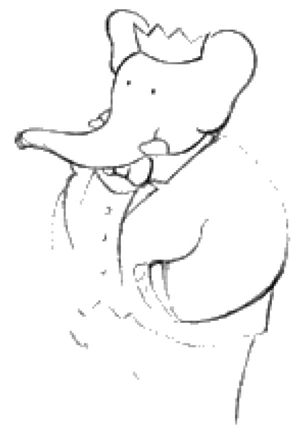
Low-energy hadronic cross sections measurements at BABAR, and implication for the $g-2$ of the muon



Denis Bernard

Laboratoire Leprince-Ringuet, Ecole polytechnique & IN2P3/CNRS, Palaiseau, France

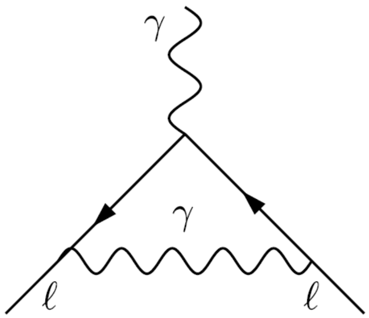
On behalf of the *BABAR* Collaboration

DIS2016,
24th workshop on Deep-Inelastic Scattering and Related Subjects.
11-15 April 2016, DESY Hamburg

The “Anomalous” Magnetic Moment of the Lepton

- Gyromagnetic factor g $\qquad \vec{\mu} = g\frac{e}{2m}\vec{s}, \qquad a = (g-2)/2$
  - (1928) Pointlike Dirac particles: $g = 2$, $a = 0$.

    $g \neq 2$ due to higher order contributions:



- (1947) Nafe measure $\qquad a_e = (2.6 \pm 0.5) \times 10^{-3}$
- (1948) Schwinger (1st order in α) $\qquad a^{(1)} = \alpha/2\pi \approx 1.2 \times 10^{-3}$

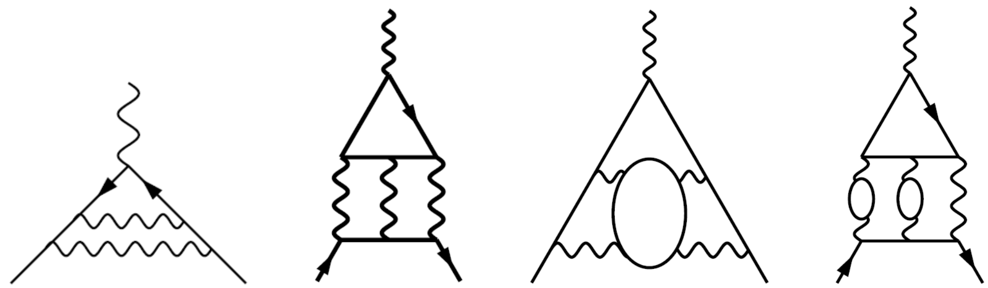
- With Tomonaga's completion of renormalized calculation of Lamb shift,

  The first basis of our belief in QED and in the gauge-theory-based SM

Higher Orders

One graph of each order given as example out of many:

2^{nd} 3^{rd} 4^{th} 5^{th}

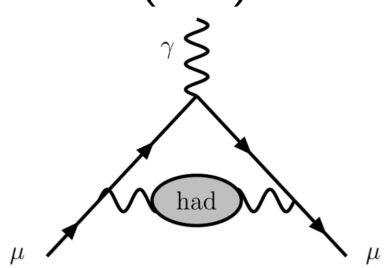


Full QED 5^{th}-order (in α) calculation

Aoyama Phys. Rev. Lett. 109 (2012) 111808

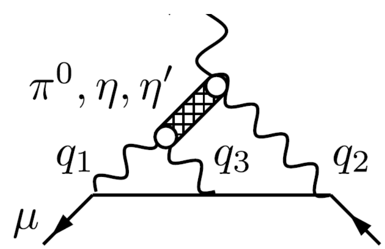
$$a = a^{\text{QED}} + a^{\text{had}} + a^{\text{weak}}$$

Hadronic Vacuum Polarisation (VP)

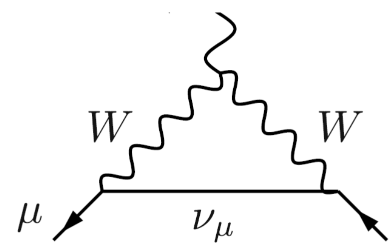


This talk

Hadronic light-by-light Scattering



Weak Interactions



a_e, α *and* a_μ

- Heavy-to-Light and Light-to-Heavy mass ratios take part differently (e/μ) in the loops (QED, QCD, weak)

$$a_e = \frac{\alpha}{2\pi} - 0.3\left(\frac{\alpha}{\pi}\right)^2 + 1.2\left(\frac{\alpha}{\pi}\right)^3 - 1.9\left(\frac{\alpha}{\pi}\right)^4 + 9.2\left(\frac{\alpha}{\pi}\right)^5 + 1.7\ 10^{-12}(\mathrm{QCD} + \mathrm{weak})$$

$$a_\mu = \frac{\alpha}{2\pi} + 0.8\left(\frac{\alpha}{\pi}\right)^2 + 24.\left(\frac{\alpha}{\pi}\right)^3 + 131.\left(\frac{\alpha}{\pi}\right)^4 + 753.\left(\frac{\alpha}{\pi}\right)^5 + 7.1\ 10^{-8}(\mathrm{QCD} + \mathrm{weak})$$

(Numbers truncated on this slide)

Corrected after the talk : Andrei Kataev wrote : I [..] found out that in the expression for a_{mu} in the $O(\alpha^4)$ term, -131 should be changed to $+131$

α from	a_μ^{QED} (10^{-10})
a_e	11 658 471.885 $\pm$ 0.004
Rubidium Rydberg constant	11 658 471.895 $\pm$ 0.008

PDG Aug. 2014.

Aoyama Phys. Rev. Lett. 109 (2012) 111808

Theoretical prediction for a_μ

- SM-to-experiment comparison [units 10^{-10}]

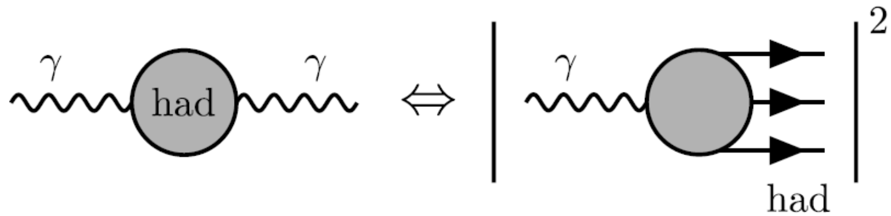
QED	11 658 471.895	$\pm$ 0.008
Leading hadronic vacuum polarization (VP)	692.3	$\pm$ 4.2
Sub-leading hadronic vacuum polarization	-9.8	$\pm$ 0.1
Hadronic light-by-light	10.5	$\pm$ 2.6
Weak (incl. 2-loops)	15.4	$\pm$ 0.1
Theory	11 659 180.3	$\pm$ 4.2 $\pm$ 2.6
Experiment [E821 @ BNL]	11 659 209.1	$\pm$ 5.4 $\pm$ 3.3
Exp. $-$ theory	$+$28.8	$\pm$ 8.0

- Exp. updated from E821 @ BNL, Bennett Phys. Rev.D73 (2006) 072003 to the newest value of the μ/p magnetic ratio Rev. Mod. Phys. 84 (2012) 1527.
- Assuming Gaussian statistics, a 3.6 σ discrepancy.

PDG Aug. 2014

uses e^+e^- input only for VP

Theoretical prediction: The Hadronic VP (1)



- Quark loops not computable from first principles – QCD.
- Vacuum polarization: energy dependent running charge:

$$e^2 \to e^2/[1 + (\Pi'(k^2) - \Pi'(0))]$$

- Dispersion relation from analyticity

$$\Pi'(k^2) - \Pi'(0) = \frac{k^2}{\pi} \int_0^\infty \frac{Im\Pi'(s)}{s(s - k^2 - i\epsilon)} \mathrm{d}s$$

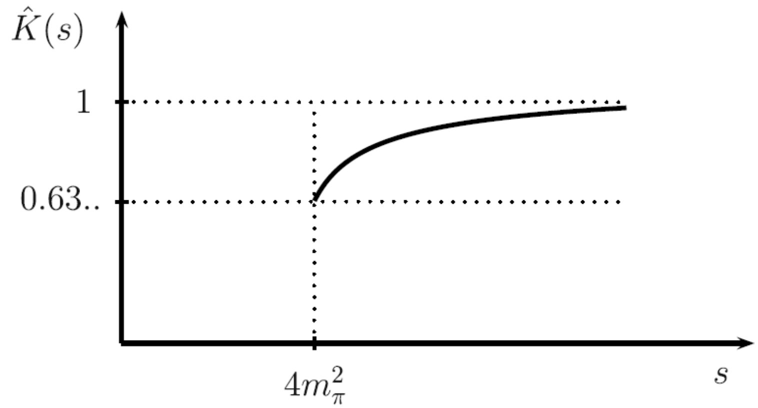
- Optical theorem (unitarity): $$Im\Pi'(s) = \frac{\alpha(s) R_{\text{had}}(s)}{3},$$

$$R_{\text{had}}(s) = \frac{\sigma_{e^+e^- \to hadrons}}{\sigma_0},$$ σ_0 pointlike muon-pair cross section.

Theoretical prediction: The Hadronic VP (2)

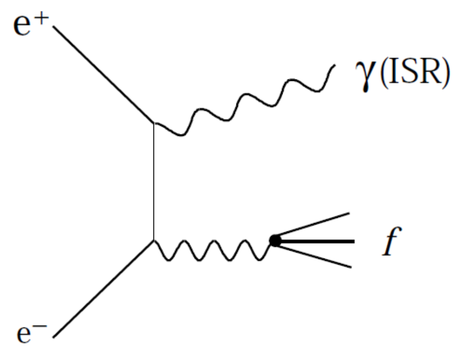
- Wrapping it up, the "dispersion integral":

$$a_\mu^{\text{had}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int \frac{R_{\text{had}}(s) \times \hat{K}(s)}{s^2} \mathrm{d}s$$



- Technically, $\int_{4m_\pi^2}^{E_{cut}^2}$ is obtained from the data, $\int_{E_{cut}^2}^{\infty}$ from pQCD.
- The estimation of the contribution with the largest uncertainty to a_μ(theory) boils down to a precise measurement of $R_{\text{had}}(s)$
- Most precision on $R_{\text{had}}(s)$ needed at low $\sqrt{s}$ (currently $E_{cut} = 1.8\,\text{GeV}$)

Initial State Radiation (ISR)



- Optimal use of the available luminosity
- Covers whole energy range with same detector condition and analysis.
- Asymmetric e^+e^- collider $\Rightarrow$ good efficiency down to threshold
- If the whole final state (γ + hadrons) is observed

  $\Rightarrow$ over-constrained kinematical fit $\Rightarrow$ powerful background noise rejection.

$$\frac{\mathrm{d}\sigma_{[e^+e^-\to f\gamma]}}{\mathrm{d}s'}(s') = \frac{2m}{s} W(s,x)\sigma_{[e^+e^-\to f]}(s') \; , \qquad x = \frac{E_\gamma}{\sqrt{s}} = 1 - \frac{s'}{s}$$

- $W(s,x)$ "radiator function", density of probability to radiate a photon with energy $E_\gamma = x\sqrt{s}$: a known function Binner, Physics Letters B 459 (1999) 279

$e^+e^- \to \pi^+\pi^-(\gamma)\gamma$

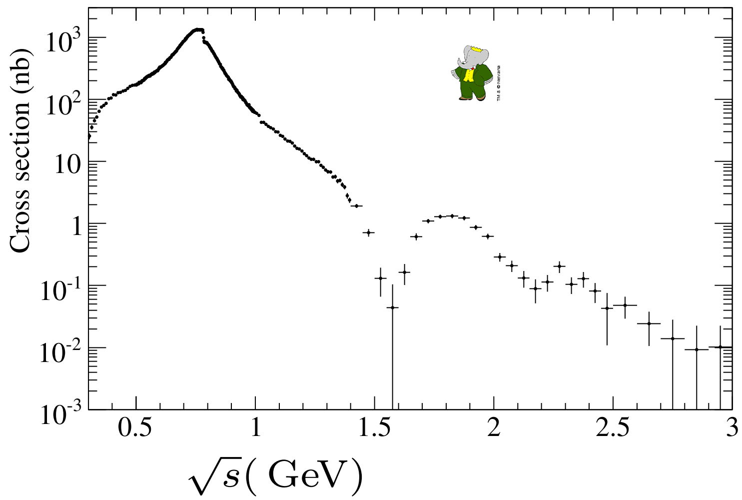
- Systematics mastered at the 10^{-3} level for the first time, to my knowledge, in BaBar
- ISR γ in EMC (thus: at large angle)
- Good quality tracks, particle identification (PID)
- Kinematic fit (using only direction of ISR γ)
  - possibly including 1 additional γ: NLO !
- All efficiencies (trigger, filter, tracking, PID, fit) from data: $\pi\pi/\mu\mu$ cross section ratio:

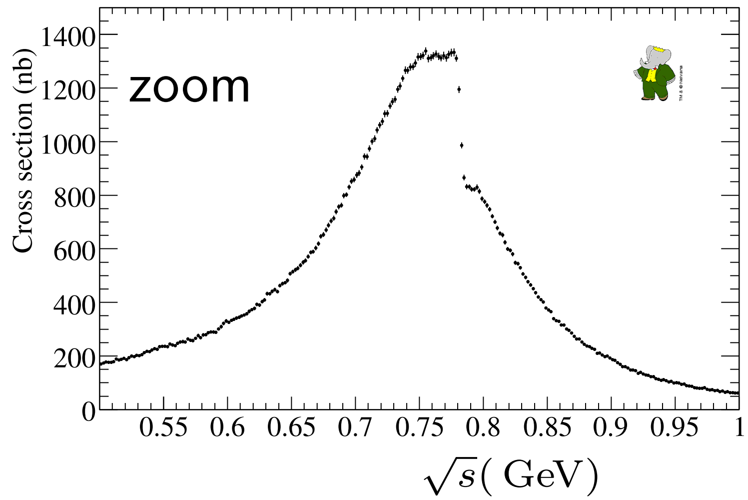
  $\Rightarrow$ Cancellation of ee luminosity, additional ISR, VP, ISR γ efficiency

- Correct for lowest order FSR in $\mu\mu$ and for ISR + additional FSR, both calc. in QED, and checked in data

$$R_{\rm exp}(s') = \frac{\sigma_{[\pi\pi\gamma(\gamma)]}(s')}{\sigma_{[\mu\mu\gamma(\gamma)]}(s')} = \frac{\sigma^0_{[\pi\pi(\gamma)]}(s')}{(1+\delta^{\mu\mu}_{\rm FSR})\sigma^0_{[\mu\mu(\gamma)]}(s')} = \frac{R(s')}{(1+\delta^{\mu\mu}_{\rm FSR})(1+\delta^{\mu\mu}_{add,FSR})}$$

$e^+e^- \to \pi^+\pi^-(\gamma)$ *Cross Section*





Bare (incl. additional FSR, VP removed), unfolded $\sigma_{e^+e^- \to \pi^+\pi^-}$ $232\,\text{fb}^{-1}$ @ $\sqrt{s} \approx 10.6\,\text{GeV}$

- Excellent precision down to threshold !

$$a_\mu^{\pi^+\pi^-}[2m_\pi, 1.8\,\text{GeV}] = (514.1 \pm 2.2 \pm 3.1) \times 10^{-10}$$

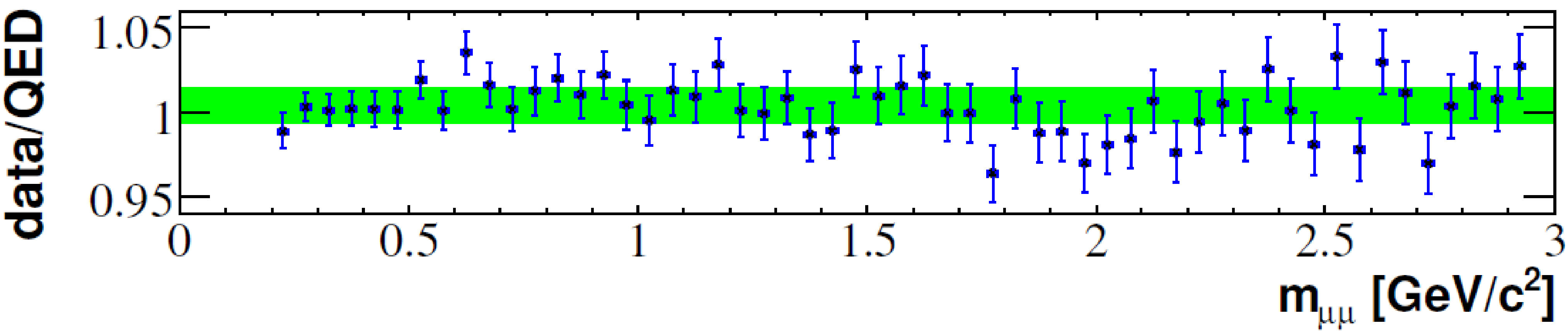
- Similar precision as combination of previous e^+e^- results.

$$a_\mu^{\pi^+\pi^-}[2m_\pi, 1.8\,\text{GeV}] = (503.5 \pm 4.5) \times 10^{-10}$$

- 1.7 σ larger than previous e^+e^- average, $\Delta = +(10.6 \pm 5.9) \times 10^{-10}$

Phys. Rev. Lett. 103 (2009) 231801, Phys. Rev. D86 (2012) 032013

BaBar: Sanity check: Comparison of the $\mu\mu$ spectrum with QED

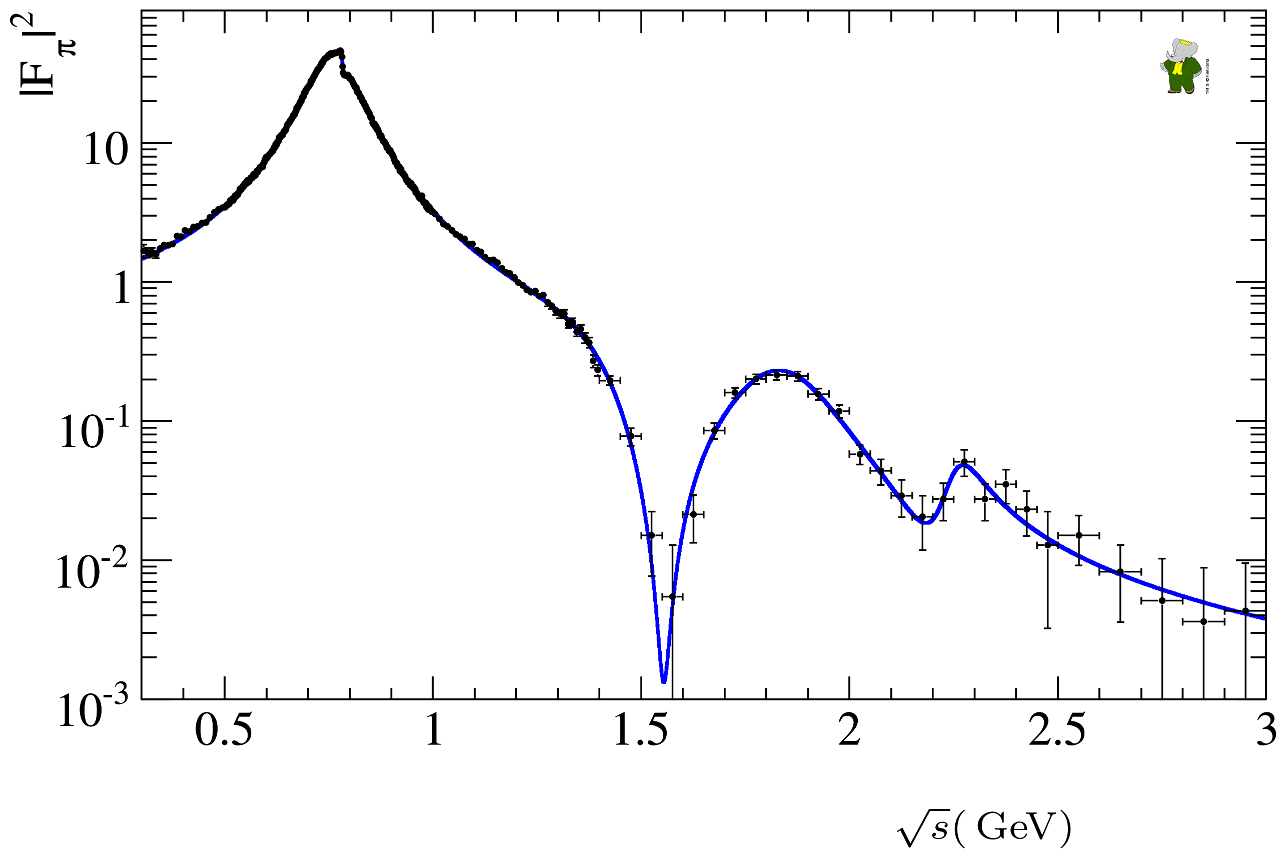


- Here the radiator function and the collider integrated luminosity are needed.
- MC simulation corrected for all known MC/data differences.
  - e.g.: ISR γ efficiency measured in data, from $\mu\mu$-only reco'ed evts.
  - MC corrected for known NLO deficiencies by comparing to PHOKHARA

Good agreement within $0.4 \pm 1.1\%$; dominated by $\mathcal{L}_{e^+e^-}$ $(\pm 0.9\%)$

Phys. Rev. Lett. 103 (2009) 231801, Phys. Rev. D86 (2012) 032013

$e^+e^- \to \pi^+\pi^-(\gamma)$: *VDM Fit of* $|F_\pi(s)|^2$

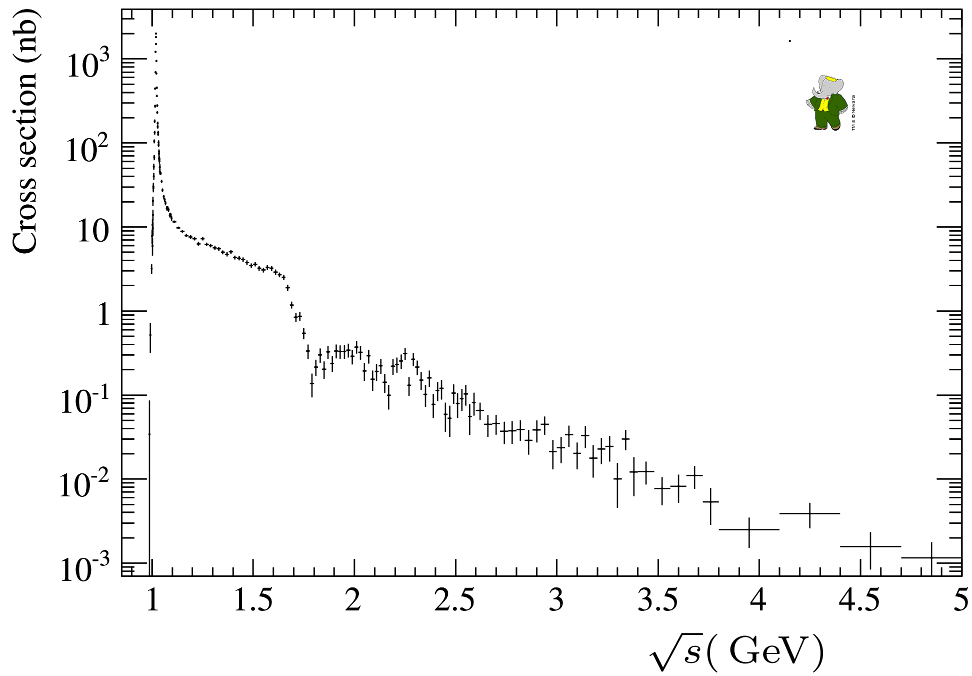


- $|\mathrm{Form\ Factor}|^2$ fitted with a vector dominance model, $\rho, \rho', \rho'', \omega$.
- ρ's described by the Gounaris-Sakurai model $\chi^2/n_{df} = 334/323$

Phys. Rev. Lett. 103 (2009) 231801, Phys. Rev. D86 (2012) 032013

$e^+e^- \to K^+K^-(\gamma)$ *Cross Section*

- Same method, as for $\pi^+\pi^-$:
  - NLO ISR, data sample, evt generators ..
  - Data driven, step-by-step evaluation of $\epsilon^{data}/\epsilon^{MC}$
  - Resolution unfolding (almost no FSR for kaons) Malaescu arXiv:0907.3791

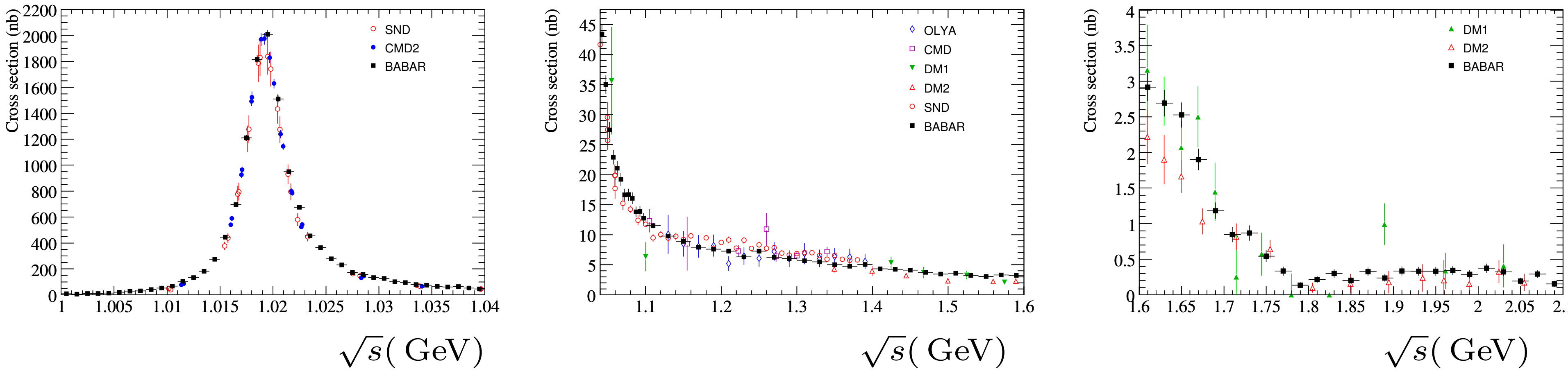


$e^+e^- \to K^+K^-(\gamma)$ bare cross section (including FSR), J/ψ and $\psi(2S)$ removed.

Phys. Rev. D88 (2013) 032013

$e^+e^- \to K^+K^-(\gamma)\gamma$

- Zooms and comparisons with previous (direct measurements) results.



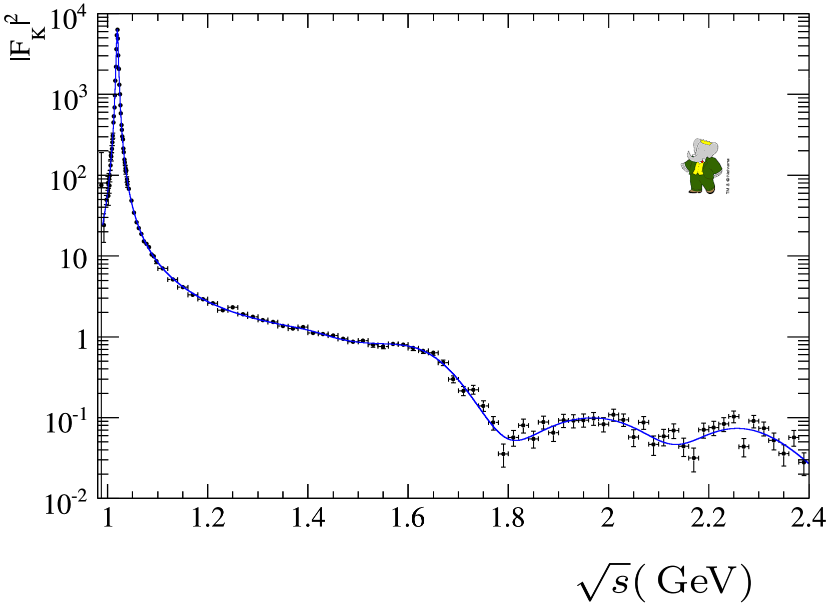
- Unprecedented precision in the ϕ region [1.01 - 1.03] GeV, of 7.3×10^{-3}.

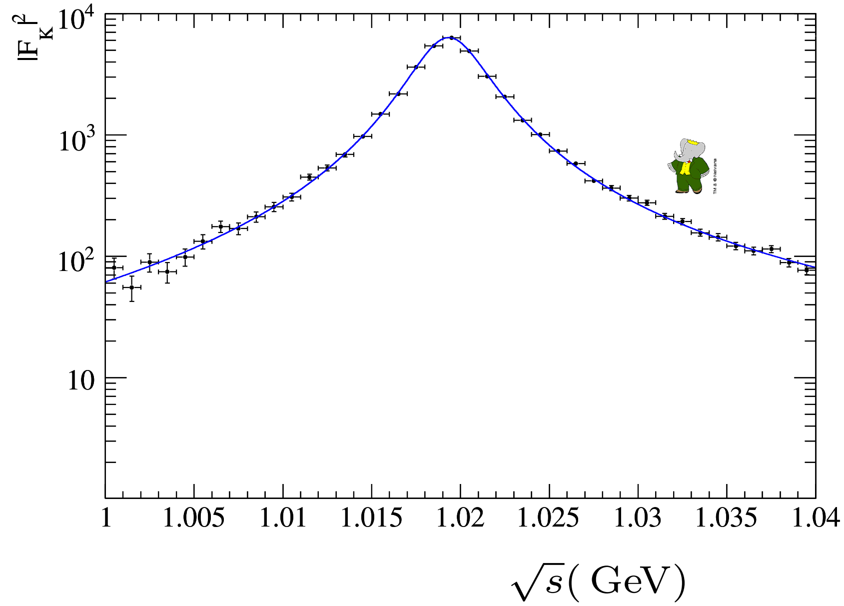
- Dispersion integral:

a_μ^{KK} $(\sqrt{s} < 1.8\,\mathrm{GeV})$	$(22.93 \pm 0.18_{\mathrm{stat}} \pm 0.22_{\mathrm{syst}} \pm 0.03_{\mathrm{VP}})\,10^{-10}$	PRD 88 (2013) 032013
Previous combination	$(21.63 \pm 0.27_{\mathrm{stat}} \pm 0.68_{\mathrm{syst}})\,10^{-10}$	EPJ C 71 (2011) 1515
Δ	$+(1.30 \pm 0.79)10^{-10}$	a 1.6 σ difference

- Precision better than combination of previous results by a factor 2.7

$e^+e^- \to K^+K^-(\gamma)$: VDM Fit of $|F_K(s)|^2$





- $F_K = \hat{\phi}/3 + \hat{\rho}/2 + \hat{\omega}/6$, with:

$$\hat{\phi} = \sum_{i=\phi}^{\phi''} a_i BW_i, \quad \hat{\rho} = \sum_{i=\rho}^{\rho'''} a_i BW_i, \quad \hat{\omega} = \sum_{i=\omega}^{\omega'''} a_i BW_i \quad \text{and}$$

$$\sum_{i=\phi}^{\phi''} a_i = 1, \quad \sum_{i=\rho}^{\rho'''} a_i = 1, \quad \sum_{i=\omega}^{\omega'''} a_i = 1.$$

		This work	PDG	
m_ϕ	=	$1019.51 \pm 0.02 \pm 0.05$	1019.455 ± 0.020	MeV/c^2
Γ_ϕ	=	$4.29 \pm 0.04 \pm 0.07$	4.26 ± 0.04	MeV
$\Gamma_\phi^{ee} \times \mathcal{B}(\phi \to K^+K^-)$	=	$0.6340 \pm 0.0070_{\rm exp} \pm 0.0037_{\rm fit} \pm 0.0013_{\rm cal}$		keV

Phys. Rev. D88 (2013) 032013

ISR: Recent LO Measurements @ BaBar

- LO: hadronic final state, ISR γ either detected ($E^*_\gamma > 3$ GeV), or undetected.
- ISR luminosity computed from MC

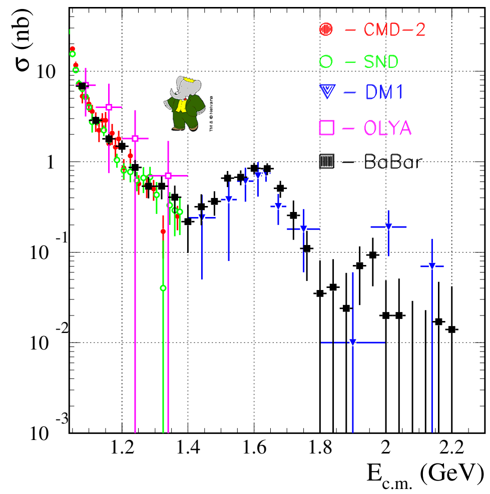
	$\mathcal{L}$ fb^{-1}	$\sqrt{s}_{\rm min}$ GeV	$\sqrt{s}_{\rm max}$ GeV	
$K^0_S K^0_L$	469		2.3	Phys. Rev. D 89, 092002 (2014)
$K^0_S K^0_L \pi^+\pi^-$			4.0	
$K^0_S K^0_S \pi^+\pi^-$			4.0	
$K^0_S K^0_S K^+K^-$			4.5	
$\bar{p}p$	454		4.5	Phys. Rev. D87 (2013) 092005
$\bar{p}p$	469	3.0	6.5	Phys. Rev. D. 88 (2013) 072009
K^+K^-	469	2.6	8.0	Phys. Rev. D 92, 072008 (2015)
$K^0_S K^+\pi^-\pi^0$	454		4.0	Preliminary
$K^0_S K^+\pi^-\eta$				

Blue: photon undetected

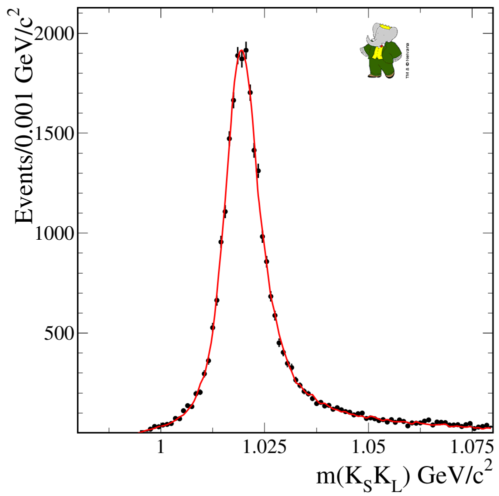
Magenta: First measurements

Channels with 2 neutral kaons

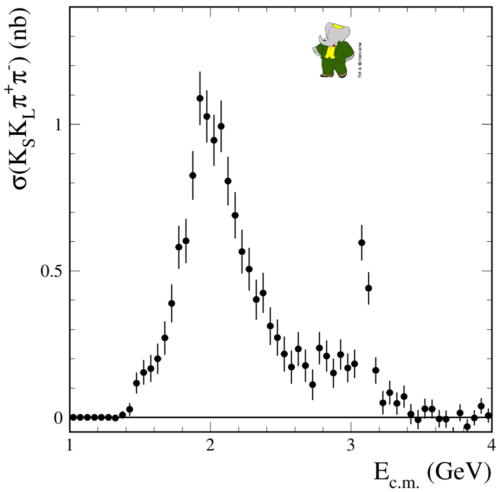
$K_S^0 K_L^0$



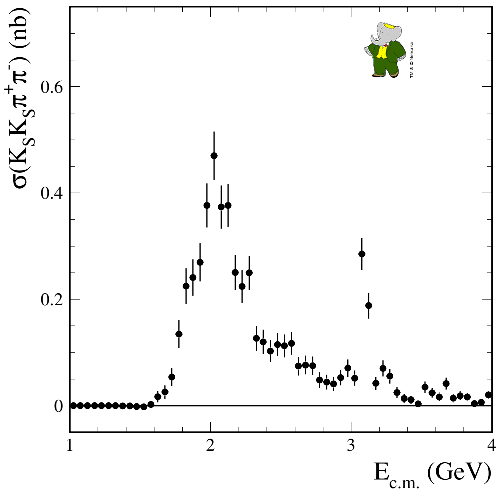
zoom $K_S^0 K_L^0$



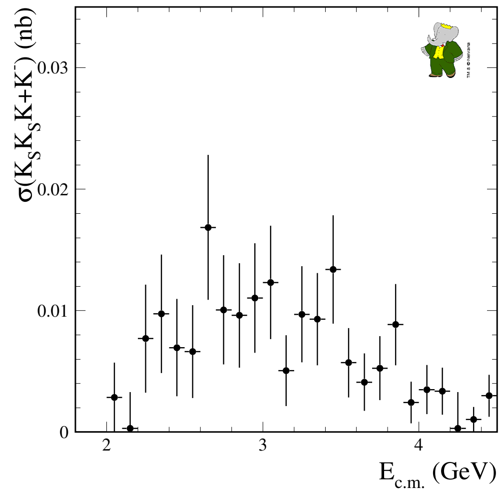
$K_S^0 K_L^0 \pi^+ \pi^-$



$K_S^0 K_S^0 \pi^+ \pi^-$



$K_S^0 K_S^0 K^+ K^-$

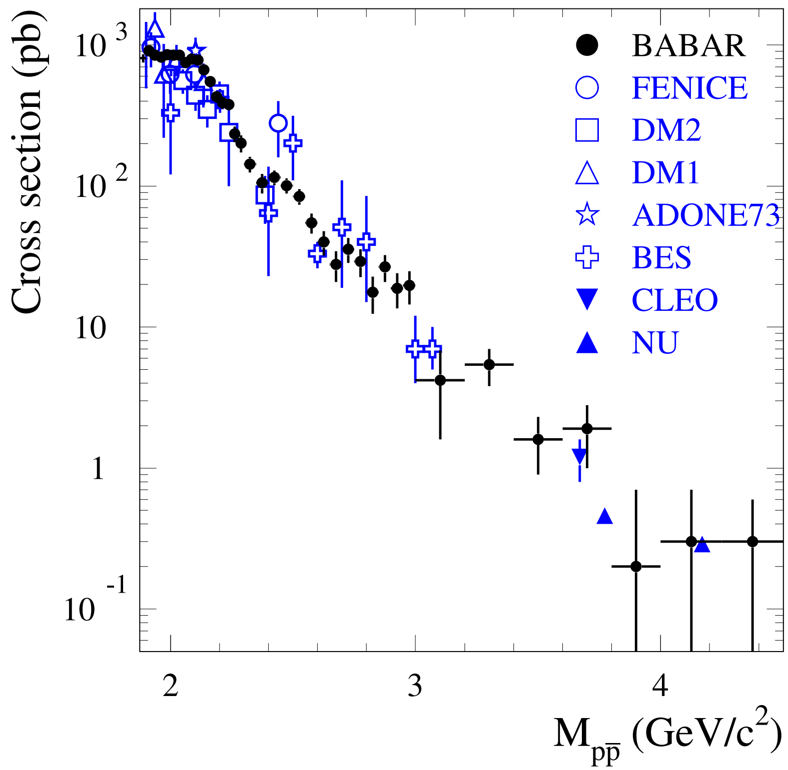


Phys. Rev. D 89, 092002 (2014)

Magenta: First measurements

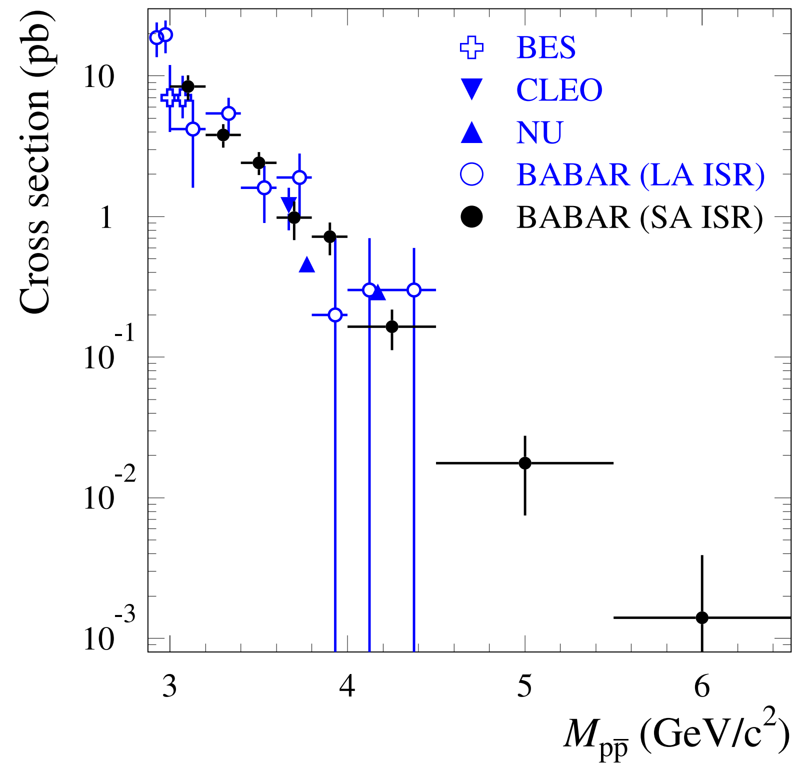
$e^+e^- \to \overline{p}p$ measurements

γ detected



Phys. Rev. D87 (2013) 092005

γ undetected

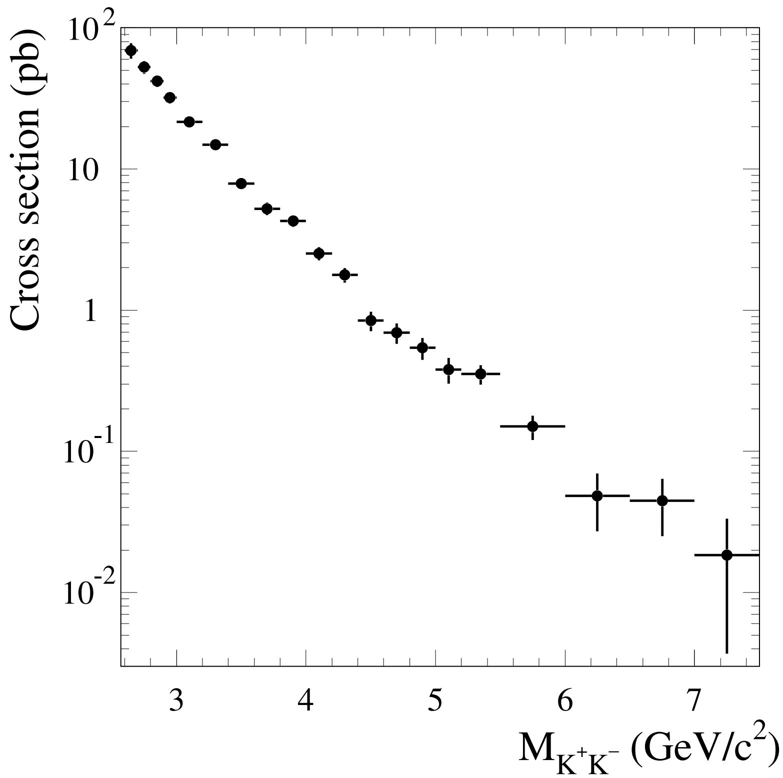


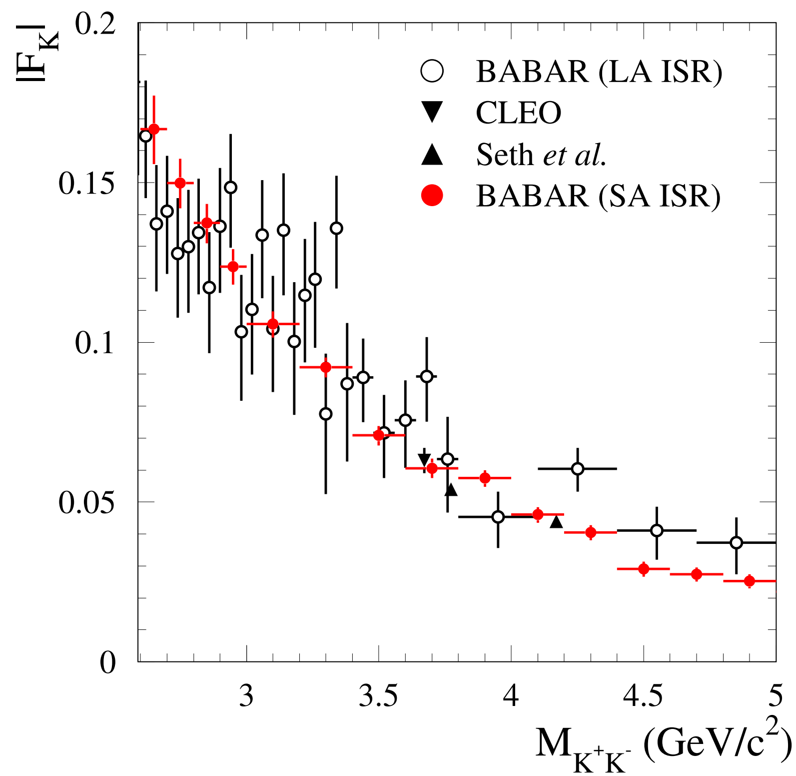
Phys. Rev. D. 88 (2013) 072009

Supersedes our Phys. Rev. D73 (2006) 012005

These $\overline{p}p$ data are not used for a_μ

$e^+e^- \to K^+K^-$, γ *undetected*



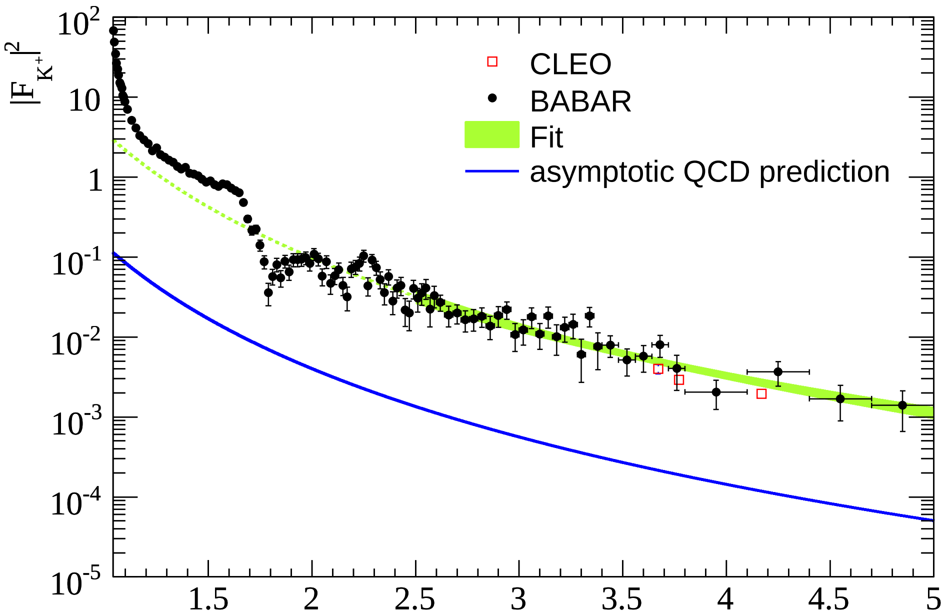


$469\,\text{fb}^{-1}$

Phys. Rev. D 92, 072008 (2015)

$e^+e^- \to K^+K^-$, comparison with QCD

γ detected

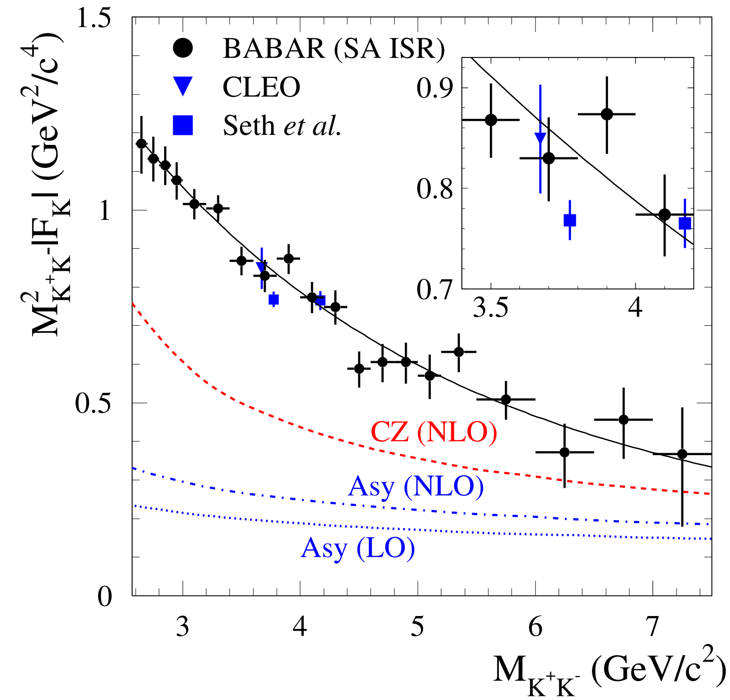


- Blue: Chernyak-Zhitnitsky NLO pQCD

Phys. Rev. D88 (2013) 032013

pQCD fails to describe the data.

γ undetected

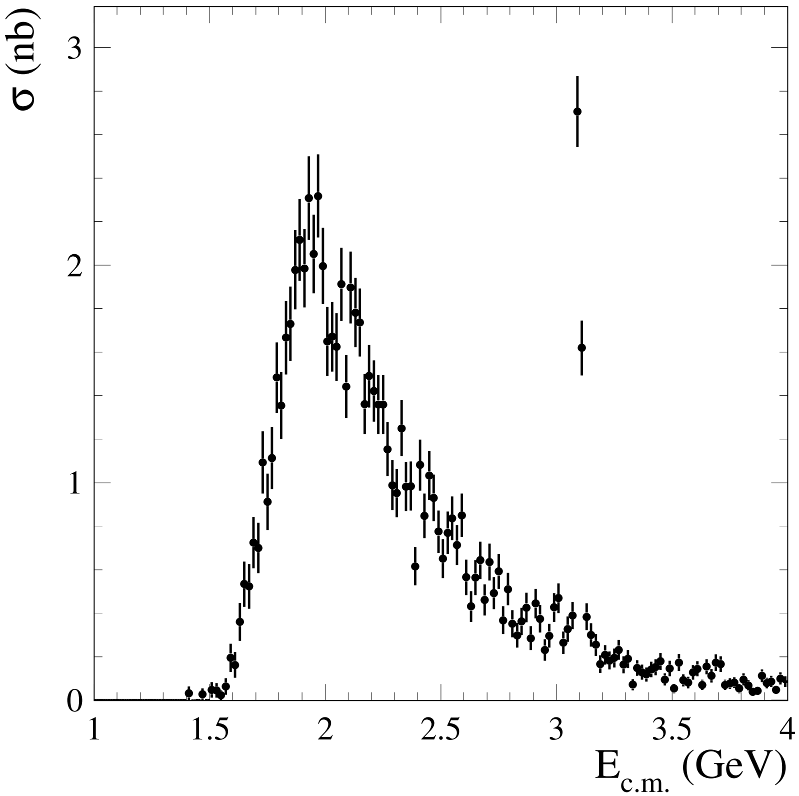


- Black: fit to BaBar data
- Red: Chernyak-Zhitnitsky NLO pQCD
- pQCD with asymptotic kaon distribution amplitude.

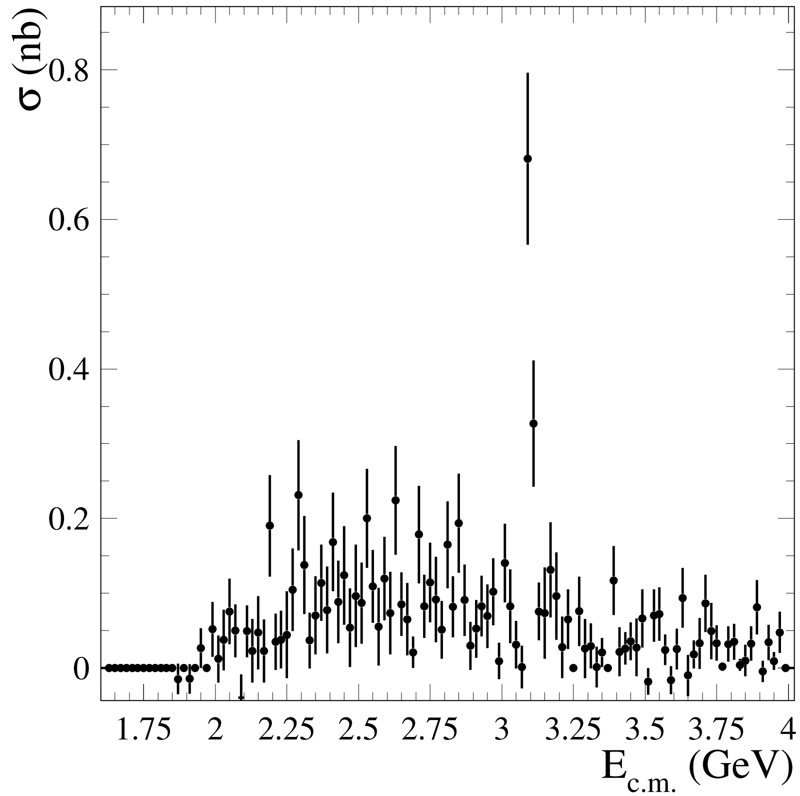
Phys. Rev. D 92, 072008 (2015)

Getting better at higher mass ?

$e^+e^- \to K_S^0 K^+ \pi^- h^0$



$K_S^0 K^+ \pi^- \pi^0$



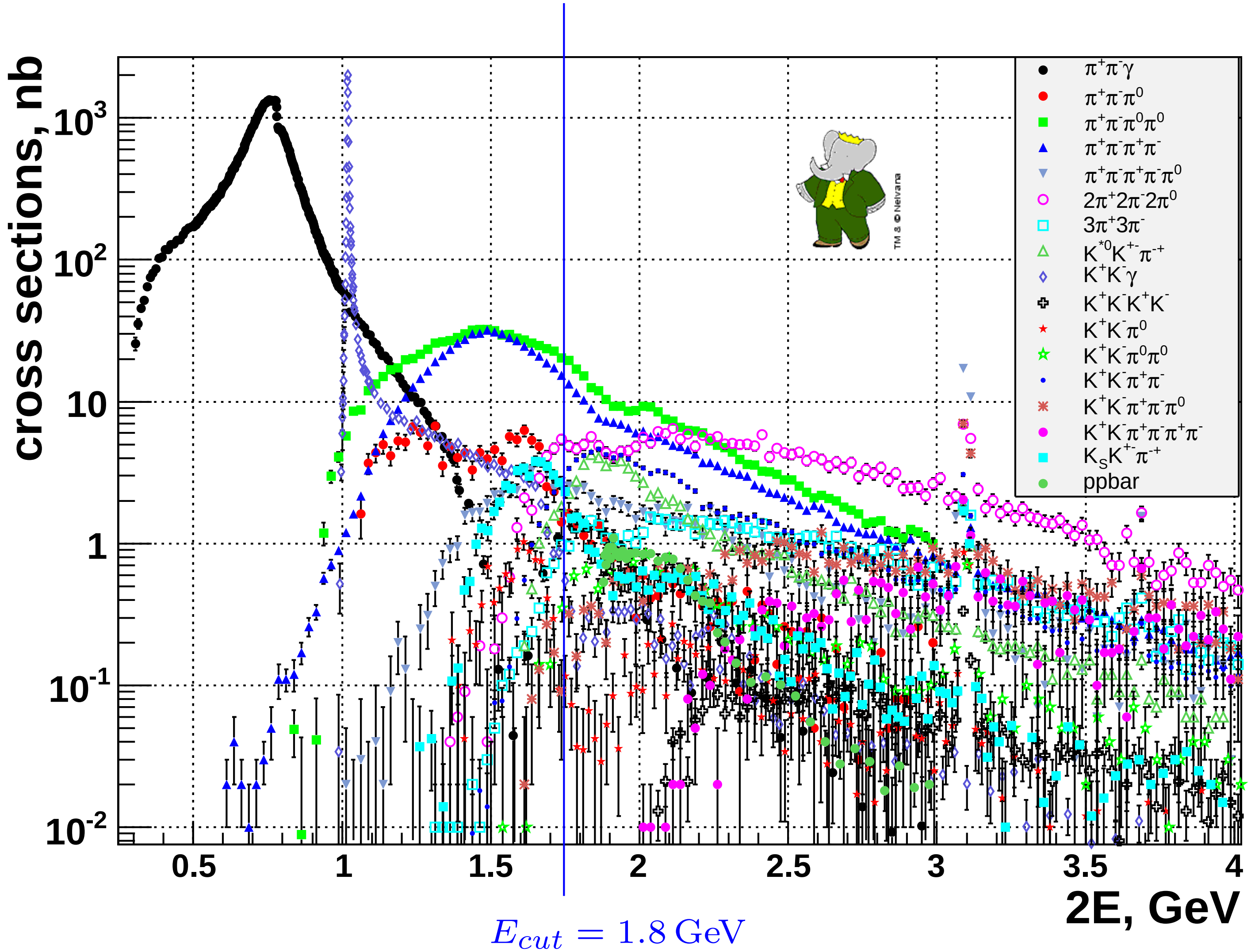
$K_S^0 K^+ \pi^- \eta$

$454\,\mathrm{fb}^{-1}$

charmonia not (yet) removed

Preliminary, publication in preparation

BaBar ISR measurements: Summary Plot



BaBar Oct 2013

Thanks Fedor V. Ignatov

The $\pi^+\pi^-\pi^0\pi^0$ entry is preliminary arXiv:0710.3455

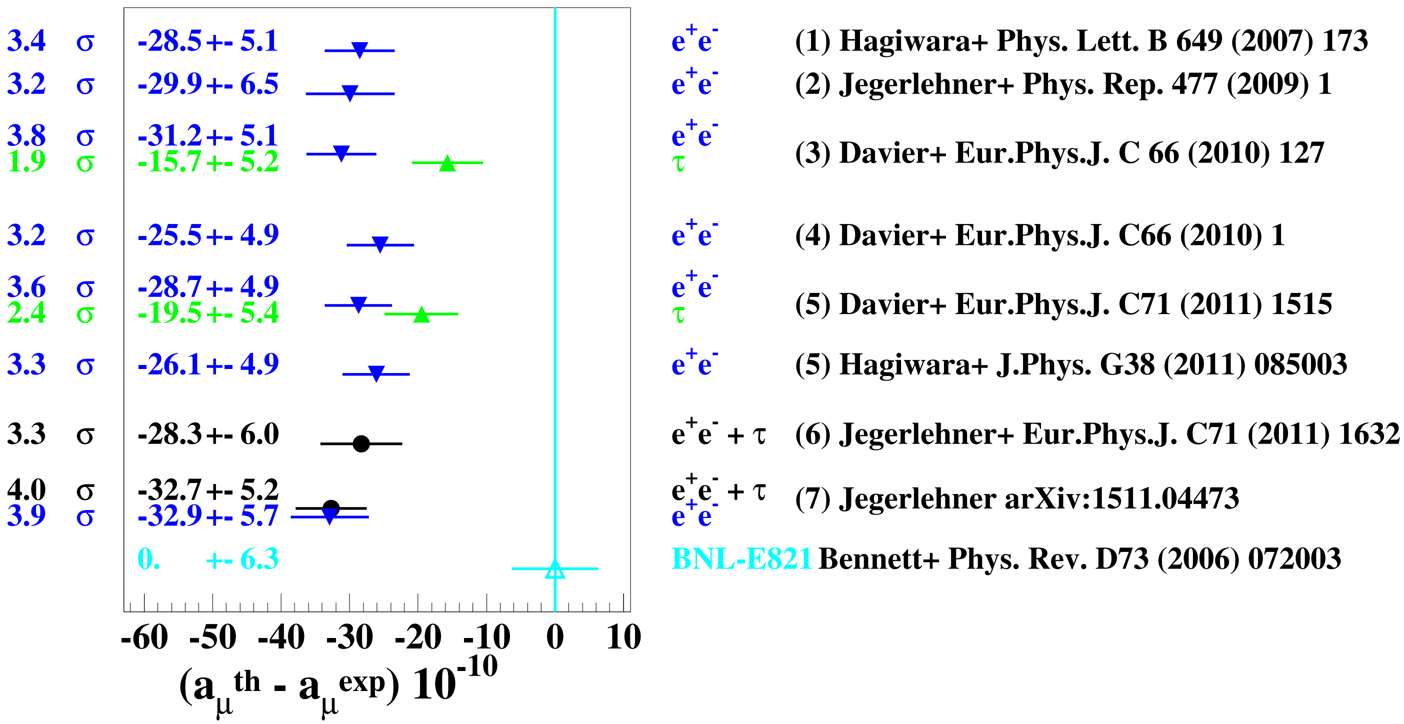
BaBar ISR measurements: present situation

- A vigorous campaign that is almost completed — @ 10.6 GeV
- radiation: LO: $e^+e^- \to X\gamma$ or NLO: $e^+e^- \to X\gamma(\gamma)$
- γ detected (lower X mass), or γ undetected (higher X mass)
- For most channels, precision improvement by a factor of ≈ 3 wrt previous averages

$K^0_S K^+\pi^-\pi^0$, $K^0_S K^+\pi^-\eta$		Preliminary
K^+K^-	γ undet.	Phys. Rev. D 92, 072008 (2015)
$K^0_S K^0_L$, $K^0_S K^0_L\pi^+\pi^-$, $K^0_S K^0_S\pi^+\pi^-$, $K^0_S K^0_S K^+K^-$		Phys. Rev. D 89, 092002 (2014)
$\bar{p}p$	γ undet.	Phys.Rev. D88 (2013) 7, 072009
$\bar{p}p$		Phys. Rev. D87 (2013) 092005
K^+K^-	NLO	Phys. Rev. D88 (2013) 032013
$\pi^+\pi^-$	NLO	Phys. Rev. Lett. 103 (2009) 231801
		Phys. Rev. D86 (2012) 032013
$2(\pi^+\pi^-)$		Phys. Rev. D85 (2012) 112009
$K^+K^-\pi^+\pi^-$, $K^+K^-\pi^0\pi^0$, $K^+K^-K^+K^-$		Phys. Rev. D86 (2012) 012008
$K^+K^-\eta$, $K^+K^-\pi^0$, $K^0K^\pm\pi^\mp$		Phys. Rev.D77 (2008) 092002
$2(\pi^+\pi^-)\pi^0$, $2(\pi^+\pi^-)\eta$, $K^+K^-\pi^+\pi^-\pi^0$, $K^+K^-\pi^+\pi^-\eta$		Phys. Rev.D76 (2007) 092005
$K^+K^-\pi^+\pi^-$, $K^+K^-\pi^0\pi^0$, $K^+K^-K^+K^-$		Phys. Rev.D76 (2007) 012008
$\Lambda\bar{\Lambda}$, $\Lambda\Sigma^0$, $\Sigma^0\Sigma^0$		Phys. Rev. D76 (2007) 092006
$3(\pi^+\pi^-)$, $2(\pi^+\pi^-\pi^0)$, $K^+K^-2(\pi^+\pi^-)$		Phys. Rev.D73 (2006) 052003
$\bar{p}p$		Phys. Rev.D73 (2006) 012005
$2(\pi^+\pi^-)$, $K^+K^-\pi^+\pi^-$, $K^+K^-K^+K^-$		Phys. Rev.D71 (2005) 052001
$\pi^+\pi^-\pi^0$		Phys. Rev.D70 (2004) 072004

Magenta: First measurements, Orange: superseded, Green: 454 - 469 fb^{-1}, Cyan: 232 fb^{-1}, Blue: 89 fb^{-1}

Exp – Theory Comparison $(a_\mu$ *units* $10^{-10})$



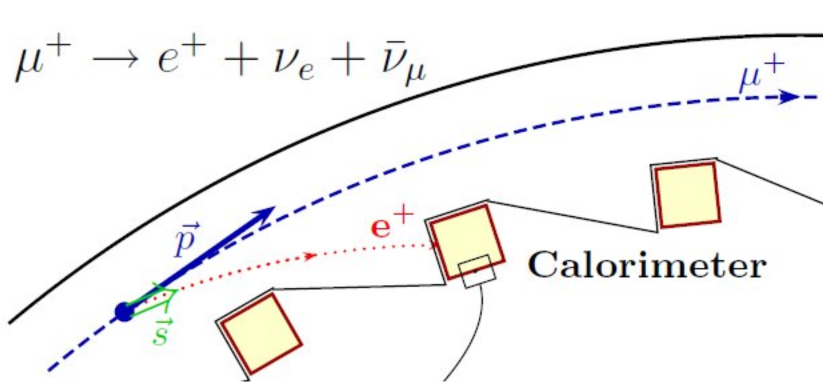
- (6) Solves $e^+e^- - \tau$ discrepancy after correcting τ for $\rho - \gamma$ mixing Jegerlehner, EPJ. C71 (2011) 1632
- (7)
  - Includes BaBar ISR data [2012 - 2014], BESIII ISR $\pi^+\pi^-$, CDM-3 $3(\pi^+\pi^-)$, SND $\omega\pi^0$
  - pQCD range restricted to [4.5 – 9.3] GeV and [13 GeV – ∞[. Rest from data.
  - Light-by-light update;
  - Addition of NNLO hadronic VP contribution.

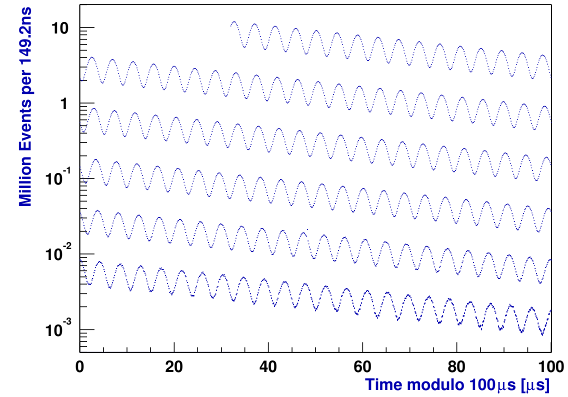
Conclusion

- BaBar ISR programme well advanced.
  Most precise $\sigma(e^+e^- \to \mathrm{had})$, from threshold to $E_{\mathrm{cut}} = 1.8\,\mathrm{GeV}$ (and above)
- a_μ: SM prediction uncertainty $\approx 5.2 \times 10^{-10}$, $a_\mu^{\mathrm{had,LO}}$- and $a_\mu^{\mathrm{l-by-l}}$- dominated
  BNL measurement uncertainty $\approx 6.3 \times 10^{-10}$
- Study of $\rho - \gamma$ mixing solves former discrepancy between e^+e^- and τ-based $a_\mu^{\pi^+\pi^-,LO}$
  Jegerlehner Eur.Phys.J. C71 (2011) 1632
- Experiment-to-SM discrepancy still at 3 – 4 σ
- New measurements of a_μ are eagerly awaited:
  - Fermilab Nucl.Phys.Proc.Suppl. 225-227 (2012) 277-281 and
  - J-PARC Nucl.Phys.Proc.Suppl. 218 (2011) 242-246

Back-up slides

a_μ *Measurement E821 @ BNL*





- $\pi \to \mu\,\nu$ violates P, μ longitudinally polarized.
- μ's at "magic momentum" [$\approx 3.1\,\mathrm{GeV}/c$], in a storage ring with constant $\vec{B}$.
  - μ rotating with freq ω_c ; μ spin precessing with freq ω_s
  - freq. difference $\omega_a = \omega_s - \omega_c = a_\mu e B / m_\mu$
- $\mu \to e\,\nu\,\overline{\nu}$ violates P, e direction (energy in lab) remembers μ polarization.
  $\Rightarrow$ Fraction of detected e above a $E_{\mathrm{threshold}}$ is modulated with freq. ω_a

$$a_\mu(\mathrm{expt}) = (11659208.0 \pm 5.4(\mathrm{stat}) \pm 3.3(\mathrm{syst})) \times 10^{-10} \qquad (0.54\ \mathrm{ppm})$$

E821 @ BNL, $\mu^+ - \mu^-$ charge average Bennett Phys. Rev.D73 (2006) 072003

a_μ *measurement: the magic* γ

- In a storage ring, uniform $\vec{B}$:

  Cyclotron frequency $\omega_c = \frac{eB}{m_\mu \gamma}$,

  Spin precession frequency $\omega_s = \frac{eB}{m_\mu \gamma} + a_\mu \frac{eB}{m_\mu}$,

  $\omega_a = \omega_s - \omega_c$ $\omega_a = a_\mu \frac{eB}{m_\mu}$

- But need quadropole $\vec{E}$ too, to prevent beam go astray; ω_a becomes:

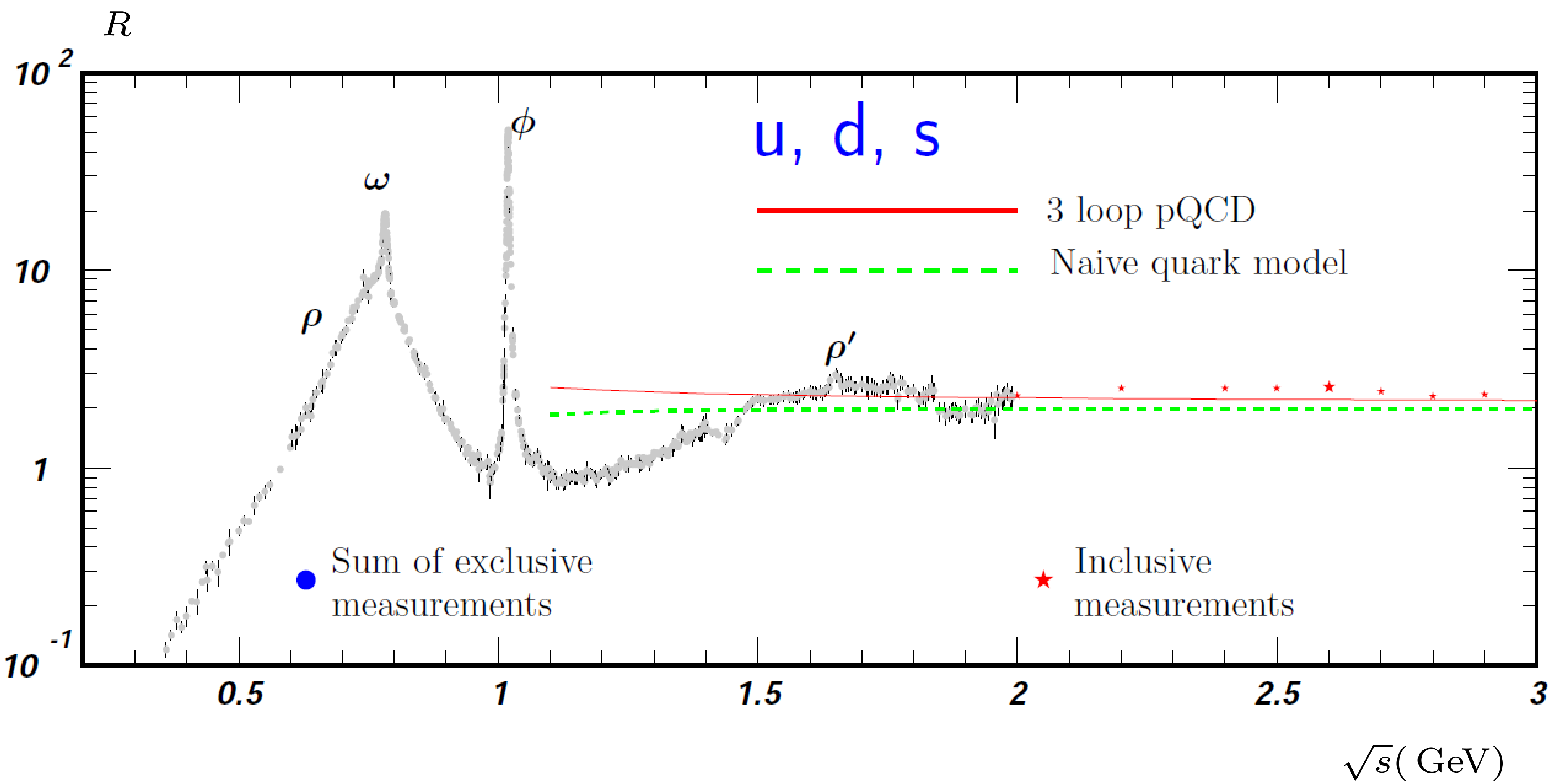
$$\vec{\omega_a} = \frac{e}{m_\mu} \left(a_\mu \vec{B} - \left[a_\mu - \frac{1}{\gamma^2 - 1} \right] \vec{v} \times \vec{E} \right)$$

- Precise value of E not needed: ω_a independent of E for "magic" γ:

$$a_\mu - 1/(\gamma^2 - 1) = 0,$$

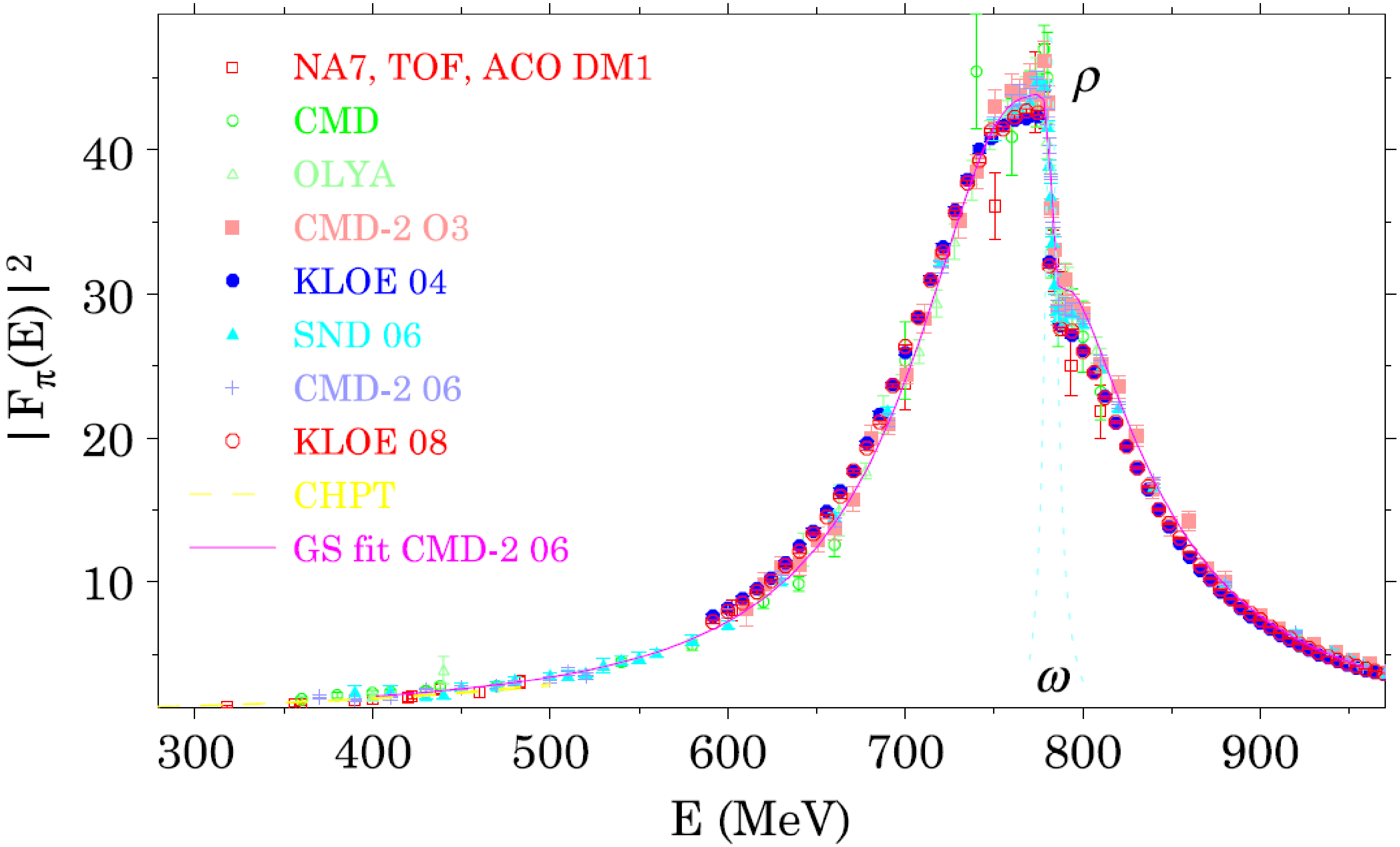
  corresponds to $\approx 3.1\,\mathrm{GeV}$ muons. Bargmann Phys. Rev. Lett. 2 (1959) 435

$R_{\rm had}(s)$: *Direct Measurements* $e^+e^- \to$ *Hadrons*



PDG 2013

$e^+e^- \to \pi^+\pi^-$: *Direct Measurements*

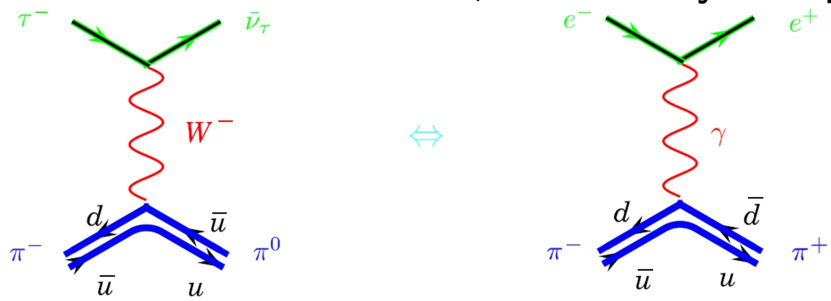


- (KLOE 08 supersedes KLOE04)
- The 3.2 σ discrepancy mentioned above is based on this input

Jegerlehner, Nyffeler / Phys Rept 477 (2009) 1110

τ *Decay Spectral Functions*

- $I=1$ part of $e^+e^- \to \mathrm{had}$ from $\tau \to \nu_\tau + \mathrm{had}$ by isospin rotation



$\pi^0\pi^- \leftrightarrow \pi^+\pi^-$, $\quad \sigma(e^+e^- \to \pi^+\pi^-) = \frac{4\pi\alpha^2}{s}\nu_0(s)$, $\quad \nu(s)$ "spectral function"

- $\frac{1}{\Gamma}\frac{\mathrm{d}\Gamma}{\mathrm{d}s} = F(s)\frac{\mathcal{B}(\tau \to e\nu_\tau\overline{\nu}_e)}{\mathcal{B}(\tau \to \pi^-\pi^0\nu_\tau)} \times \nu_-(s)$, where $F(s)$ is a known function of s
- CVC: $\nu_0(s) = \nu_-(s)$, isospin breaking (IB) corrections $\cdots$

	$a_\mu^{\pi^+\pi^-}[2m_\pi, 1.8\,\mathrm{GeV}/c^2]$ (10^{-10})	
ALEPH, CLEO, OPAL	$520.1 \pm 2.4_{\mathrm{exp}} \pm 2.7_{\mathrm{Br.}} \pm 2.5_{\mathrm{IB}}$	Eur. Phys. J. C 27, 497 (2003).
Belle (72 fb^{-1})	$523.5 \pm 1.5_{\mathrm{exp}} \pm 2.6_{\mathrm{Br.}} \pm 2.5_{\mathrm{IB}}$	Fujikawa Phys. Rev.D78:072006,2008.

Hadron form factor, bare and dressed cross sections

A hadron h,

$$|F_h|^2(s') = \frac{3s'}{\pi\alpha^2(0)\beta_h^3}\frac{\sigma_{\overline{h}h}(s')}{C_{\mathrm{FS}}}\ , \qquad \sigma_{\overline{h}h}(s') = \sigma^0_{\overline{h}h(\gamma)}(s') \ \left(\frac{\alpha(s')}{\alpha(0)}\right)^2$$

F_h	hadron form factor	
$\sigma_{\overline{h}h}(s')$	dressed cross section	$e^+e^- \to \overline{h}h$
$\sigma^0_{\overline{h}h(\gamma)}$	bare cross section	$e^+e^- \to \overline{h}h$
$\beta_h = \sqrt{1 - 4m_h^2/s'}$	hadron velocity	
$C_{\mathrm{FS}} = 1 + \frac{\alpha}{\pi}\eta_h(s')$	final-state correction (Coulomb $\cdots$	