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Perturbative QCD, resummation and non-perturbative aspects in SIDIS processes

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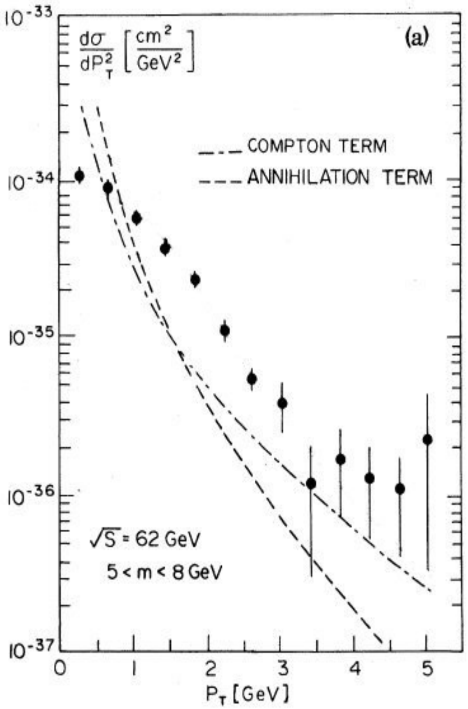
Naive TMD approach

- Calculating a cross section which describes a hadronic process over the whole q_T range is a highly non-trivial task

  Let's consider Drell Yan processes (for historical reasons):

- Fixed order calculations cannot describe correctly DY data at small q_T:

  At Born Level the cross section is vanishing
  At order α_s the cross section is divergent...



$$q_T \to 0$$

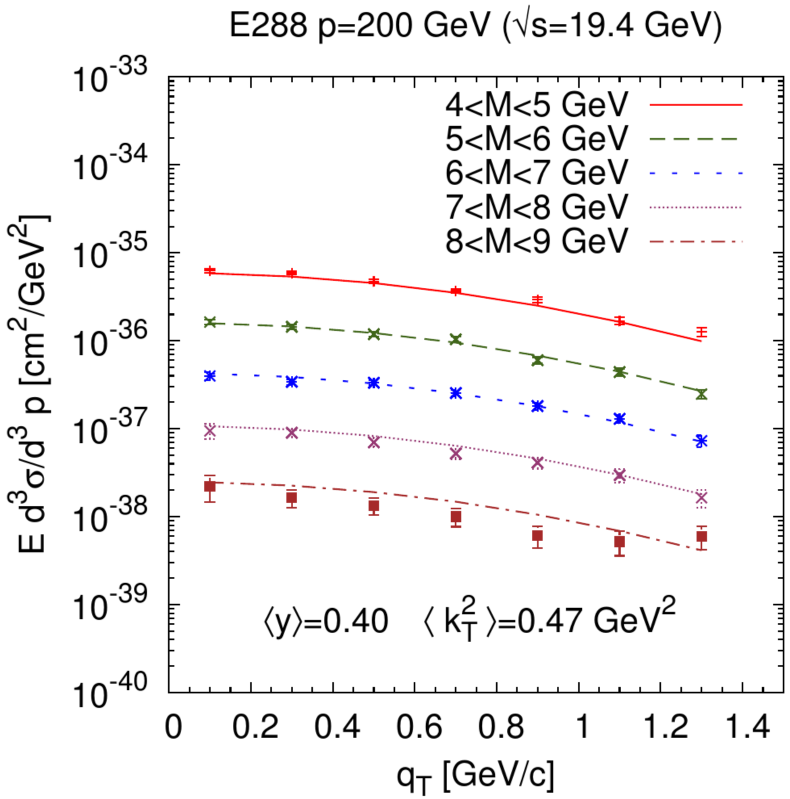
$$\frac{1}{\sigma_0}\frac{d\sigma}{dq_T^2} = \frac{2C_F}{2\pi q_T^2}\alpha_s \ln\left(\frac{M^2}{q_T^2} - \frac{3}{2}\right)$$

Naive TMD approach

- Low energy data

$$\frac{d\sigma}{dP_T^2} \propto \frac{\alpha_{em}}{M^2} \sum_q f_{q/h_1}(x_1) \bar{f}_{q/h_2}(x_2) \frac{\exp(-P_T^2/\langle P_T^2 \rangle)}{\pi \langle P_T^2 \rangle}$$

$\langle P_T^2 \rangle = 2\langle k_\perp^2 \rangle$

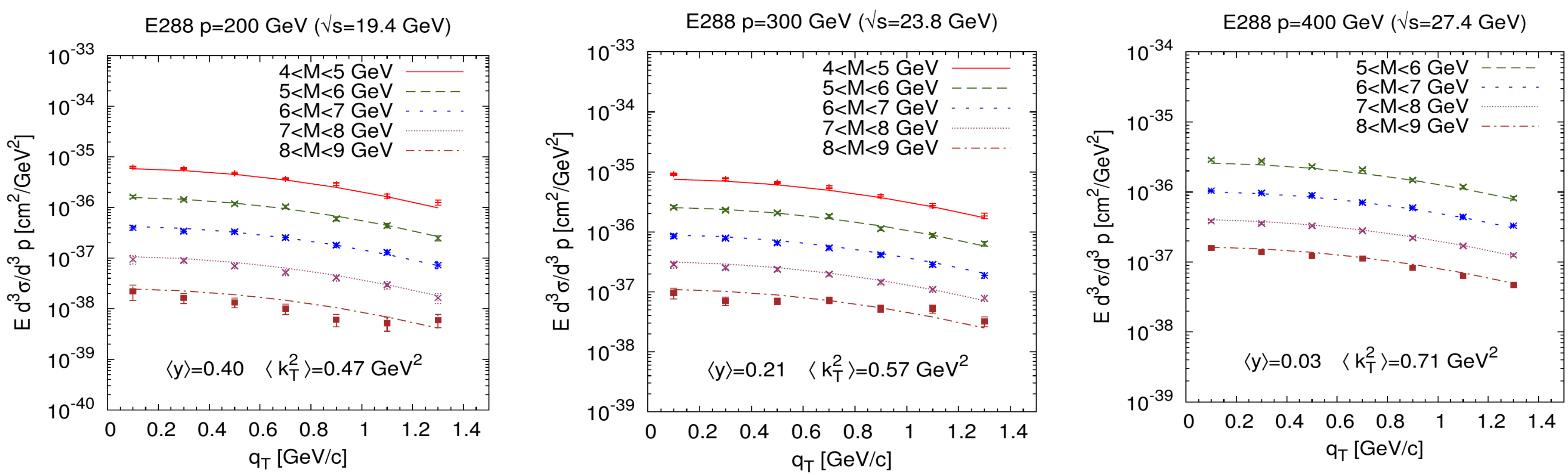


The M^2 dependence is described by the Gaussian model, and it is given by the interplay between the $1/M^2$ born cross section, DGLAP evolution and kinematics

Naive TMD approach

$$\frac{d\sigma}{dP_T^2} \propto \frac{\alpha_{em}}{M^2} \sum_q f_{q/h_1}(x_1) \bar{f}_{q/h_2}(x_2) \frac{\exp(-P_T^2/\langle P_T^2 \rangle)}{\pi \langle P_T^2 \rangle}$$

- Considering the same DY process at different energies:

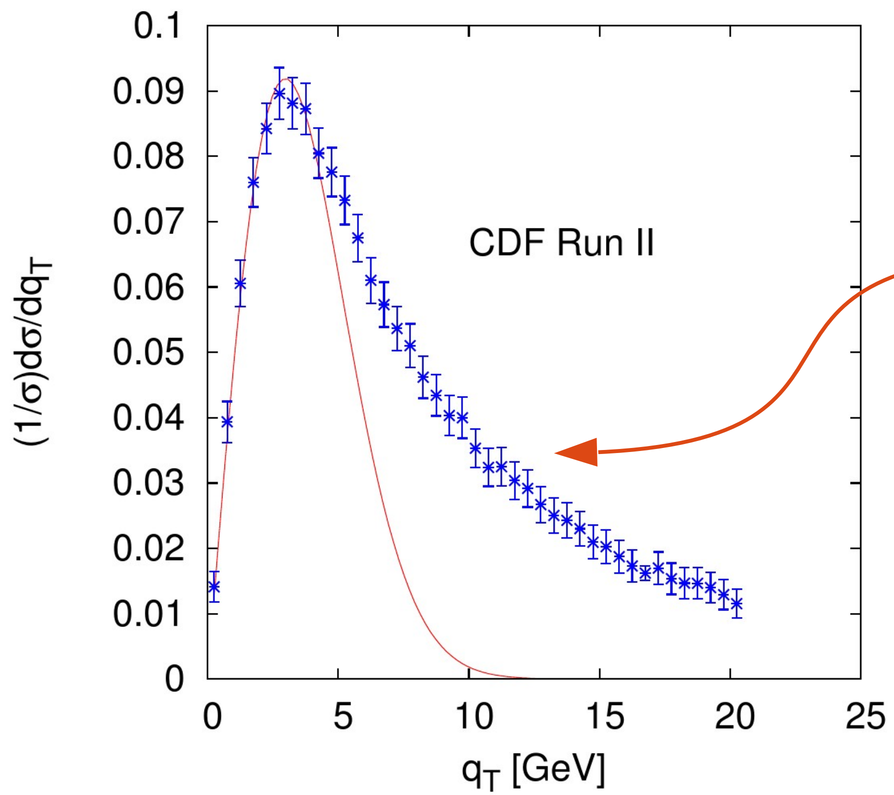


Each data set is Gaussian but with a different width

Drell-Yan phenomenology

- Does the q_T distribution behave like a Gaussian ?

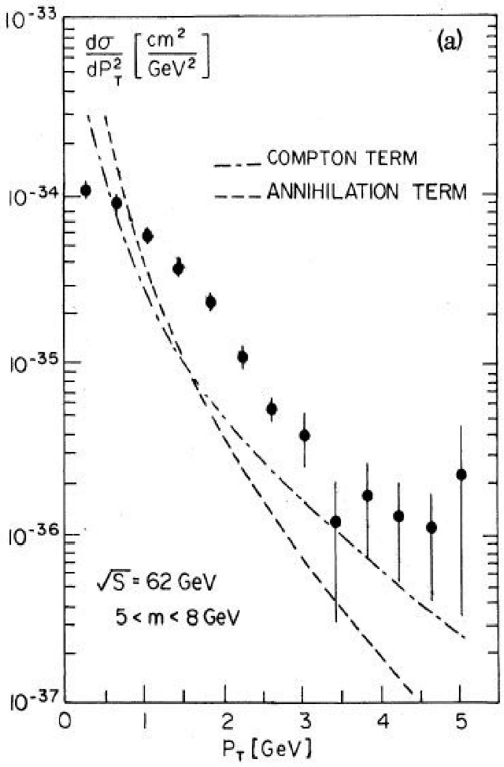
$$\frac{d\sigma}{dP_T^2} \propto \frac{\alpha_{em}}{M^2} \sum_q f_{q/h_1}(x_1) \bar{f}_{q/h_2}(x_2) \frac{\exp(-P_T^2/\langle P_T^2 \rangle)}{\pi \langle P_T^2 \rangle}$$



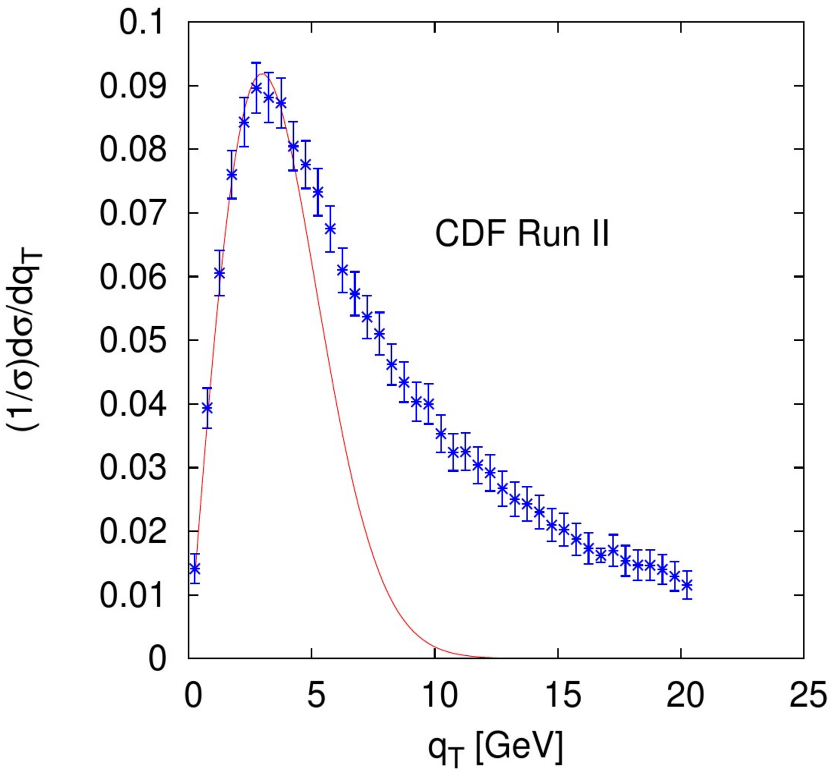
Clearly this is not a Gaussian tail !

Drell-Yan phenomenology

Fixed order calculations cannot describe correctly DY/SIDIS data at small q_T



DY/SIDIS cross sections do not show a Gaussian behaviour at large q_T

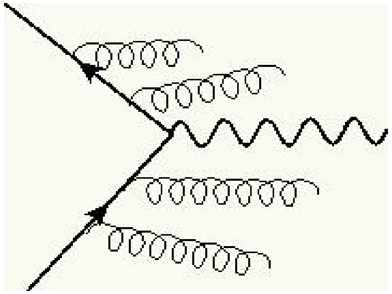


Resummation/TMD evolution

- Fixed order calculations cannot describe correctly DY/SIDIS data at small q_T

$$\frac{1}{\sigma_0}\frac{d\sigma}{dq_T^2} = \frac{2C_F}{2\pi q_T^2}\alpha_s \ln\left(\frac{M^2}{q_T^2} - \frac{3}{2}\right)$$

- These divergencies are cured by TMD evolution/resummation



The cross section is written in b_T space:

$$\frac{1}{\sigma_0}\frac{d\sigma}{dQ^2 dy dq_T^2} = \int \frac{d^2\boldsymbol{b}_T e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}_T}}{(2\pi)^2}\sum_j e_j^2 \overbrace{W_j(x_1,x_2,b_T,Q)}^{\text{Resummed term}} + \overbrace{Y(x_1,x_2,q_T,Q)}^{\text{Finite term}}$$

Resummation/TMD evolution

- We lose the direct connection with q_T. Instead we have to deal with b_T ...
  Example in the CSS resummation scheme:

$$W_j(x_1, x_2, b_T, Q) = \exp\left[S_j(b_T, Q)\right] \sum_{i,k} C_{ji} \otimes f_i(x_1, C_1^2/b_T^2)\, C_{\bar{j}k} \otimes f_k(x_2, C_1^2/b_T^2)$$

$$S_j(b_T, Q) = -\int_{C_1^2/b_T^2}^{Q^2} \frac{d\kappa^2}{\kappa^2}\left[A_j(\alpha_s(\kappa)) \ln\left(\frac{Q^2}{\kappa^2}\right) + B_j(\alpha_s(\kappa))\right]$$

$$\mu = \frac{C_1}{b_T}$$

At large b_T the scale μ becomes too small!

Not trivially connected to the physical region: $Q^2 \gg q_T^2 \simeq \Lambda_{QCD}^2$

- All TMD evolution schemes require a model to deal with the non-perturbative region

Non perturbative region

- This is a perturbative scheme.
  All the scales are frozen when reaching the non perturbative region:

$$b_T \longrightarrow b_* = \frac{b_T}{\sqrt{1 + b_T^2 / b_{max}^2}} \qquad\qquad \mu = \frac{C_1}{b_T} \longrightarrow \mu_b = C_1 / b_*$$

And then we define a non perturbative function for large b_T:

$$\frac{W_j(x_1, x_2, b_T, Q)}{W_j(x_1, x_2, b_*, Q)} = F_{NP}(x_1, x_2, b_T, Q)$$

$$W_j(x_1, x_2, b_T, Q) = \sum_{i,k} \exp\left[S_j(b_*, Q)\right] \left[C_{ji} \otimes f_i\,(x_1, \mu_b)\right] \left[C_{\bar{j}k} \otimes f_k\,(x_2, \mu_b)\right] F_{NP}(x_1, x_2, b_T, Q)$$

$b_*,\ \mu_b$ $\qquad\qquad$ b_T

$C_1 = 2\exp(-\gamma_E)$

Collins, Soper, Sterman, Nucl. Phys. B250, 199 (1985)

CSS for DY processes

To perform phenomenological studies we need a non perturbative function.

$$F_{NP}(x_1, x_2, b_T, Q)$$

Davies-Webber-Stirling (DWS) $\exp\left[-g_1 - g_2 \ln\left(\frac{Q}{2Q_0}\right)\right] b^2;$

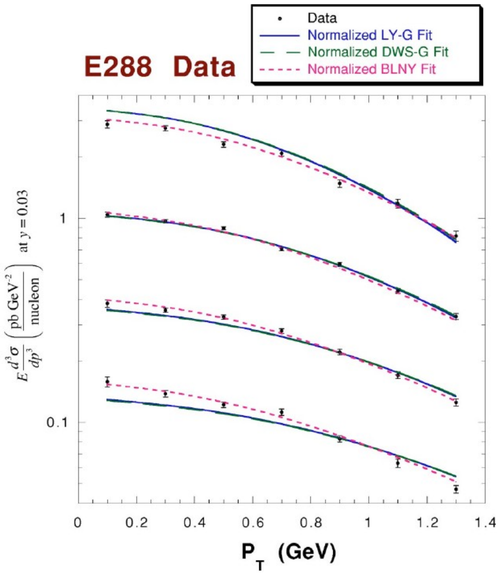
Ladinsky-Yuan (LY) $\exp\left\{\left[-g_1 - g_2 \ln\left(\frac{Q}{2Q_0}\right)\right] b^2 - [g_1 g_3 \ln(100 x_1 x_2)] b\right\};$

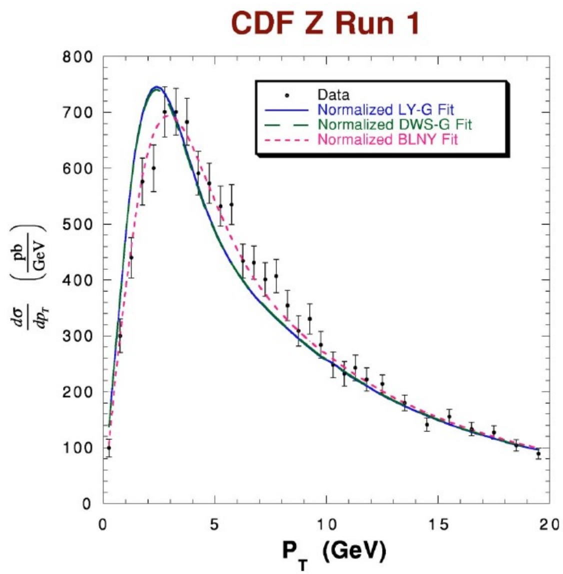
Brock-Landry-Nadolsky-Yuan (BLNY) $\exp\left[-g_1 - g_2 \ln\left(\frac{Q}{2Q_0}\right) - g_1 g_3 \ln(100 x_1 x_2)\right] b^2$

Nadolsky et al., Phys.Rev. D67,073016 (2003)

CSS for DY processes

Nadolsky et al.* analyzed successfully
low energy DY data and Z_0 production data
using different parametrizations





Parameter	DWS-G fit	LY-G fit	BLNY fit
g_1	0.016	0.02	0.21
g_2	0.54	0.55	0.68
g_3	0.00	-1.50	-0.60
CDF Z Run-0 N_{fit}	1.00 (fixed)	1.00 (fixed)	1.00 (fixed)
R209 N_{fit}	1.02	1.01	0.86
E605 N_{fit}	1.15	1.07	1.00
E288 N_{fit}	1.23	1.28	1.19
DØ Z Run-1 N_{fit}	1.01	1.01	1.00
CDF Z Run-1 N_{fit}	0.89	0.90	0.89
χ^2	416	407	176
χ^2/DOF	3.47	3.42	1.48

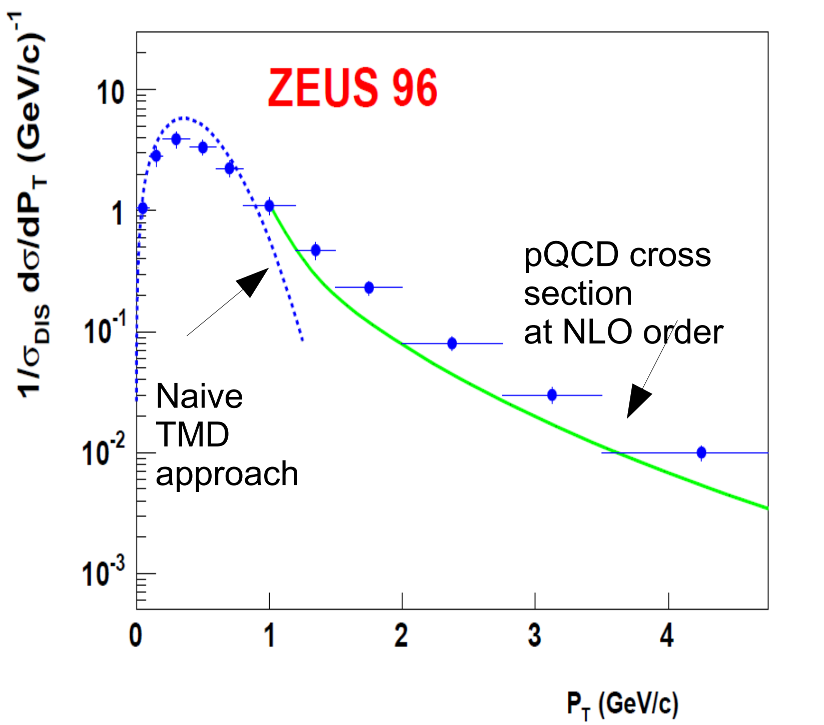
$b_{max} = 0.5\ \text{GeV}^{-1}$

**Nadolsky et al., Phys.Rev. D67,073016 (2003)*

SIDIS

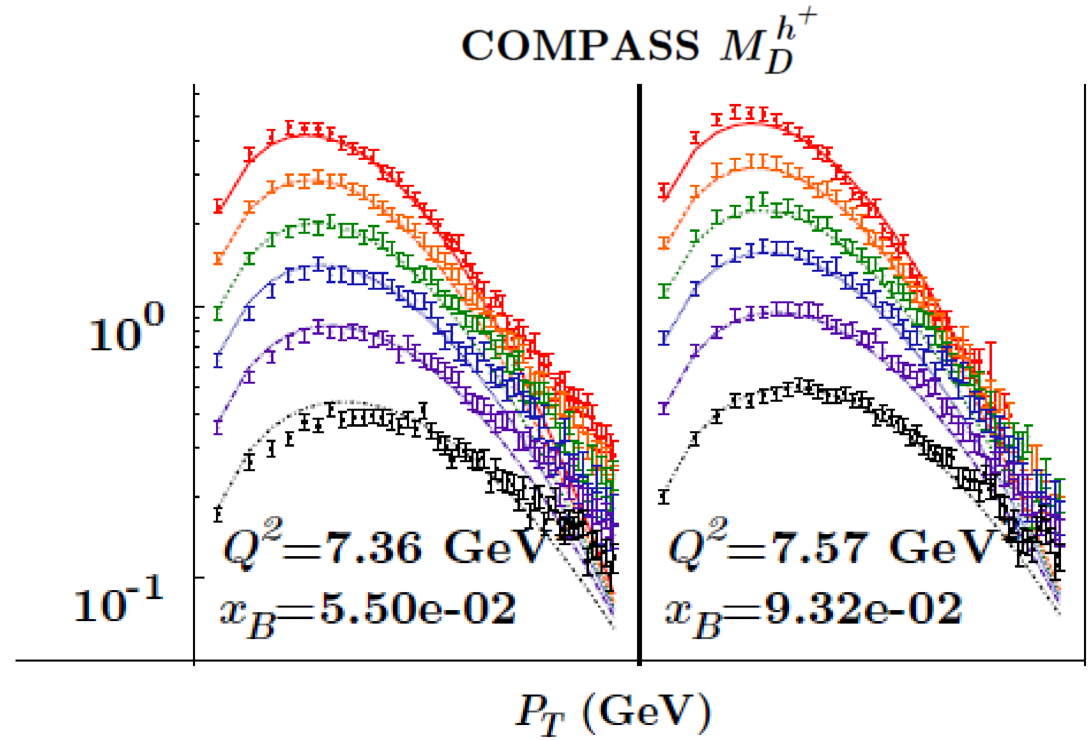
Resummation in SIDIS

- As mentioned above
  - fixed order pQCD calculation fail to describe the SIDIS cross sections at small q_T,
  - the cross section tail at large q_T is clearly non-Gaussian.



Anselmino, Boglione, Prokudin, Turk, Eur.Phys.J. A31 (2007) 373-381

ZEUS Collaboration (M. Derrick), Z. Phys. C 70, 1 (1996)



Anselmino, Boglione, Gonzalez, Melis, Prokudin, JHEP 1404 (2014) 005

COMPASS, Adolph et al., Eur. Phys. J. C 73 (2013) 2531

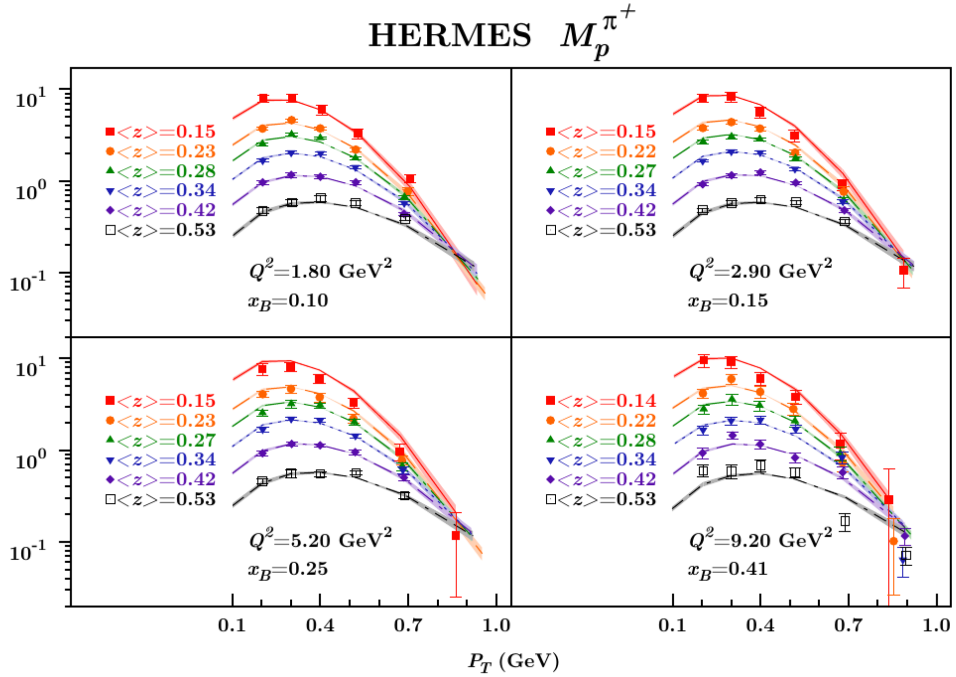
Need resummation of large logs and matching perturbative to non-perturbative contributions

Naive TMD approach

- Simple phenomenological ansatz can reproduce low q_T data

$$f_{q/p}(x,k_\perp) = f(x)\,\frac{e^{-k_\perp^2/\langle k_\perp^2\rangle}}{\pi\langle k_\perp^2\rangle} \qquad D_{h/q}(z,p_\perp) = D_{h/q}(z)\,\frac{e^{-p_\perp^2/\langle p_\perp^2\rangle}}{\pi\langle p_\perp^2\rangle}$$

$$F_{UU} = \sum_q e_q^2\, f_{q/p}(x_B)\, D_{h/q}(z_h)\,\frac{e^{-P_T^2/\langle P_T^2\rangle}}{\pi\langle P_T^2\rangle} \qquad \langle P_T^2\rangle = \langle p_\perp^2\rangle + z_h^2\langle k_\perp^2\rangle$$



Airapetian et al, Phys. Rev. D 87 (2013) 074029

$$\langle k_\perp^2\rangle = 0.57 \pm 0.08 \text{ GeV}^2$$

$$\langle p_\perp^2\rangle = 0.12 \pm 0.01 \text{ GeV}^2$$

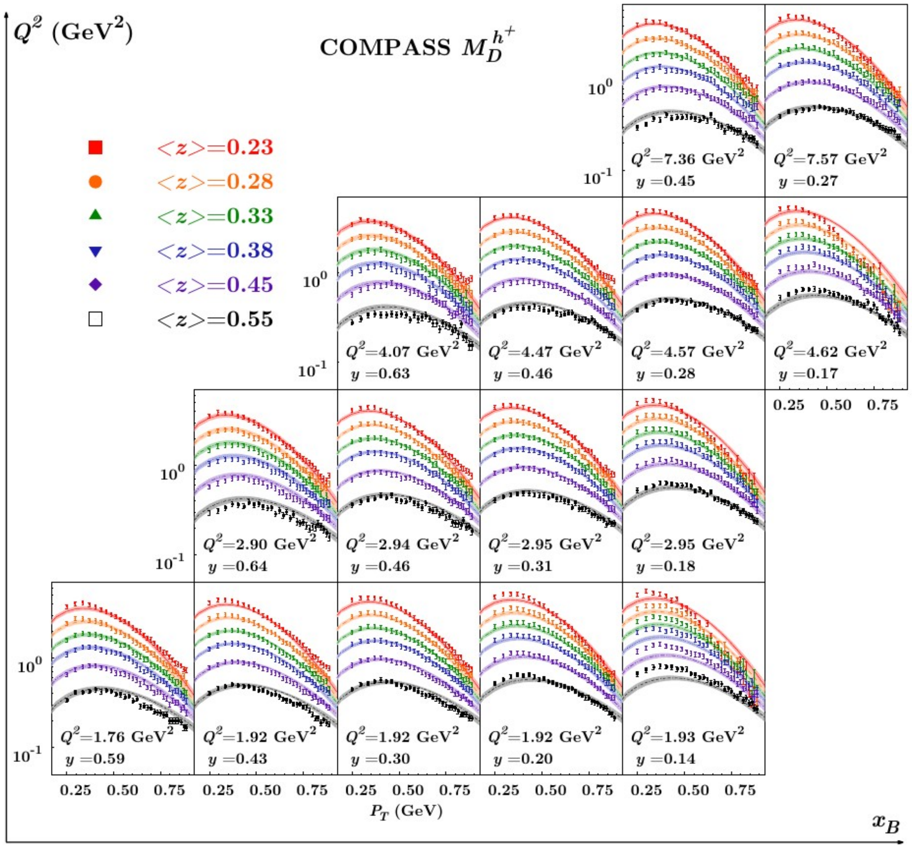
$$\chi^2_{\rm dof} = 1.69$$

Anselmino et al. JHEP 1404 (2014) 005

Naive TMD approach

$$F_{UU} = \sum_q e_q^2 \, f_{q/p}(x_B) \, D_{h/q}(z_h) \frac{e^{-P_T^2/\langle P_T^2\rangle}}{\pi\langle P_T^2\rangle}$$

$$\langle P_T^2\rangle = \langle p_\perp^2\rangle + z_h^2 \langle k_\perp^2\rangle$$



Anselmino et al. JHEP 1404 (2014) 005

$$\langle k_\perp^2\rangle = 0.60 \pm 0.14 \text{ GeV}^2$$

$$\langle p_\perp^2\rangle = 0.20 \pm 0.02 \text{ GeV}^2$$

$$\chi^2_{\rm dof} = 3.42$$

$$N_y = A + B\,y$$

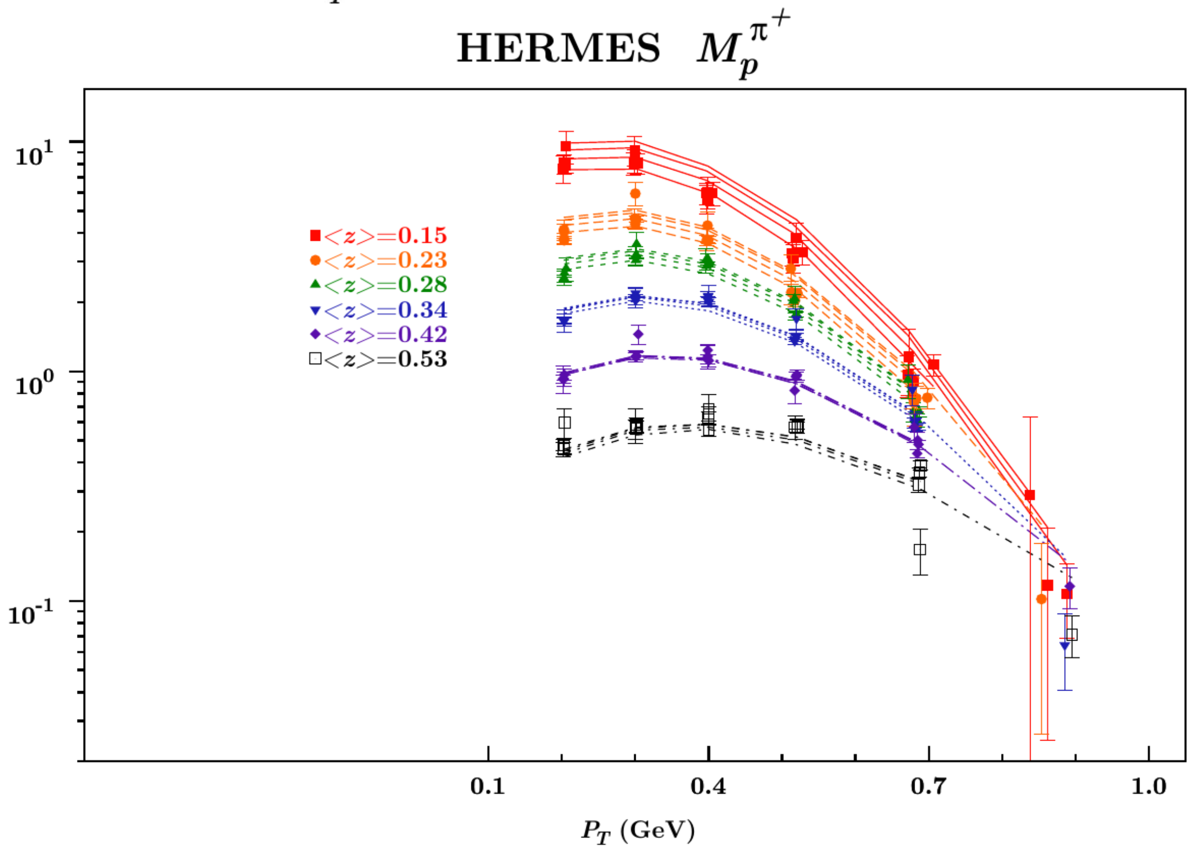
"The point-to-point systematic uncertainty in the measured multiplicities as a function of p_T^2 is estimated to be 5% of the measured value. The systematic uncertainty in the overall normalization of the p_T^2-integrated multiplicities depends on z and y and can be as large as 40%".

Erratum Eur.Phys.J. C75 (2015) 2, 94

Q^2 dependence of the data...

$$F_{UU} = \sum_q e_q^2 \, f_{q/p}(x_B) \, D_{h/q}(z_h) \frac{e^{-P_T^2/\langle P_T^2 \rangle}}{\pi \langle P_T^2 \rangle}$$

$$\langle P_T^2 \rangle = \langle p_\perp^2 \rangle + z_h^2 \langle k_\perp^2 \rangle$$



$$\langle k_\perp^2 \rangle = 0.57 \pm 0.08 \text{ GeV}^2$$

$$\langle p_\perp^2 \rangle = 0.12 \pm 0.01 \text{ GeV}^2$$

$$\chi^2_{\rm dof} = 1.69$$

All four bins have been overlapped in the same panel

Anselmino et al. JHEP 1404 (2014) 005

Resummation of large logarithms

- To ensure momentum conservation, analyse the cross section in the Fourier conjugate space

$$\delta^2(\boldsymbol{q}_T - \boldsymbol{k}_{1T} - \boldsymbol{k}_{2T} - \ldots\ldots - \boldsymbol{k}_{nT} + \ldots) = \int \frac{d^2\boldsymbol{b}_T}{(2\pi)^2} e^{-i\boldsymbol{b}_T\cdot(\boldsymbol{q}_T - \boldsymbol{k}_{1T} - \boldsymbol{k}_{2T} - \ldots\ldots - \boldsymbol{k}_{nT} + \ldots)}$$

$$\boxed{\frac{1}{\sigma_0}\frac{d\sigma}{dQ^2 dy dq_T^2} = \left[\int \frac{d^2\boldsymbol{b}_T e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}_T}}{(2\pi)^2} X_{div}(b_T)\right] + Y_{reg}(q_T)}$$

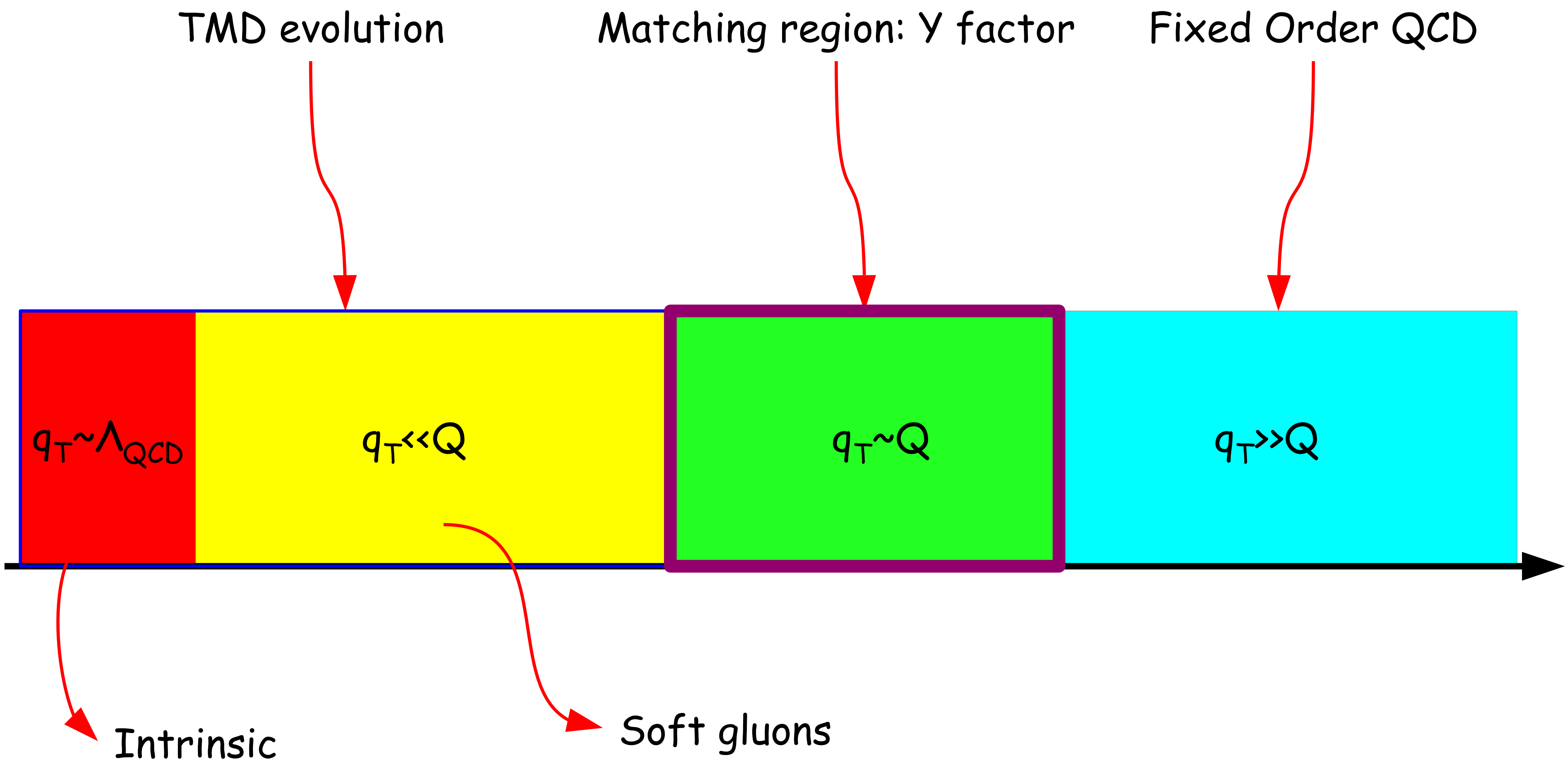
$$X_{div}(b_T) \longrightarrow W(b_T) = \exp\left[S(b_T)\right] \times (\text{PDFs and Hard coefficients})$$

$$\boxed{\frac{d\sigma^{total}}{dx\, dy\, dz\, dq_T^2} = \pi\sigma_0^{DIS} \int \frac{d^2\boldsymbol{b}_T e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}_T}}{(2\pi)^2} W^{SIDIS}(x, z, b_T, Q) + Y^{SIDIS}(x, z, q_T, Q)}$$

Resummed part

Regular part

TMD regions

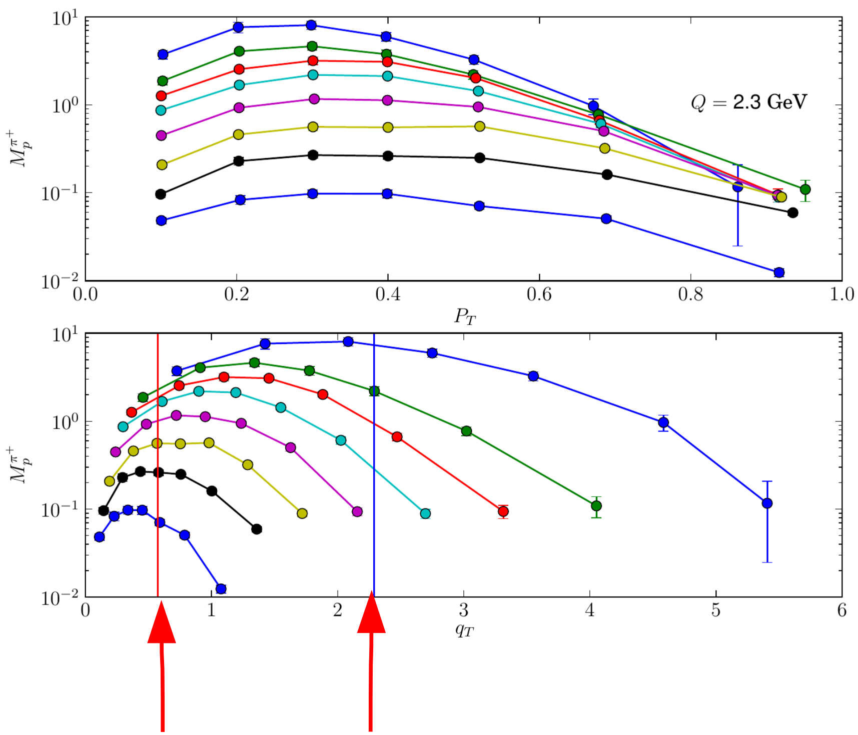

TMD evolution
Matching region: Y factor
Fixed Order QCD
$q_T \sim \Lambda_{QCD}$
$q_T \ll Q$
$q_T \sim Q$
$q_T \gg Q$
Intrinsic
Soft gluons

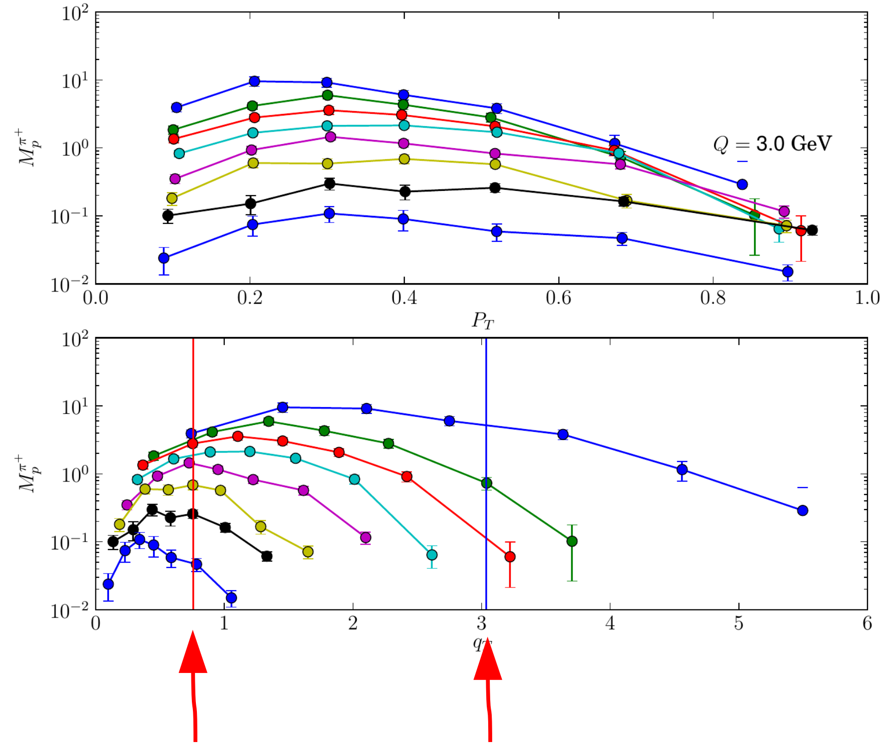
TMD regions

- TMD regions are defined in terms of q_T and noot in terms of P_T

P_T

$q_T=P_T/z$

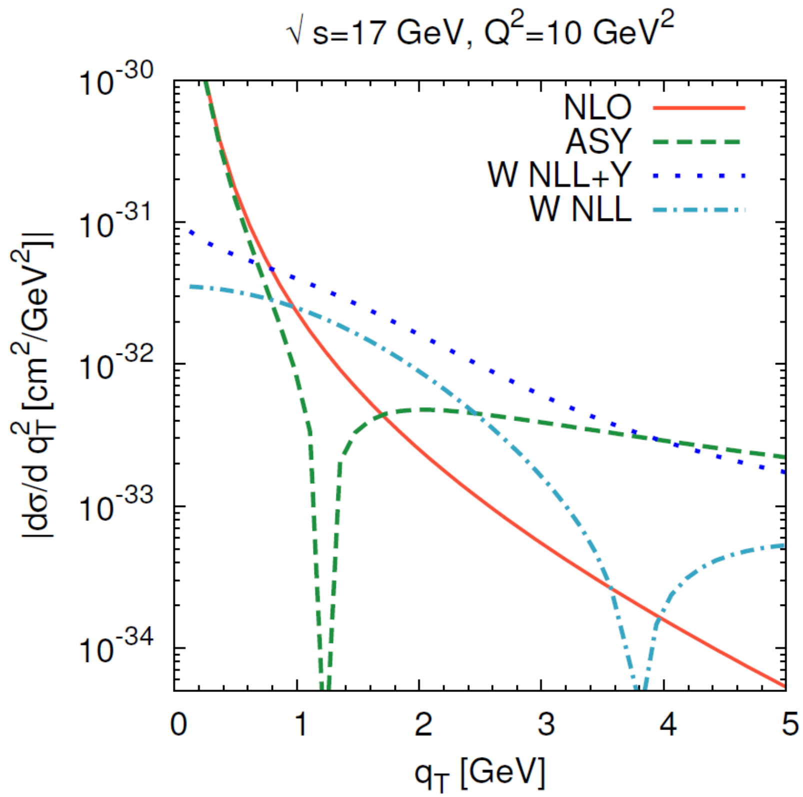




$q_T=Q/4$ $q_T=Q$

$q_T=Q/4$ $q_T=Q$

SIDIS - Y factor



The Y factor is very large and could be affected by large theoretical uncertainties

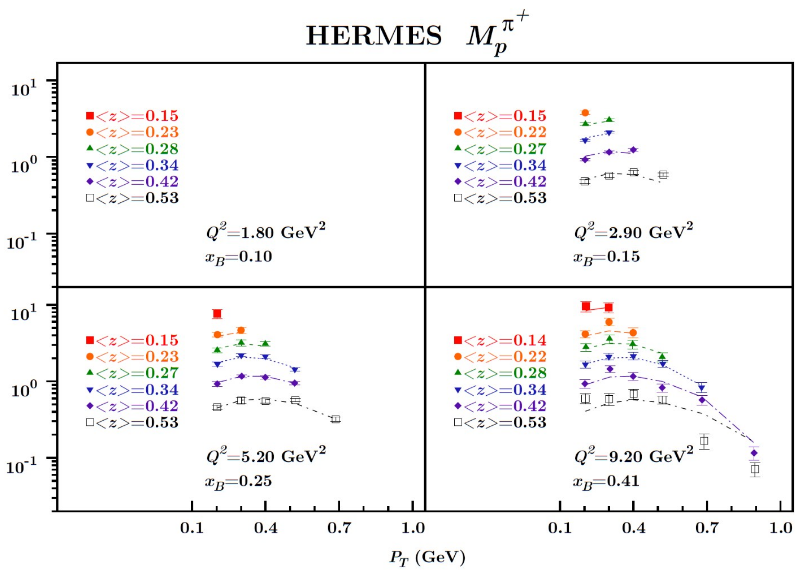
Boglione, Gonzalez,Melis, Prokudin, JHEP 02 (2015) 095

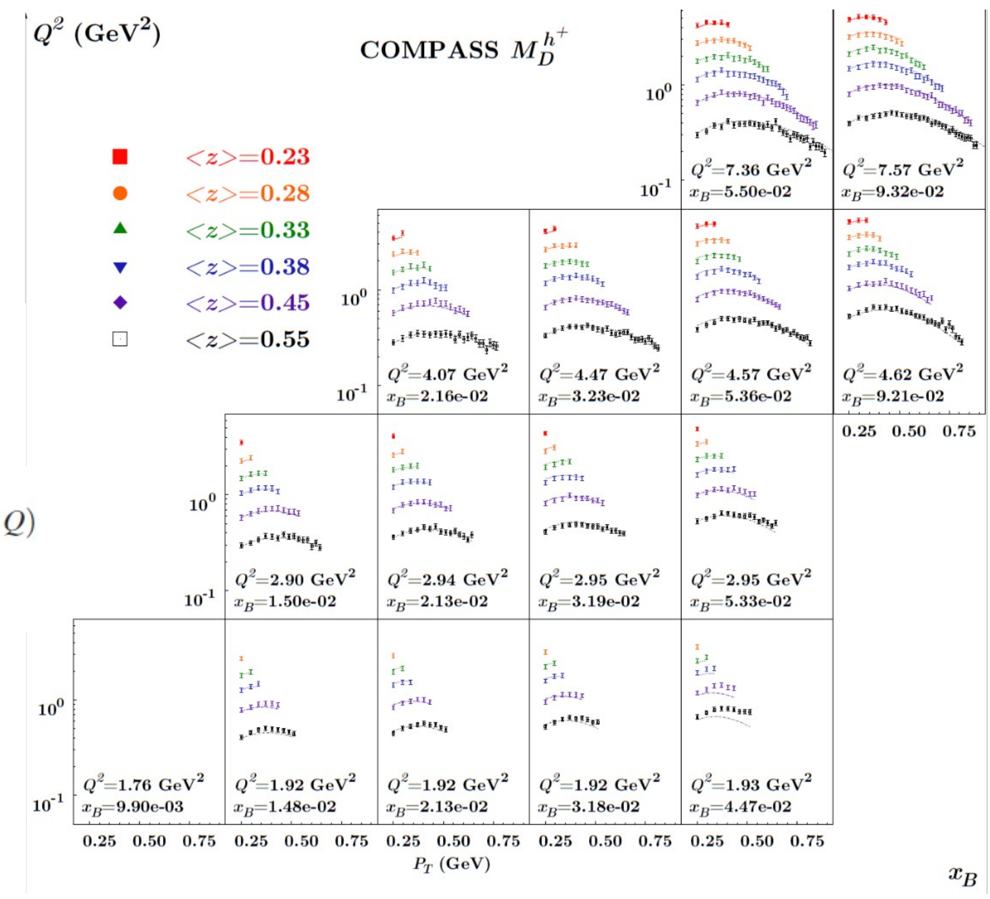
See also: Sun, Isac, Yuan, arXiv:1406.3073

$$\boxed{\frac{d\sigma^{\rm NLO}}{dx\,dy\,dz\,dq_T^2} = \frac{d\sigma^{\rm ASY}}{dx\,dy\,dz\,dq_T^2} + Y}$$

Fit of HERMES and COMPASS data
Attempting “Resummation” in SIDIS …

J. Osvaldo Gonzalez Hernandez, work in progress





$$\frac{d\sigma}{dx\,dy\,dz\,dq_T^2} = \pi\sigma_0^{DIS}\left\{\int \frac{d^2\mathbf{b_T}e^{i\mathbf{q_T}\cdot\mathbf{b_T}}}{(2\pi)^2} W^{SIDIS}(x,z,b_*,Q,C_1,C_2,C_3)F_{NP}^{SIDIS}(x,z,b_T,Q)\right.$$
$$\left. \cancel{+Y^{SIDIS}(x,z,q_T,Q,C_4)}\right\},$$

$$F_{NP}^{SIDIS}(x,z,Q) = \exp\left\{\left[-\frac{g_1+g_{1f}/z^2}{2} - g_2\ln(Q/(2Q_0)) - g_1 g_3\ln(10x)\right]b_T^2\right\}$$

N ~ 2 (One overall normalization parameter is required)
g1 ~ 0.5
g2 ~ 0.5
g3 ~ - 0.03

Drell-Yan and SIDIS global fits

- Next step on the road is to attempt a global fit of Drell-Yan and SIDIS data
- These fit will test the robustness of the TMD evolution scheme

Theoretical uncertainties and dependence on the C_1, C_2, C_3 parameters in the CSS formalism in Drell-Yan and SIDIS

Theoretical uncertainties in pQCD

- Perturbative, fixed order, calculations are affected by theoretical uncertainties due, for instance, to the choice of the factorization **scale**.
  The cross section depends on logs like:

$$\ln(Q/\mu_F)$$

- To "optimize" the expansion the factorization scale is set to be equal to the hard scale

$$\ln(Q/\mu_F) \longrightarrow \mu_F = Q$$

- The theoretical error is built changing the value of the factorization scale.

  Usually:

$$Q/2 < \mu_F < 2Q$$

Theoretical uncertainties in resummation

- Similarly, in resummation several scales appear.
  For instance, using the standard CSS nomenclature we have:

$$C_1/b_T \qquad\qquad C_2 Q \qquad\qquad C_3/b_T$$

- Studying the theoretical uncertainties in resummation is important, as it gives us a measure of how much we know of the perturbative part of the cross section and, correspondingly, how much we have to model.

- This is particularly important for low energy SIDIS data that, contrary to Drell-Yan data, are difficult to describe with resummation.

Cross section with scales in the CSS formalism

- Drell-Yan cross section

$$\frac{d\sigma}{dQ^2dydq_T^2} = \sigma_0\left\{\int\frac{d^2\boldsymbol{b}_T e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}_T}}{(2\pi)^2}\sum_j e_j^2 \underbrace{W_j(x_1,x_2,C_1/b_*,C_2Q,C_3/b_*)}_{\text{Perturbative resummed part of the cross section}}\underbrace{F_{NP}(x,b_T,Q)}_{\text{Non-perturbative function}}\right\}$$

$$\underbrace{+Y(x_1,x_2,q_T,C_4Q)}_{\text{Y-factor (matching function)}}$$

$$W_j(x_1,x_2,Q,C_1/b_*,C_2Q,C_3/b_*) = \sum_{i,k}\underbrace{\exp\left[S(b_*,C_1/b_*,C_2Q)\right]}_{\text{Perturbative Sudakov}}\underbrace{\left[C_{ji}\otimes f_i\right]\left[C_{\bar{j}k}\otimes f_k\right]}_{\text{Convolutions of PDFs and Wilson Coefficients}}$$

$$S(b_T,Q,C_1,C_2) = -\int_{C_1^2/b_T^2}^{C_2^2Q^2}\frac{d\kappa^2}{\kappa^2}\left[A(\alpha_s(\kappa),C_1)\ln\left(\frac{C_2^2Q^2}{\kappa^2}\right)+B(\alpha_s(\kappa),C_1,C_2)\right]$$

$$[C_{ji}\otimes f_i] = \int_x^1\frac{dz}{z}C_{ji}(z,b,\mu=\frac{C_3}{b},C_1,C_2)f_i(x/z,\mu=\frac{C_3}{b})$$

Evaluation of the theoretical errors

- Our choice:
  we change the value of C_1, C_2, C_3 at fixed values of the parameters in the ranges

$$b_0/2 < C_1 < 2\,b_0$$
$$1/2 < C_2 < 2$$
$$b_0/2 < C_3 < 2\,b_0$$

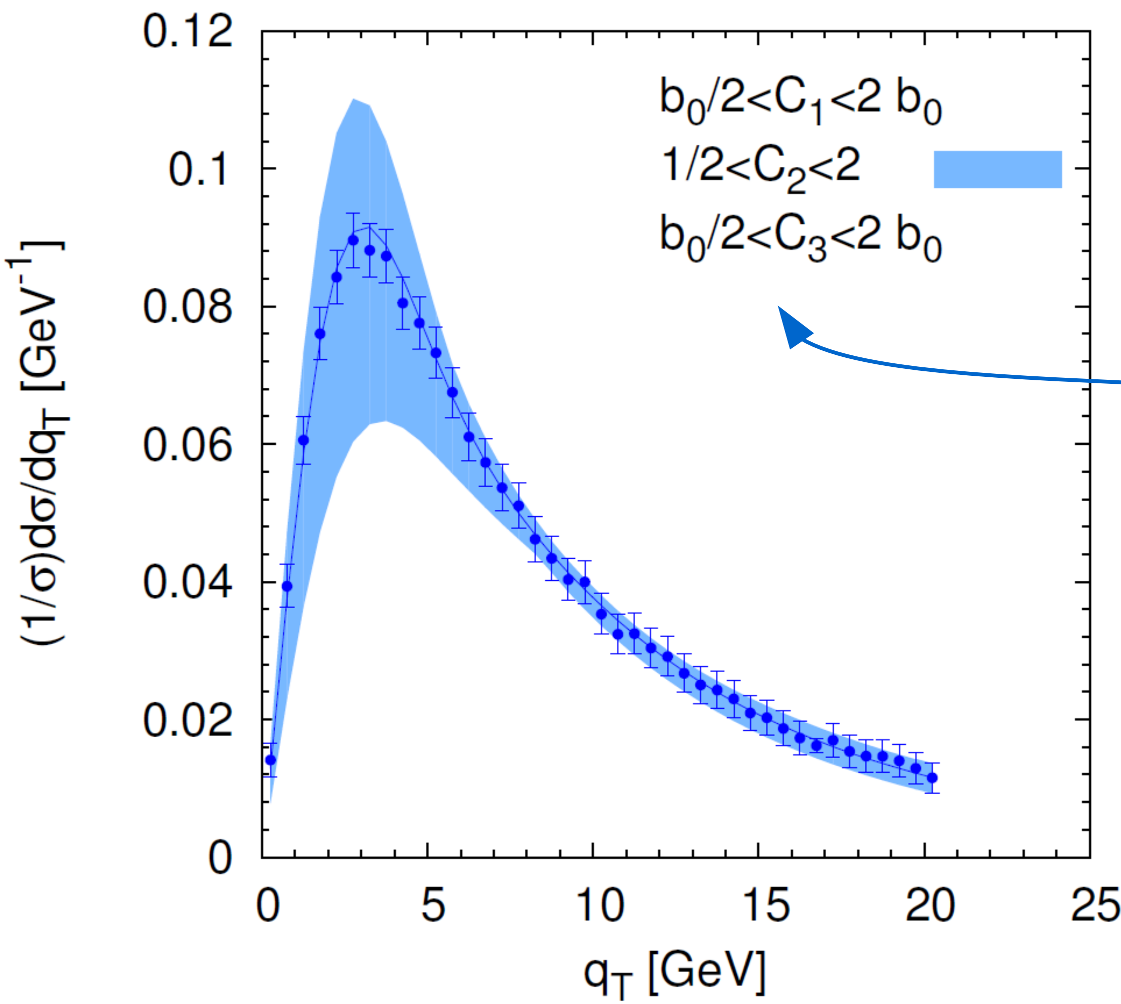
NLL BLNY parametrization, b_{max}=0.5 GeV^{-1}

$$\boxed{F_{NP}^{BLNY} = \exp\left\{\left[-\frac{g_1}{2} - g_2\ln(Q/(2Q_{0L})) - g_1 g_3 \ln(10x)\right] b_T^2\right\}}$$

$$g_1 = 0.21\,\mathrm{GeV}^2 \qquad g_2 = 0.68\,\mathrm{GeV}^2 \qquad g_3 = -0.6$$

Drell-Yan

NLL BLNY parametrization, b_{max}=0.5 GeV^{-1}



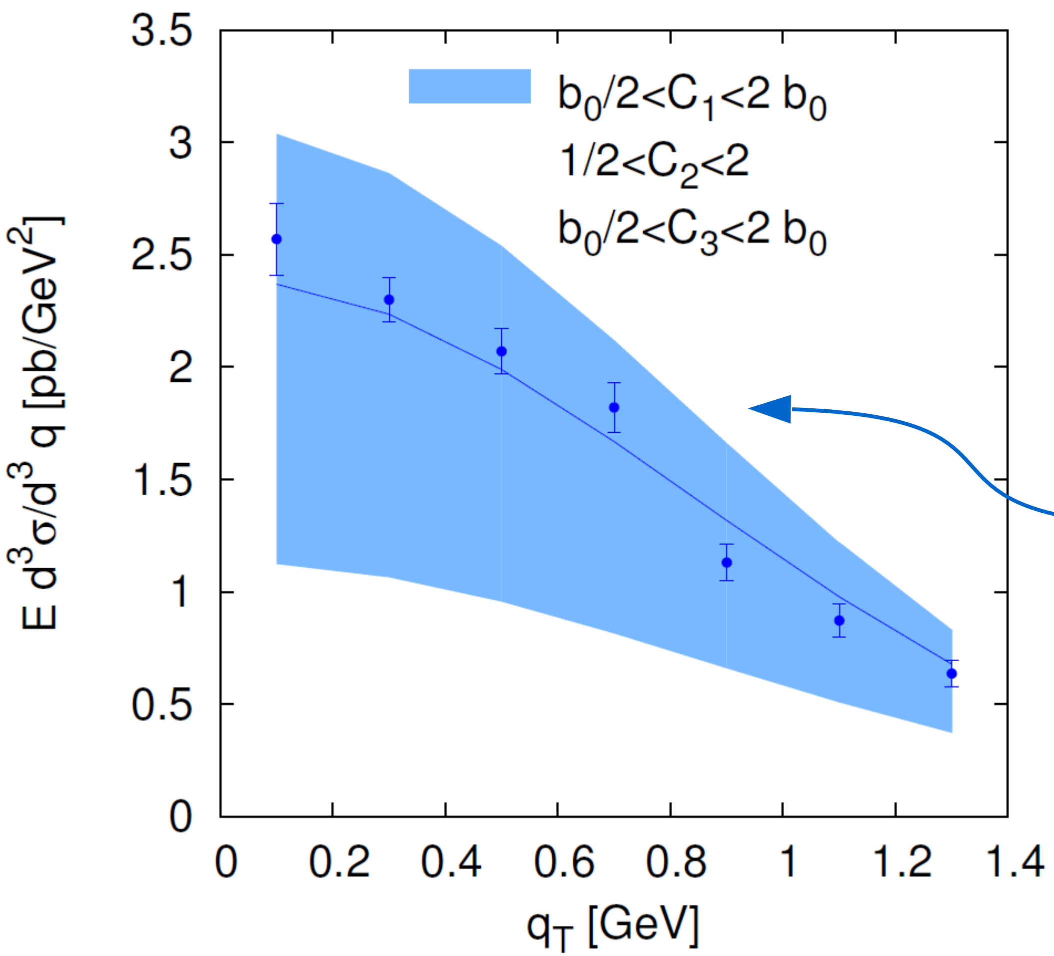
CDF Run II
(High energy exp)

We explore the correlation between the scales.
Warning: the band is an envelope of all possible curves in that range.
It overestimates the errors

Theoretical uncertainties are under control in the perturbative region.
However, they become large as we enter the non-perturbative region.

Drell-Yan

NLL BLNY parametrization, b_{max}=0.5 GeV^{-1}



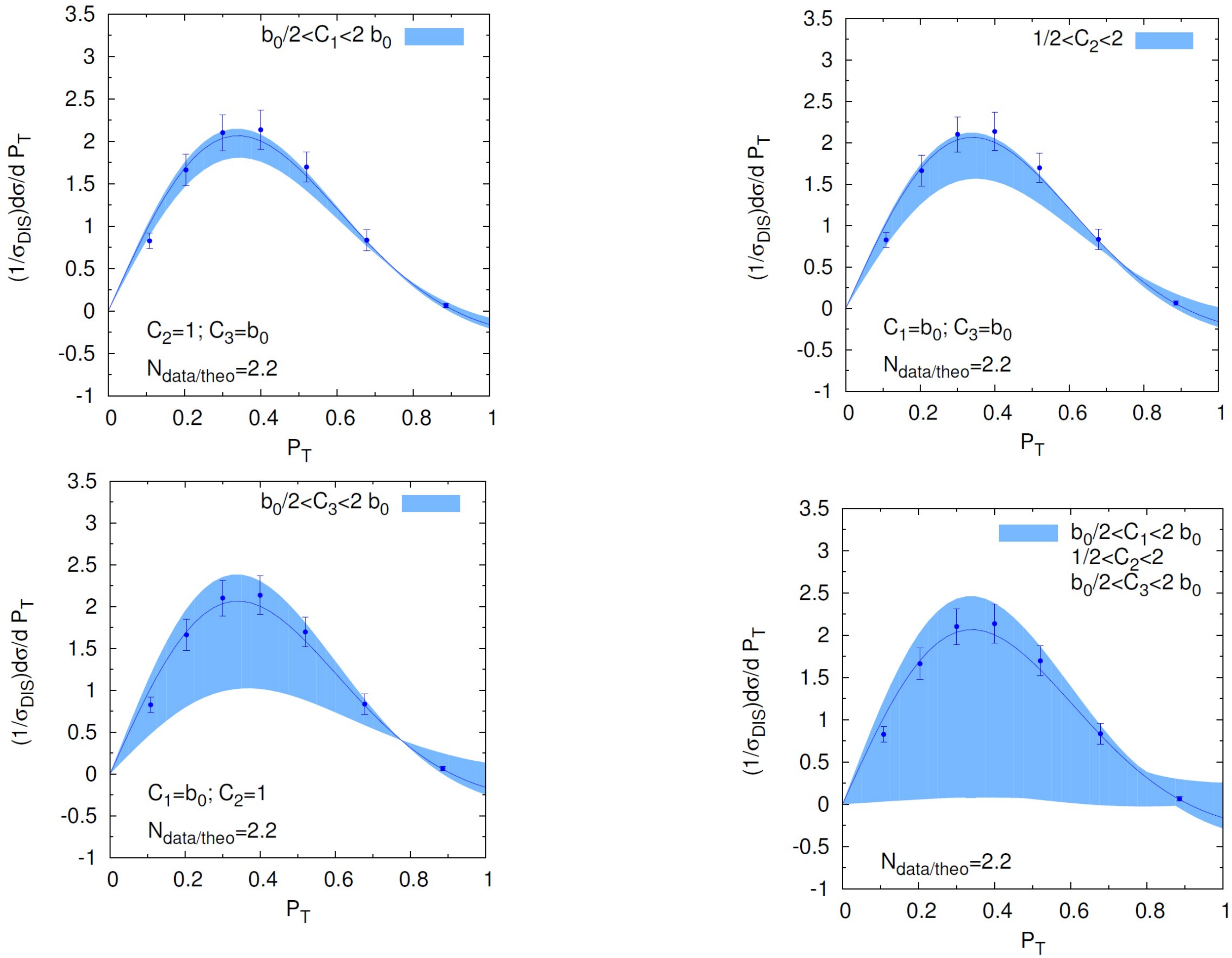
E288 √s=23.8 GeV
(Low energy exp)

Q=5 GeV

Warning: the band is an envelope of all possible curves in that range. It overestimates the errors

As we enter the non-perturbative region, the theoretical uncertainties grow larger

HERMES BLNY (b_{max}=0.5 GeV^{-1})



We explore the correlation between the scales.
Warning: the band is an envelope of all possible curves in that range.
Errors are overestimated

Summary

- High energy processes are affected by reasonable theoretical errors.

- Low energy processes are instead affected by large uncertainties.
  Different choices of the scale would give very different sets of parameters.

- It is possible that a NNLL calculation could help to shrink the bands.

- For low energy SIDIS experiments (HERMES/COMPASS) the Y factor is large... but in principle it could be affected by the same uncertainties which affect the resummed cross section.

- Is a simultaneous fit of Drell_Yan and SIDIS data possible within this picture ?