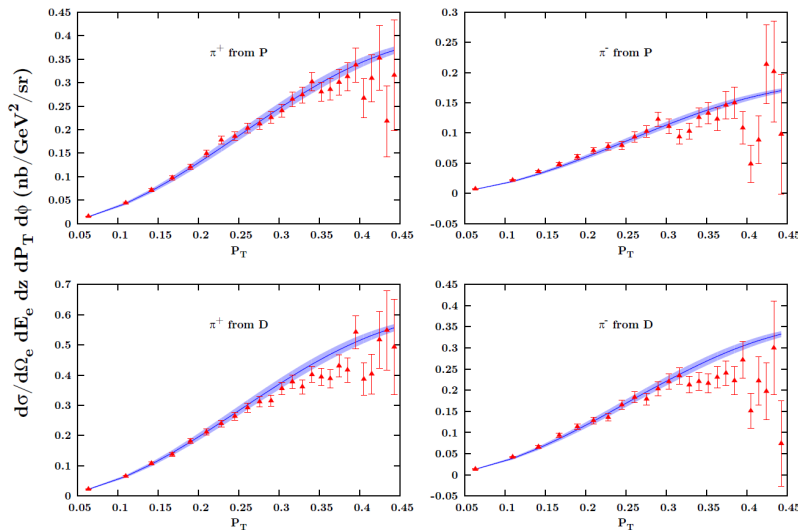
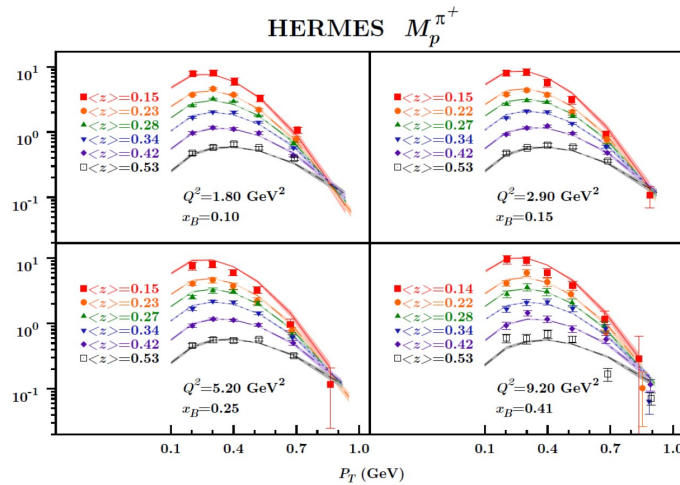
Unpolarized
TMDF
& TMDPDF

TMD Transversity
& Collins function

Ingredients for extraction of Collins function.

Unpolarized TMDFF & TMDPDF
from previous Analysis of SIDIS data

SIDIS



$$\frac{d\sigma^{\ell(S_e)+p(S)\rightarrow\ell' hX}}{dx_B dQ^2 dz_h d^2\mathbf{P}_T d\phi_S} = \frac{2\alpha^2}{Q^4} \left\{ \frac{1+(1-y)^2}{2} F_{UU} + \dots + S_T(1-y)(\sin(\phi_h + \phi_S) F_{UT}^{\sin(\phi_h + \phi_S)}) \right\}$$

$$A_{UT}^{\sin(\phi_h + \phi_S)} \sim \frac{F_{UT}^{\sin(\phi_h + \phi_S)}}{F_{UU}}$$

Unpolarized
TMDFF
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TMD Transversity
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Ingredients for extraction of Collins function.

$e^+e^- \rightarrow \pi\pi X$

SIDIS

Unpolarized TMDFF

Collins TMDFF

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$$D^U = D_{\pi^+\pi^-} + D_{\pi^-\pi^+}$$

$$N^U = N_{\pi^+\pi^-} + N_{\pi^-\pi^+}$$

$$D^L = D_{\pi^+\pi^+} + D_{\pi^-\pi^-}$$

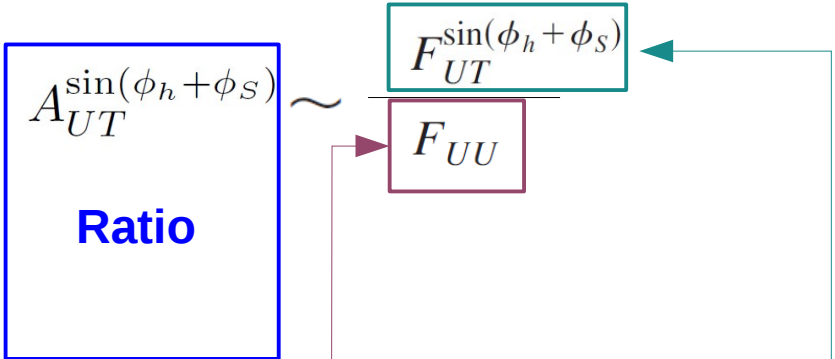
$$N^L = N_{\pi^+\pi^+} + N_{\pi^-\pi^-}$$

$$D^C = D^U + D^L$$

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Unpolarized TMDFF & TMDPDF

TMD Transversity & Collins function

Ingredients for extraction of Collins function.

Transversity

$$\Delta_T q(x, k_\perp) = \Delta_T q(x) \frac{e^{-k_\perp^2 / \langle k_\perp^2 \rangle_T}}{\pi \langle k_\perp^2 \rangle_T}$$

$$\Delta_T q(x, Q_0^2) = \mathcal{N}_q^T(x, Q_0^2) \frac{1}{2} [f_{q/p}(x, Q_0^2) + \Delta q(x, Q_0^2)]$$

$$\mathcal{N}_q^T(x) = N_q^T x^\alpha (1-x)^\beta \frac{(\alpha + \beta)^{\alpha + \beta}}{\alpha^\alpha \beta^\beta}$$

$$(q = u, d, v)$$

Collins function

$$\Delta^N D_{h/q^\uparrow}(z, p_\perp) = \tilde{\Delta}^N D_{h/q^\uparrow}(z) h(p_\perp) \frac{e^{-p_\perp^2 / \langle p_\perp^2 \rangle}}{\pi \langle p_\perp^2 \rangle}$$

$$\tilde{\Delta}^N D_{h/q^\uparrow}(z, Q_0^2) = 2 \mathcal{N}_q^C(z, Q_0^2) D_{h/q}(z, Q_0^2)$$

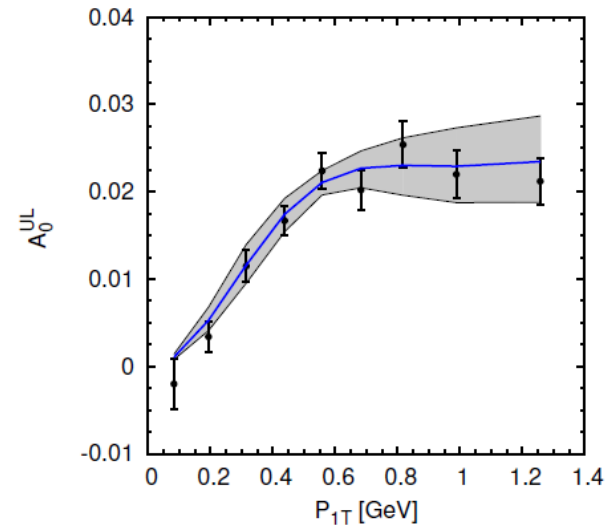
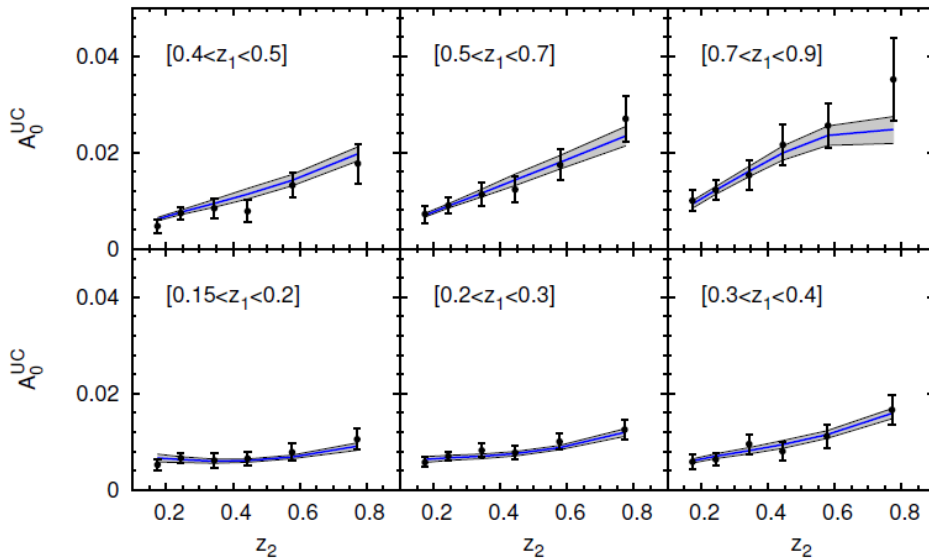
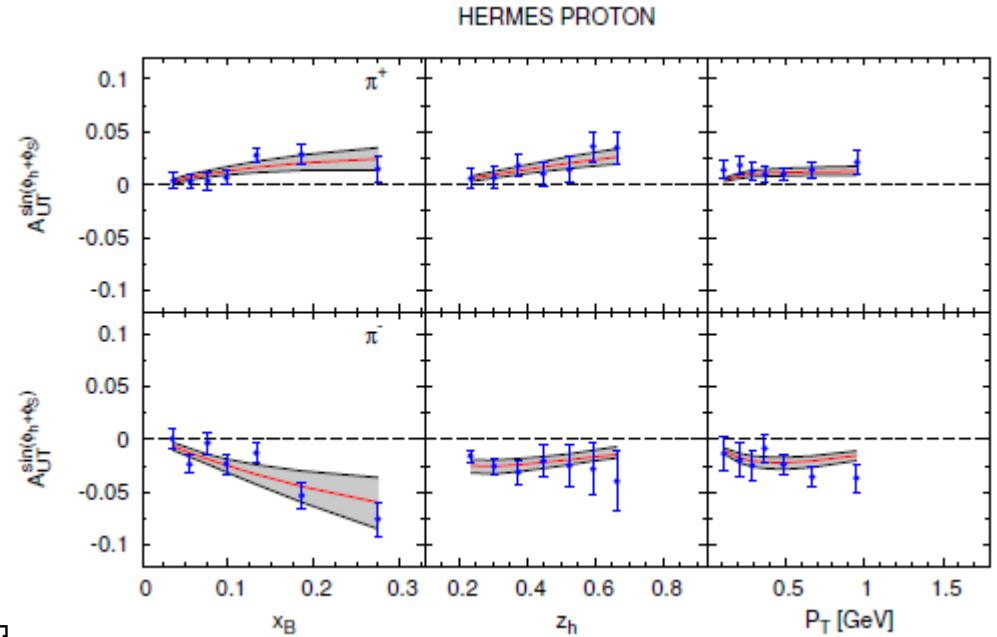
$$h(p_\perp) = \sqrt{2} e \frac{p_\perp}{M_C} e^{-p_\perp^2 / M_C^2}$$

$$\mathcal{N}_{\text{fav}}^C(z) = N_{\text{fav}}^C z^\gamma (1-z)^\delta \frac{(\gamma + \delta)^{\gamma + \delta}}{\gamma^\gamma \delta^\delta}$$

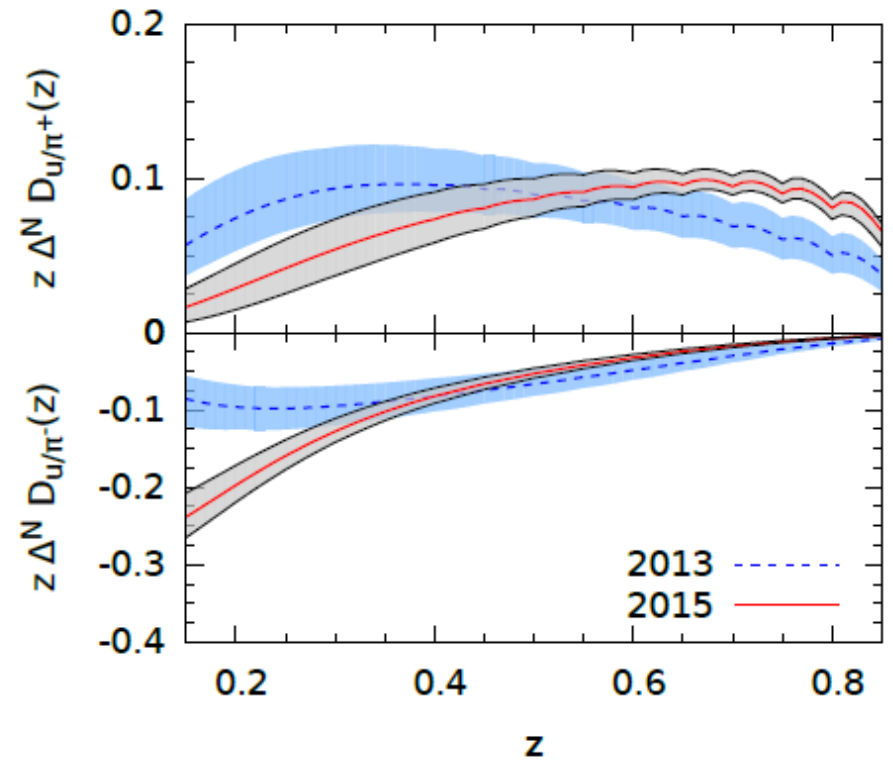
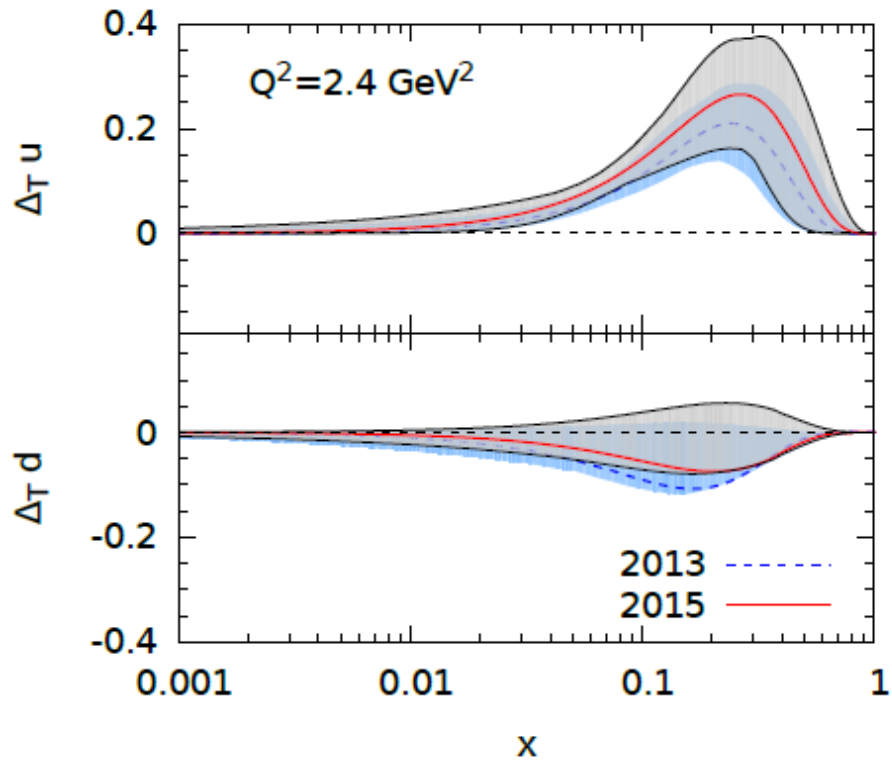
$$\mathcal{N}_{\text{dis}}^C(z) = N_{\text{dis}}^C$$

Results on pion Collins function

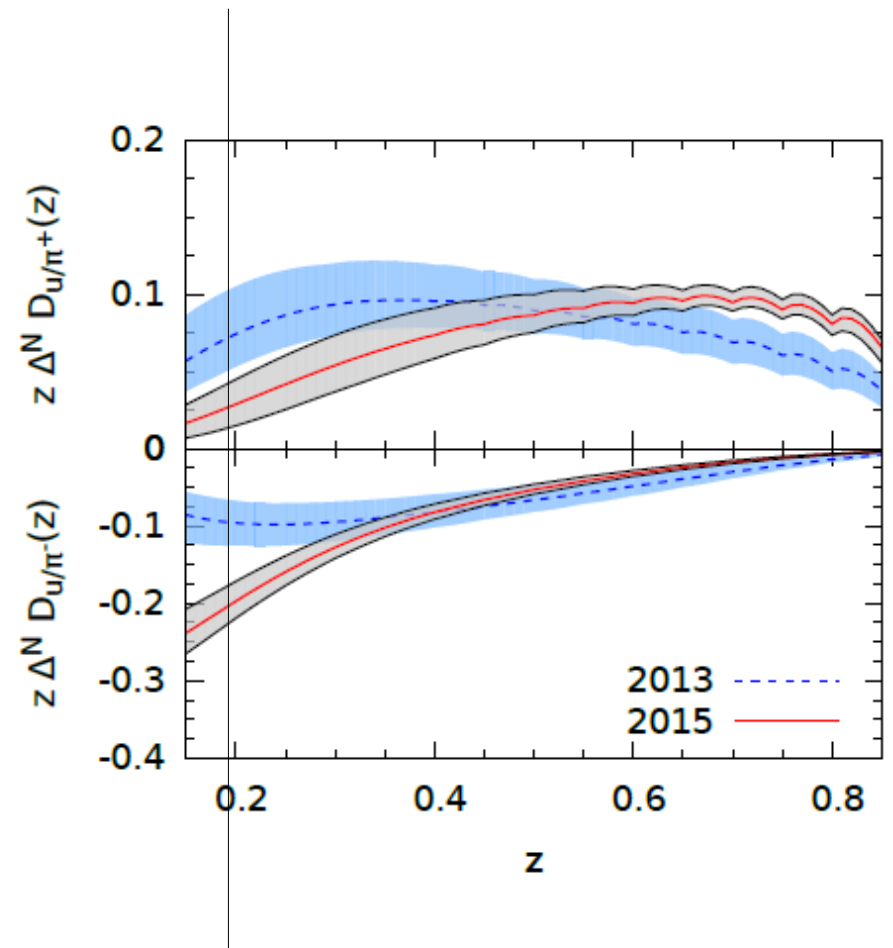
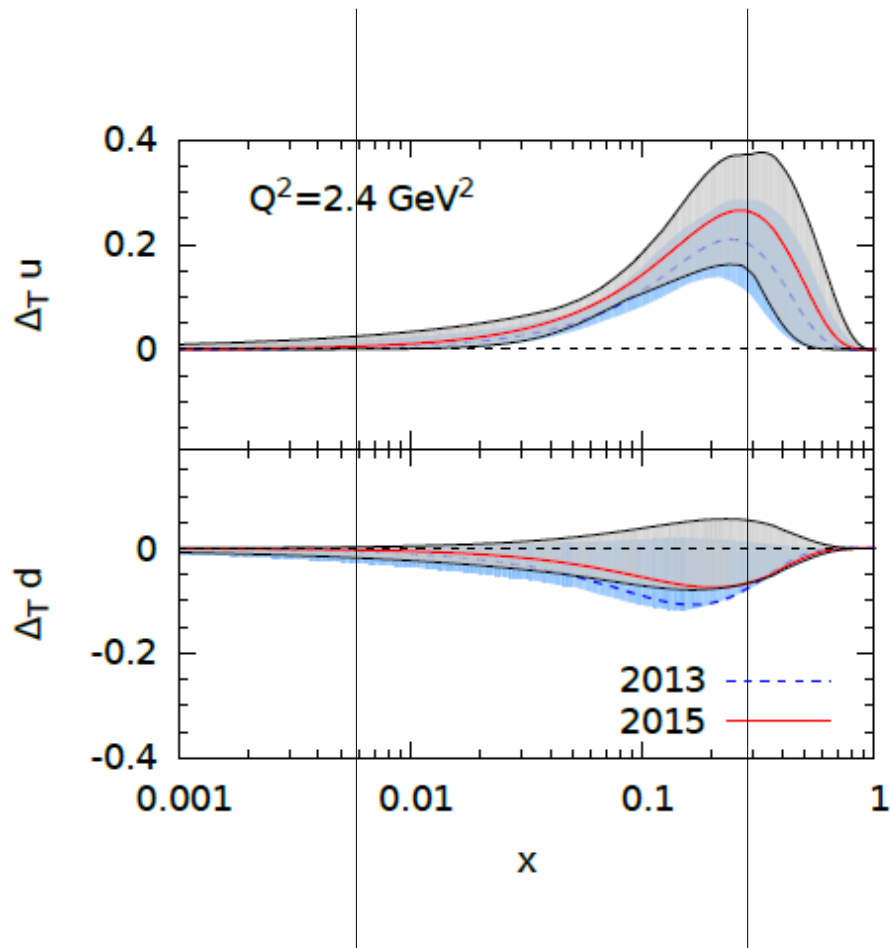
$N_{u_v}^T = 0.61^{+0.39}_{-0.23}$	$N_{d_v}^T = -1.00^{+1.86}_{-0.00}$
$\alpha = 0.70^{+1.31}_{-0.63}$	$\beta = 1.80^{+7.60}_{-1.80}$
$N_{\text{fav}}^C = 0.90^{+0.09}_{-0.34}$	$N_{\text{dis}}^C = -0.37^{+0.05}_{-0.05}$
$\gamma = 2.02^{+0.83}_{-0.33}$	$\delta = 0.00^{+0.42}_{-0.00}$
$M_C^2 = 0.28^{+0.20}_{-0.09} \text{ GeV}^2$	



Results on pion Collins function

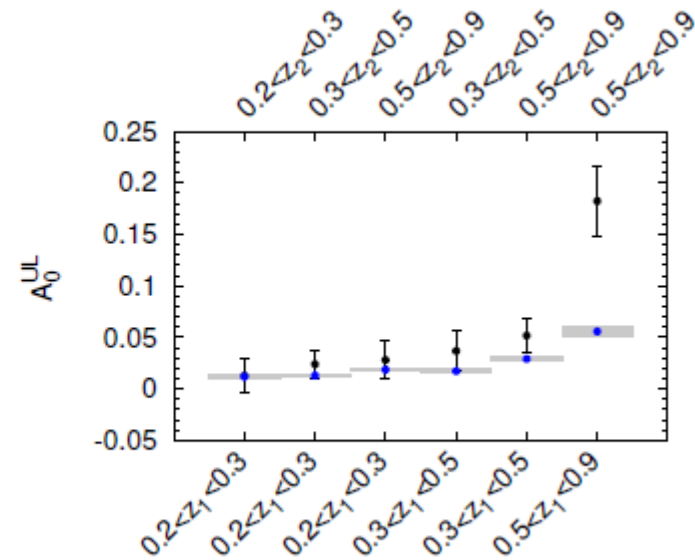
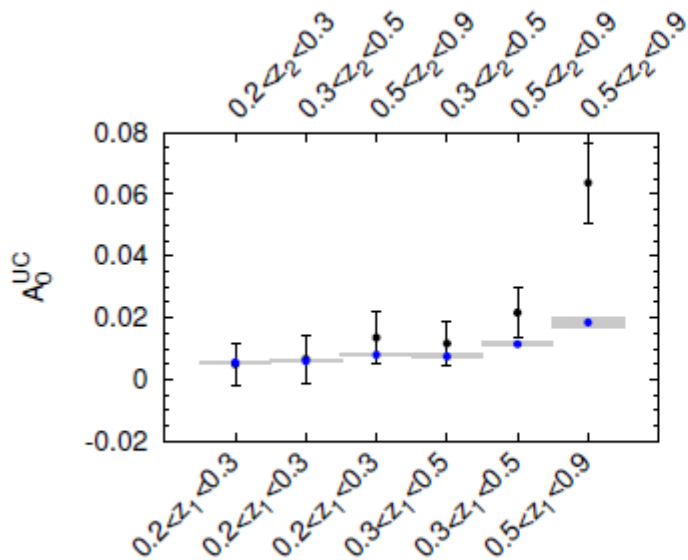
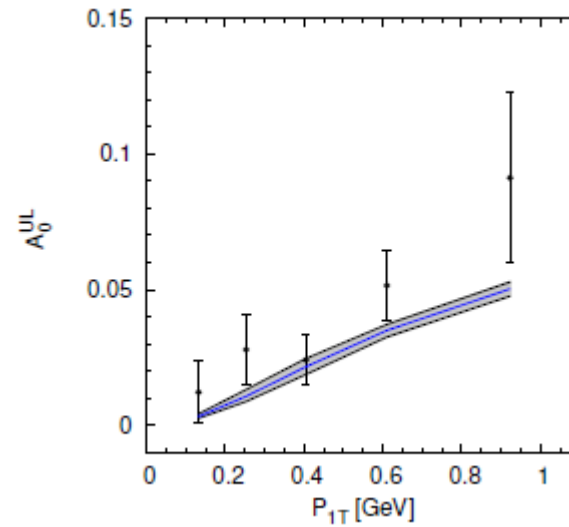
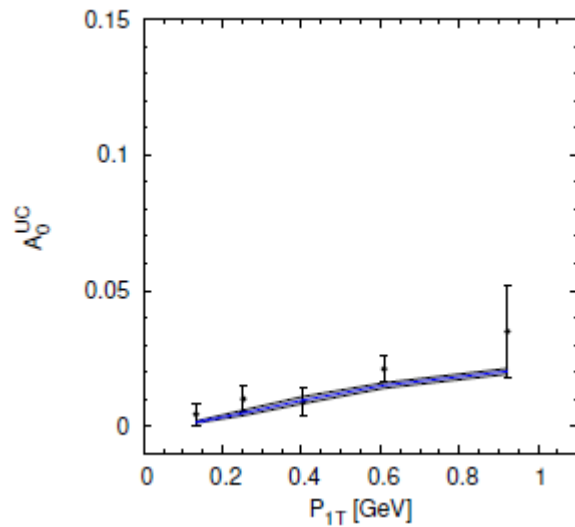


Results on pion Collins function



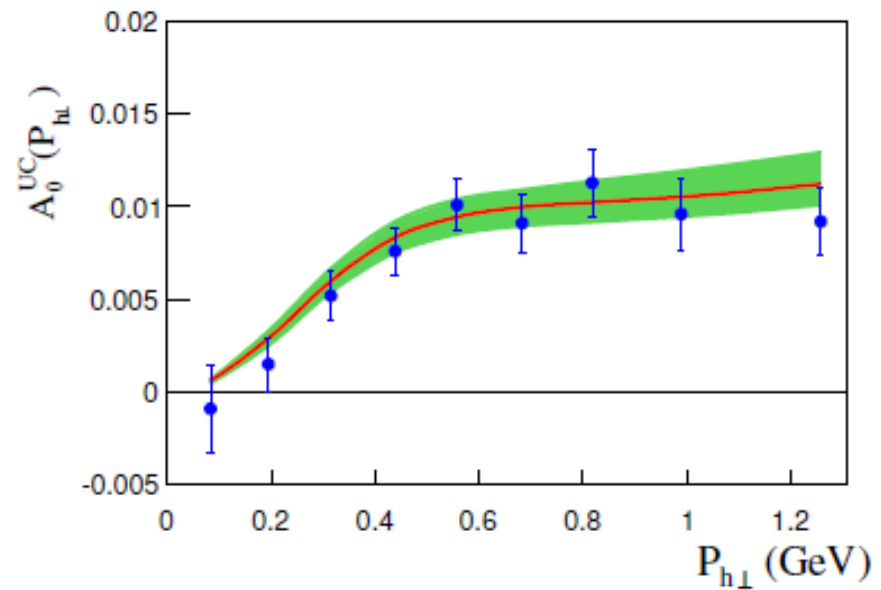
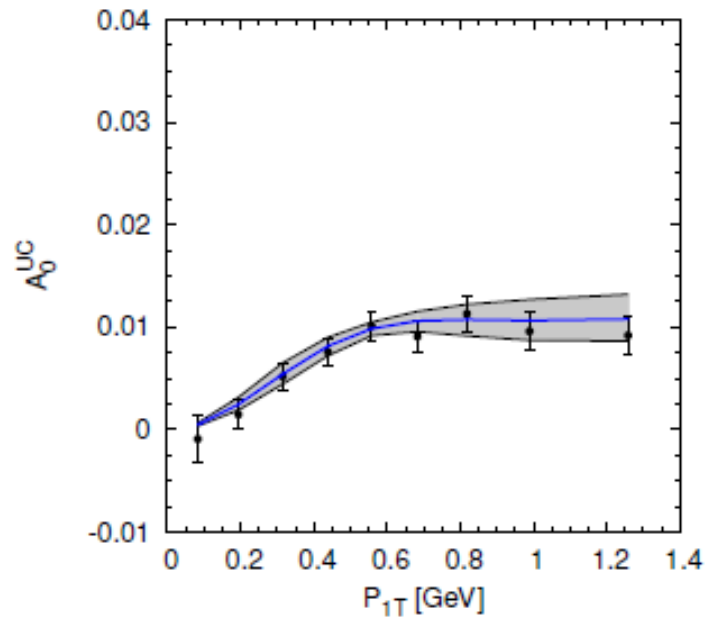
Comparison to BESIII

$Q^2 = 13 \text{ GeV}^2$



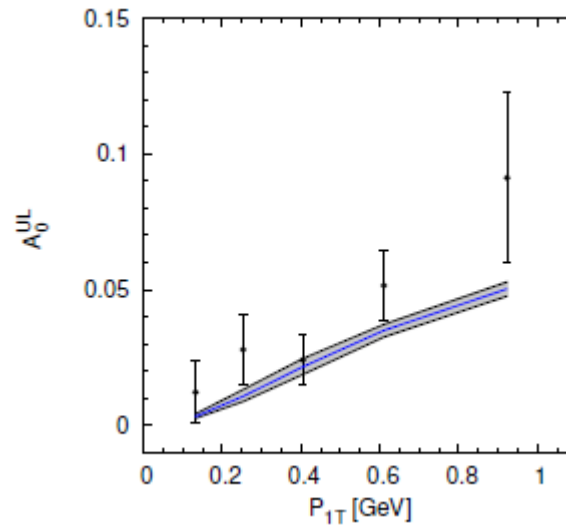
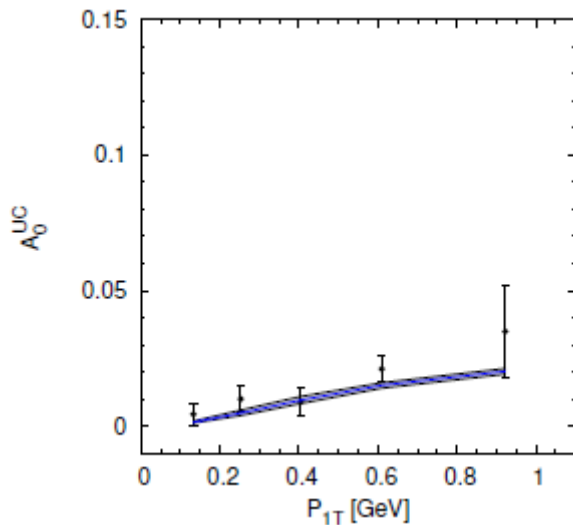
TMD evolution?

$$Q^2 = 13 \text{ GeV}^2$$

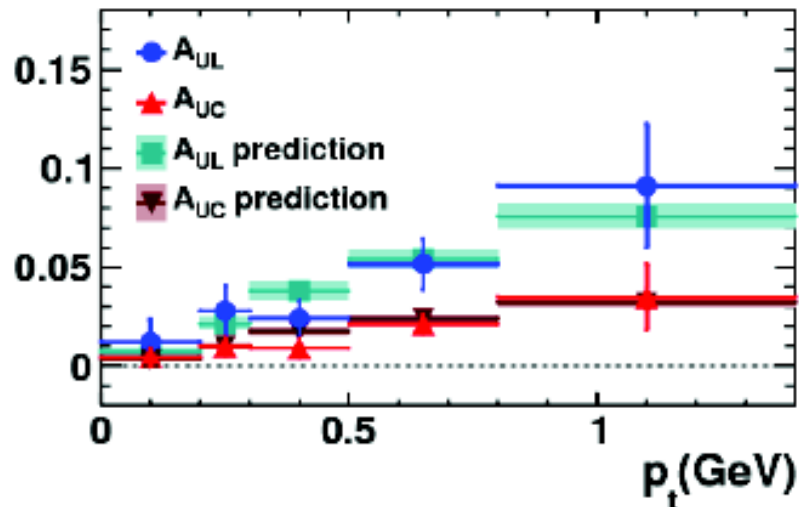


TMD evolution?

$$Q^2 = 13 \text{ GeV}^2$$



Are current data suitable for TMD evolution studies?



Kang, Prokudin, Sun, Yuan

Phys.Rev. D93 (2016) no.1, 014009

arXiv:1505.05589 [hep-ph] JLAB-THY-15-2044

Ingredients for extraction of Collins function.

$e^+e^- \rightarrow \pi\pi X$

SIDIS

Unpolarized TMDFF

Collins TMDFF

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$$\boxed{A_{UT}^{\sin(\phi_h + \phi_S)}} \sim \frac{\boxed{F_{UT}^{\sin(\phi_h + \phi_S)}}}{\boxed{F_{UU}}}$$

Ratio

Unpolarized TMDFF & TMDPDF

TMD Transversity & Collins function

Results on kaon Collins function.

$$e^+e^- \rightarrow h_1 h_2 X$$

Unpolarized TMDFF

Collins TMDFF

Ratio

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$$P_0^{U,L,C} = \frac{N^{U,L,C}}{D^{U,L,C}}$$

$$P_{0L}^{\pi\pi} \equiv \frac{N_L^{\pi\pi}}{D_L^{\pi\pi}} = \frac{N^{\pi^+\pi^+} + N^{\pi^-\pi^-}}{D^{\pi^+\pi^+} + D^{\pi^-\pi^-}}$$

$$P_{0U}^{\pi\pi} \equiv \frac{N_U^{\pi\pi}}{D_U^{\pi\pi}} = \frac{N^{\pi^+\pi^-} + N^{\pi^-\pi^+}}{D^{\pi^+\pi^-} + D^{\pi^-\pi^+}}$$

$$P_{0C}^{\pi\pi} \equiv \frac{N_C^{\pi\pi}}{D_C^{\pi\pi}} = \frac{N_L^{\pi\pi} + N_U^{\pi\pi}}{D_L^{\pi\pi} + D_U^{\pi\pi}}.$$

$$P_{0L}^{KK} \equiv \frac{N_L^{KK}}{D_L^{KK}} = \frac{N^{K^+K^+} + N^{K^-K^-}}{D^{K^+K^+} + D^{K^-K^-}}$$

$$P_{0U}^{KK} \equiv \frac{N_U^{KK}}{D_U^{KK}} = \frac{N^{K^+K^-} + N^{K^-K^+}}{D^{K^+K^-} + D^{K^-K^+}}$$

$$P_{0C}^{KK} \equiv \frac{N_C^{KK}}{D_C^{KK}} = \frac{N_L^{KK} + N_U^{KK}}{D_L^{KK} + D_U^{KK}},$$

$$P_{0L}^{\pi K} \equiv \frac{N_L^{\pi K}}{D_L^{\pi K}} = \frac{N^{\pi^+K^+} + N^{\pi^-K^-} + N^{K^+\pi^+} + N^{K^-\pi^-}}{D^{\pi^+K^+} + D^{\pi^-K^-} + D^{K^+\pi^+} + D^{K^-\pi^-}}$$

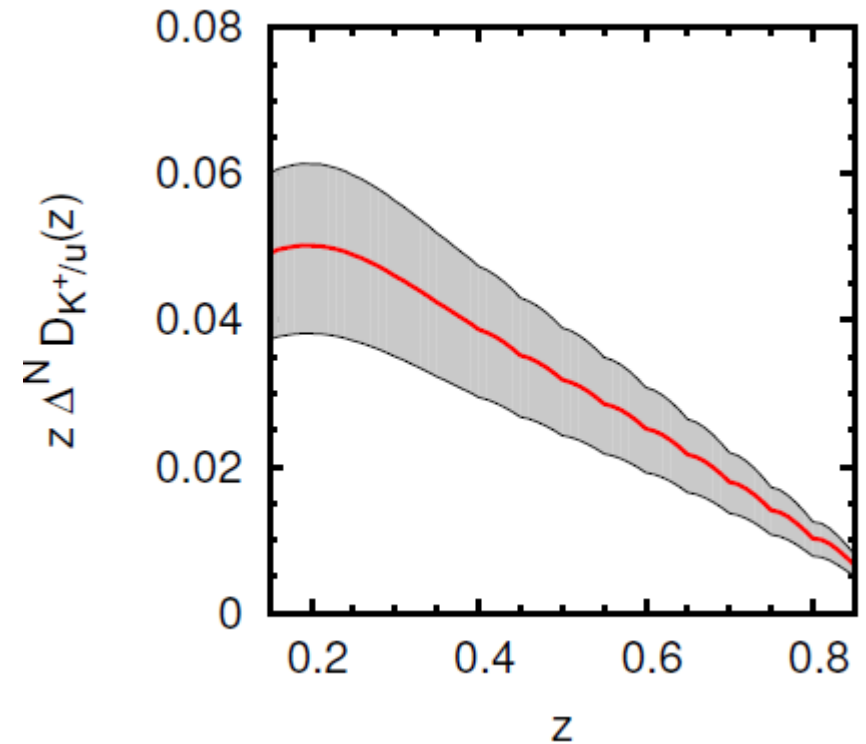
$$P_{0U}^{\pi K} \equiv \frac{N_U^{\pi K}}{D_U^{\pi K}} = \frac{N^{\pi^+K^-} + N^{\pi^-K^+} + N^{K^+\pi^-} + N^{K^-\pi^+}}{D^{\pi^+K^-} + D^{\pi^-K^+} + D^{K^+\pi^-} + D^{K^-\pi^+}}$$

$$P_{0C}^{\pi K} \equiv \frac{N_C^{\pi K}}{D_C^{\pi K}} = \frac{N_L^{\pi K} + N_U^{\pi K}}{D_L^{\pi K} + D_U^{\pi K}}.$$

Newest BaBar data allow access to kaon Collins functions.

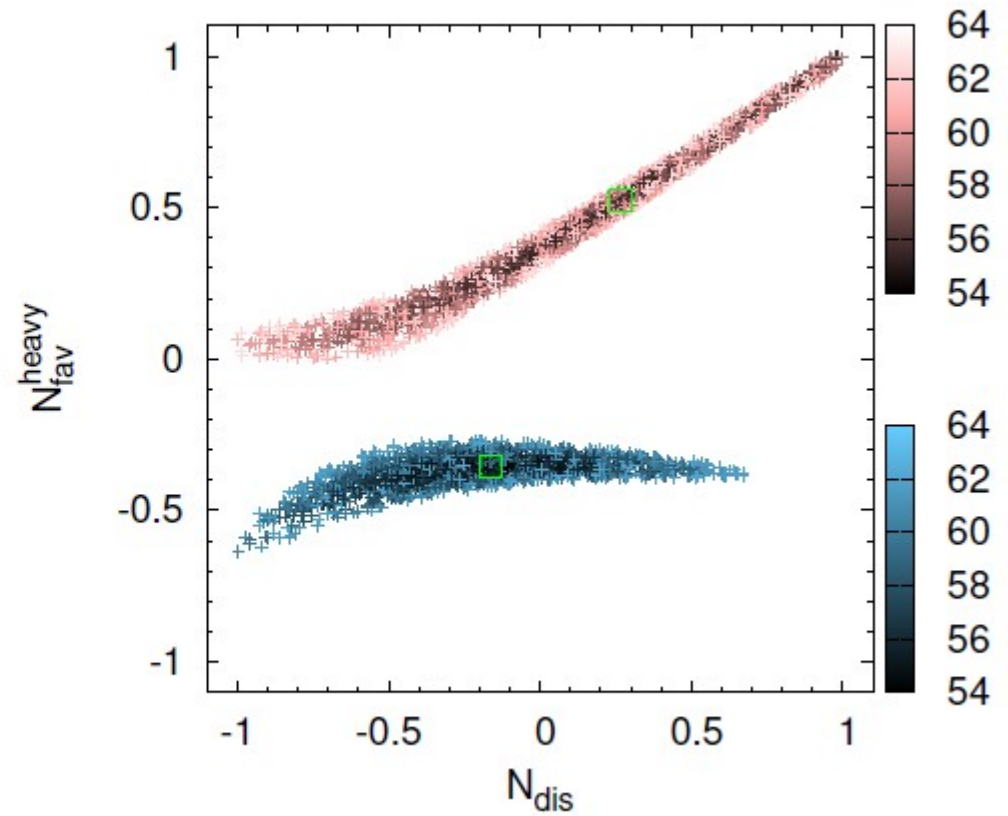
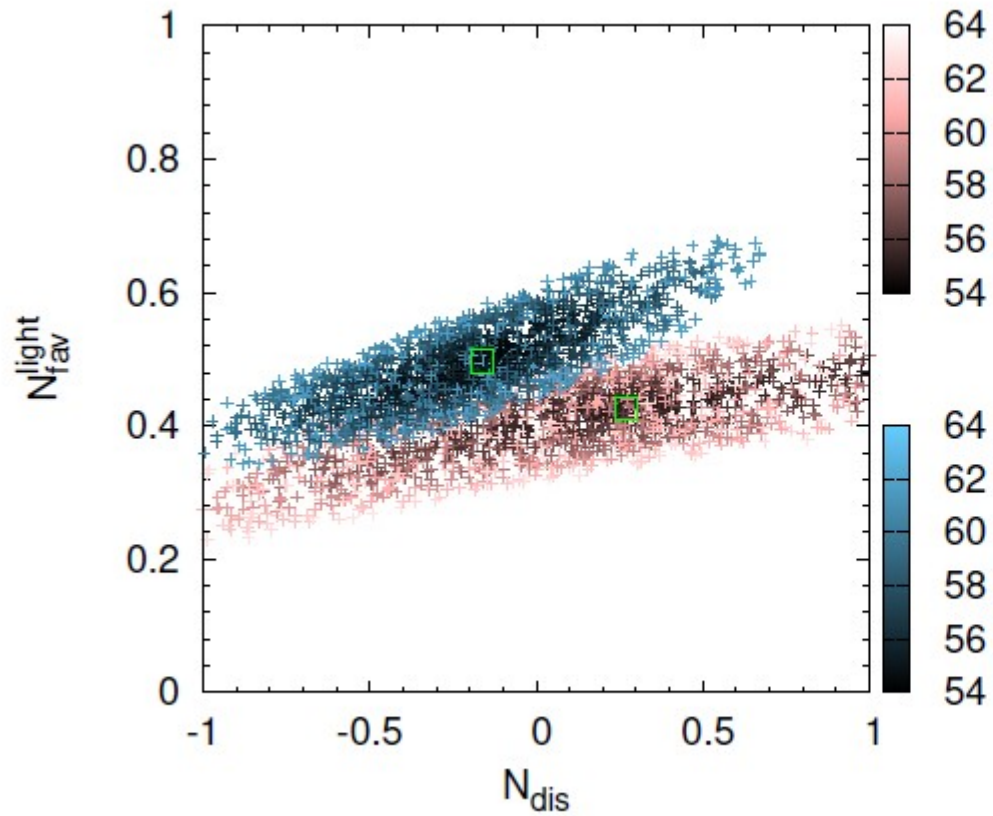
Results on kaon Collins function.

$N_{\text{fav}}^{\text{light}}$	$N_{\text{fav}}^{\text{heavy}} > 0$	$N_{\text{fav}}^{\text{heavy}} < 0$	N_{dis}	$\chi_{\text{d.o.f.}}^2$
●	○	○	○	1.83
○	●	○	○	3.32
○	○	●	○	5.68
○	○	○	●	3.94
●	●	○	○	0.89
●	○	●	○	0.88
●	○	○	●	0.98
○	●	○	●	2.00
○	○	●	●	4.00
●	●	○	●	0.90
●	○	●	●	0.89



Only $u \rightarrow K^+$ can be constrained.

Results on kaon Collins function.



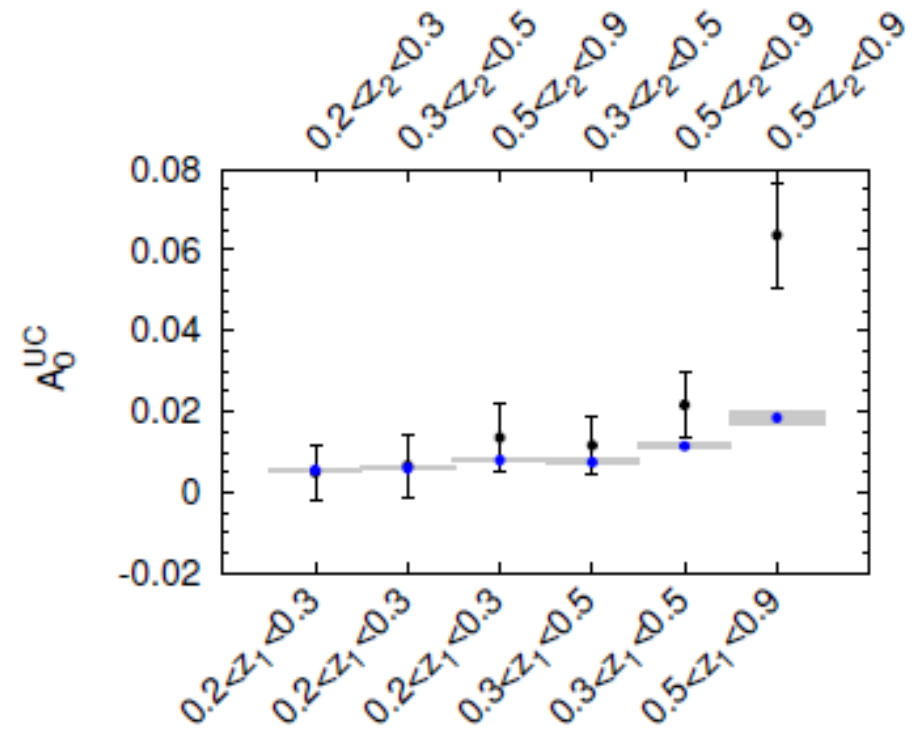
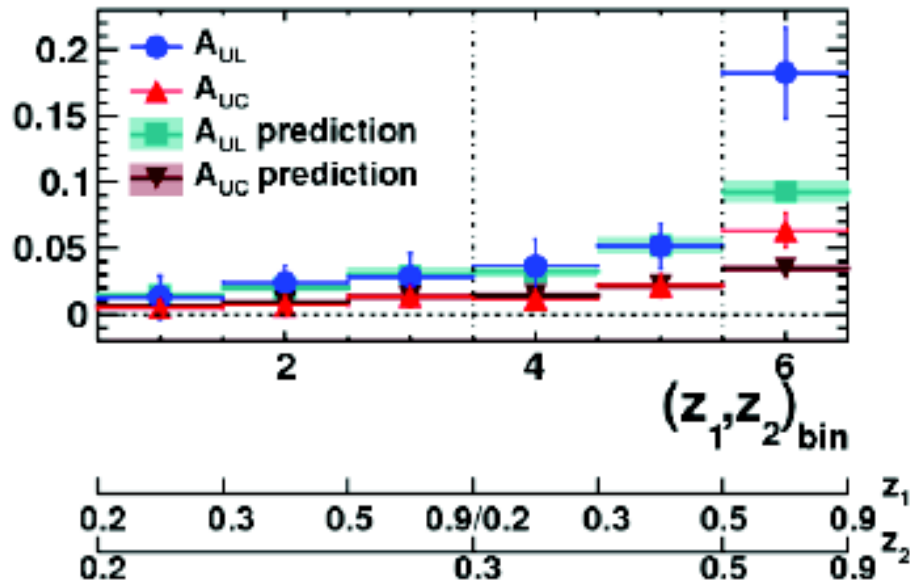
Final notes, summary.

- **Collins function extraction update (pT-dependence somehow constrained)**
- **Update consistent with previous extraction.**
- **Transversity for d-valence still compatible with zero**
- **Collins function for kaons still hard to extract: can only constrain $u \rightarrow K^+$ ($\bar{u} \rightarrow K^-$)**
- **TMD evolution are mild in current data: Collins asymmetry and double ratios.**

Thanks

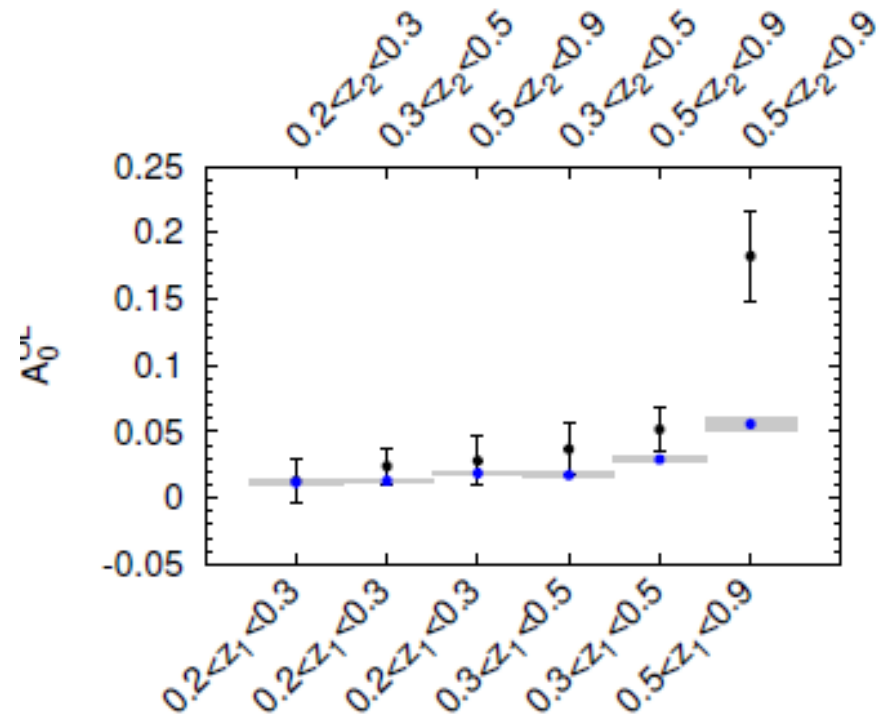
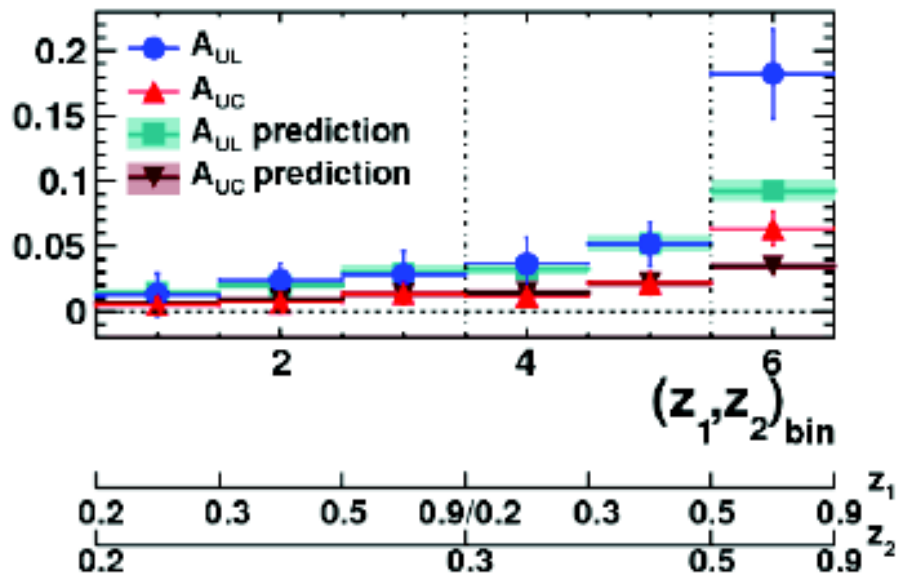
Back up

$Q^2 = 13 \text{ GeV}^2$

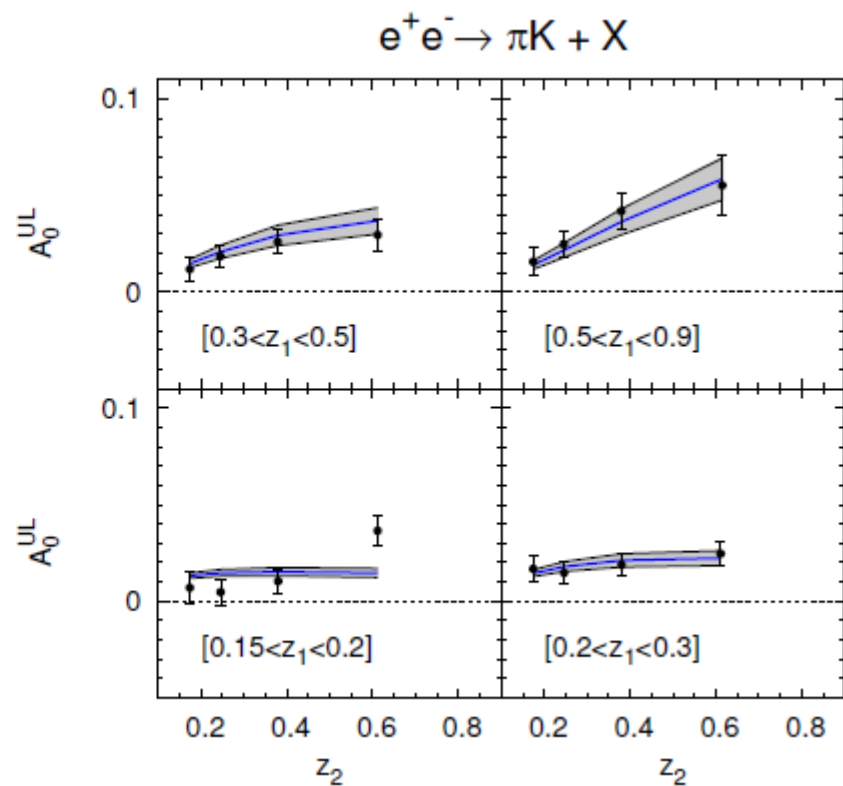
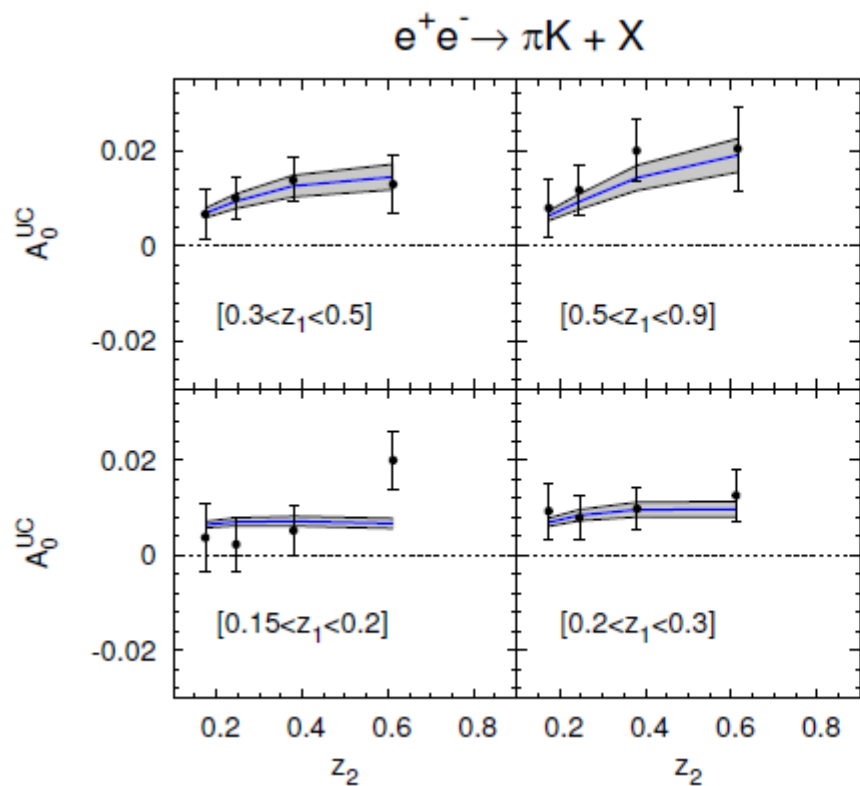


Back up

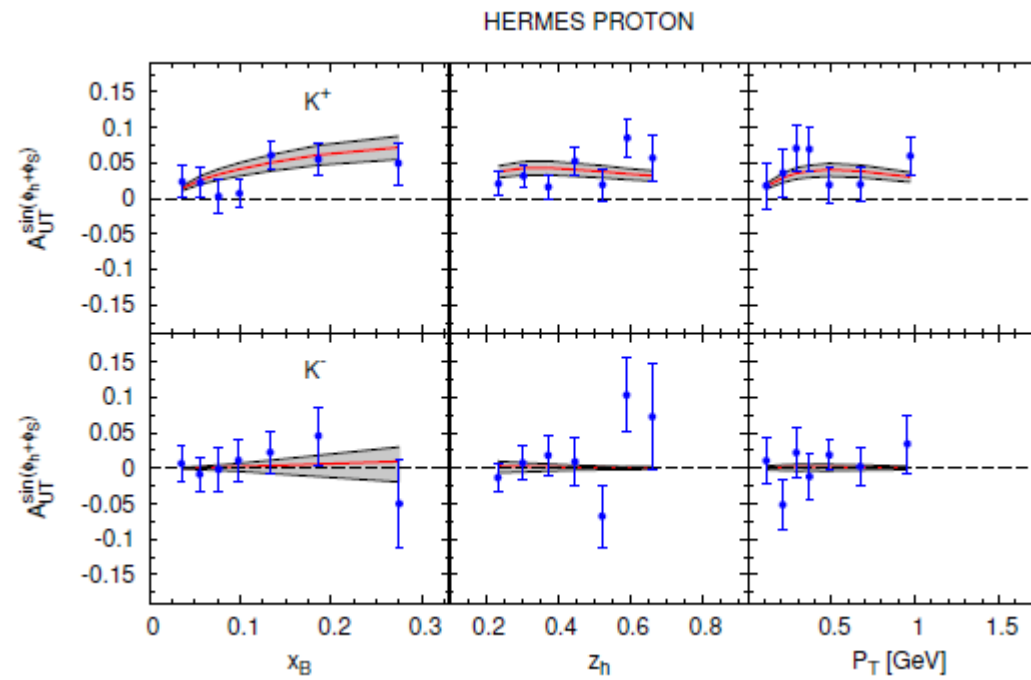
$Q^2 = 13 \text{ GeV}^2$



Back up



Back up



Back up

