

# NLO merging: overview and recent developments

Johannes Bellm (IPPP), DIS16 Hamburg  
13.4.2016

LO+PS

(standard Generators like Herwig, Pythia, Sherpa)

LO merging

CKKW(-L)

MLM

MEPS

ULOPS

NLO matching

MC@NLO

POWHEG

MEPS@NLO

NLO merging

GENEVA

See talk by S. Alioli

FxFx

UNLOPS

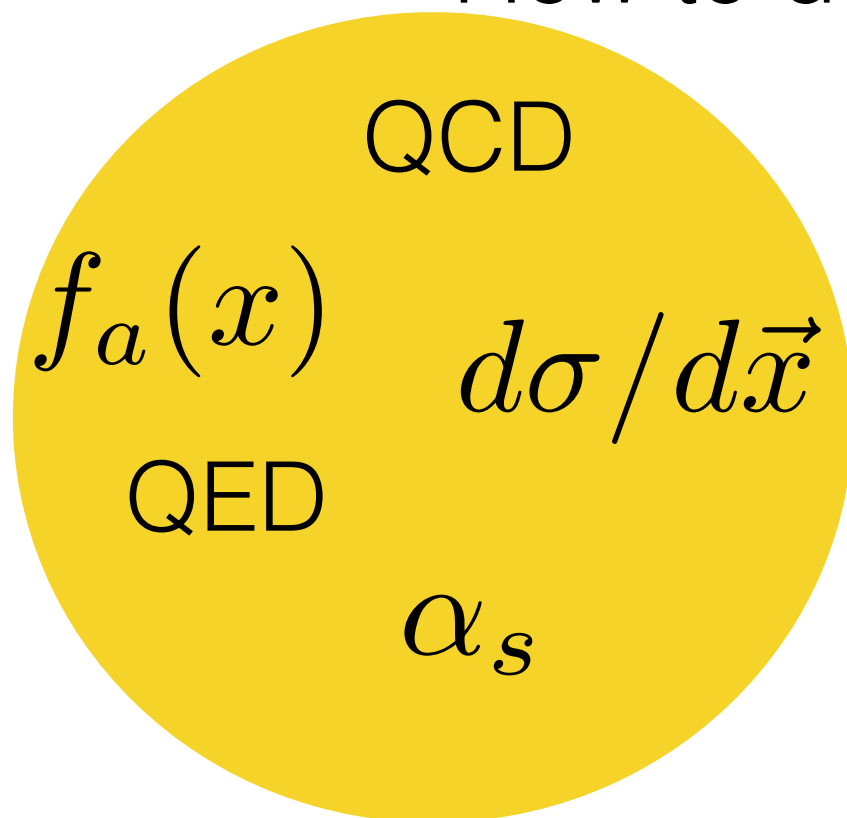
Herwig7.1

See talk by S. Alioli for:

UNNLOPS

NNLOPS

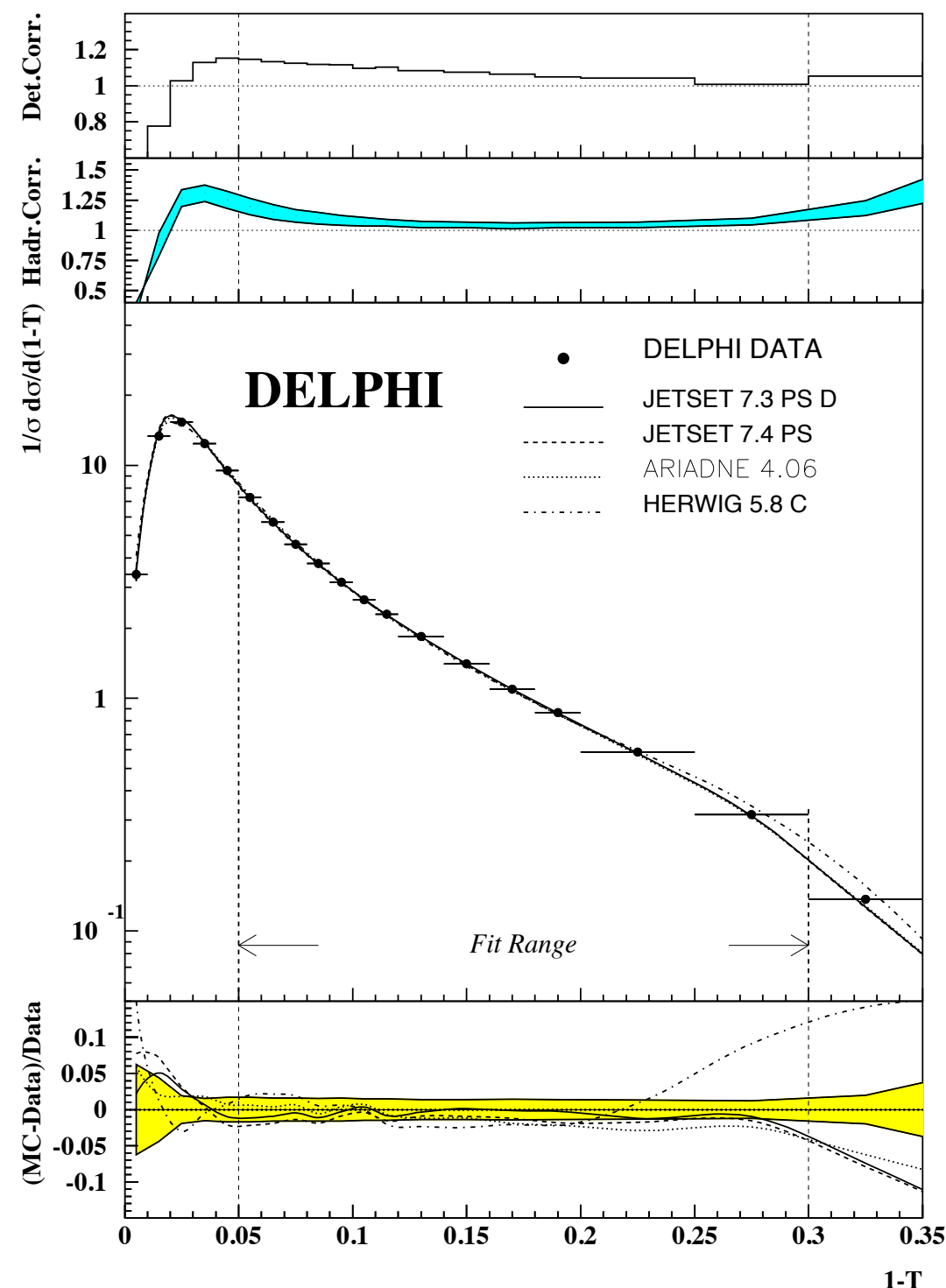
How to describe exclusive/differential observables?



$PS[\phi_i(Q)]$  Parton Shower

$$\Delta_{\mu}^Q = e^{-\int \sum_j P_j(z) \theta_j d\phi_{n+1}}$$

emission/no emission



Event generators try to describe data in a full simulation.  
So starting from hard processes to the hadronisation and decays.

Parton showers are needed to get to high multiplicities and describe fully exclusive observables.

Approximation breaks for large angle or hard emissions.

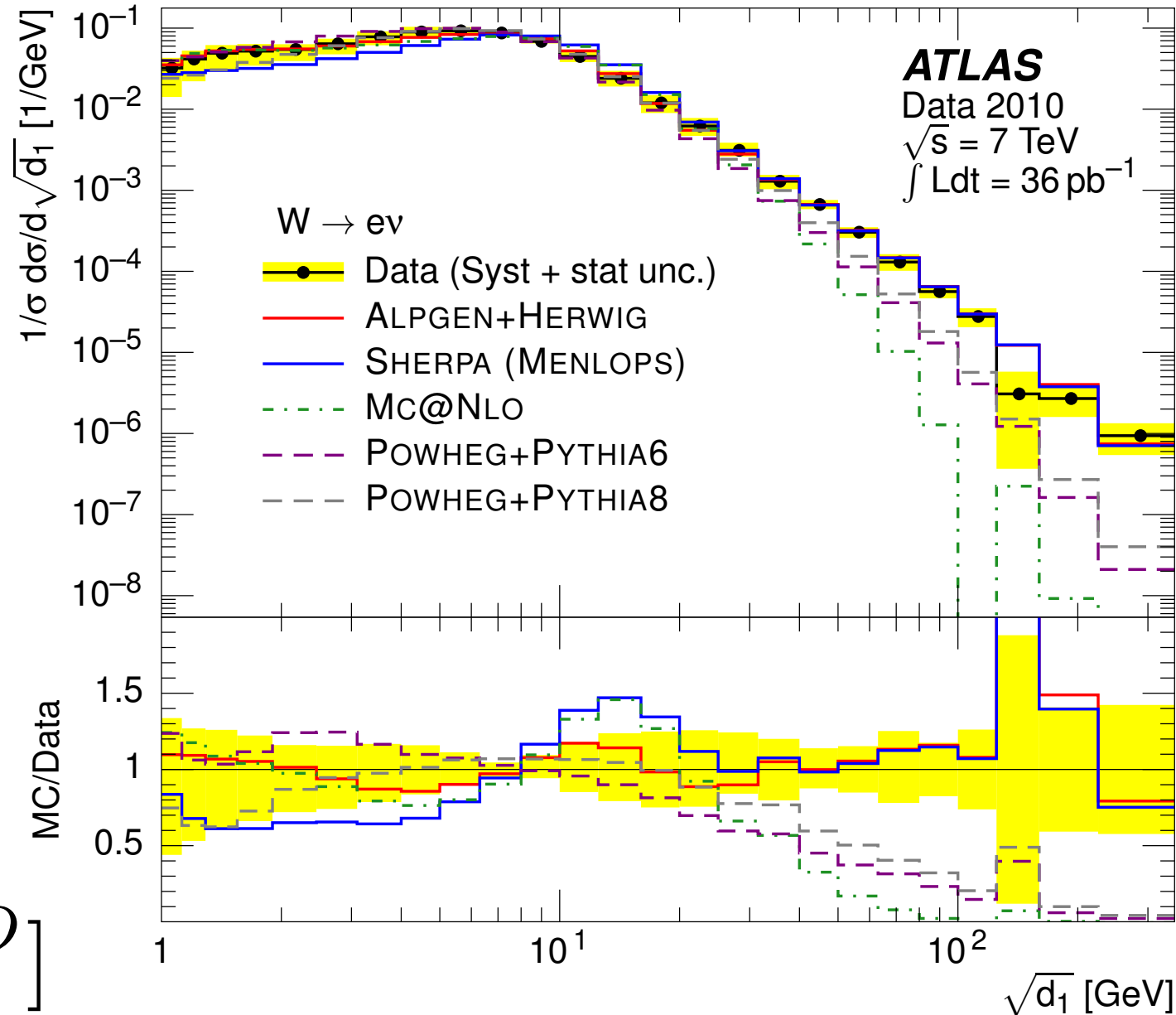
Merging of LO predictions helps to correct for these emission but are still LO.

Aim: Full simulation with NLO corrections for inclusive cross section and hard emissions.

Solving the double counting problem by subtraction.

Expand the shower to  $\mathcal{O}(\alpha_S)$  in emissions and no emissions.

$$\begin{aligned}
 PS[d\sigma^{\text{matched}}] &= PS_0[d\sigma^{LO}] \\
 &+ PS_0[d\sigma^V + \int d\phi_1 P(z) d\sigma^{LO}] \\
 &+ PS_1[d\sigma^R - \int d\phi_1 P(z) d\sigma^{LO}]
 \end{aligned}$$

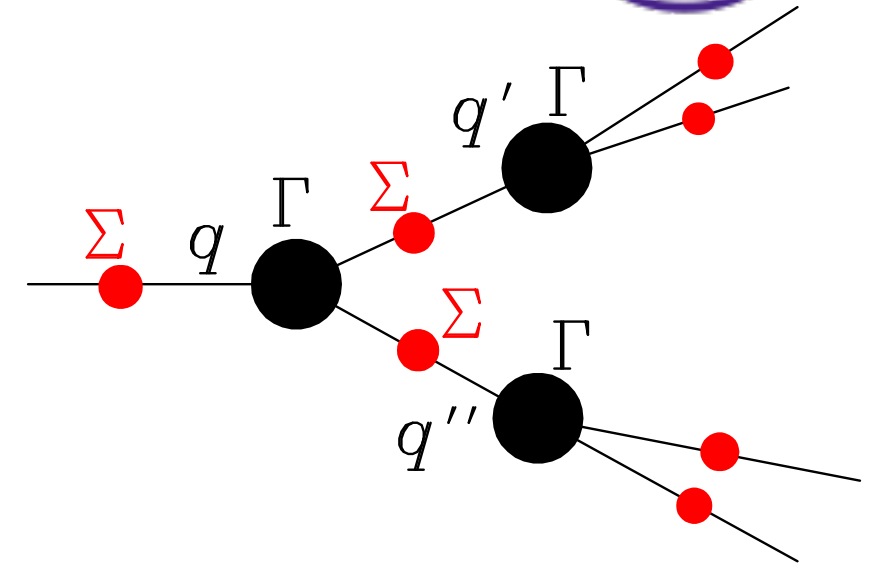


Merging several exclusive LO cross sections needs:

1. Find a shower history for different multiplicities.
2. Reweight with no emission probabilities for intermediate scales between the emissions and from last scale to shower cutoff (except highest mult.) .
3. Reweight the emissions with appropriate running couplings.

$$R_2(Q_1, Q) = [\Delta_q(Q_1, Q)]^2 ,$$

$$R_3(Q_1, Q) = 2 [\Delta_q(Q_1, Q)]^2 \int_{Q_1}^Q dq \Gamma_q(q, Q) \Delta_g(Q_1, q)$$



(picture from MiNLO, see next slide)

$$\Gamma_q(q, Q) = \frac{2C_F}{\pi} \frac{\alpha_s(q)}{q} \left( \ln \frac{Q}{q} - \frac{3}{4} \right)$$

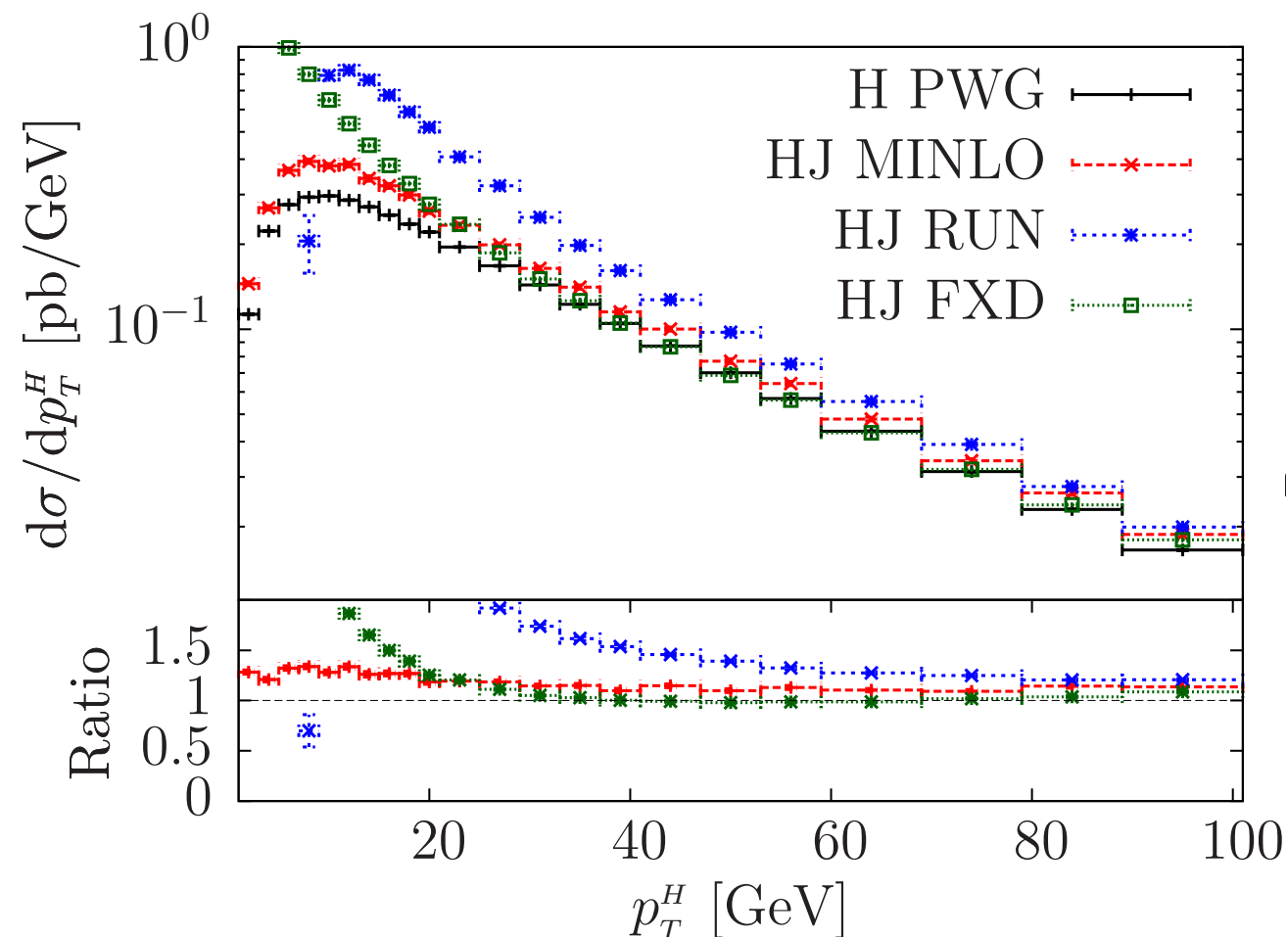
$$\Gamma_g(q, Q) = \frac{2C_A}{\pi} \frac{\alpha_s(q)}{q} \left( \ln \frac{Q}{q} - \frac{11}{12} \right)$$

$$\Gamma_f(q) = \frac{N_f}{3\pi} \frac{\alpha_s(q)}{q} ,$$

# MiNLO (Multi-scale improved NLO)

Combining fixed order NLO for e.g. H+jet with ideas of LO merging/resummation.

Reweight with (analytic) shower history.



$$\bar{B} = \alpha_S^2(M_H^2) \alpha_S(q_T^2) \Delta_g^2(M_H, q_T) \left[ B \left( 1 - 2\Delta_g^{(1)}(M_H, q_T) \right) + V + \int d\Phi_{\text{rad}} R \right]$$

weighted with Sudakov suppression (e.g. Higgs):

$$F = \alpha_S^2(M_H) \alpha_S(p_T) \left\{ \exp \left[ -\frac{C_A}{\pi b_0} \left\{ \log \frac{\log \frac{Q^2}{\Lambda^2}}{\log \frac{Q_0^2}{\Lambda^2}} \left( \frac{1}{2} \log \frac{Q^2}{\Lambda^2} - \frac{\pi b_0}{C_A} \right) - \frac{1}{2} \log \frac{Q^2}{Q_0^2} \right\} \right] \right\}^2$$

Same problems as matching and LO merging (combined).

Idea: Use LO merging and expand to  $\mathcal{O}(\alpha_S)$   
as it was done in the matching.

Lowest multiplicity basically behaves as in NLO matching??

$$\begin{aligned} PS[d\sigma^{merged}] &= PS_0^V[d\sigma^0 \Delta_\mu^0] \\ &+ PS_1^V[d\sigma^1 \Delta_1^0 \Delta_\mu^1] \\ &+ PS_2[d\sigma^2 \Delta_1^0 \Delta_2^1 \Delta_\mu^1] \end{aligned}$$

The expanded version constructs  
the subtraction expression as in NLO matching



Same problems as matching and LO merging (combined).

Idea: Use LO merging and expand to  $\mathcal{O}(\alpha_S)$  as it was done in the matching.

Lowest multiplicity basically behaves as in NLO matching??

$$\begin{aligned}
 PS[d\sigma^{merged}] &= PS_0^V[d\sigma^0 \Delta_\mu^0] \cdots \Delta_\mu^0 = 1 - \int d\phi_1 P(z) + \mathcal{O}(\alpha_S^2) \\
 &\quad + PS_1^V[d\sigma^1 \Delta_1^0 \Delta_\mu^1] \\
 &\quad + PS_2[d\sigma^2 \Delta_1^0 \Delta_2^1 \Delta_\mu^1] \\
 PS[d\sigma^{matched}] &= PS_0[d\sigma^{LO}] \\
 d\sigma^1 &= d\sigma^R + \mathcal{O}(\alpha_S^2) \cdots + PS_0[d\sigma^V + \int d\phi_1 P(z) d\sigma^{LO}] \\
 &\quad \cdots + PS_1[d\sigma^R - d\phi_1 P(z) d\sigma^{LO}]
 \end{aligned}$$

Same problems as matching and LO merging (combined).

Idea: Use LO merging and expand to  $\mathcal{O}(\alpha_s)$  as it was done in the matching.

Weight in LO merging:

$$B_1 \Delta_1^0 \theta_{ME} = f_1(Q_f) \alpha_s^n(Q_r) \tilde{B}_1 \cdot \Delta_{q_1}^0 \frac{\alpha_s(q_1)}{\alpha_s(Q_r)} \frac{f_0(Q_f)}{f_0(q_1)} \frac{f_1(q_1)}{f_1(Q_f)} \theta_{ME}$$

history weight

also needs to be expanded up to  $\mathcal{O}(\alpha_s)$

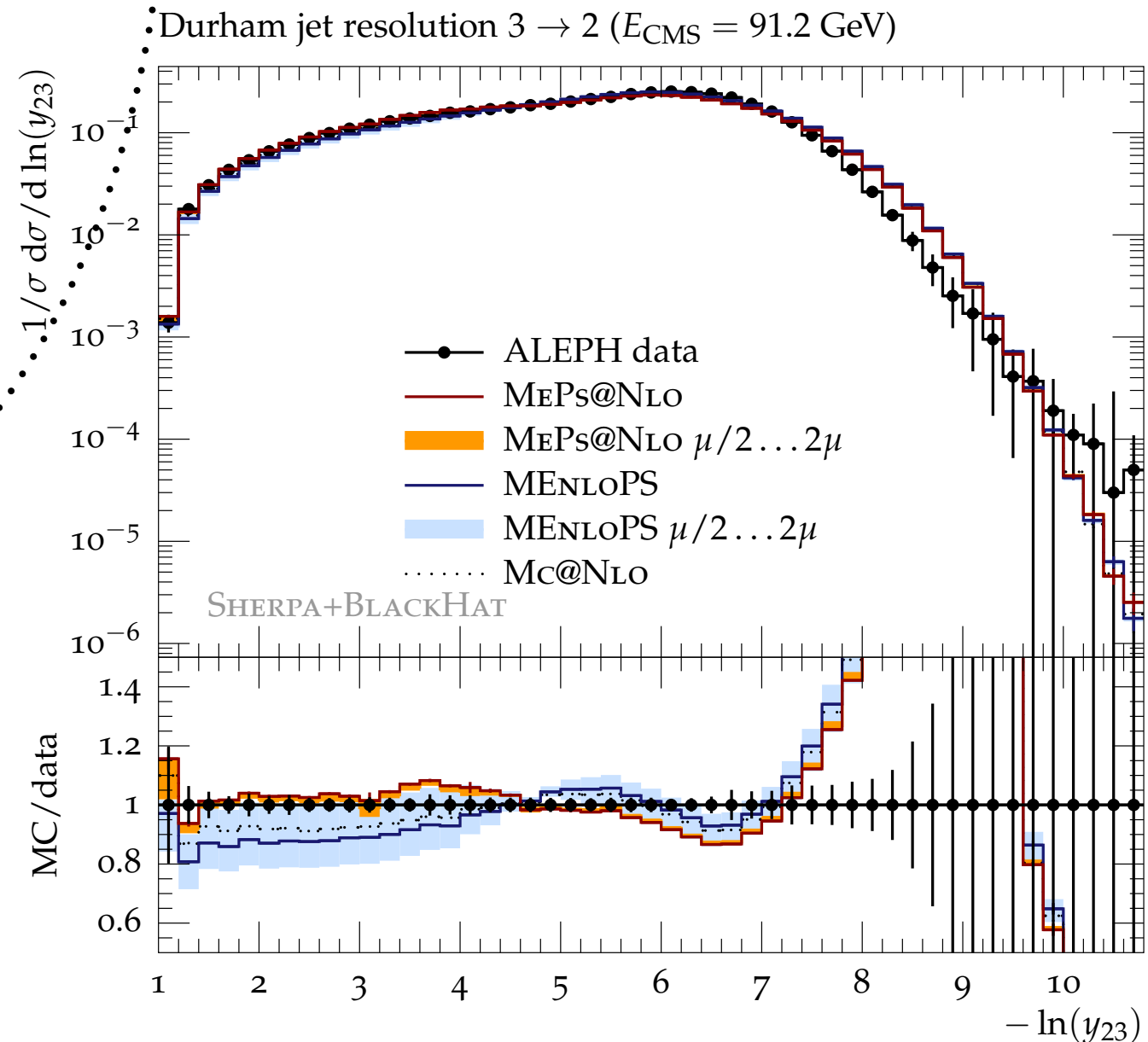
$$B_1 \Delta_1^0 \theta_{ME} = f_1(Q_f) \alpha_s^n(Q_r) \tilde{B}_1 \cdot \Delta_{q_1}^0 \frac{\alpha_s(q_1)}{\alpha_s(Q_r)} \frac{f_0(Q_f)}{f_0(q_1)} \frac{f_1(q_1)}{f_1(Q_f)} \theta_{ME}$$

$$\alpha_s(\tilde{\mu}_R^2)^n \left( 1 - \frac{\alpha_s(\tilde{\mu}_R^2)}{2\pi} \beta_0 \sum_{i=1}^n \log \frac{\mu_i^2}{\tilde{\mu}_R^2} \right)$$

Expanding (naturally)  
produces expressions used in  
the MiNLO ansatz.

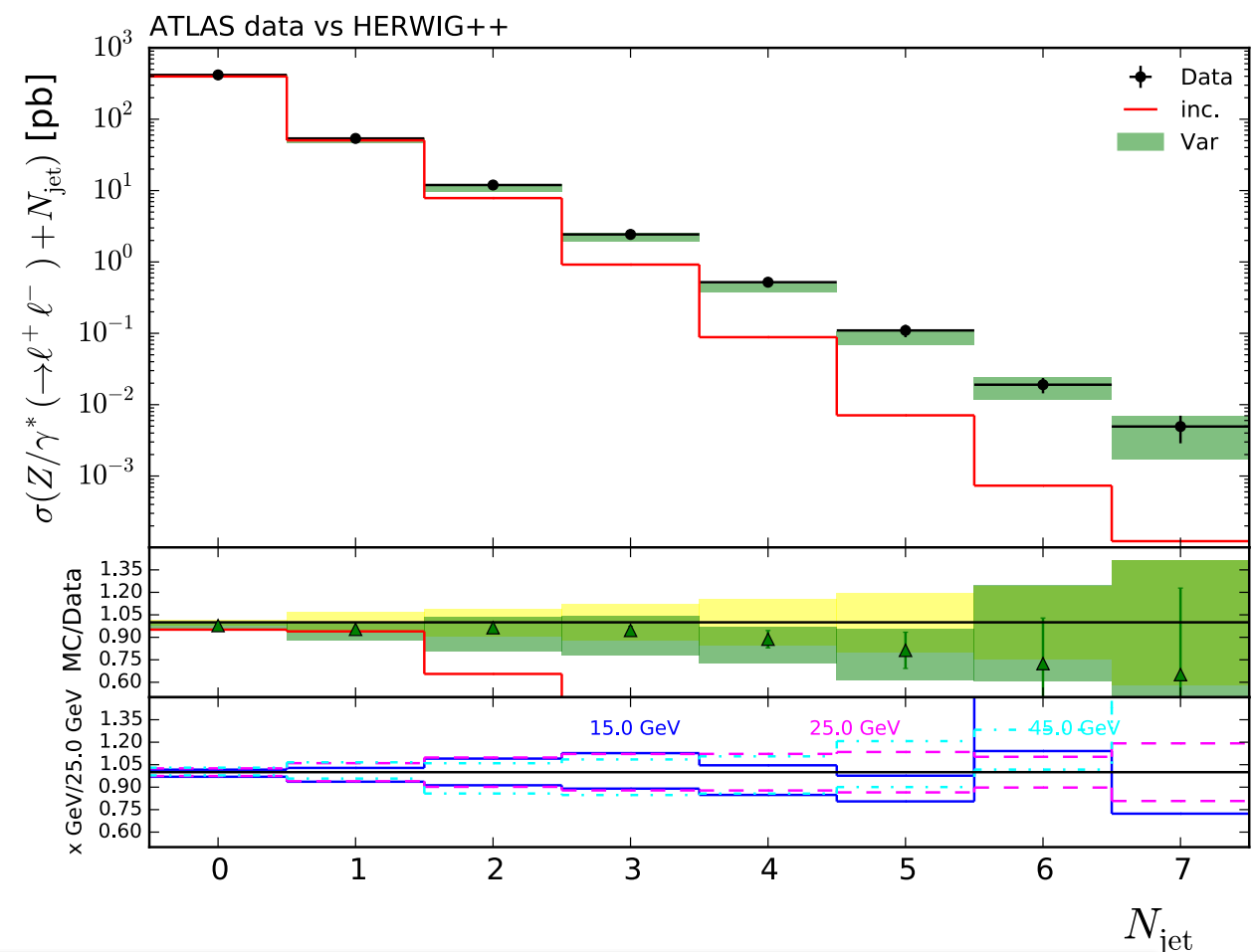
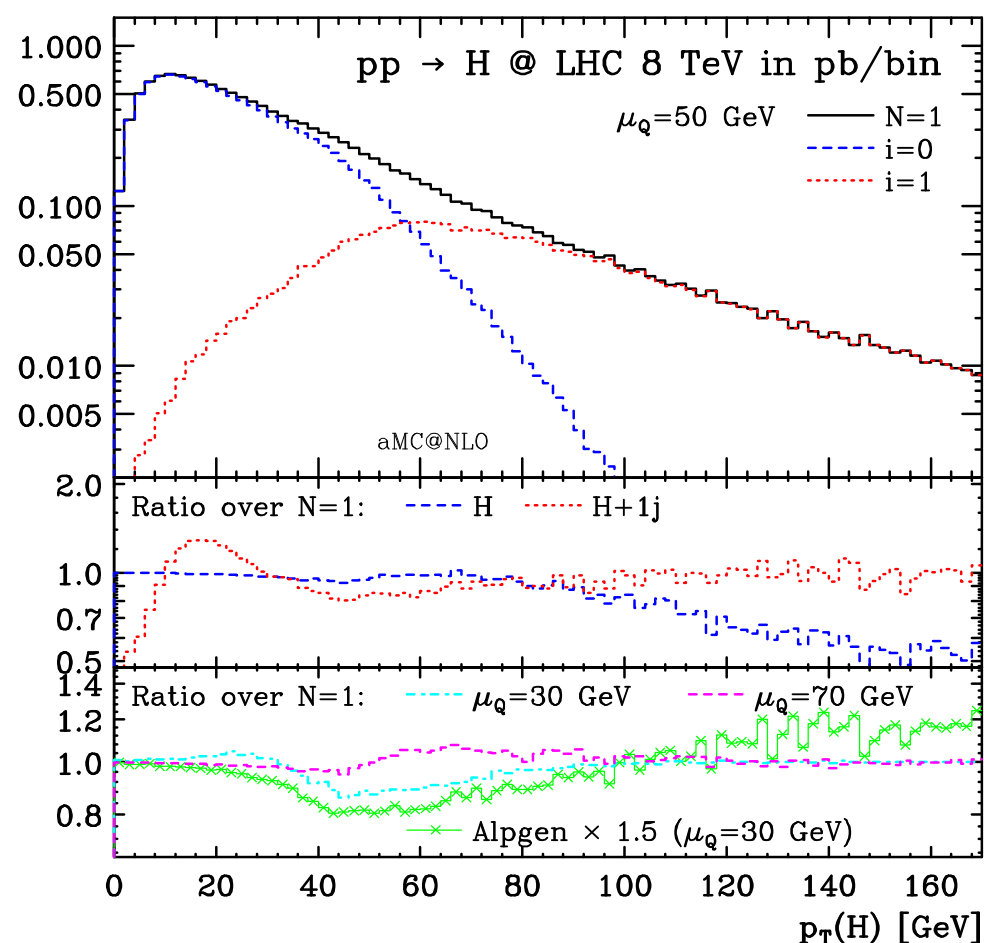
$$\bar{B}_{n+1}^{(A)} \left( 1 + \frac{B_{n+1}}{\bar{B}_{n+1}^{(A)}} \int_{t_{n+1}}^{\mu_Q^2} d\Phi_1 K_n \right)$$

In addition one needs  
to expand the intermediate  
Sudakov factors.



Staging multiple MC@NLO expressions  
with phase space restrictions on emissions.

Analytical Sudakov suppression, MiNLO-like scale choice  
and introduction of a smooth transition function  
between shower and ME region



The „subtract-what-you-add“-method of merging NLO.

Inspired by the LoopSim method for nNLO

Rubin, Salam, Sapeta  
JHEP 1009 (2010) 084

Major Difference to other merging schemes:

$$B_n \Delta_n^0 \Delta_\mu^n \rightarrow B_n \Delta_n^0 - \int_\mu^{q_n} B_{n+1} \Delta_{n+1}^0$$

Less influence on inclusive observables.

**UNLOPS:**

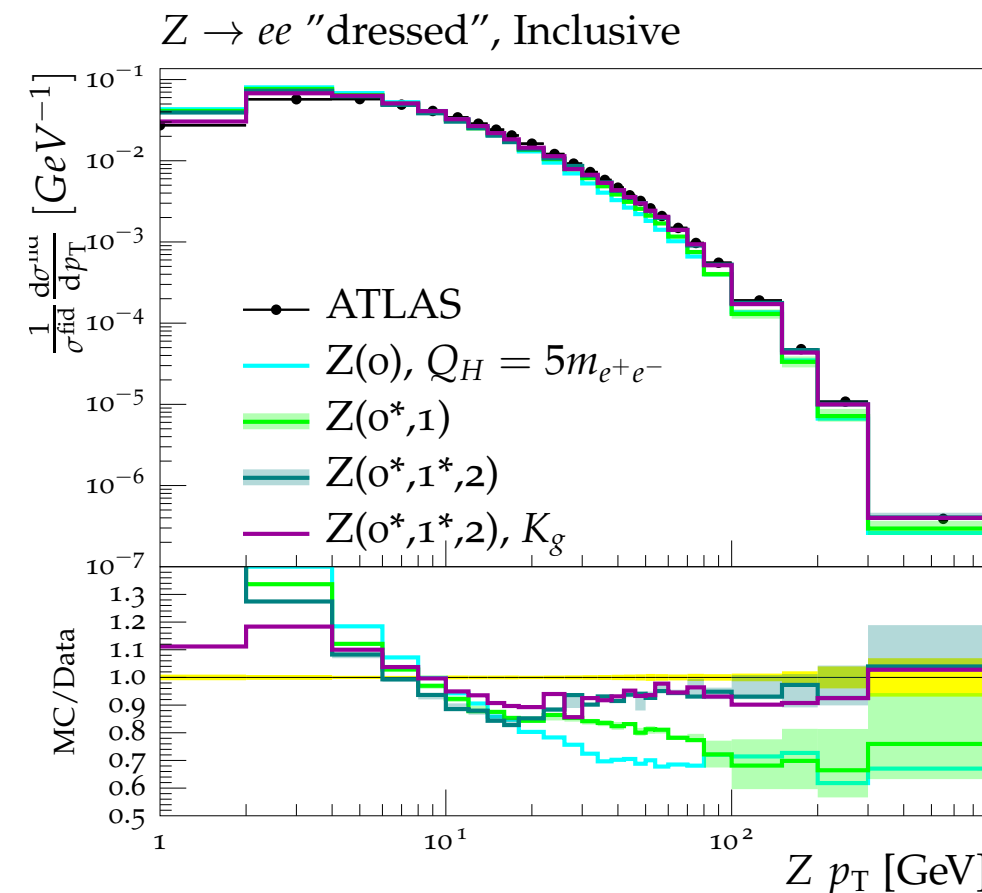
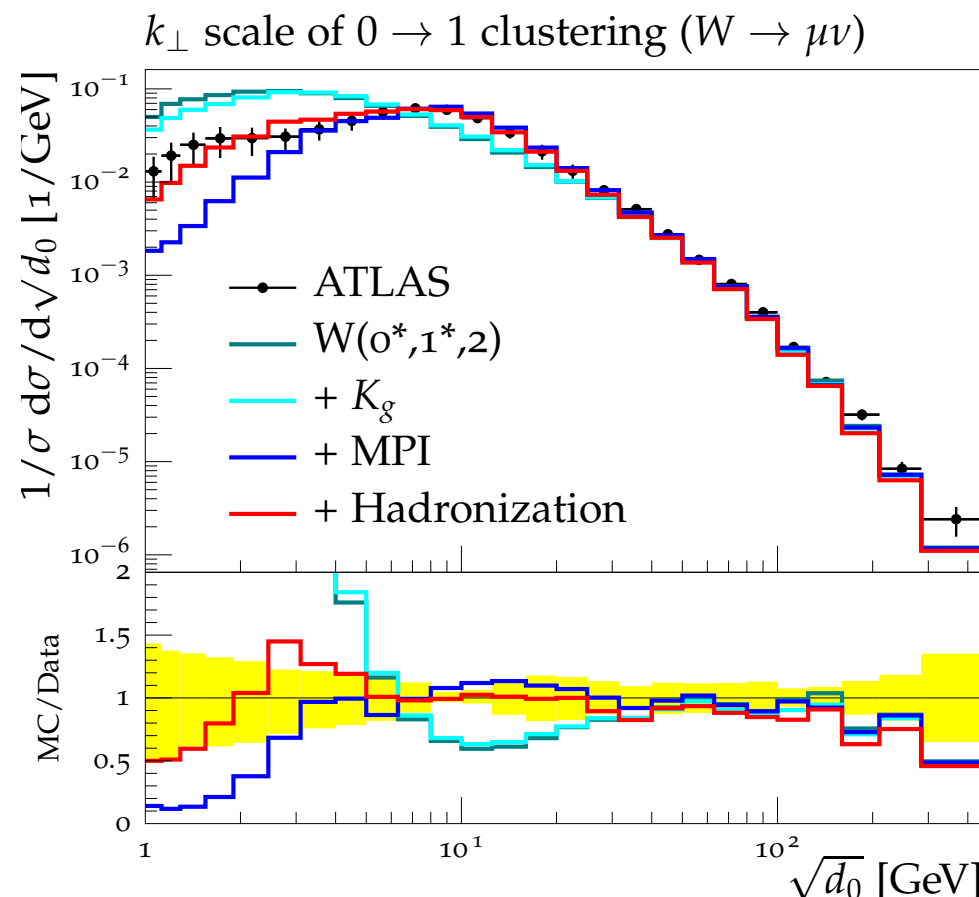
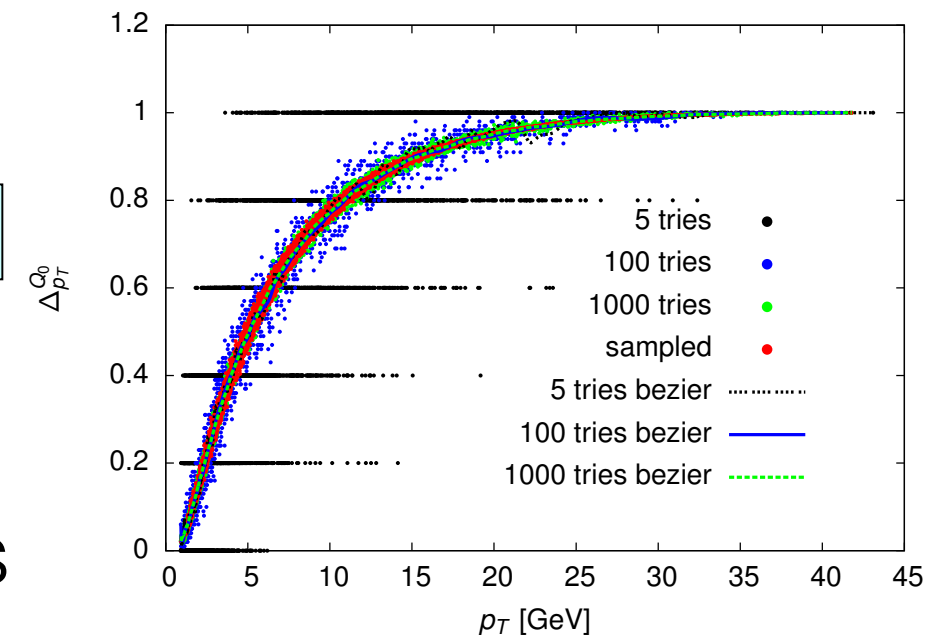
$$\frac{x_{i-1}^\pm f_{i-1}^\pm(x_{i-1}^\pm, \rho_{i-1})}{x_{i-1}^\pm f_{i-1}^\pm(x_{i-1}^\pm, \rho_i)} = 1 + \frac{\alpha_s(\mu_R)}{2\pi} \ln \left\{ \frac{\rho_{i-1}}{\rho_i} \right\} \int_{x_{i-1}^\pm}^1 \frac{dy}{y} \frac{x_{i-1}^\pm \hat{f}_{i-1}^\pm(\frac{x_{i-1}^\pm}{y}, \mu_F)}{x_{i-1}^\pm f_{i-1}^\pm(x_{i-1}^\pm, \mu_F)} + \mathcal{O}(\alpha_s^2(\mu_R))$$

Based on developments in Herwig7 and publicly available in Herwig 7.1.

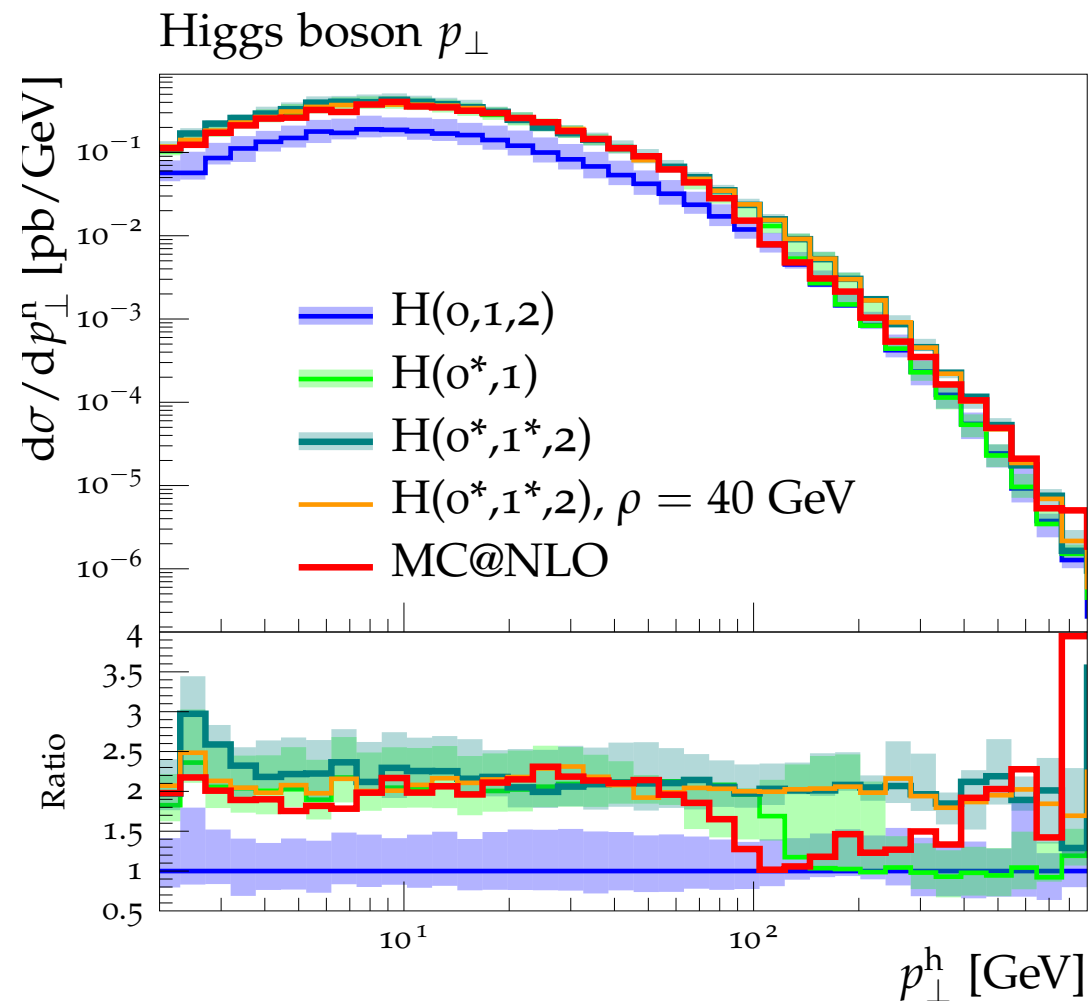
$(\bar{V}_1 + IPK_1 - (\partial_{\alpha_s}^1 \Delta_1^0) B_1) \theta_{ME}^1$	$\int (B_2 - D_2) \theta_{ME}^2 \theta_{ME}^1$	$\int (P_2 B_1 - D_2) \theta_{ME}^2 \theta_{ME}^1$
	$(B_2 \Theta \leftarrow P_2 B_1) \theta_{ME}^2 \theta_{ME}^1$	

$\mathcal{O}(\alpha_s)$  expansion of parton shower weights      cuts on multiple emissions

MC@NLO-like contributions  
below the merging scale



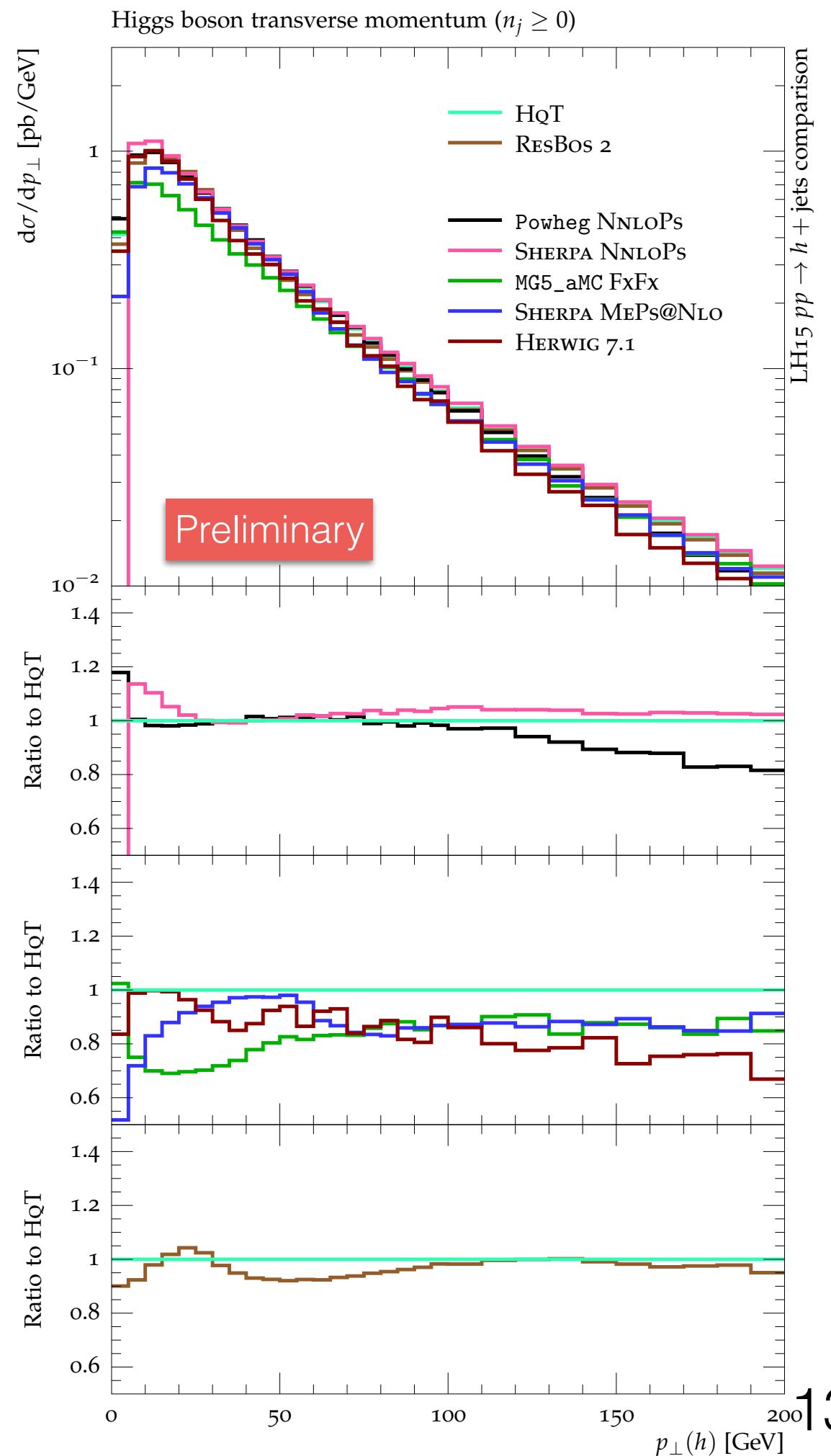
# Herwig NLO Merging



MC@NLO and merging 1 NLO agree.

NLO merging of 2 NLO show smooth transition at the merging scale.  
(remember the large K-factor)

Right: Up to 3 NLO merged.



# Herwig NLO Merging

Comment on  $\alpha_S$  and scale choice:

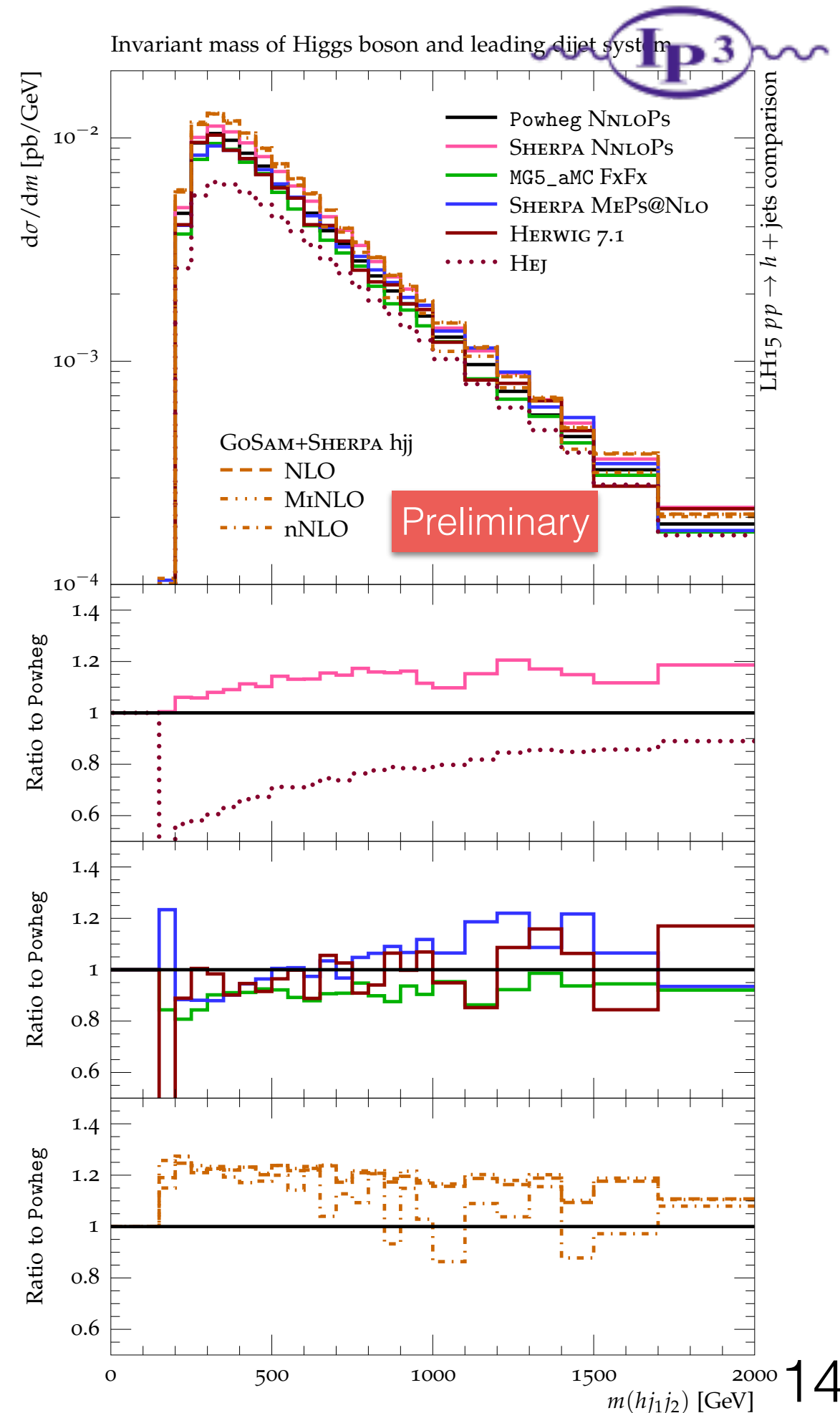
The value of  $\alpha_S$  is usually tuned to data.

Motivated by CMW scheme:

$$K_g = \left( \frac{67}{18} - \frac{\pi^2}{6} \right) C_A - \frac{10}{9} T_R n_f$$

Approximates already NLO to first emission.

Need to take this into account to describe data.





Gave an overview on various merging schemes.

Methods are used but still need to be improved.

The merging should not spoil the NLO accuracy of the inclusive cross section.

Choice of couplings and pdf scales can improve the prediction.