

# Study of three-body charmonium decays in *BABAR*

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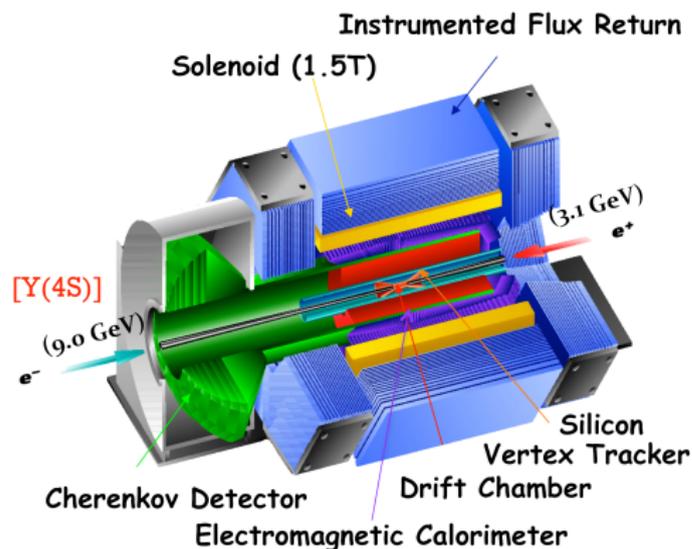
INFN Ferrara  
On behalf of the *BABAR* Collaboration

DIS 2016 - Hamburg  
11- 15 April 2016



- The *BABAR* experiment
- Measurement of the  $I = 1/2$   $K\pi$   $S$ -wave amplitude from Dalitz plot analyses of  $\eta_c \rightarrow K\bar{K}\pi$  in two photon interactions.  
PRD 93, 012005 (2016)
- Dalitz plot analysis of  $J/\psi \rightarrow \pi^+\pi^-\pi^0$  and  $J/\psi \rightarrow K^+K^-\pi^0$  produced in  $e^+e^-$  annihilation via Initial State Radiation.  
Preliminary results
- Summary

# The *BABAR* experiment and data sample



## Features

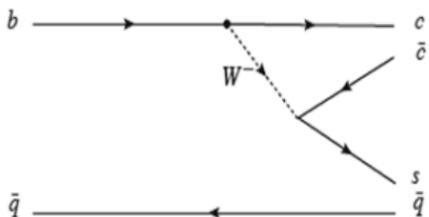
- Data taking: 1999-2008
- Asymmetric beam energies; at the  $\Upsilon(4S)$  ( $\sqrt{s}=10.58$  GeV):  
 $E_{e^-}=9$  GeV,  $E_{e^+}=3.1$  GeV;  
 $\mathcal{L}=431 \text{ fb}^{-1} + 45 \text{ fb}^{-1}$  at 10.54 GeV.  
Primarily for B physics.
- Additional data-taking at the
  - $\Upsilon(3S)$  [ $30 \text{ fb}^{-1}$ ]
  - $\Upsilon(2S)$  [ $14 \text{ fb}^{-1}$ ] $\Rightarrow \sim 520 \text{ fb}^{-1}$  e.g. for two-photon physics, ISR studies
  - Scan above  $\Upsilon(4S)$  [ $3.9 \text{ fb}^{-1}$ ]

*BABAR* is a **B factory**: 467 million  $B\bar{B}$  pairs in the total data sample.

*BABAR* is also a **c factory**: 1.3 million charm events per  $\text{fb}^{-1}$  (676 million charm events with  $520 \text{ fb}^{-1}$ ).

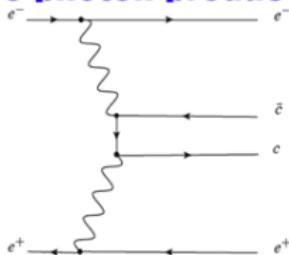
# Experimental methods for charmonium production at the B-factories

## B meson decays



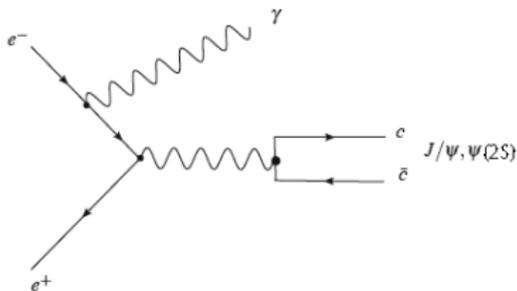
States of any quantum numbers can be produced

## Two-photon production



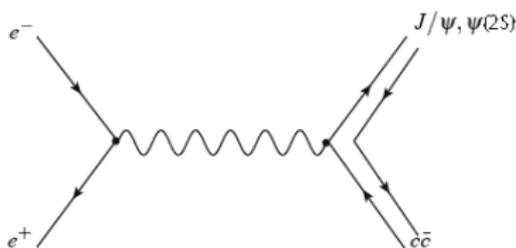
Only states with  $J^{PC} = 0^{\pm+}, 2^{\pm+}, 4^{\pm+}, \dots, 3^{++}, 5^{++}, \dots$  can be produced

## Initial State Radiation (ISR)

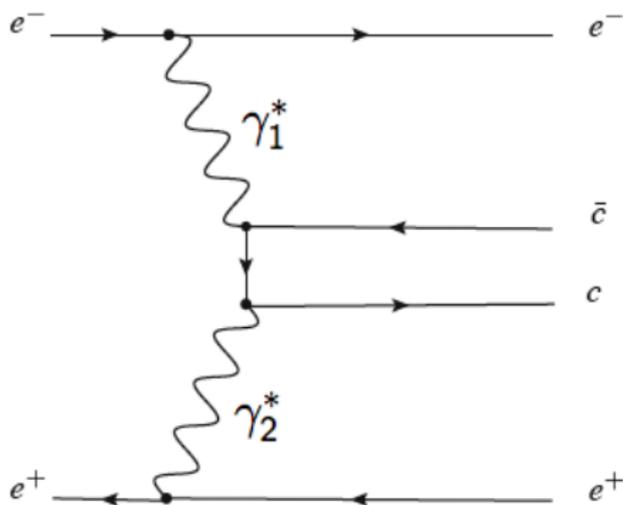


Only states with  $J^{PC} = 1^{--}$  can be produced

## Double charmonium production



Only charmonium states with  $C=+1$  are allowed to be produced in association with the J/psi or the psi(2S)



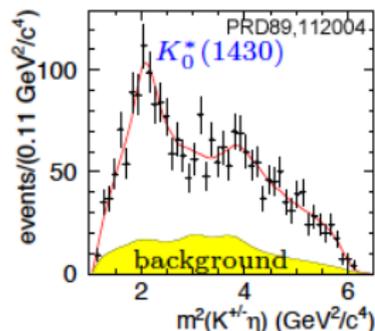
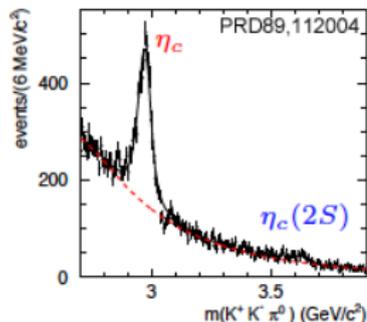
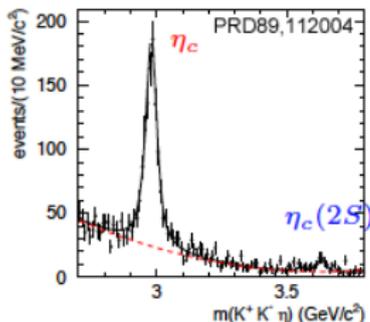
Measurement of the  $l = 1/2$   $K\pi$   $S$ -wave amplitude  
 from Dalitz plot analyses of  $\eta_c \rightarrow K\bar{K}\pi$  in two  
 photon interactions.

**PRD 93, 012005 (2016)**

The use of charge-conjugate reactions is implied throughout

# Introduction

- The *BABAR* Dalitz plot analyses for the decays  $\eta_c \rightarrow K^+ K^- \eta$  and  $\eta_c \rightarrow K^+ K^- \pi^0$  have provided the first clear observations of the  $K_0^*(1430)$  as a Breit-Wigner peak; the observation of the decay mode  $K_0^*(1430) \rightarrow K \eta$  with a  $\text{BF} \sim 5\%$  was a surprise, but is consistent with measurements of  $K\pi$  scattering (Phys. Rev. D 89, 112004 (2014))



- We measured the  $K_0^*(1430)$  branching ratio:

$$\frac{\mathcal{B}(K_0^*(1430) \rightarrow \eta K)}{\mathcal{B}(K_0^*(1430) \rightarrow \pi K)} = 0.092 \pm 0.025^{+0.010}_{-0.025}$$

- We also found that the  $\eta_c$  three-body hadronic decays proceed almost entirely through  $\eta_c \rightarrow \text{pseudoscalar} + \text{scalar}$
- Therefore three-body decays of the  $\eta_c$  provide a unique window through which to study the properties of the scalar mesons

# Selection of $\gamma\gamma \rightarrow K\bar{K}\pi$ - PRD 93, 012005 (2016)

- We study, using  $519 \text{ fb}^{-1}$ , the reactions:

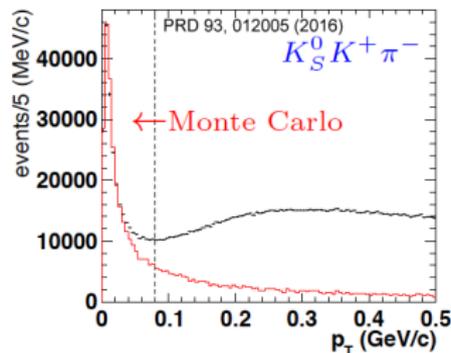
$$\gamma\gamma \rightarrow K_S^0 K^+ \pi^-$$

$$\gamma\gamma \rightarrow K^+ K^- \pi^0$$

Details will be given only for the  $K_S^0 K^+ \pi^-$  final state

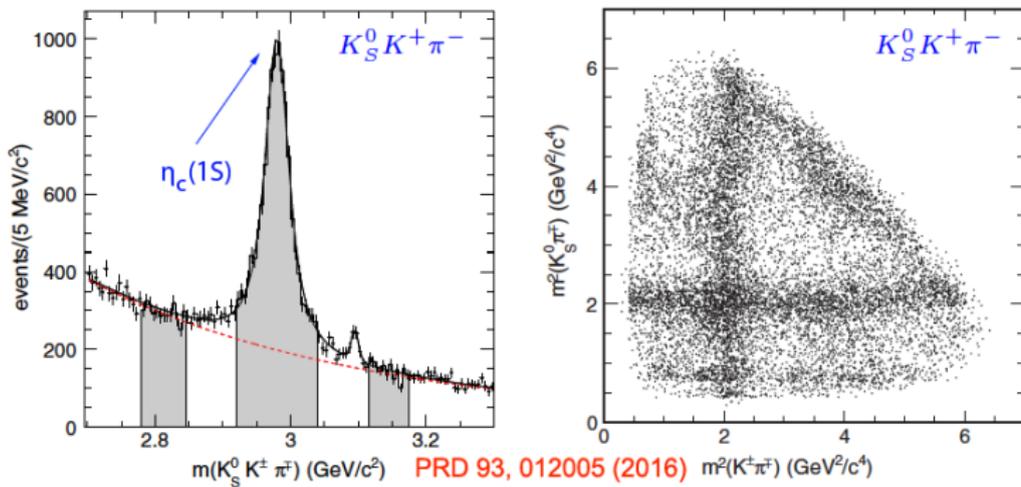
- Select events having only four tracks

- The signal at low  $p_T$  indicates the presence of two-photon events. We require  $p_T < 0.08 \text{ GeV}/c$   
 $p_T$ : transverse momentum of the  $K_S^0 K^+ \pi^-$  system with respect to the  $e^+e^-$  collision axis.



- We remove ISR events by requiring  $M_{rec}^2 > 10 \text{ GeV}^2/c^4$ ;  
We define  $M_{rec}^2 = (p_{e^+e^-} - p_{rec})^2$ ;  
where  $p_{e^+e^-}$  is the four-momentum of the initial state and  $p_{rec}$  is the four-momentum of the  $K_S^0 K^+ \pi^-$  system.

# The $K\bar{K}\pi$ mass spectra in the $\eta_c$ region - PRD 93, 012005 (2016)



- $\eta_c \rightarrow K_S^0 K^+ \pi^-$ , 12849 events with  $(64.3 \pm 0.4)\%$  purity
- $\eta_c \rightarrow K^+ K^- \pi^0$ , 6710 events with  $(55.2 \pm 0.6)\%$  purity
- Small residual  $J/\psi$  signals from ISR
- The Dalitz plots are dominated by the presence of  $K_0^*(1430)$
- $\text{Purity} = N_{sig} / (N_{sig} + N_{back})$

# Dalitz plot analysis of $\eta_c \rightarrow K\bar{K}\pi$ - PRD 93, 012005 (2016)

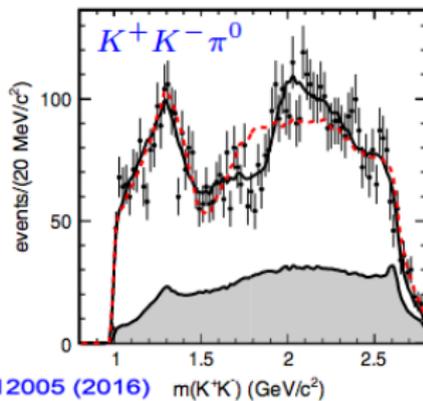
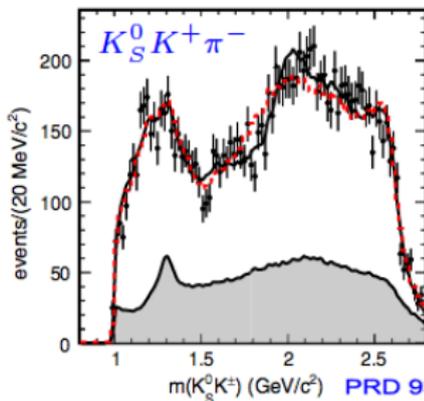
- We perform Dalitz plot analyses of the  $K\bar{K}\pi$  systems in the  $\eta_c$  mass region using unbinned maximum likelihood fits.
- Fits performed using:
  - **Isobar model**: resonances described by Breit-Wigner functions  
D. Asner, Review of Particle Physics; Phys. Lett. B 592, 1 (2004)
  - **Model-Independent Partial Wave Analysis (MIPWA)**  
Phys. Rev. D 73, 032004 (2006);  $D^+ \rightarrow K^-\pi^+\pi^+$  (FNAL E791)  
Phys. Rev. D 79, 032003 (2009);  $D_s^+ \rightarrow \pi^+\pi^-\pi^+$  (BABAR)

## Model-Independent Partial Wave Analysis (MIPWA)

- The  $K\pi$  mass spectrum is divided into 30 equally spaced mass intervals 60 MeV wide, and for each interval we add to the fit two new free parameters, the amplitude and the phase of the  $K\pi$  S-wave (constant inside the bin)
- The interference between the two  $K\pi$  modes is positive for  $\eta_c$  decays (confirmed empirically)
- The  $K_2^*(1430)$ ,  $a_0(980)$ ,  $a_0(1450)$ ,  $a_2(1320)$ , ... contributions are modeled as relativistic Breit-Wigner functions multiplied by the corresponding angular functions.
- Backgrounds are fitted separately and interpolated into the  $\eta_c$  signal region.

- The fits improve when a new high-mass  $a_0(1950) \rightarrow K\bar{K} \text{ } l=1$  resonance is included with free parameters in both  $\eta_c$  decay modes.

Red curve: no  $a_0(1950)$   
 Shaded regions indicate the contributions from the background



Statistical significance for the  $a_0(1950)$  effect (including systematics) are:

$$2.5\sigma \text{ for } \eta_c \rightarrow K_S^0 K^+ \pi^-$$

$$4.2\sigma \text{ for } \eta_c \rightarrow K^+ K^- \pi^0$$

$a_0(1950)$

Final state	Mass (MeV/ $c^2$ )	Width (MeV)
$\eta_c \rightarrow K_S^0 K^\pm \pi^\mp$	$1949 \pm 32 \pm 76$	$265 \pm 36 \pm 110$
$\eta_c \rightarrow K^+ K^- \pi^0$	$1927 \pm 15 \pm 23$	$274 \pm 28 \pm 30$
Weighted mean	$1931 \pm 14 \pm 22$	$271 \pm 22 \pm 29$

Good agreement between the two  $\eta_c$  decay modes

# Fit fractions from the MIPWA. Comparison with the Isobar Model -

PRD 93, 012005 (2016)

Amplitude	$\eta_c \rightarrow K_S^0 K^\pm \pi^\mp$		$\eta_c \rightarrow K^+ K^- \pi^0$	
	Fraction (%)	Phase (rad)	Fraction (%)	Phase (rad)
$(K\pi S\text{-wave}) \bar{K}$	$107.3 \pm 2.6 \pm 17.9$	fixed	$125.5 \pm 2.4 \pm 4.2$	fixed
$a_0(980)\pi$	$0.8 \pm 0.5 \pm 0.8$	$1.08 \pm 0.18 \pm 0.18$	$0.0 \pm 0.1 \pm 1.7$	...
$a_0(1450)\pi$	$0.7 \pm 0.2 \pm 1.4$	$2.63 \pm 0.13 \pm 0.17$	$1.2 \pm 0.4 \pm 0.7$	$2.90 \pm 0.12 \pm 0.25$
$a_0(1950)\pi$	$3.1 \pm 0.4 \pm 1.2$	$-1.04 \pm 0.08 \pm 0.77$	$4.4 \pm 0.8 \pm 0.8$	$-1.45 \pm 0.08 \pm 0.27$
$a_2(1320)\pi$	$0.2 \pm 0.1 \pm 0.1$	$1.85 \pm 0.20 \pm 0.20$	$0.6 \pm 0.2 \pm 0.3$	$1.75 \pm 0.23 \pm 0.42$
$K_2^*(1430)\bar{K}$	$4.7 \pm 0.9 \pm 1.4$	$4.92 \pm 0.05 \pm 0.10$	$3.0 \pm 0.8 \pm 4.4$	$5.07 \pm 0.09 \pm 0.30$
Total	$116.8 \pm 2.8 \pm 18.1$		$134.8 \pm 2.7 \pm 6.4$	
$-2 \log \mathcal{L}$	$-4314.2$		$-2339$	
$\chi^2/N_{\text{cells}}$	$301/254 = 1.17$		$283.2/233 = 1.22$	

## ISOBAR MODEL

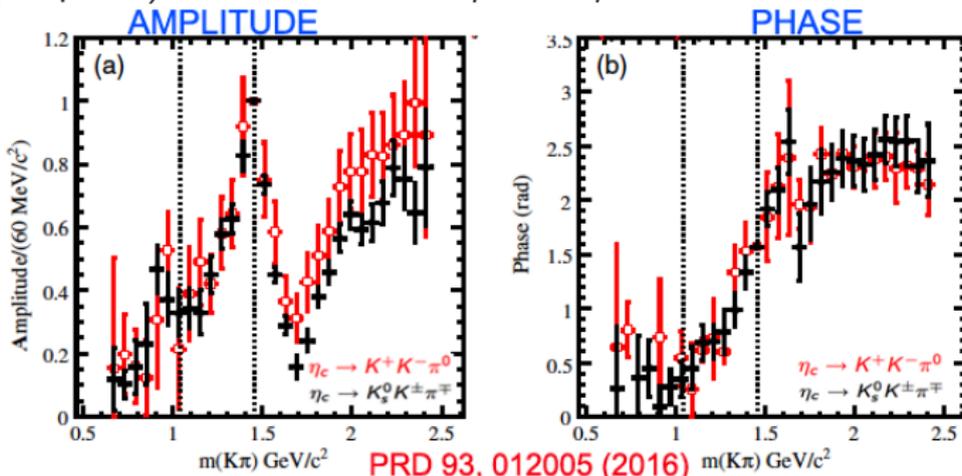
Amplitude	Fraction %	Phase (rad)
$K_0^*(1430)\bar{K}$	$40.8 \pm 2.2$	0.
$K_0^*(1950)\bar{K}$	$14.8 \pm 1.7$	$-1.00 \pm 0.07$
NR	$18.0 \pm 2.5$	$1.94 \pm 0.09$

$$\chi^2/N_{\text{cells}} = 467/256 = 1.82$$

- For the MIPWA, good agreement between the two  $\eta_c$  decay modes
- $(K\pi S\text{-wave}) \bar{K}$  amplitude is dominant with small contributions from  $K_2^*(1430)^0 \bar{K}$  and  $a_0(1950)\pi$  amplitudes.
- Good description of the data with the MIPWA
- Poorer description of the data with the Isobar Model

# The $K - \pi$ S-wave amplitude and phase - PRD 93, 012005 (2016)

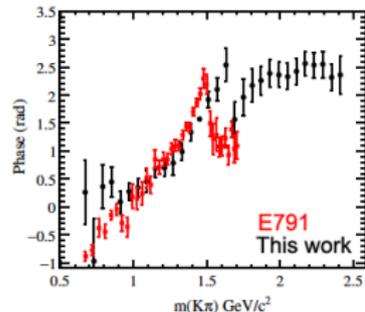
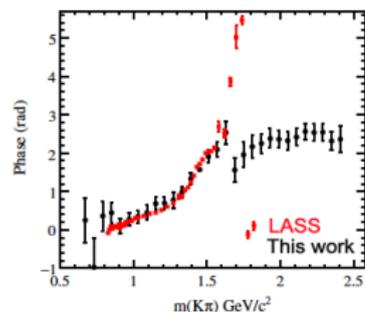
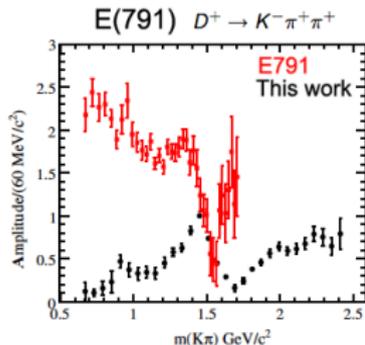
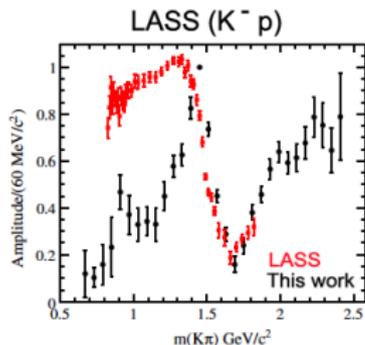
$K\pi$  S-wave amplitude (a) and phase (b) from  $\eta_c \rightarrow K_s^0 K^\pm \pi^\mp$  (black points) and  $\eta_c \rightarrow K^+ K^- \pi^0$  (red points). Dashed lines are  $K\eta$  and  $K\eta'$  thresholds.



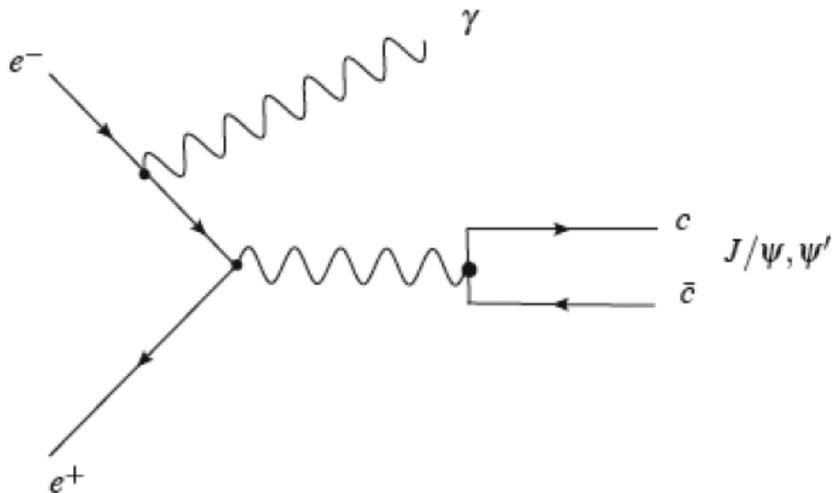
- **Amplitude:** clear peak related to the  $K_0^*(1430)$  resonance which shows a rapid drop around 1.7  $\text{GeV}/c^2$  where a broad structure is present which can be related to the  $K_0^*(1950)$  resonance
- **Phase:** We see the expected behavior for the resonance phase which varies by about  $\pi$  in the  $K_0^*(1430)$  resonance region. The phase shows a drop around 1.7  $\text{GeV}/c^2$  related to interference with the  $K_0^*(1950)$  resonance

Good agreement between the two  $\eta_c$  decay modes

- Black is  $\eta_c \rightarrow K_S^0 K^+ \pi^-$  from the present analysis
- Phase before the  $K\eta'$  threshold are similar, as expected from Watson theorem (PR88,1163(1952))
- Amplitudes are very different



- LASS: Nucl. Phys. B 296, 493 (1988); E791: PRD 73, 032004 (2006)



**Dalitz plot analyses of  $J/\psi \rightarrow \pi^+\pi^-\pi^0$  and  $J/\psi \rightarrow K^+K^-\pi^0$  produced via  $e^+e^-$  annihilation with Initial State Radiation.**  
**Preliminary results**

- Only an unpublished Mark III Dalitz plot analysis exists for the decay  $J/\psi \rightarrow \pi^+\pi^-\pi^0$  (SLAC-PUB-5674, (1991))
- Although large samples of  $J/\psi$  decays exist, some branching fractions remain poorly measured. In particular the  $J/\psi \rightarrow K^+K^-\pi^0$  branching fraction has been measured by Mark II using only 25 events (Phys. Rev. Lett. 51, 963 (1983)).
- The BESIII experiment has performed an angular analysis of  $J/\psi \rightarrow K^+K^-\pi^0$ . The analysis requires the presence of a broad  $J^{PC} = 1^{--}$  state in the  $K^+K^-$  threshold region, which is interpreted as a multi-quark state (Phys. Rev. Lett. 97, 142002 (2006))

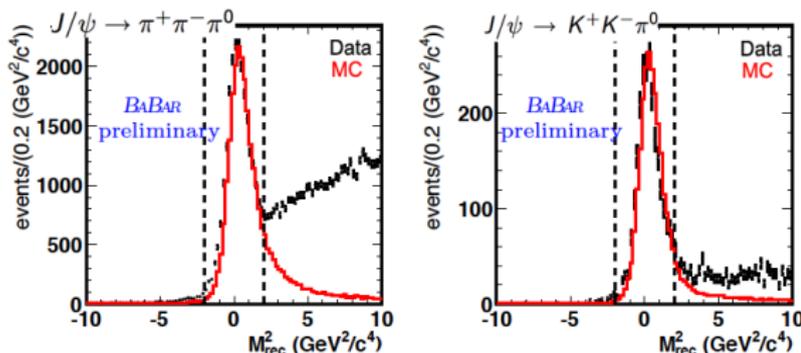
# Data selection

- We study, using  $519 \text{ fb}^{-1}$ , the reactions:

$$e^+e^- \rightarrow \gamma_{ISR} \pi^+\pi^-\pi^0$$
$$e^+e^- \rightarrow \gamma_{ISR} K^+K^-\pi^0$$

where  $\gamma_{ISR}$  is the undetected ISR photon

- We select events having only two tracks and one mass-constrained  $\pi^0$
- We compute  $M_{rec}^2 = (p_{e^-} + p_{e^+} - p_{h^+} - p_{h^-} - p_{\pi^0})^2$  where  $h = \pi/K$
- This quantity should peak near zero for ISR events



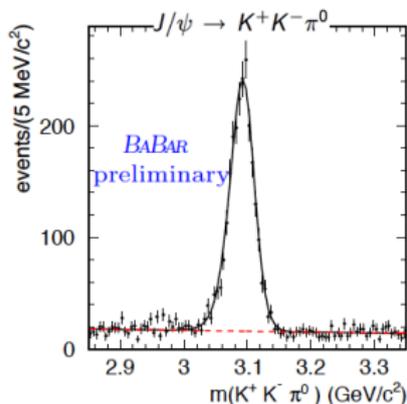
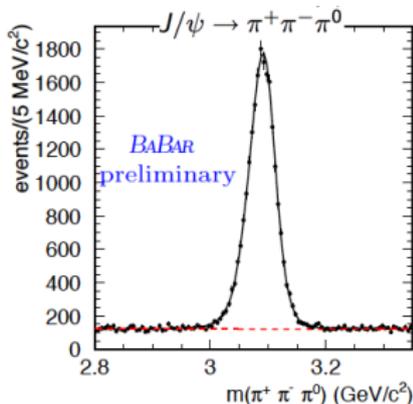
Plot of  $M_{rec}^2$  in the  $J/\psi$  signal region. In red are Monte Carlo simulations.

- We select events in the ISR region by requiring  $|M_{rec}^2| < 2 \text{ GeV}^2/c^4$

# $J/\psi$ signals and yields

We fit the  $J/\psi$  invariant mass distribution using the Monte Carlo resolution functions described by Crystal Ball\* + Gaussian functions

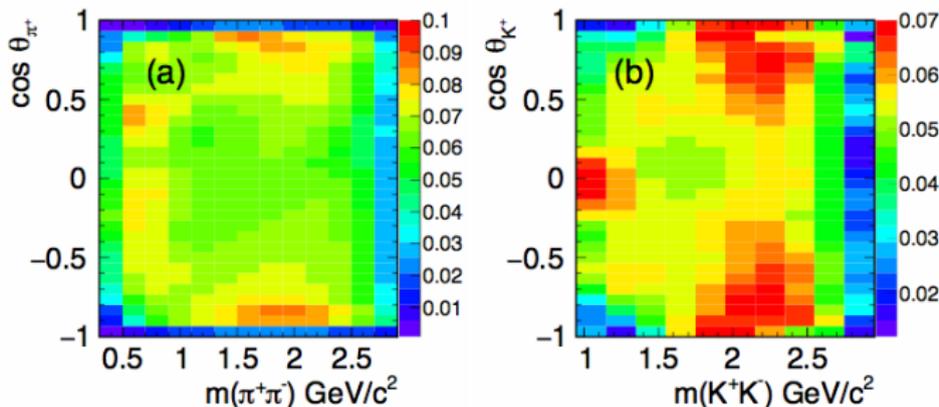
$J/\psi$ decay mode	$\chi^2/NDF$	$J/\psi$ mass (MeV)	Fitted events	weight
$J/\psi \rightarrow \pi^+\pi^-\pi^0$	84/115	$3099.8 \pm 0.2$	$19916 \pm 141$	$15.223 \pm 0.079$
$J/\psi \rightarrow K^+K^-\pi^0$	111/95	$3101.0 \pm 0.2$	$2201 \pm 47$	$18.110 \pm 0.184$



\* M.J.Oreglia, Ph.D Thesis, SLAC-236(1980), Appendix D  
J. E. Gaiser, Ph.D Thesis, SLAC-255 (1982), Appendix E

# Efficiency and Branching fractions

The efficiency is mapped and fitted on the  $(m(h^+h^-), \cos\theta_h)$  plane, where the helicity angle  $\theta_h$  is the angle in the  $h^+h^-$  rest frame between the directions of the  $h^+$  and the boost from the  $J/\psi$  rest frame ( $h=\pi/K$ )



We obtain the weighted efficiencies:

$$\epsilon_{h^+h^-\pi^0} = \frac{\sum_{i=1}^N f_i}{\sum_{i=01}^N f_i / \epsilon(m_i, \cos\theta_i)}$$

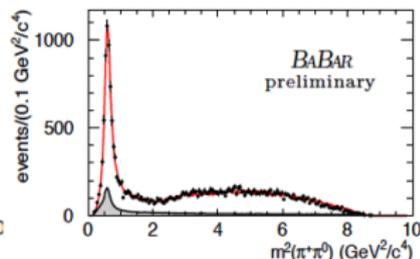
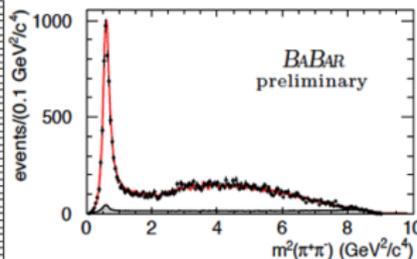
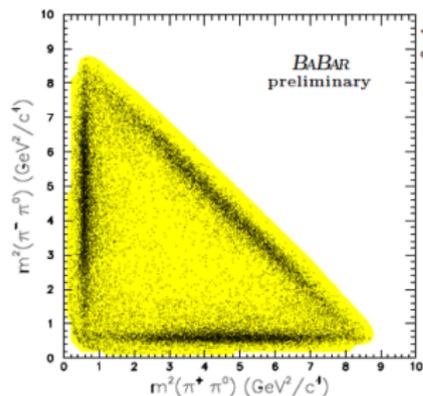
where  $f = 1$  for events in the signal region and  $f = -1$  for events in the sideband regions.

- We obtain:

$$\mathcal{R} = \frac{\mathcal{B}(J/\psi \rightarrow K^+ K^- \pi^0)}{\mathcal{B}(J/\psi \rightarrow \pi^+ \pi^- \pi^0)} = 0.0929 \pm 0.002 \pm 0.002$$

- In PRL 51, 963 (1983), the value  $\mathcal{B}(J/\psi \rightarrow K^+ K^- \pi^0) = (2.8 \pm 0.8) \times 10^{-3}$  is reported on the basis of 25 events.  
In addition, the value  $\mathcal{B}(J/\psi \rightarrow \pi^+ \pi^- \pi^0) = (2.11 \pm 0.07) \times 10^{-2}$  is reported in PDG 2014.  
These values then yield  $\mathcal{R} = 0.133 \pm 0.038$ , which differs by only  $\sim 1\sigma$  from our result.
- We note also that in PRD 15, 1814 (1977) the value  $\mathcal{B}(J/\psi \rightarrow K \bar{K} \pi) = (7.8 \pm 2.1) \times 10^{-3}$  is reported on the basis of a sample of events for the final states  $K_S K^\pm \pi^\mp$ . From isospin invariance, this would yield the value  $\mathcal{B}(J/\psi \rightarrow K^+ K^- \pi^0) = (1.30 \pm 0.35) \times 10^{-3}$  and the corresponding ratio-value  $\mathcal{R} = 0.062 \pm 0.017$ .
- The weighted average of these two measurements is  $\mathcal{R} = 0.074 \pm 0.016$ , which again differs only by  $\sim 1\sigma$  from our measurement.
- Our value is more precise than this average by a factor of  $\sim 7$ .

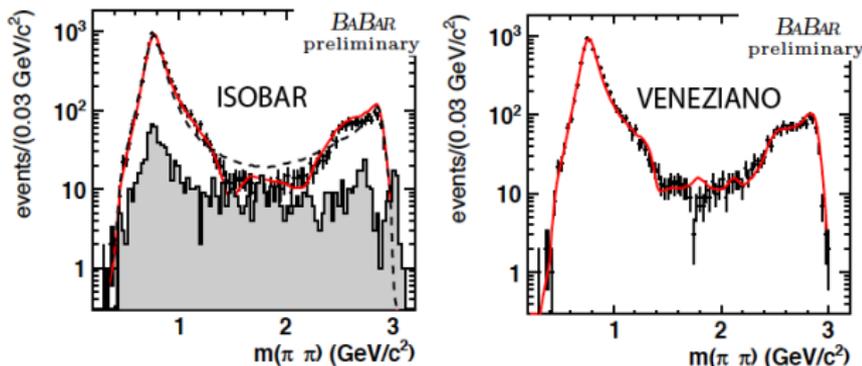
# $J/\psi \rightarrow \pi^+ \pi^- \pi^0$ : Dalitz plot and projections



Shaded regions indicate the contributions from the background

- Dominated by  $\rho(770)\pi$  contributions
- Dalitz plot analyses performed using:
  - **Isobar model** using Zemach tensors [C.Zemach, Phys. Rev. 133, B1201 (1964)]
  - **Veneziano model** [A.P.Szczepaniak, M.R. Pennington, Phys. Lett. B 737, 283 (2014)]: the dynamical assumptions behind this model are the resonance dominance of the low-energy spectrum and resonance-Regge duality. The complexity of the model is related to  $n$ , the number of Regge trajectories included in the fit (the fit requires  $n=5$ ).

# $J/\psi \rightarrow \pi^+\pi^-\pi^0$ : Dalitz plot analysis



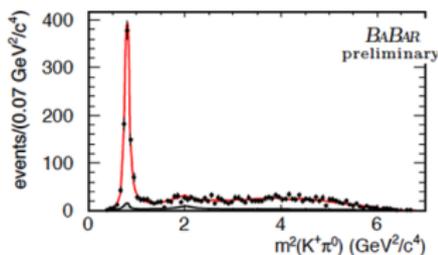
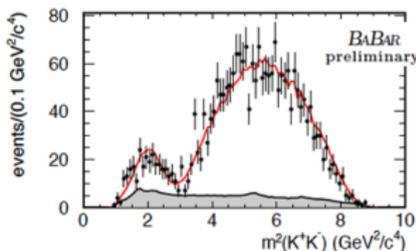
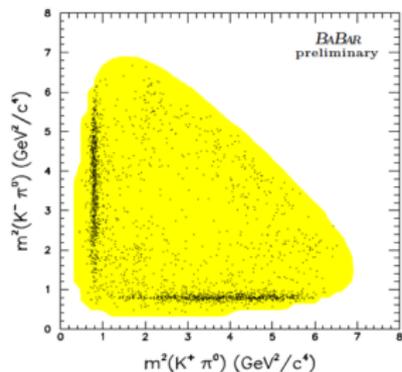
Shaded region indicates the contribution from the background

Isobar model: the dashed curve is the result of the fit with only the  $\rho(770)$  resonance.

Final state	Isobar fraction %			Phase (radians)	Veneziano fraction %
$\rho(770)\pi$	$119.0 \pm 1.1$	$\pm 3.3$		0.	$120.0 \pm 1.9$
$\rho(1450)\pi$	$16.9 \pm 2.0$	$\pm 3.1$		$3.92 \pm 0.05 \pm 0.11$	$1.53 \pm 0.13$
$\rho(1700)\pi$	$0.1 \pm 0.1$	$\pm 0.2$		$1.01 \pm 0.35 \pm 0.79$	$0.84 \pm 0.08$
$\rho(2150)\pi$	$0.04 \pm 0.05$	$\pm 0.02$		$1.89 \pm 0.30 \pm 0.48$	$2.03 \pm 0.17$
$\rho_3(1690)\pi$					$0.09 \pm 0.02$
Sum	$136.0 \pm 2.3$	$\pm 4.3$			$124.5 \pm 2.3$
$\chi^2/\nu$		764/552			780/554

The two models give similar data representation, but different fractions.

# $J/\psi \rightarrow K^+ K^- \pi^0$ : Dalitz plot analysis



Shaded regions indicate the contribution from the background

- The decay is dominated by  $K^*(892)\bar{K}$  amplitudes
- We make use of the Isobar model only
- Broad enhancement in the  $K^+ K^-$  mass spectrum attributed to the presence of the  $\rho(1450)$  resonance:  
 $m(\rho(1450))=1361\pm 43$  MeV/ $c^2$   
 $\Gamma(\rho(1450))=479\pm 63$  MeV

Final state	fraction %	phase
$K^*(892)K$	$87.8 \pm 2.0 \pm 1.7$	0.
$\rho(1450)^0 \pi^0$	$11.5 \pm 2.1 \pm 2.1$	$-2.81 \pm 0.25 \pm 0.36$
$K^*(1410)K$	$1.7 \pm 0.7 \pm 1.1$	$2.89 \pm 0.35 \pm 0.08$
$K_2^*(1430)K$	$3.8 \pm 1.4 \pm 0.5$	$-2.42 \pm 0.22 \pm 0.07$
$\rho(1700)^0 \pi^0$	$0.9 \pm 1.0 \pm 0.6$	$1.06 \pm 0.20 \pm 0.7$
Total	$105.6 \pm 3.4 \pm 3.0$	
$\chi^2/\nu = 94/92$		

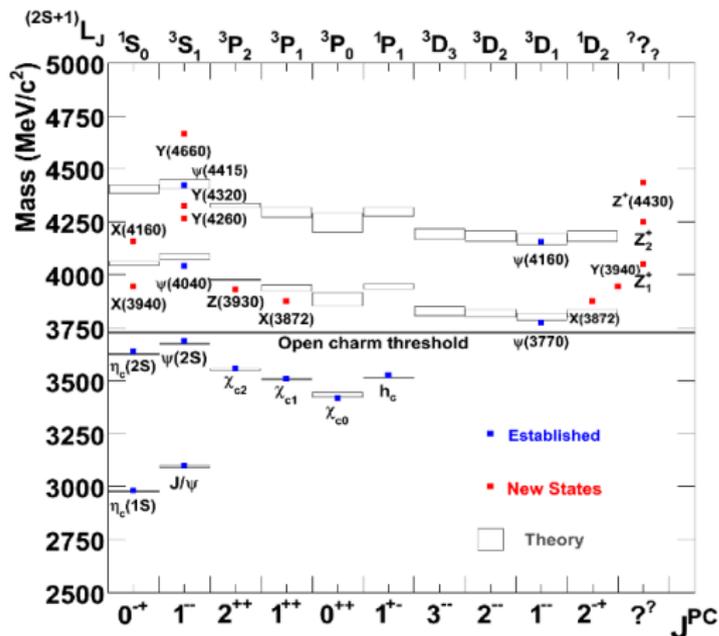
- We perform a measurement of the  $\rho(1450)$  relative branching fraction using the isobar model only
- We have measured the ratio:  
$$\mathcal{R} = \mathcal{B}(J/\psi \rightarrow K^+K^-\pi^0)/\mathcal{B}(J/\psi \rightarrow \pi^+\pi^-\pi^0) = 0.0929 \pm 0.002 \pm 0.002$$
- From the Dalitz plot analysis of  $J/\psi \rightarrow \pi^+\pi^-\pi^0$  we obtain:  
$$\mathcal{B}_1 = \mathcal{B}(J/\psi \rightarrow \rho(1450)^0\pi^0) = [(16.9 \pm 2.0 \pm 3.1)/3]\% = (5.63 \pm 0.67 \pm 1.03)\%$$
- From the Dalitz plot analysis of  $J/\psi \rightarrow K^+K^-\pi^0$  we obtain:  
$$\mathcal{B}_2 = \mathcal{B}(J/\psi \rightarrow \rho(1450)^0\pi^0) = (11.5 \pm 2.1 \pm 2.1)\%$$
- We therefore obtain: 
$$\frac{\mathcal{B}(\rho(1450)^0 \rightarrow K^+K^-)}{\mathcal{B}(\rho(1450)^0 \rightarrow \pi^+\pi^-)} = \frac{\mathcal{B}_2}{\mathcal{B}_1} \cdot \mathcal{R} = 0.190 \pm 0.042 \pm 0.049$$

- We show the results on the Dalitz plot analyses of  $\eta_c \rightarrow K_s^0 K^+ \pi^-$  and  $\eta_c \rightarrow K^+ K^- \pi^0$  produced in two-photon interactions ([PRD 93, 012005 \(2016\)](#))
  - We extract for the first time the  $K\pi$  S-wave amplitude and phase using a MIPWA.
- We show preliminary results on Dalitz plot analyses of Initial State Radiation data on the decays  $J/\psi \rightarrow \pi^+ \pi^- \pi^0$  and  $J/\psi \rightarrow K^+ K^- \pi^0$ ; the analyses make use of Isobar and Veneziano models.
  - $\mathcal{R} = \frac{\mathcal{B}(J/\psi \rightarrow K^+ K^- \pi^0)}{\mathcal{B}(J/\psi \rightarrow \pi^+ \pi^- \pi^0)} = 0.0929 \pm 0.002 \pm 0.002$   
This value represents an improvement in precision by a factor of  $\sim 7$  over the previous measurements.

**THANKS FOR YOUR ATTENTION!**

# BACKUP SLIDES

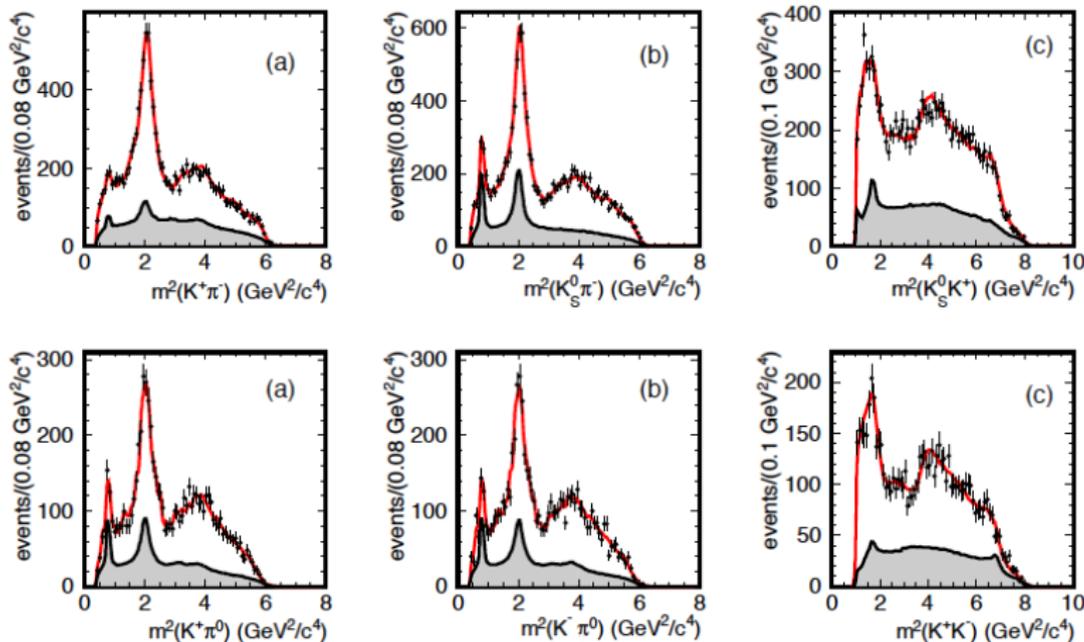
# Charmonium spectrum



- Below the  $D\bar{D}$  threshold, all expected states have been observed, with properties in good agreement with theory; there are no additional states.
- Many unexpected states have been reported above the  $D\bar{D}$  threshold, seemingly too many with  $J^{PC} = 1^{--}$ . Several exotic hypotheses as to their nature: tetraquarks, hadronic molecules, hybrids, glueballs, hadro-quarkonia.
- These result mainly from Belle and *BABAR*, with significant contributions also from CDF, D0, CLEO, LHCb and BES.

# Dalitz plot mass projections

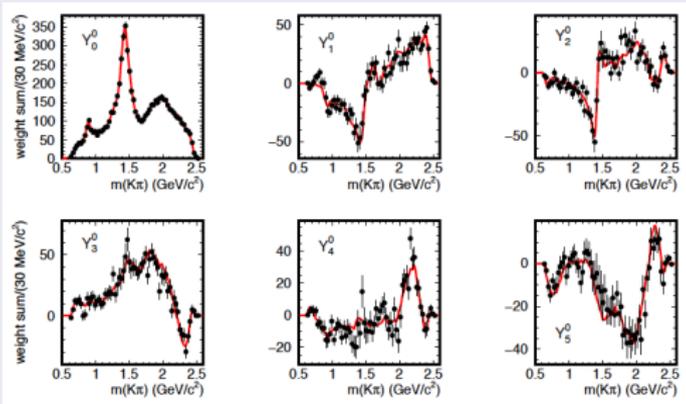
Dalitz plot projections with fit results for  $\eta_c \rightarrow K_S^0 K^+ \pi^-$  (top) and  $\eta_c \rightarrow K^+ K^- \pi^0$  (bottom)



Shaded is contribution from the interpolated background  
 $K^*(890)$  contributions entirely from background

# Legendre polynomial moments: $\eta_c \rightarrow K_S^0 K^+ \pi^-$

Mass projections weighted by  $Y_L^0$  moments and compared with fit results.  
 $m(K^+ \pi^-) + m(K_S^0 \pi^-)$  projections



$m(K_S^0 K^+)$  projections

Good agreement in all the projections  
 Similarly for  $\eta_c \rightarrow K^+ K^- \pi^0$

