Charged-Current Deep Inelastic Scattering at Third Order

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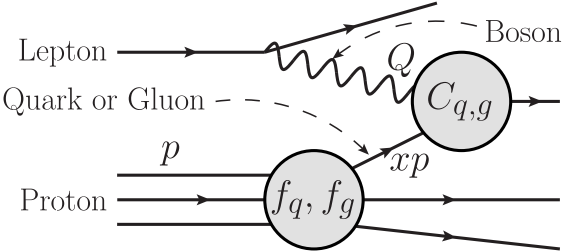
Deep Inelastic Scattering 2016, DESY Hamburg

INTRODUCTION

Deep Inelastic Scattering – lepton scatters from a proton:



Boson: γ, H, Z^0 (Neutral Current) or $W^{\pm}$ (Charged Current)

Cross-section $\sigma \sim \sum_a F_a = \sum_a \left[C_{a,q} \otimes f_q + C_{a,g} \otimes f_g\right]$.

$C_{a,j}$ – *"Coefficient Function"*
f_j – *"Parton Distribution Function"*
F_a – *"Structure Function"*

Introduction

Anomalous dimensions/Splitting functions – scale evolution of PDFs:

$$\frac{\mathrm{d}}{\mathrm{d}\ln Q^2} f_j = \sum_k \gamma_{jk} f_k$$

Series expansions in α_s:

$$C_{a,j} = c_{a,j}^{(0)} + \alpha_s c_{a,j}^{(1)} + \alpha_s^2 c_{a,j}^{(2)} + \alpha_s^3 c_{a,j}^{(3)} + \cdots$$

$$\gamma_{jk} = \alpha_s \gamma_{jk}^{(0)} + \alpha_s^2 \gamma_{jk}^{(1)} + \alpha_s^3 \gamma_{jk}^{(2)} + \cdots$$

KNOWN SO FAR

Natural to consider *combinations* $F_a^{W^+ + W^-}$ and $F_a^{W^+ - W^-}$.

Known, to third-order (massless DIS):

- γ probe: $C_{2,q}, C_{2,g}, C_{L,q}, C_{L,g}$ [hep-ph/0504.242]
- H probe: $C_{\phi,q}, C_{\phi,g}$ [hep-ph/0912.0369]
- $W^\pm$ probe (+): $C_{2,q}^+, C_{2,g}^+, C_{L,q}^+, C_{L,g}^+, C_{3,ns}^+$ [hep-ph/0812.4168]

Unknown/Incomplete:

- Z^0 probe: $C_{2,q}, C_{2,g}, C_{L,q}, C_{L,g}, C_{3,q}$
- $W^\pm$ probe (−): $C_{2,ns}^-, C_{L,ns}^-, C_{3,ns}^-$

Charged-Current DIS

Previous work at α_s^3. Moch, Rogal: [hep-ph/0704.1740]

- $N = 3, 5, ..., 11$ of $C^-_{2,ns}$, $C^-_{L,ns}$.
- $N = 2, 4, ..., 10$ of $C^-_{3,ns}$.

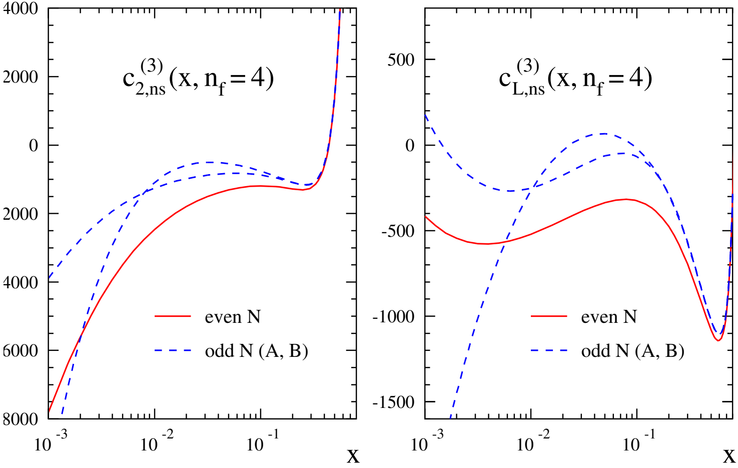
Used to produce approximations to full analytic result.

(We compute in Mellin space. Nth Mellin Moment:

$$F_a(N, Q^2) = \int_0^1 x^{N-1} F_a(x, Q^2) \mathrm{d}x.$$

For $F_a^{W^+ - W^-}$: F_2, F_L live on odd moments, F_3 on even moments.)

CHARGED-CURRENT DIS



even N: $W^+ + W^-$, odd N: $W^+ - W^-$ [hep-ph/0704.1740]

Approximations unconstrained below $x \approx 10^{-2}$.

COMPUTATION

Diagram preparation:

- QGRAF + FORM: produce diagram database.

Mellin moment computation:

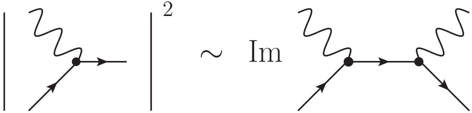
- MINCER (FORM): verify Moch, Rogal's moments and compute more.

Analytic computation:

- All-N FORM code: used in MVV's calculation, directly produces analytic results in N.

THE OPTICAL THEOREM

Relates structure functions to the *forward scattering amplitude*:



Number of forward scattering "diagrams":

- F_2, F_L: 209
- F_3: 197

Big improvement on diagram database of Moch, Rogal (1300)
→ faster moment computation.

Check: gauge parameter in propagators, ensure cancellation.

RENORMALIZATION

Forward scattering amplitudes T_a will contain poles in Dim.-Reg. parameter ε (where $D = 4 - 2\varepsilon$), due to loop integrals (UV) and collinear momenta.

F_a are physically measurable – renormalize!

- UV poles: coupling constant renormalization
- Collinear poles: mass factorization

We can write:

$$F_a = T_a\left(\varepsilon^{-1}, \varepsilon\right)\hat{f} = C_a\left(\varepsilon\right)Z\left(\varepsilon^{-1}\right)\hat{f} = C_a\left(\varepsilon\right)f.$$

Absorb poles into a redefinition of the non-perturbative parton distribution function: $f = Z \cdot \hat{f}$.

RENORMALIZATION

Z is a function of the (already known) anomalous dimensions γ_{ns} since

$$\frac{\mathrm{d}Z}{\mathrm{d}\ln Q^2} = -\gamma_{ns} Z$$

gives

$$Z = 1 + \alpha_s \frac{1}{\epsilon}\gamma_{ns}^{(0)} + \alpha_s^2 \left[\frac{1}{2\epsilon^2}\left(\gamma_{ns}^{(0)} - \beta_0\right)\gamma_{ns}^{(0)} + \frac{1}{2\epsilon}\gamma_{ns}^{(1)}\right] + \mathcal{O}\left(\alpha_s^3\right).$$

Setting $\gamma_{ns}^{(i)}$ order-by-order removes remaining poles from forward scattering amplitude, fixes coefficient function.

$$T_a\left(\varepsilon^{-1}, \varepsilon\right) = C_a\left(\varepsilon\right) Z\left(\varepsilon^{-1}\right)$$

Reproducing $\gamma_{ns}^{(i)}$ is a strong consistency check of the calculation.

MOMENT CALCULATION

Verify that the new diagram database yields the same moments. Compute with MINCER.

Extend from first 5 to first 14 moments: (non trivial!)

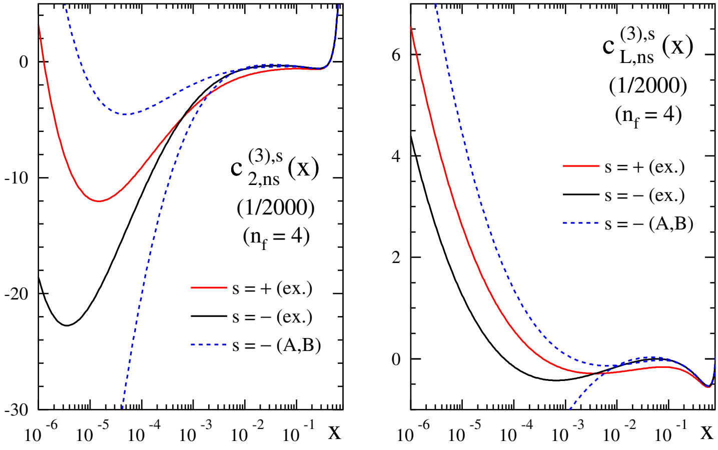
- $N = 3, 5, ..., 29$ of $C^{-}_{2,ns}$, $C^{-}_{L,ns}$.
- $N = 2, 4, ..., 28$ of $C^{-}_{3,ns}$.

Agree with Moch, Rogal's moments ✓

(Possibility to reconstruct analytic result from moments in the style of MVV's recent calculation of polarized anomalous dimension: [hep-ph/1409.5131]
$\rightarrow$ unsuccessful).

Analytic expressions computed with all-N code reproduce these moments ✓

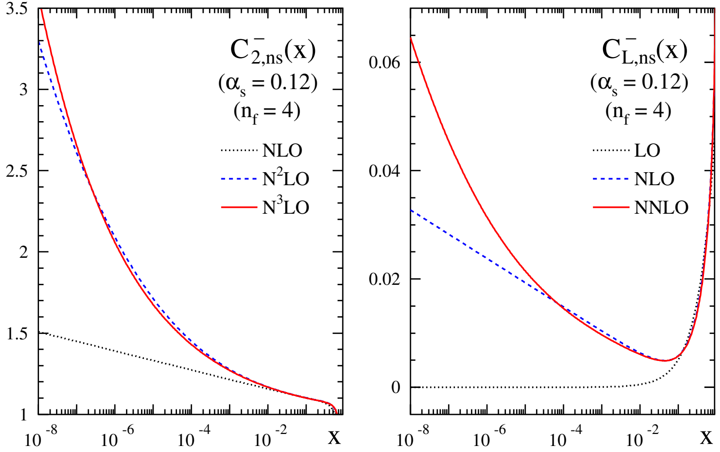
COMPARISON WITH APPROXIMATIONS



$s = +$: $W^+ + W^-$, $\quad s = -$: $W^+ - W^-$.

Approximation error $> 5\%$ below $x \approx 0.1$.

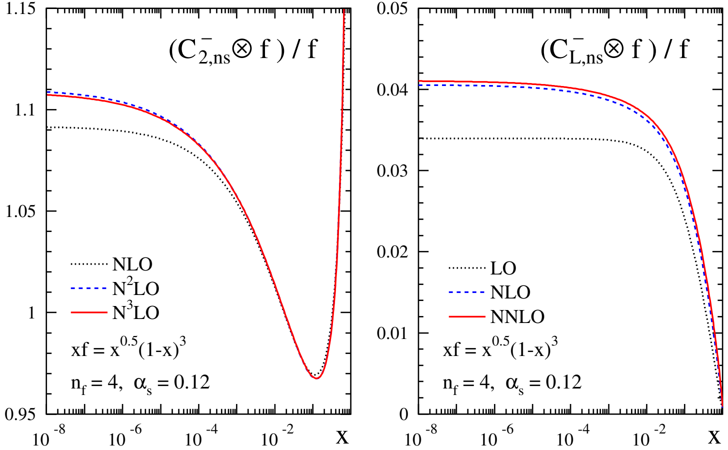
PERTURBATIVE STABILITY



N^{3}LO correction to $C^-_{2,ns}$: $< 3\%$ for $x \in (6 \times 10^{-8}, 0.75)$

N^{2}LO correction to $C^-_{L,ns}$: $< 5\%$ for $x \in (0.007, 0.9)$

Perturbative Stability

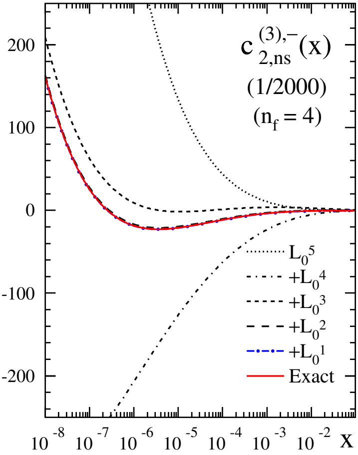


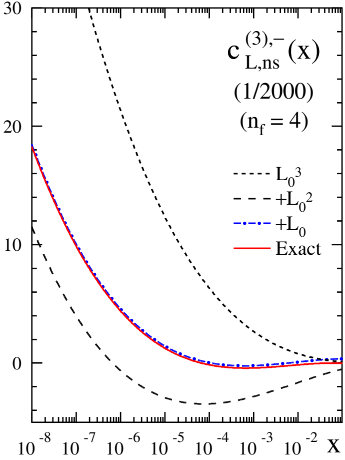
Converge better:

N^{3}LO correction to $C^-_{2,ns}$: $< 1\%$ for $x \in (10^{-8}, 0.8)$

N^{2}LO correction to $C^-_{L,ns}$: $< 3\%$ for $x \in (10^{-8}, 0.12)$

SMALL-x BEHAVIOUR





Error $< 3\%$ for $x < 3 \times 10^{-4}$. Error $< 3\%$ for $x < 9 \times 10^{-10}$.

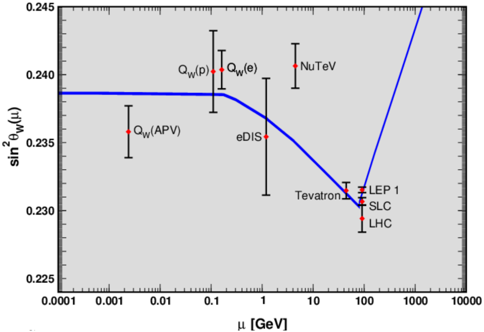
Small-x logarithms ($L_0 = \ln(x)$) are a poor approximation!

Paschos-Wolfenstein Relation

Ratio between NC and CC cross-sections:

$$R^{-} = \frac{\sigma(\nu P \to \nu X) - \sigma(\bar{\nu} P \to \bar{\nu} X)}{\sigma(\nu P \to \mu^{-} X) - \sigma(\bar{\nu} P \to \mu^{+} X)}$$

Determines $\sin^2 \theta_W$. NuTeV anomaly?



[PDG 2015]

PASCHOS-WOLFENSTEIN RELATION

Can now provide exact α_s^3 correction.
($N = 2$ of $\delta C_{2,ns}^{(3)}$, $\delta C_{L,ns}^{(3)}$ where $\delta C_{2,ns}^{(3)} = C_{2,ns}^{(3)W^+ + W^-} - C_{2,ns}^{(3)W^+ - W^-}$)

Exact results correct Moch, Rogal's approximations to $\delta C_{2,ns}$ and $\delta C_{L,ns}$ by just 0.5% and 0.05%.
$\rightarrow$ Don't expect much change in PW relation.

$$R^- = \left(\frac{1}{2} - \sin^2\theta_W\right) + \frac{u^- - d^- + c^- - s^-}{u^- + d^-}\left\{1 - \frac{7}{3}\sin^2\theta_W + \left(\frac{1}{2} - \sin^2\theta_W\right)\frac{8\alpha_s}{9\pi}\left[1 + 1.689\alpha_s + 3.661\alpha_s^2\right]\right\}$$

Including the third-order result:
16% correction of [ ], 1% correction of { } over α_s^2 contribution.

Conclusion

Charged-Current DIS in $F_i^{W^+-W^-}$ combination computed analytically at third order.

Exact expressions differ greatly from previous approximations at small x.

Coefficient functions largely stable in α_s expansion over physically relevant x range.

Paper to come soon:

- Full discussion of all results (inc. polarized $\Delta C_{g1}^{(3)}$)
- 0.1% accurate parametrizations for numerics
- FORM files for analytic results
- Fortran and C++ (GiNaC) files for numerics