



Recent Results from BaBar on Charmed Mesons **DIS**16 DESY11-15th April 2016





Outline

- Physics motivation
- PEP II and the BaBar Detector
- Data sets
- Results:
 - Mixing parameters for the time-dependent amplitude analysis of $D^0 \rightarrow \pi^+ \pi^- \pi^0$. arXiv:1604.00857, submitted to PRD
 - Differential branching fraction, form factor and CKM element extraction from $D^0 \to \pi^- e^+ \nu_{e^*}$

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Physics motivation

- The charm quark is the heaviest up type quark that forms a bound state: D⁰, D[±]_(s).
- Unique laboratories to study nature: e.g.
 - (differential) Branching fractions;
 - Neutral meson mixing, and CP violation;
 - CKM matrix element determination;
 - Form factor measurements.





BaBar & PEP-II

Asymmetric energy e⁺e⁻ collider operating primarily at





B Factories are charm factories

- BaBar recorded 524fb⁻¹ during the operational lifetime.
 - ~500M B pairs (σ =1.2nb for bb production).
 - ~500M D mesons (σ =1.3nb for $c\overline{c}$ production).
- Reconstruct events using a D* tag via:

$$e^+e^- \to c\overline{c} \to D^{*+}X; \ D^{*+} \to D^0\pi_s^+$$



- Charge of slow pion tags the flavour of the D⁰ at point of creation.
- Δm=m(D*)-m(D) is a powerful background suppression variable.
- B background and other D decay background may also need to be suppressed.



- RESULTS: $D^0 \to \pi^+ \pi^- \pi^0$
 - Using 468.1 fb⁻¹ of on- and off-peak data.



 Study mixing in charm using the small ΔM and ΔΓ approximations:



x and y are O(%) level parameters given by:

$$x = rac{\Delta M}{\Gamma}$$
 and $y = rac{\Delta \Gamma}{2\Gamma}$

Assume |q/p|=1 (no CP violation in mixing).



- Measurements of x and y teach us about $D^0 \leftrightarrow \overline{D}^0$ mixing.
- Final state is analysed as a function of the Dalitz plot coordinates $s_{\pm} = m^2 (\pi^{\pm} \pi^0)$:

$$\begin{split} |\mathcal{M}(D^{0})|^{2} \propto \frac{1}{2} e^{-\Gamma_{D}t} \left\{ |A_{f}|^{2} \left[\cosh\left(y\Gamma_{D}t\right) + \cos\left(x\Gamma_{D}t\right) \right] \right\} \\ &+ |\frac{q}{p} \overline{A}_{f}|^{2} \left[\cosh\left(y\Gamma_{D}t\right) - \cos\left(x\Gamma_{D}t\right) \right] \\ &- 2 \left[\operatorname{Re} \left(\frac{q}{p} A_{f}^{*} \overline{A}_{f} \right) \sinh\left(y\Gamma_{D}t\right) \\ &- \operatorname{Im} \left(\frac{q}{p} A_{f}^{*} \overline{A}_{f} \right) \sin\left(x\Gamma_{D}t\right) \right] \right\}, \end{split}$$
(1)

 Dalitz analysis uses Isobar model description as a sum over many intermediate partial waves

$$\overline{A}_f(s_-, s_+) = A_f(s_+, s_-) = \sum_k c_k \mathcal{W}_k(s_+, s_-)$$

Unbinned extended ML fit using GooFit.

R. Andreassen et al., IEEE Access 2, 160 (2014).R. Andreassen et al., J. Phys.: Conf. Ser. 513 052003, 2014.

http://github.com/GooFit/GooFit



$$D^0 \to \pi^+ \pi^- \pi^0$$

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474 fb⁻¹ data

Background types:



- Measurements of x and y teach us about $D^0 \leftrightarrow \overline{D}^0$ mixing.
- Final state is analysed as a function of the Dalitz plot coordinates $s_{\pm} = m^2 (\pi^{\pm} \pi^0)$: MeV

$$\begin{split} |\mathcal{M}(D^{0})|^{2} \propto \frac{1}{2} e^{-\Gamma_{D}t} \left\{ |A_{f}|^{2} \left[\cosh\left(y\Gamma_{D}t\right) + \cos\left(x\Gamma_{D}t\right) \right] \right\}^{2} \\ &+ |\frac{q}{p} \overline{A}_{f}|^{2} \left[\cosh\left(y\Gamma_{D}t\right) - \cos\left(x\Gamma_{D}t\right) \right]^{2} \\ &- 2 \left[\operatorname{Re}\left(\frac{q}{p} A_{f}^{*} \overline{A}_{f}\right) \sinh\left(y\Gamma_{D}t\right) \\ &- \operatorname{Im}\left(\frac{q}{p} A_{f}^{*} \overline{A}_{f}\right) \sin\left(x\Gamma_{D}t\right) \right] \right\}, \end{split}$$
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$D^0 \to \pi^+ \pi^- \pi^0$



		Resonance pa	rameters	Fit to data results					
State	J^{PC}	Mass (MeV)	Width (MeV)	Magnitude	Phase $(^{\circ})$	Fraction f_r (%)			
$ ho(770)^+$	1	775.8	150.3	1	0	$66.4{\pm}0.5$			
$ ho(770)^0$	1	775.8	150.3	$0.55 {\pm} 0.00$	$16.10 {\pm} 0.43$	$23.9{\pm}0.3$			
$ ho(770)^-$	1	775.8	150.3	$0.73 {\pm} 0.01$	$-1.58{\pm}0.51$	$35.6{\pm}0.4$			
$ ho(1450)^+$	1	1465	400	$0.55 {\pm} 0.07$	$-7.69{\pm}8.17$	$1.1{\pm}0.3$			
$ ho(1450)^{0}$	1	1465	400	$0.19{\pm}0.07$	$-70.39{\pm}15.91$	$0.1{\pm}0.1$			
$ ho(1450)^-$	1	1465	400	$0.53 {\pm} 0.06$	$8.15 {\pm} 6.66$	$1.0{\pm}0.2$			
$ ho(1700)^+$	1	1720	250	$0.91 {\pm} 0.15$	$-23.34{\pm}10.27$	$1.5{\pm}0.5$			
$ ho(1700)^{0}$	1	1720	250	$0.60 {\pm} 0.13$	$-56.32{\pm}16.03$	$0.7{\pm}0.3$			
$ ho(1700)^-$	1	1720	250	$0.98{\pm}0.17$	$78.88 {\pm} 8.48$	$1.7{\pm}0.6$			
$f_{0}(980)$	0++	980	44	$0.06 {\pm} 0.00$	$-58.75{\pm}2.89$	$0.3{\pm}0.0$			
$f_0(1370)$	0++	1434	173	$0.20 {\pm} 0.03$	$-19.63 {\pm} 9.45$	$0.3{\pm}0.1$			
$f_0(1500)$	0++	1507	109	$0.18{\pm}0.02$	$7.41{\pm}7.40$	$0.3{\pm}0.1$			
$f_0(1710)$	0++	1714	140	$0.40 {\pm} 0.08$	$42.92 {\pm} 8.84$	$0.3{\pm}0.1$			
$f_2(1270)$	$ 2^{++} $	1275.4	185.1	$0.25 {\pm} 0.01$	$8.84{\pm}2.61$	$0.9{\pm}0.0$			
$f_0(500)$	0++	500	400	$0.26{\pm}0.01$	$-4.12{\pm}3.67$	$0.9{\pm}0.1$			
NR				0.43 ± 0.07	-22.10 ± 11.70	0.4 ± 0.1			

Choice of amplitudes comes from: PRL 99, 251801 (2007).



Adrian Bevan

Results:

 $x = (1.50 \pm 1.17 \pm 0.56)\%$

 $y = (0.19 \pm 0.89 \pm 0.46)\%$

• Fit for τ_D as a crosscheck: good agreement found with PDG:

Measure $\tau_D = 410.2 \pm 3.8$ fs vs PDG value of 410.1±1.5 fs

Statistically limited measurement; systematics are:

Source	$x \ [\%]$	$y \ [\%]$
"Lucky" false slow pion fraction	0.01	0.01
Time resolution dependence on reconstructed D^0 mass	0.03	0.02
Amplitude-model variations	0.31	0.12
Resonance radius	0.02	0.10
DP efficiency parametrization	0.03	0.03
DP normalization granularity	0.03	0.04
Background DP distribution	0.21	0.11
Decay time window	0.18	0.19
$\sigma_t ext{ cutoff}$	0.01	0.01
Number of σ_t ranges	0.11	0.26
σ_t parametrization	0.05	0.03
Background-model MC time distribution parameters	0.06	0.11
Fit bias correction	0.29	0.02
SVT misalignment	0.20	0.23
Total	0.56	0.46

dominant for x dominant for y



• Results: $x = (1.50 \pm 1.17 \pm 0.56)\%$ $y = (0.19 \pm 0.89 \pm 0.46)\%$

(Consistent with WA)

- Proof of principle time-dependent Dalitz plot mixing analysis for charm.
- c.f. World Average as of Charm 2015.





- RESULTS: $D^0 \to \pi^- e^+ \nu_e$
 - Uses 347 fb⁻¹ on-peak and 36.6 fb⁻¹ off-peak data.



- Measurements of:
 - Differential branching fraction;
 - CKM matrix elements: |V_{cd}| and impact on |V_{ub}|;
 - Form factors.
- Semi-leptonic rate is:

$$\frac{d\Gamma}{dq^2 d\cos\theta_e} = \frac{G_F^2}{32\pi^3} \left(|V_{cd}| \times |f_{+,D}^{\pi}(q^2)| \right)^2 p_{\pi}^{*3}(q^2) \sin^2\theta_e$$

 $p_{\pi}^{*}: \pi$ momentum in *D* rest frame $q^{2}: (p_{D}-p_{\pi})^{2}$ θ_{e} : angle between e^{+} in the *ev* rest frame and the π in the D^{0} rest frame. $f_{+,D}^{\pi}(q^{2})$: form factor



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Differential rate provides a baseline measurement to interpret underlying physical properties of these decays.

Also report the product $|V_{cd}| \times f^{\pi}_{+,D}(q^2)$



- Measurements of:
 - Differential branching fraction;
 - CKM matrix elements: |V_{cd}| and impact on |V_{ub}|;
 - Form factors.
- Semi-leptonic rate is:

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Using Lattice QCD we can extract $|V_{cd}|$.



- Measurements of:
 - Differential branching fraction;
 - CKM matrix elements: |V_{cd}| and impact on |V_{ub}|;
 - Form factors.
- Semi-leptonic rate is:

$$\frac{d\Gamma}{dq^2 d\cos\theta_e} = \frac{G_F^2}{32\pi^3} \left(|V_{cd}| \times |f_{+,D}^{\pi}(q^2)| \right)^2 p_{\pi}^{*3}(q^2) \sin^2\theta_e$$

Using CKM Unitarity: $|V_{cd}| = |V_{us}| = \lambda$, we can extract $f^{\pi}_{+,D}(q^2)$. $|V_{us}| = 0.2252 \pm 0.0009$; PDG



- Use Fisher discriminants to suppress backgrounds from $e^+e^- \to B\overline{B}$ and $e^+e^- \to c\overline{c}$.
- Obtain 9926 events with an estimated background of 4623 events.





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• Normalise signal to $D^0 \rightarrow K^-\pi^+$:

$$R_D = \frac{\mathcal{B}(D^0 \to \pi^- e^+ \nu_e)}{\mathcal{B}(D^0 \to K^- \pi^+)}$$
$$R_D = 0.0702 \pm 0.0017 \pm 0.0023$$
$$\mathcal{B}(D^0 \to \pi^- e^+ \nu_e) = (2.770 \pm 0.068 \pm 0.092 \pm 0.037) \times 10^{-3}$$
$$\underset{(\text{stat.})}{\text{Adrian Bevan}}$$



DIS

- Use Fisher discriminants to suppress backgrounds from $e^+e^- \to B\overline{B}$ and $e^+e^- \to c\overline{c}$.
- Obtain 9926 events with an estimated background of 4623 events.



$$\mathcal{B}(D^{0} \to K^{-}\pi^{+}) \qquad \text{DR}^{-}(2.05 \pm 0.000) \times 10^{-10}$$

$$R_{D} = 0.0702 \pm 0.0017 \pm 0.0023$$

$$\mathcal{B}(D^{0} \to \pi^{-}e^{+}\nu_{e}) = (2.770 \pm 0.068 \pm 0.092 \pm 0.037) \times 10^{-3}$$

$$(300) \times 10^{-3}$$



$$D^0 \to \pi^- e^+ \nu_e : |\mathbf{V}_{cd}|$$

Phys. Rev. D**91** 052022 arXiv:1412.5502



■ From Lattice QCD f^T_{+,D}(0)=0.666±0.029.

H. Na, C.T. Davies, E. Follana, J. Koponen, G.P. Lepage, et al., Phys. Rev. D 84, 114505 (2011).

• Hence:

$$|V_{cd}| = 0.206 \pm 0.007_{EXP} \pm 0.009_{LQCD}$$
.



 $D^0 \to \pi^- e^+ \nu_e : f^{\pi}_{+,D}(0)$

Phys. Rev. D91 052022 arXiv:1412.5502



Assume CKM Unitarity: $|V_{cd}| = |V_{us}| = 0.2252 \pm 0.0009$.

Thus:

$$f_{+,D}^{\pi}(0)=0.610\pm0.020\pm0.005.$$



Phys. Rev. D**91** 052022 arXiv:1412.5502

• Unfolded differential decay rate translates into a differential measurement of $|V_{cd}| f^{\pi}_{+,D}(q^2)$:



- Good agreement with z-pole expansion model over all q².
- Good agreement with HFAG.



Application: V_{ub} **extraction**

• Comparison with other V_{ub} determinations:



- BaBar systematics of different origin, expected to be reduced by Lattice calculations:
 - \rightarrow f_B(q²)/f_D (q²) form factor ratio as function of E_{π} (or w)
 - \rightarrow **g**_{H*H π} couplings

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Arantza Oyanguren

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- First time-dependent mixing parameter amplitude analysis in $D^0 \rightarrow \pi^+ \pi^- \pi^0$ that yields $x = (1.50 \pm 1.17 \pm 0.56)\%$ $y = (0.19 \pm 0.89 \pm 0.46)\%$
 - Results consistent with World Average.
 - Used GooFit (fitting package based on CUDA).
 - Proof of principle measurement: Belle II will be able to take this much further with their expected larger data samples.
- Measured the branching ratio of $D^0 \to \pi^- e^+ \nu_e$ and used this to determine

$$|V_{cd}| = 0.206 \pm 0.007_{EXP} \pm 0.009_{LQCD};$$

 $f^{\pi}_{+,D}(0) = 0.610 \pm 0.020_{EXP} \pm 0.005_{EXT}$

Phys. Rev. D**91** 052022, arXiv:1412.5502



- Time evolution of charm decays.
- More on the accelerator.
- Background suppression and differential branching fraction for $D^0 \to \pi^- e^+ \nu_e$.
- Where to find out about the broader physics programme of the B Factories.



Time-evolution of a D⁰ meson

• Reconstruct the signal via $e^+e^- \to c\overline{c} \to D^{*+}X; D^{*+} \to D^0\pi_s^+$



$$|D_{1,2}\rangle = p|D^0\rangle \pm q|\overline{D}^0$$
$$\Delta M = m_1 - m_2$$
$$\Delta \Gamma = \Gamma_1 - \Gamma_2$$
$$\Gamma = (\Gamma_1 + \Gamma_2)/2$$

$$\begin{split} & \Gamma(P^0 \to f) \propto e^{-\Gamma_1 t} \left[\frac{\left(1 + e^{\Delta \Gamma t}\right)}{2} + \frac{Re(\lambda_f)}{1 + |\lambda_f|^2} \left(1 - e^{\Delta \Gamma t}\right) + e^{\Delta \Gamma t/2} \left(\frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2} \cos \Delta M t - \frac{2Im(\lambda_f)}{1 + |\lambda_f|^2} \sin \Delta M t\right) \right] \\ & \Gamma(\overline{P}^0 \to f) \propto e^{-\Gamma_1 t} \left[\frac{\left(1 + e^{\Delta \Gamma t}\right)}{2} + \frac{Re(\lambda_f)}{1 + |\lambda_f|^2} \left(1 - e^{\Delta \Gamma t}\right) + e^{\Delta \Gamma t/2} \left(-\frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2} \cos \Delta M t + \frac{2Im(\lambda_f)}{1 + |\lambda_f|^2} \sin \Delta M t\right) \right] \end{split}$$

• Neglect CP violation (small) and use the fact that $\Delta\Gamma$ and ΔM are small to analyse in terms of x and y.

$$x = \frac{\Delta M}{\Gamma} \text{ and } y = \frac{\Delta \Gamma}{2\Gamma} \qquad \begin{array}{c} \lambda_f = \left|\frac{q}{p}\right| e^{i\phi_{MIX}} \left|\frac{A}{A}\right| e^{i\phi_{CP}} \\ \downarrow \\ 1 \end{array} \qquad \begin{array}{c} 0 \end{array}$$



BaBar & PEP-II

- Data was taken from 1999 through 2008
 - Mostly at the Y(4S);
 - Some "off-peak" just below the Y(4S); at the Y(2S), Y(3S) and a high energy scan.
 - 524fb⁻¹ recorded in total.
 - High operational efficiency throughout data taking.
- Re: charm: large cross section $\sigma(e^+e^- \to c\overline{c})_{10.58\,GeV} = 1.3nb$





PEP-II

Parameters		PEP-II
Beam energy	(GeV)	$9.0\;(e^-),3.1\;(e^+)$
Beam current	(A)	$1.8\;(e^-),2.7\;(e^+)$
Beam size at IP a	(μm)	140
y	(μm)	3
2	(mm)	8.5
Luminosity	$({ m cm^{-2}s^{-1}})$	$1.2 imes10^{34}$
Number of beam bu	nches	1732
Bunch spacing	(m)	1.25
Beam crossing angle	e (mrad)	0 (head-on)

• Y(4S) boost: βγ=0.56.

• Two separate rings brought together at BaBar IP for collisions.









$D^0 \rightarrow \pi^- e^+ \nu_e$ background suppression

1.3nb for ccbar; 0.525nb for B⁺B⁻ and B⁰B⁰bar; 2.09nb for uds continuum events



For B suppression we use:

- R_2 ; the ratio of 2^{nd} to 0^{th} FW moments;
- Total multiplicity of charged and neutral particles detected;
- Momentum of the slow pion: π[±]_s used to reconstruct the D*.

For D suppression we use:

- D momentum;
- mass of spectators in the signal hemisphere (i.e. no signal);
- Angle between momentum sum and thrust;
- |p| for leading spectator;
- Angle between leading spectator and D0 directions;
- Angle between leading spectator and thrust axis;
- Direction of lepton relative to pion in ev frame;
- Charged lepton momentum in CM frame.





 Efficiency varies linearly with q² from 1.6% at low q2 to 1.0% at high q².





Relate D decay to B decay via:

$$\frac{d\mathcal{B}^B}{dw} = \left. \frac{d\mathcal{B}^D}{dw} \right|_{\text{meas.}} \frac{m_B \tau_B}{m_D \tau_D} \left(\frac{|V_{ub}|}{|V_{cd}|} \right)^2 \underbrace{R_{BD}^2}_{\text{R}_{BD}} \\ R_{BD} = 1.8 \pm 0.2 \\ \text{(average of available calculations)}$$



Scaling the differential spectrum from this result using the exclusive V_{ub} result from BaBar shows good agreement between the two.

Required as the same V_{ub} is being used for both parts of the comparison.

This plot uses: $|V_{ub}|^{excl.} = (3.23 \pm 0.31) \times 10^{-3}$

• Fitting the D decay spectrum with a 3-pole model allows us to obtain: $|V_{ub}| = (3.65 \pm 0.18_{exp.} \pm 0.40_{R_{BD}}) \times 10^{-3}$



(2) Impact on V_{ub}

Relate D decay to B decay via residues in the 3-pole anzatz:

$$f_{+,B}^{\pi}(q^2) = Res(f_{+,B}^{\pi})_{B^*} \left(\frac{1}{m_{B^*}^2 - q^2} - \frac{d_2}{m_{B_1^*}^2 - q^2} - \frac{d_3}{m_{B_2^*}^2 - q^2}\right), \text{ with } d_3 = 1 - d_2. \quad (:$$

The ratio of residues is:

$$\frac{\operatorname{Res}(f_{+,B}^{\pi})_{B^*}}{\operatorname{Res}(f_{+,D}^{\pi})_{D^*}} = \frac{m_{B^*}^{3/2} m_B^{1/2}}{m_{D^*}^{3/2} m_D^{1/2}} \frac{f_{B^*}}{f_B} \frac{f_D}{f_{D^*}} \frac{\hat{g}_B}{\hat{g}_D} = 6.0 \pm 0.2 \pm 1.0$$

where:

$$\begin{split} m(B^*) &= 5.941 \text{ GeV} \\ f_{D^*}/f_D &= 1.02 \pm 0.02 \text{ (lattice)} \\ f_{B^*}/f_B &= 1.06 \pm 0.01 \text{ (lattice + data)} \\ \text{effective mass of the third pole is (7.4 \pm 0.4) GeV} \end{split}$$

• Yields:

$$|V_{ub}| = (2.6 \pm 0.2_{\text{exp.}} \pm 0.4_{\text{theory}}) \times 10^{-3}$$



\mathcal{Q} $D^0 \to \pi^- e^+ \nu_e$ differential branching fraction

TABLE VI: Differential branching fractions $(\Delta \mathcal{B}(D^0 \to \pi^- e^+ \nu_e))$ in ten bins in q^2 , spanning from 0 to q^2_{max} in GeV² (second row), with separate statistical and systematic uncertainties and correlation matrices below. The second row lists the values of the differential branching fraction integrated over 0.3 GeV² intervals (quoted in the first row). The off-diagonal elements of the correlation matrices are provided for both the statistical (upper half) and systematic (lower half) uncertainties. The diagonal elements refer to the uncertainties (×10³). The uncertainty on the normalization channel (see Eq. (15)) must be added when evaluating the total uncertainty.

q^2 bin (GeV ²)	[0.0, 0.3]	[0.3, 0.6]	[0.6, 0.9]	[0.9, 1.2]	[1.2, 1.5]	[1.5, 1.8]	[1.8, 2.1]	[2.1, 2.4]	[2.4, 2.7]	$[2.7, q_{\max}^2]$
$\Delta \mathcal{B} \times 10^3$	0.5037	0.4672	0.4551	0.3827	0.3037	0.2664	0.2110	0.1235	0.0477	0.0090
stat.	0.0257	-0.3345	-0.1429	0.0732	0.0121	-0.0097	-0.0024	0.0004	0.0004	0.0003
uncert.		0.0315	-0.1420	-0.2417	0.0401	0.0311	-0.0034	-0.0050	-0.0007	0.0003
and			0.0290	-0.0852	-0.2376	0.0205	0.0368	0.0034	-0.0062	-0.0062
correl.				0.0283	-0.0110	-0.2395	-0.0223	0.0330	0.0119	0.0034
					0.0263	0.0702	-0.2221	-0.0600	0.0281	0.0382
						0.0254	0.2619	-0.1551	-0.1050	-0.0614
							0.0239	0.3904	-0.1211	-0.2012
								0.0200	0.5148	0.2643
									0.0174	0.9233
										0.0057
syst.	0.0133	0.7488	0.7239	0.6568	0.6321	0.3769	0.0735	0.0309	0.1667	0.2194
uncert.		0.0174	0.8281	0.3433	0.6907	0.4597	0.1576	0.1800	0.3585	0.4216
and			0.0136	0.4608	0.6949	0.6524	0.3740	0.3482	0.4333	0.4196
correl.				0.0119	0.7096	0.4462	0.2939	0.2055	0.2310	0.1772
					0.0103	0.7076	0.4513	0.4597	0.6588	0.6371
						0.0120	0.8772	0.8344	0.7076	0.5088
							0.0181	0.9644	0.6184	0.3135
								0.0156	0.7439	0.4539
									0.0087	0.9345
										0.0025

This result:

$$|V_{cd}| \times f^{\pi}_{+,D}(0) = 0.1374 \pm 0.0038 \pm 0.0022 \pm 0.0009$$

TABLE VII: Measurements of the normalization factor $|V_{cd}| \times f^{\pi}_{+,D}(0)$ and of the parameters r_1 and r_2 used in the z-expansion parameterization of the hadronic form factor. The two sets of values for the CLEO-c (2008) untagged analysis correspond to the $\pi^- e^+ \nu_e$ and $\pi^0 e^+ \nu_e$ channels, respectively. Predictions based on four LQCD calculations, obtained using $|V_{cd}| = |V_{us}|$, are listed at the bottom.

Experiment	ref.	$ V_{cd} imes f^{\pi}_{+,D}(0)$	r_1	r_2
Belle (2006)	[6]	$0.140 \pm 0.004 \pm 0.007$		
CLEO-c untagged (2008)	[7]	$0.140 \pm 0.007 \pm 0.003$	-2.1 ± 0.7	-1.2 ± 4.8
CLEO-c untagged (2008)	[7]	$0.138 \pm 0.011 \pm 0.004$	-0.22 ± 1.51	-9.8 ± 9.1
CLEO-c tagged (2009)	[8]	$0.150 \pm 0.004 \pm 0.001$	$-2.35 \pm 0.43 \pm 0.07$	3 ± 3
BESIII (2012) (prel.)	[42]	$0.144 \pm 0.005 \pm 0.002$	$-2.73 \pm 0.48 \pm 0.08$	$4.2\pm3.1\pm0.4$
HFAG average (2012)	[40]	0.146 ± 0.003	-2.69 ± 0.32	4.18 ± 2.16
BESIII (2014)(prel.)	[9]	$0.1420 \pm 0.0024 \pm 0.0010$	$-1.84 \pm 0.22 \pm 0.07$	$-1.4\pm1.5\pm0.5$
This analysis		$0.137 \pm 0.004 \pm 0.002 \pm 0.001$	$-1.31 \pm 0.70 \pm 0.43$	$-4.2\pm4.0\pm1.9$
LQCD Predictions	ref.	$ V_{cd} imes f^{\pi}_{+,D}(0)$	r_1	r_2
FNAL/MILC (2004)	[43]	0.144 ± 0.016		
ETMC (2011)	[44]	0.146 ± 0.020		
HPQCD (2011)	[41]	0.150 ± 0.007		
HPQCD (2013)	[45]	0.153 ± 0.009	-1.93 ± 0.20	0.37 ± 0.93



