Quark and Gluon collinear and TMD parton distributions from HERA DIS data

• Ola Lelek [1] Francesco Hautmann [2] Hannes Jung [1]

[1] Deutsches Elektronen-Synchrotron (DESY)

[2] University of Oxford

14.04.2016

DIS 2016, Hamburg

Table of Contents

Motivation

Why Transverse Momentum Dependent PDFs?

Goal: **full TMD PDFs**

TDMs are important in studies on:

- resummation at all orders in the QCD coupling to many observables in high-energy hadronic collisions,
- nonperturbative information on hadron structure at very low k_T,
- perturbative region where QCD evolution equations (DGLAP, BFKL, CCFM) describe processes
- a proper and consistent simulation of parton showers,
- multi-scale problems in hadronic collisions,
- ...

Acta Physica Polonica B, Vol. 46 (2015), Transverse Momentum Dependent (TMD) Parton Distribution Functions: Status and Prospects

Important processes: Drell-Yan hadroproduction of electroweak gauge bosons, Higgs production...

Why Transverse Momentum Dependent PDFs?

Goal: **full TMD PDFs**

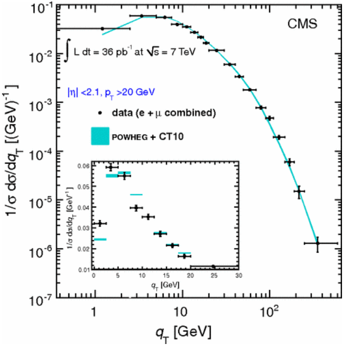
TDMs are important in studies on:

- resummation at all orders in the QCD coupling to many observables in high-energy hadronic collisions,
- nonperturbative information on hadron structure at very low k_T,
- perturbative region where QCD evolution equations (DGLAP, BFKL, CCFM) describe processes
- a proper and consistent simulation of parton showers,
- multi-scale problems in hadronic collisions,
- ...

Acta Physica Polonica B, Vol. 46 (2015), Transverse Momentum Dependent (TMD) Parton Distribution Functions: Status and Prospects

Important processes: Drell-Yan hadroproduction of electroweak gauge bosons, Higgs production...

Example:
The Z-boson transverse momentum q_T spectrum in pp collisions at the LHC:



S. Chatrchyan et al., Phys. Rev. D 85, 032002 (2012)

Evolution using MC method:

In this presentation: MC results obtained with updated and improved uPDFevolv code:

old version:

- ccfm evolution,
- only gluon and valence quark evolution (separately)
- all loop $P(z)$,
- 1- or 2-loop-α_s.
- $f(x,t)$,

Evolution applicable only for small x, not very high Q^2 and all kinematically allowed k_T.

new version:

- full coupled quark and gluon DGLAP evolution (gluon, sea and valence evolution)
- fixed flavour number scheme (only u,d,s)
- LO in $P(z)$ (we plan to have NLO in $P(z)$),
- 1-loop-α_s (but also 2-loop-α_s implemented).
- $xf(x,t)$,

Evolution over the whole range in x, Q^2 and all kinematically allowed k_T.

https://updfevolv.hepforge.org/

Evolution using MC method:

In this presentation: MC results obtained with updated and improved uPDFevolv code:

old version:

- ccfm evolution,
- only gluon and valence quark evolution (separately)
- all loop $P(z)$,
- 1- or 2-loop-α_s.
- $f(x,t)$,

Evolution applicable only for small x, not very high Q^2 and all kinematically allowed k_T.

new version:

- full coupled quark and gluon DGLAP evolution (gluon, sea and valence evolution)
- fixed flavour number scheme (only u,d,s)
- LO in $P(z)$ (we plan to have NLO in $P(z)$),
- 1-loop-α_s (but also 2-loop-α_s implemented).
- $xf(x,t)$,

Evolution over the whole range in x, Q^2 and all kinematically allowed k_T.

https://updfevolv.hepforge.org/

Similar codes exist (use similar formalism):

evolution code EvolFMC by Cracow group
S. Jadach et al., Markovian Monte Carlo program EvolFMC v.2 for solving QCD evolution equations, Comput.Phys.Commun. 181 (2010) 393-412

Advantages of updfevolv:

- the structure of the code suitable for usage in xFitter (to have full TMDs): structure of grids $\rightarrow$ fitting method fast (see slide 13)
- different options for studying large z behaviour (see slide 15)

Introduction to the method

Sudakov formalism

Evolution equation for parton density

$$t\frac{\partial f(x,t)}{\partial t} = \frac{\alpha_s}{2\pi}\int \frac{dz}{z}P(z)_+ f\left(\frac{x}{z},t\right) = \frac{\alpha_s}{2\pi}\int \frac{dz}{z}P(z)f\left(\frac{x}{z},t\right) - \frac{\alpha_s}{2\pi}f(x,t)\int dzP(z), \quad (1)$$

where $\int_0^1 f(x)g(x)_+ dx = \int_0^1 f(x)g(x)dx - \int_0^1 f(1)g(x)dx$.

Sudakov formalism

Evolution equation for parton density

$$t\frac{\partial f(x,t)}{\partial t} = \frac{\alpha_s}{2\pi}\int \frac{dz}{z}P(z)_+ f\left(\frac{x}{z},t\right) = \frac{\alpha_s}{2\pi}\int \frac{dz}{z}P(z) f\left(\frac{x}{z},t\right) - \frac{\alpha_s}{2\pi} f(x,t)\int dz P(z), \qquad (1)$$

where $\int_0^1 f(x)g(x)_+ dx = \int_0^1 f(x)g(x)dx - \int_0^1 f(1)g(x)dx$.

Introducing *Sudakov form factor*

$$\Delta_s(t,t_0) \equiv \Delta_s(t) = exp\Big(-\int_0^{z_{max}} dz \int_{t_0}^{t} \frac{\alpha_s}{2\pi}\frac{dt'}{t'}P(z)\Big) \qquad (2)$$

we can rewrite (1)

$$t\frac{\partial f(x,t)}{\partial t} = \frac{\alpha_s}{2\pi}\int \frac{dz}{z}P(z)f(\frac{x}{z},t) + f(x,t)\frac{t}{\Delta_s(t)}\frac{\partial \Delta_s(t)}{\partial t}. \qquad (3)$$

Sudakov formalism

Evolution equation for parton density

$$t\frac{\partial f(x,t)}{\partial t} = \frac{\alpha_s}{2\pi}\int \frac{dz}{z}P(z)_+ f\left(\frac{x}{z},t\right) = \frac{\alpha_s}{2\pi}\int \frac{dz}{z}P(z)f\left(\frac{x}{z},t\right) - \frac{\alpha_s}{2\pi}f(x,t)\int dzP(z), \qquad (1)$$

where $\int_0^1 f(x)g(x)_+dx = \int_0^1 f(x)g(x)dx - \int_0^1 f(1)g(x)dx$.

Introducing *Sudakov form factor*

$$\Delta_s(t,t_0) \equiv \Delta_s(t) = exp\Big(-\int_0^{z_{max}} dz \int_{t_0}^{t} \frac{\alpha_s}{2\pi}\frac{dt'}{t'}P(z)\Big) \qquad (2)$$

we can rewrite (1)

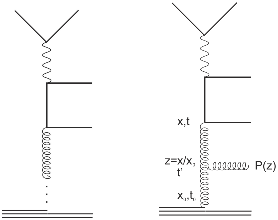
$$t\frac{\partial f(x,t)}{\partial t} = \frac{\alpha_s}{2\pi}\int \frac{dz}{z}P(z)f(\frac{x}{z},t) + f(x,t)\frac{t}{\Delta_s(t)}\frac{\partial \Delta_s(t)}{\partial t}. \qquad (3)$$

After integration

$$f(x,t) = f(x,t_0)\Delta_s(t) + \frac{\alpha_s}{2\pi}\int \frac{dt'}{t'}\frac{\Delta_s(t)}{\Delta_s(t')}\int \frac{dz}{z}P(z)f(\frac{x}{z},t'). \qquad (4)$$

Sudakov formalism

$$f(x,t) = f(x,t_0)\Delta_s(t) + \frac{\alpha_s}{2\pi}\int \frac{dt'}{t'}\frac{\Delta_s(t)}{\Delta_s(t')}\int \frac{dz}{z}P(z)f(\frac{x}{z},t'). \tag{5}$$



Sudakov: probability of evolving from t_0 to t without any resolvable branching.

iterative solution:

$$f(x,t) = \lim_{n\to\infty} f_n(x,t) = \lim_{n\to\infty}\sum_n \frac{1}{n!}log^n(\frac{t}{t_0})A^n \otimes \Delta_s(t)f(\frac{x}{z},t_0), \tag{6}$$

where $A = \frac{\alpha_s}{2\pi}\int \frac{dz}{z}P(z)$.

Momentum weighted parton densities & momentum sum rule

More details:
Not true that for every splitting function : $P(z) = P(z)_+$
$\rightarrow$ the Sudakov form factor formalism more complicated, in $\Delta_s(t)$ the virtual part of $P(z)$.
But
convenient to have the same splitting functions:

$$t\frac{\partial f(x,t)}{\partial t} = \frac{\alpha_s}{2\pi}\int \frac{dz}{z}\widehat{P}(z) f(\frac{x}{z},t) + f(x,t)\frac{t}{\Delta_s(t)}\frac{\partial \Delta_s(t)}{\partial t}$$

P - regularized splitting function (can be with *plus prescription*),
$\widehat{P}$ - unregularized splitting function (without *plus prescription*)

To include all flavours in the evolution & use $\Delta_s(t)$ with unregularised splitting functions we need to
switch from $f(x,t)$ to $xf(x,t)$ & use momentum sum rule:

$$\sum_a \int_0^1 zP_{ab}(\alpha_s, z)dz = 0. \quad (7)$$

Advantage: get rid of $\frac{1}{z}$ term in P_{gg} and P_{gq}.

Momentum weighted parton densities & momentum sum rule

Example for gluon:
P - regularized splitting function (can be with *plus prescription*),
$\widehat{P}$ - unregularized splitting function (without *plus prescription*)

$$t\frac{\partial xg(x,t)}{\partial t} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z}\left[P_{gq}(z)xq\left(\frac{x}{z},t\right) + P_{gg}(z)xg\left(\frac{x}{z},t\right)\right] \tag{8}$$

Momentum weighted parton densities & momentum sum rule

Example for gluon:
P - regularized splitting function (can be with *plus prescription*),
$\widehat{P}$ - unregularized splitting function (without *plus prescription*)

$$t\frac{\partial xg(x,t)}{\partial t} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z}\left[P_{gq}(z)xq\left(\frac{x}{z},t\right) + P_{gg}(z)xg\left(\frac{x}{z},t\right)\right] - \frac{\alpha_s}{2\pi}xg(x,t)\underbrace{\int_0^1 dzz\left[\sum_i P_{q_ig} + P_{gg}\right]}_{0} \tag{8}$$

Momentum weighted parton densities & momentum sum rule

Example for gluon:
P - regularized splitting function (can be with *plus prescription*),
$\widehat{P}$ - unregularized splitting function (without *plus prescription*)

$$t\frac{\partial xg(x,t)}{\partial t} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z}\left[P_{gq}(z)xq\left(\frac{x}{z},t\right) + P_{gg}(z)xg\left(\frac{x}{z},t\right)\right] - \frac{\alpha_s}{2\pi}xg(x,t)\underbrace{\int_0^1 dzz\left[\sum_i P_{q_ig} + P_{gg}\right]}_{0}$$

$$\rightarrow \frac{\alpha_s}{2\pi}\int_x^{z_{max}} \frac{dz}{z}\left[\widehat{P}_{gq}(z)xq\left(\frac{x}{z},t\right) + \widehat{P}_{gg}(z)xg\left(\frac{x}{z},t\right)\right] - \frac{\alpha_s}{2\pi}xg(x,t)\int_x^{z_{max}} dzz\left(2n_f\widehat{P}_{qg} + z\widehat{P}_{gg}\right) \tag{8}$$

Momentum weighted parton densities & momentum sum rule

Example for gluon:
P - regularized splitting function (can be with *plus prescription*),
$\widehat{P}$ - unregularized splitting function (without *plus prescription*)

$$t\frac{\partial xg(x,t)}{\partial t} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z}\left[P_{gq}(z)xq\left(\frac{x}{z},t\right) + P_{gg}(z)xg\left(\frac{x}{z},t\right)\right] - \frac{\alpha_s}{2\pi}xg(x,t)\underbrace{\int_0^1 dzz\left[\sum_i P_{q_ig} + P_{gg}\right]}_{0}$$

$$\rightarrow \frac{\alpha_s}{2\pi}\int_x^{z_{max}} \frac{dz}{z}\left[\widehat{P}_{gq}(z)xq\left(\frac{x}{z},t\right) + \widehat{P}_{gg}(z)xg\left(\frac{x}{z},t\right)\right] - \frac{\alpha_s}{2\pi}xg(x,t)\int_x^{z_{max}} dzz\left(2n_f\widehat{P}_{qg} + z\widehat{P}_{gg}\right) \tag{8}$$

$\rightarrow$

$$t\frac{\partial xg(x,t)}{\partial t} = \frac{\alpha_s}{2\pi}\int \frac{dz}{z}\sum_j \widehat{P}_{gj}(z)xf_j\left(\frac{x}{z},t\right) + xg(x,t)\frac{t}{\Delta_s(t)_g}\frac{\partial \Delta_s(t)_g}{\partial t}. \tag{9}$$

Momentum weighted parton densities & momentum sum rule

Example for gluon:
P - regularized splitting function (can be with *plus prescription*),
$\widehat{P}$ - unregularized splitting function (without *plus prescription*)

$$t\frac{\partial xg(x,t)}{\partial t} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z}\left[P_{gq}(z)xq\left(\frac{x}{z},t\right) + P_{gg}(z)xg\left(\frac{x}{z},t\right)\right] - \underbrace{\frac{\alpha_s}{2\pi}xg(x,t)\int_0^1 dzz\left[\sum_i P_{q_ig} + P_{gg}\right]}_{0}$$

$$\to \frac{\alpha_s}{2\pi}\int_x^{z_{max}} \frac{dz}{z}\left[\widehat{P}_{gq}(z)xq\left(\frac{x}{z},t\right) + \widehat{P}_{gg}(z)xg\left(\frac{x}{z},t\right)\right] - \frac{\alpha_s}{2\pi}xg(x,t)\int_x^{z_{max}} dzz\left(2n_f\widehat{P}_{qg} + z\widehat{P}_{gg}\right) \tag{8}$$

$\to$

$$t\frac{\partial xg(x,t)}{\partial t} = \frac{\alpha_s}{2\pi}\int \frac{dz}{z}\sum_j \widehat{P}_{gj}(z)xf_j\left(\frac{x}{z},t\right) + xg(x,t)\frac{t}{\Delta_s(t)_g}\frac{\partial \Delta_s(t)_g}{\partial t}. \tag{9}$$

where Sudakov is defined with unregularised splitting functions:

$$\Delta_s(t)_g = \exp\left(-\int_x^{z_{max}} dz\int_{t_0}^t \frac{\alpha_s}{2\pi}\frac{dt'}{t'}z\sum_j \widehat{P}(z)_{jg}\right) \tag{10}$$

MC solution of the evolution equation

Purpose of MC method: solve integrals, generate branchings where explicitly energy momentum conservation is applied

MC solution of the evolution equation

Purpose of MC method: solve integrals, generate branchings where explicitly energy momentum conservation is applied

First branching:
evolve from t_0 to t' obtained from $\Delta_s(t')$:

$$R_1 = \Delta_s(t'), \tag{11}$$

where R_1 is a random number in the interval $(0, 1)$.

Check:
if $t' > t$ evolution is stopped without any branching,
if $t' < t$ branching is generated according to $P(z)$

$$\int_{z_{min}}^{z} dz' P(z') = R_2 \int_{z_{min}}^{z_{max}} dz' P(z') \tag{12}$$

and the evolution continues.

MC solution of the evolution equation

Purpose of MC method: solve integrals, generate branchings where explicitly energy momentum conservation is applied

First branching:
evolve from t_0 to t' obtained from $\Delta_s(t')$:

$$R_1 = \Delta_s(t'), \tag{11}$$

where R_1 is a random number in the interval $(0, 1)$.

Check:
if $t' > t$ evolution is stopped without any branching,
if $t' < t$ branching is generated according to $P(z)$

$$\int_{z_{min}}^{z} dz' P(z') = R_2 \int_{z_{min}}^{z_{max}} dz' P(z') \tag{12}$$

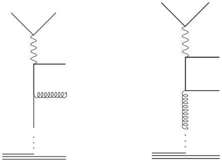
and the evolution continues.

Second branching:
evolve from t' to t'' generated according to $\Delta_s(t'', t')$.

check:
If $t'' > t$ evolution is stopped only with one branching.
If $t'' < t$ branching is generated according to $P(z)$ and the evolution continues...etc.

Evolution in the code

We consider ep collisions in which we can measure different pdfs:



Forward evolution: final parton is not specified when the evolution begins.

Four different situations:

- gluon at the beginning and at the end,
- quark at the beginning and gluon at the end,
- quark at the beginning and quark at the end,
- gluon at the beginning and quark at the end.

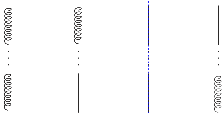
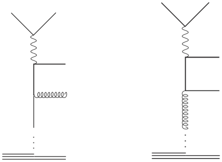


Figure : K_{gg}, K_{gq}, K_{qq},K_{qg}

Evolution in the code

We consider ep collisions in which we can measure different pdfs:



Forward evolution: final parton is not specified when the evolution begins.

Four different situations:

- gluon at the beginning and at the end,
- quark at the beginning and gluon at the end,
- quark at the beginning and quark at the end,
- gluon at the beginning and quark at the end.

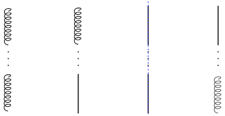


Figure : K_{gg}, K_{gq}, K_{qq},K_{qg}

The initial distributions for $u, d, s, \overline{u}, \overline{d}, \overline{s}$ and gluon come from QCDNum17 (but any other parametrization can be used).

Fitting method- grids

New approach to fitting method

Two different evolution grids are defined:

- initial quark $\rightarrow$ quark grid,
- initial gluon $\rightarrow$ gluon grid.

Kernels for evolution initiated by gluons and quarks are calculated separately only once per run of the code and combined at the end $\rightarrow$ fitting procedure is fast.

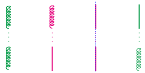


Figure : K_{gg}, K_{gq}, K_{qq},K_{qg}: gluon grid , quark grid

Fitting method- grids

New approach to fitting method

Two different evolution grids are defined:

- initial quark $\rightarrow$ quark grid,
- initial gluon $\rightarrow$ gluon grid.

Kernels for evolution initiated by gluons and quarks are calculated separately only once per run of the code and combined at the end $\rightarrow$ fitting procedure is fast.

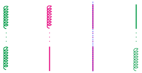


Figure : K_{gg}, K_{gq}, K_{qq}, K_{qg}: gluon grid, quark grid

To get the final pdf: evolution kernel is folded with starting distribution. For example for gluon:

$$xf(x,t)_g = x \int dx_0 \int dz \left(f_{0g}(x_0) K_{gg} + f_{0q}(x_0) K_{gq} \right) \delta(zx_0 - x), \tag{13}$$

Integrated PDFs from TMD evolution using MC method

Avoiding divergences in $P(z)$ at $z \to 1$

Some of the splitting functions are divergent for $z \to 1$.

To avoid divergences:

$$\begin{aligned}\frac{\partial xf(x,t)}{\partial t} &= \frac{\alpha_s}{2\pi}\frac{1}{t}\int_x^1 dzP(z)\frac{x}{z}f\left(\frac{x}{z},t\right) - \frac{\alpha_s}{2\pi}xf(x,t)\frac{1}{t}\int_x^1 dzP(z) \approx \\ &\approx \frac{\alpha_s}{2\pi}\frac{1}{t}\int_x^{z_{max}} dzP(z)\frac{x}{z}f\left(\frac{x}{z},t\right) - \frac{\alpha_s}{2\pi}xf(x,t)\frac{1}{t}\int_x^{z_{max}} dzP(z).\end{aligned} \tag{14}$$

Avoiding divergences in $P(z)$ at $z \to 1$

Some of the splitting functions are divergent for $z \to 1$.

To avoid divergences:

$$\begin{aligned}\frac{\partial xf(x,t)}{\partial t} &= \frac{\alpha_s}{2\pi}\frac{1}{t}\int_x^1 dzP(z)\frac{x}{z}f\left(\frac{x}{z},t\right) - \frac{\alpha_s}{2\pi}xf(x,t)\frac{1}{t}\int_x^1 dzP(z) \approx \\ &\approx \frac{\alpha_s}{2\pi}\frac{1}{t}\int_x^{z_{max}} dzP(z)\frac{x}{z}f\left(\frac{x}{z},t\right) - \frac{\alpha_s}{2\pi}xf(x,t)\frac{1}{t}\int_x^{z_{max}} dzP(z).\end{aligned} \tag{14}$$

it can be shown that terms $\int_{zmax}^{1}$ skipped in the integral in eq. (14) are of order $\mathcal{O}(1 - z_{max})$ multiplied by $xf(x,t)$ or $x\frac{df(x,t)}{dt}$

Avoiding divergences in $P(z)$ at $z \to 1$

Some of the splitting functions are divergent for $z \to 1$.

To avoid divergences:

$$\begin{aligned}\frac{\partial xf(x,t)}{\partial t} &= \frac{\alpha_s}{2\pi}\frac{1}{t}\int_x^1 dzP(z)\frac{x}{z}f\left(\frac{x}{z},t\right) - \frac{\alpha_s}{2\pi}xf(x,t)\frac{1}{t}\int_x^1 dzP(z) \approx \\ &\approx \frac{\alpha_s}{2\pi}\frac{1}{t}\int_x^{z_{max}} dzP(z)\frac{x}{z}f\left(\frac{x}{z},t\right) - \frac{\alpha_s}{2\pi}xf(x,t)\frac{1}{t}\int_x^{z_{max}} dzP(z).\end{aligned} \tag{14}$$

it can be shown that terms $\int_{z_{max}}^1$ skipped in the integral in eq. (14) are of order $\mathcal{O}(1-z_{max})$ multiplied by $xf(x,t)$ or $x\frac{df(x,t)}{dt}$

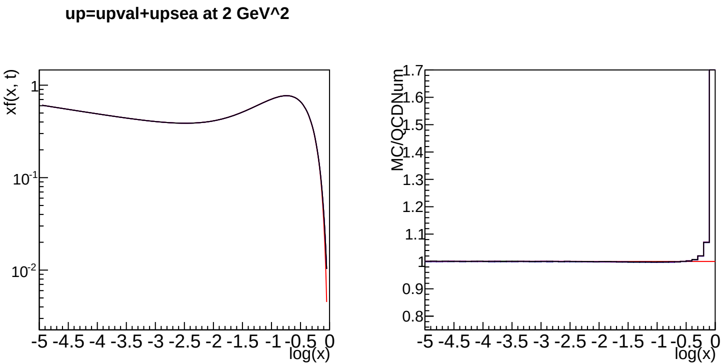
Different choices of z_{max}:

- z_{max} - fixed
- z_{max} - can change dynamically with the scale, for example:
  angular ordering: $z_{max} = 1 - \left(\frac{Q_0}{Q}\right)^2$

In this presentation: results from fixed z_{max} (dynamical will come in the future).

up quarks

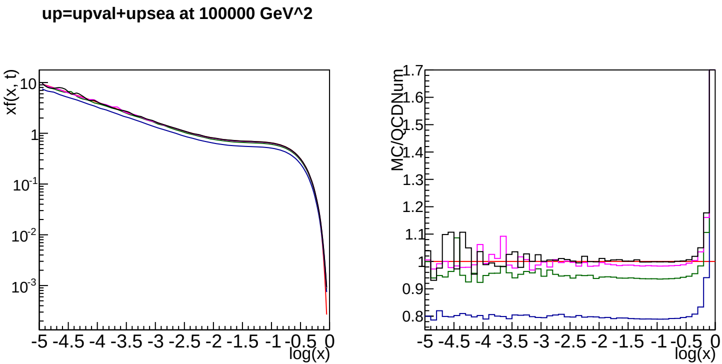
QCDNum, 1-zmax=10^{-3}, 1-zmax=10^{-5}, 1-zmax=10^{-7}, 1-zmax=10^{-9}



MC results close to the QCDNum results.
The differences between MC and QCDNum at large x are an artefact of the histogram binning.

up quarks

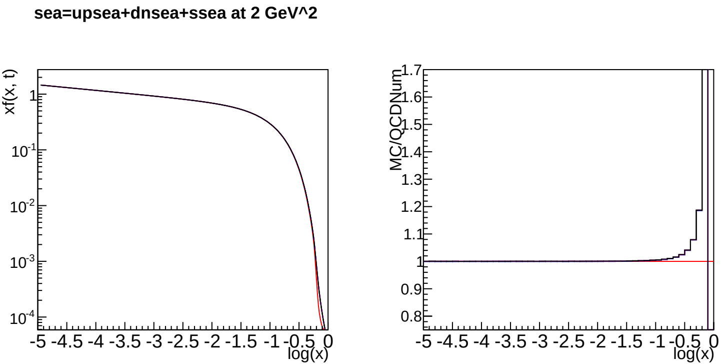
QCDNum, 1-zmax=10^{-3}, 1-zmax=10^{-5}, 1-zmax=10^{-7}, 1-zmax=10^{-9}



MC results close to the QCDNum results.
Effects on z_{max} observed: z_{max} values closer to 1 give better results:
for higher scale more splittings: effect of z_{max} being far away from 1 accumulated.
The differences between MC and QCDNum at large x are an artefact of the histogram binning.

sea quarks

QCDNum, 1-zmax=10^{-3}, 1-zmax=10^{-5}, 1-zmax=10^{-7}, 1-zmax=10^{-9}

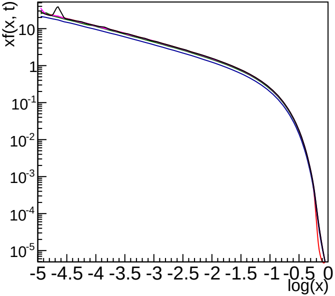


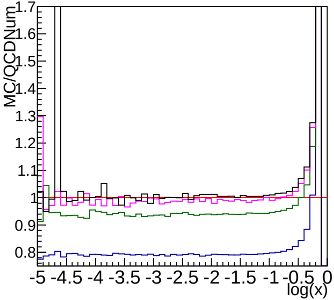
MC results close to the QCDNum results.
The differences between MC and QCDNum at large x are an artefact of the histogram binning.

sea quarks

QCDNum, 1-zmax=10^{-3}, 1-zmax=10^{-5}, 1-zmax=10^{-7}, 1-zmax=10^{-9}

sea=upsea+dnsea+ssea at 100000 GeV^2



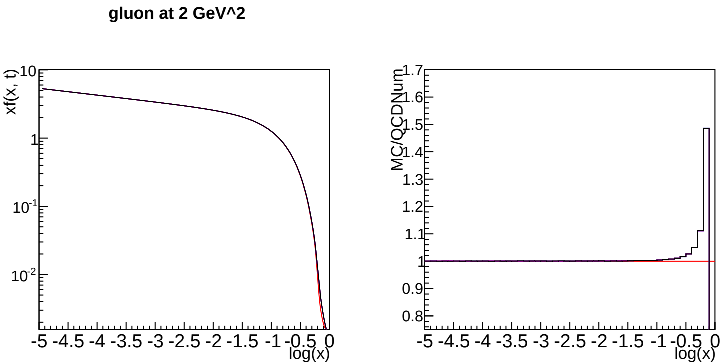


MC results close to the QCDNum results.

Effects on z_{max} observed: z_{max} values closer to 1 give better results:
for higher scale more splittings: effect of z_{max} being far away from 1 accumulated.
The differences between MC and QCDNum at x are an artefact of the histogram binning.

gluon

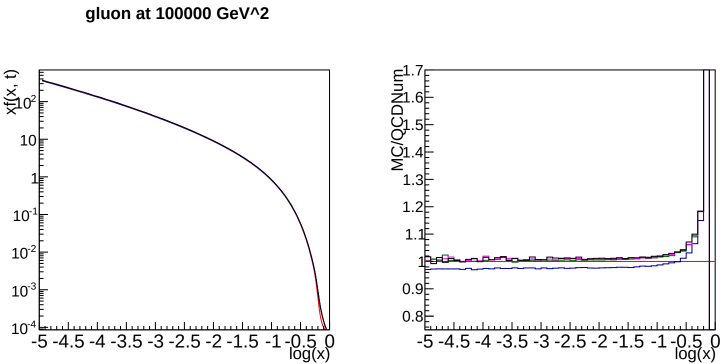
QCDNum, 1-zmax=10^{-3}, 1-zmax=10^{-5}, 1-zmax=10^{-7}, 1-zmax=10^{-9}



MC results close to the QCDNum results.
The differences between MC and QCDNum at x are an artefact of the histogram binning.

gluon

QCDNum, 1-zmax=10^{-3}, 1-zmax=10^{-5}, 1-zmax=10^{-7}, 1-zmax=10^{-9}

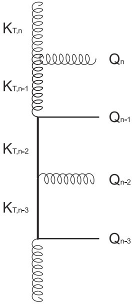


MC results close to the QCDNum results.
Effects on z_{max} observed: z_{max} values closer to 1 give better results:
for higher scale more splittings: effect of z_{max} being far away from 1 accumulated.
The differences between MC and QCDNum at x are an artefact of the histogram binning.

Results for TMDs

k_T dependence

MC method: for every branching Q is generated and Q_x and Q_y are calculated
→ The information about k_T is available for every branching.

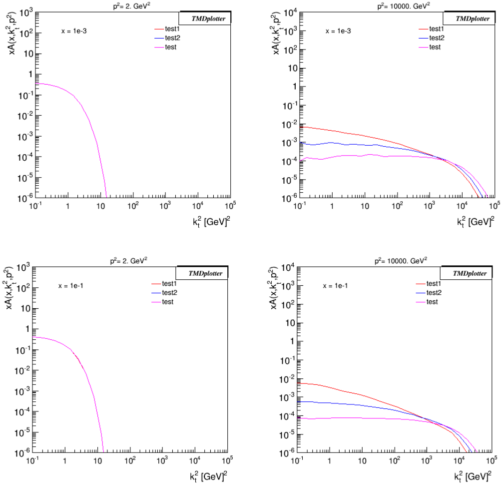


k_T contains the whole history of the evolution:

$$\vec{k}_{T,n} = \vec{k}_{T,n-1} + \vec{Q}_{T,n-1}.$$

up quarks

1-zmax=10^{-3}, 1-zmax=10^{-5}, 1-zmax=10^{-9}



- Initial scale: intrinsic k_t distribution.
- Different choices of z_{max} lead to different uTMDs, especially different large k_T tails.

First fit of full integrated TMDs to HERA DIS data with xFitter

TMDs and xFitter

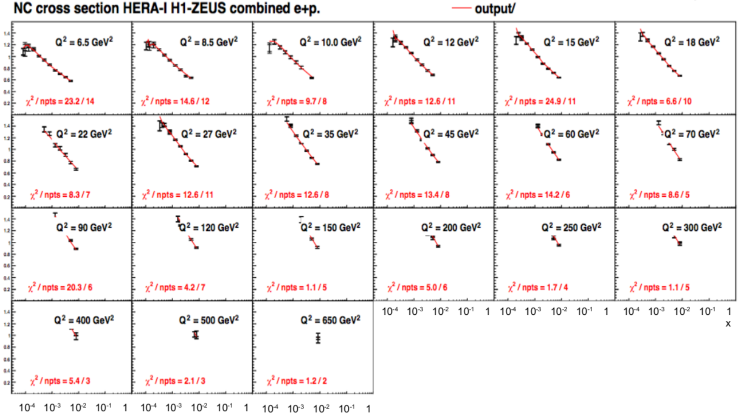
xFitter - an open source QCD fit framework to extract PDFs.

https://wiki-zeuthen.desy.de/xFitter/

What was done within xFitter with TMDs:

gluon TMD pdf fit to F_2 from H1/Zeus data for $Q^2 > 5\mathrm{GeV}^2$ for $x < 0.01$

from `herafitter` output

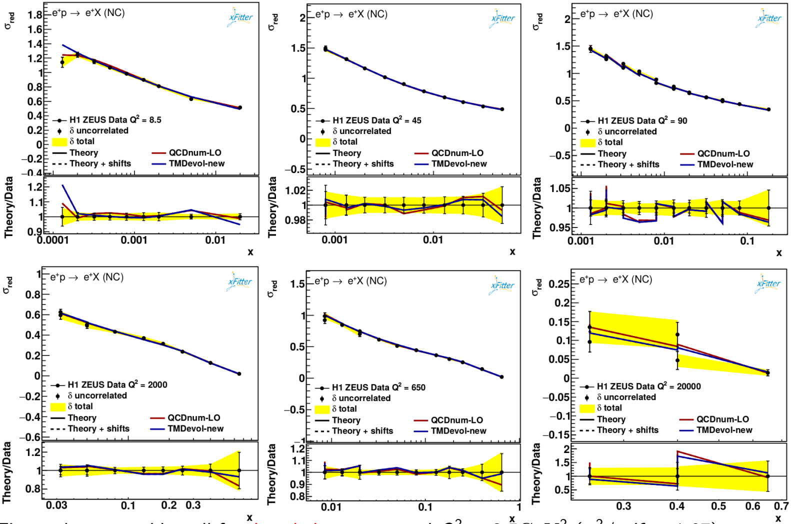


Nuclear Physics B 883 (2014) 1-19

First fit of full integrated TMDs to HERA H1 and Zeus data

WORK IN PROGRESS

Integrated TMDs for gluon, valence and sea from updfevolv were used in xFitter to fit F_2.
QCDNum convolution of integrated TMDs with collinear ME was used to obtain the structure function.



Fits works reasonably well for the whole x range and $Q^2 > 8.5\,\mathrm{GeV}^2$ ($\chi^2/ndf \approx 1.07$).

Summary

Summary

New approach to solve coupled gluon and quark DGLAP evolution equation with MC method was shown.

Advantages:

- a full TMD pdf evolution including gluon, sea and valence quarks over the full range in x and Q^2 with the k_T dependence in the whole kinematically available range (not limited to the small k_T),
- reproduce semi-analytical solution (results consistent with QCDNum),
- direct usage in PS matched calculation.

TMDs are implemented in the preliminary version of xFitter.

New results of fitting integrated TMD pdfs to F_2 with xFitter were shown:
gluon and quark are fitted for $Q^2 > 8.5\,\mathrm{GeV}^2$ for all x with $\chi^2/ndf \approx 1.07$

Prospects:

- include NLO in $P(z)$,
- development of full TMD MC - CASCADE.

Thank you!

Back up

Ordering dependence- z_{max} origin

Some of the splitting functions are divergent for $z \to 1$.

To avoid divergences:

$$\frac{\partial xf(x,t)}{\partial t} = \frac{\alpha_s}{2\pi}\frac{1}{t}\int_x^1 dzP(z)\frac{x}{z}f\left(\frac{x}{z},t\right) - \frac{\alpha_s}{2\pi}xf(x,t)\frac{1}{t}\int_x^1 dzP(z) \approx$$
$$\approx \frac{\alpha_s}{2\pi}\frac{1}{t}\int_x^{z_{max}} dzP(z)\frac{x}{z}f\left(\frac{x}{z},t\right) - \frac{\alpha_s}{2\pi}xf(x,t)\frac{1}{t}\int_x^{z_{max}} dzP(z). \tag{15}$$

Ordering dependence- z_{max} origin

Some of the splitting functions are divergent for $z \to 1$.

To avoid divergences:

$$\frac{\partial xf(x,t)}{\partial t} = \frac{\alpha_s}{2\pi}\frac{1}{t}\int_x^1 dzP(z)\frac{x}{z}f\left(\frac{x}{z},t\right) - \frac{\alpha_s}{2\pi}xf(x,t)\frac{1}{t}\int_x^1 dzP(z) \approx$$
$$\approx \frac{\alpha_s}{2\pi}\frac{1}{t}\int_x^{z_{max}} dzP(z)\frac{x}{z}f\left(\frac{x}{z},t\right) - \frac{\alpha_s}{2\pi}xf(x,t)\frac{1}{t}\int_x^{z_{max}} dzP(z). \tag{15}$$

Using the form of the splitting functions:

$$P_{ab}(\alpha_s,z) = A_{ab}(\alpha_s)\delta(1-z) + K_{ab}(\alpha_s)\frac{1}{(1-z)_+} + R_{ab}(\alpha_s) \tag{16}$$

and the expansion of $xf(\frac{x}{z},t)$

$$xf\left(\frac{x}{z},t\right) = xf(x,t) + x^2 f'(x,t)(1-z) + \mathcal{O}(1-z_{max}) \tag{17}$$

it can be shown that terms $\int_{zmax}^1$ skipped in the integral in eq. (15) are of order $\mathcal{O}(1-z_{max})$ multiplied by $xf(x,t)$ or $x\frac{df(x' t)}{dt}$

Ordering dependence- z_{max} origin

Some of the splitting functions are divergent for $z \to 1$.

To avoid divergences:

$$\frac{\partial xf(x,t)}{\partial t} = \frac{\alpha_s}{2\pi}\frac{1}{t}\int_x^1 dz P(z)\frac{x}{z}f\left(\frac{x}{z},t\right) - \frac{\alpha_s}{2\pi}xf(x,t)\frac{1}{t}\int_x^1 dz P(z) \approx$$
$$\approx \frac{\alpha_s}{2\pi}\frac{1}{t}\int_x^{z_{max}} dz P(z)\frac{x}{z}f\left(\frac{x}{z},t\right) - \frac{\alpha_s}{2\pi}xf(x,t)\frac{1}{t}\int_x^{z_{max}} dz P(z). \tag{15}$$

Using the form of the splitting functions:

$$P_{ab}(\alpha_s, z) = A_{ab}(\alpha_s)\delta(1-z) + K_{ab}(\alpha_s)\frac{1}{(1-z)_+} + R_{ab}(\alpha_s) \tag{16}$$

and the expansion of $xf(\frac{x}{z},t)$

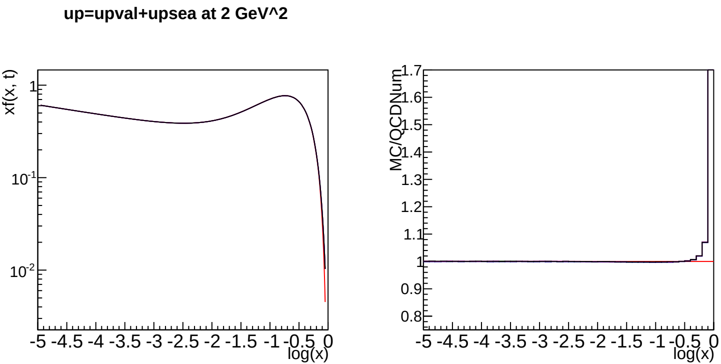
$$xf\left(\frac{x}{z},t\right) = xf(x,t) + x^2 f'(x,t)(1-z) + \mathcal{O}(1-z_{max}) \tag{17}$$

it can be shown that terms $\int_{z_{max}}^1$ skipped in the integral in eq. (15) are of order $\mathcal{O}(1-z_{max})$ multiplied by $xf(x,t)$ or $x\frac{df(x' t)}{dt}$
example:

$$\frac{\alpha_s}{2\pi}\frac{1}{t}\int_{z_{max}}^1 dz K_{ab}(\alpha_s)\frac{1}{1-z}\frac{x}{z}f\left(\frac{x}{z},t\right) - \frac{\alpha_s}{2\pi}xf(x,t)\frac{1}{t}\int_{z_{max}}^1 dz K_{ab}(\alpha_s)\frac{1}{1-z} =$$
$$= \frac{\alpha_s}{2\pi}\frac{1}{t}K_{ab}(\alpha_s)\left[x^2 f'(x,t)(1-z_{max}) + \mathcal{O}(1-z_{max})\right] \tag{18}$$

up quarks

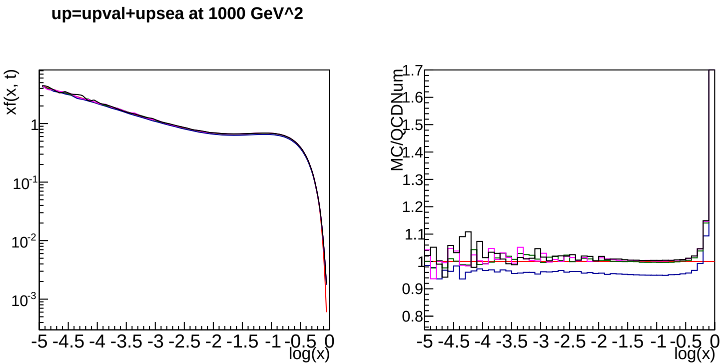
QCDNum, 1-zmax=10^{-3}, 1-zmax=10^{-5}, 1-zmax=10^{-7}, 1-zmax=10^{-9}



MC results close to the QCDNum results.
The differences between MC and QCDNum at x are an artefact of the histogram binning.

up quarks

QCDNum, 1-zmax=10^{-3}, 1-zmax=10^{-5}, 1-zmax=10^{-7}, 1-zmax=10^{-9}

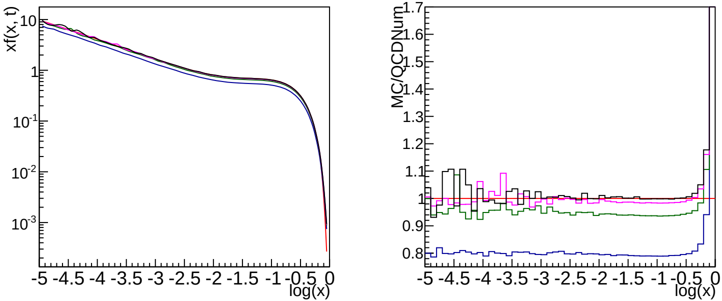


MC results close to the QCDNum results.
Effects on z_{max} observed: z_{max} values closer to 1 give better results.
The differences between MC and QCDNum at x are an artefact of the histogram binning.

up quarks

QCDNum, 1-zmax=10^{-3}, 1-zmax=10^{-5}, 1-zmax=10^{-7}, 1-zmax=10^{-9}

up=upval+upsea at 100000 GeV^2



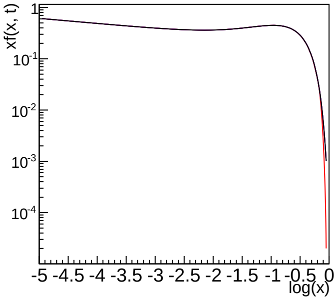
MC results close to the QCDNum results.
Effects on z_{max} observed: z_{max} values closer to 1 give better results.

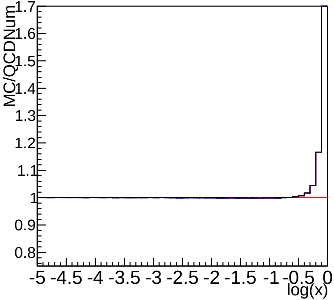
The differences between MC and QCDNum at x are an artefact of the histogram binning.

down quarks

QCDNum, 1-zmax=10^{-3}, 1-zmax=10^{-5}, 1-zmax=10^{-7}, 1-zmax=10^{-9}

dn=dnval+dnsea at 2 GeV^2

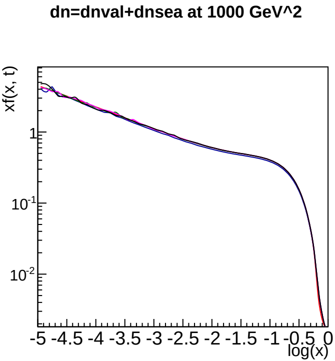


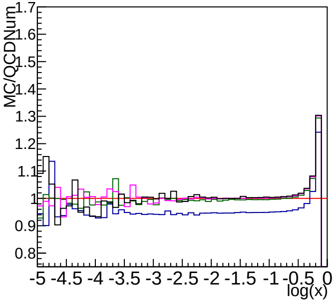


MC results close to the QCDNum results.
The differences between MC and QCDNum at x are an artefact of the histogram binning.

down quarks

QCDNum, 1-zmax=10^{-3}, 1-zmax=10^{-5}, 1-zmax=10^{-7}, 1-zmax=10^{-9}



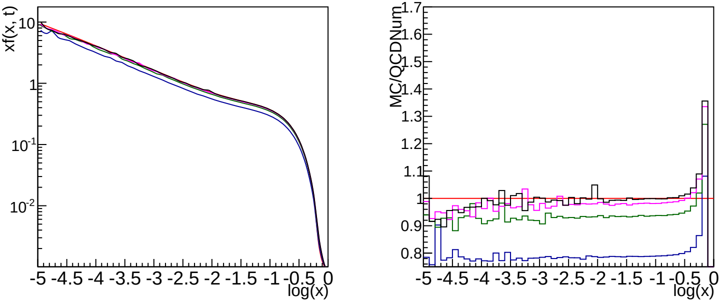


MC results close to the QCDNum results.
Effects on z_{max} observed: z_{max} values closer to 1 give better results.
The differences between MC and QCDNum at x are an artefact of the histogram binning.

down quarks

QCDNum, 1-zmax=10^{-3}, 1-zmax=10^{-5}, 1-zmax=10^{-7}, 1-zmax=10^{-9}

dn=dnval+dnsea at 100000 GeV^2

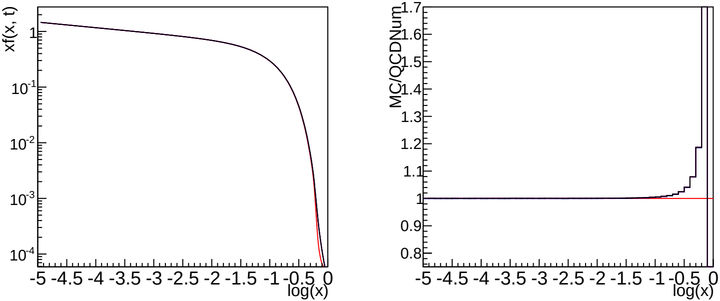


MC results close to the QCDNum results.
Effects on z_{max} observed: z_{max} values closer to 1 give better results.
The differences between MC and QCDNum at x are an artefact of the histogram binning.

sea quarks

QCDNum, 1-zmax=10^{-3}, 1-zmax=10^{-5}, 1-zmax=10^{-7}, 1-zmax=10^{-9}

sea=upsea+dnsea+ssea at 2 GeV^2

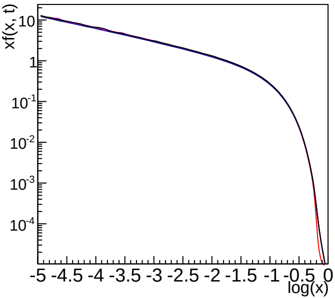


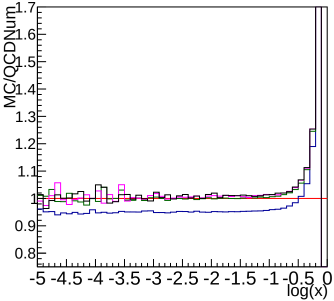
MC results close to the QCDNum results.
The differences between MC and QCDNum at x are an artefact of the histogram binning.

sea quarks

QCDNum, 1-zmax=10^{-3}, 1-zmax=10^{-5}, 1-zmax=10^{-7}, 1-zmax=10^{-9}

sea=upsea+dnsea+ssea at 1000 GeV^2

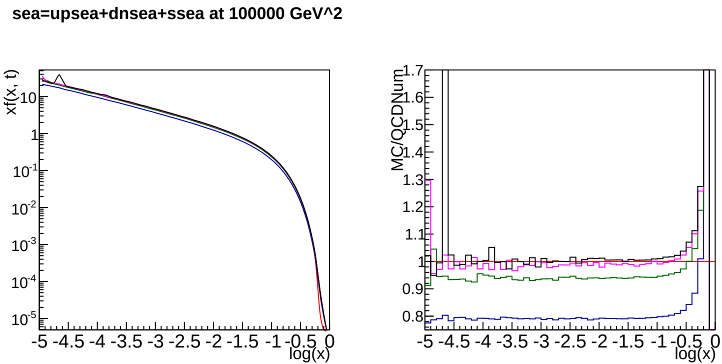




MC results close to the QCDNum results.
Effects on z_{max} observed: z_{max} values closer to 1 give better results:
for higher scale more splittings: effect of z_{max} being far away from 1 accumulated.
The differences between MC and QCDNum at x are an artefact of the histogram binning.

sea quarks

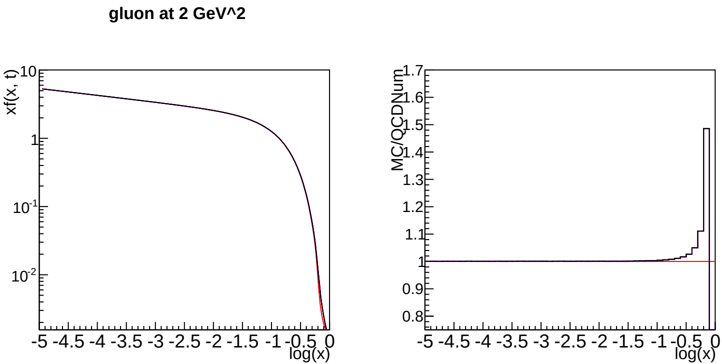
QCDNum, 1-zmax=10^{-3}, 1-zmax=10^{-5}, 1-zmax=10^{-7}, 1-zmax=10^{-9}



MC results close to the QCDNum results.
Effects on z_{max} observed: z_{max} values closer to 1 give better results:
for higher scale more splittings: effect of z_{max} being far away from 1 accumulated.
The differences between MC and QCDNum at x are an artefact of the histogram binning.

gluon

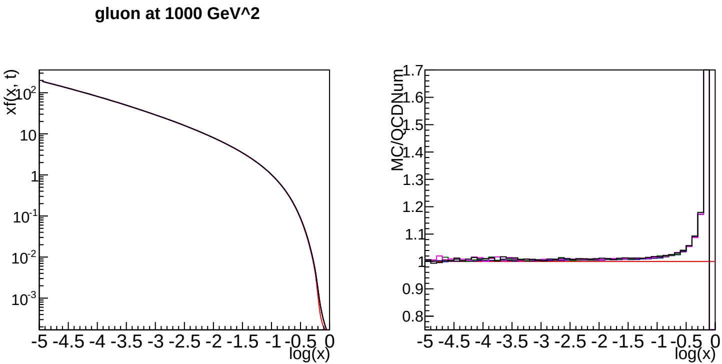
QCDNum, 1-zmax=10^{-3}, 1-zmax=10^{-5}, 1-zmax=10^{-7}, 1-zmax=10^{-9}



MC results close to the QCDNum results.
The differences between MC and QCDNum at x are an artefact of the histogram binning.

gluon

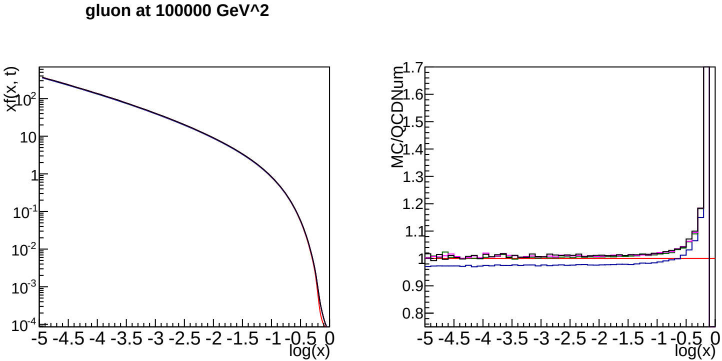
QCDNum, 1-zmax=10^{-3}, 1-zmax=10^{-5}, 1-zmax=10^{-7}, 1-zmax=10^{-9}



MC results close to the QCDNum results.
Effects on z_{max} observed: z_{max} values closer to 1 give better results:
for higher scale more splittings: effect of z_{max} being far away from 1 accumulated.
The differences between MC and QCDNum at x are an artefact of the histogram binning.

gluon

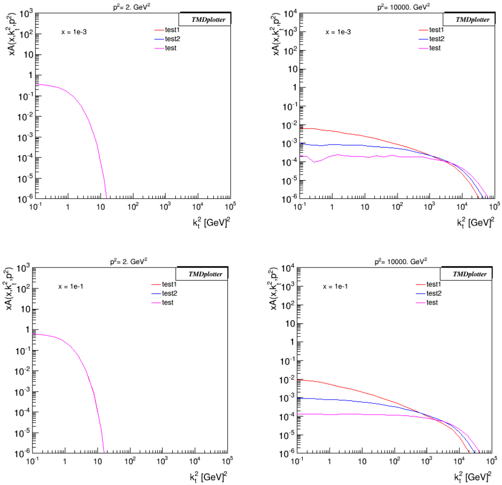
QCDNum, 1-zmax=10^{-3}, 1-zmax=10^{-5}, 1-zmax=10^{-7}, 1-zmax=10^{-9}



MC results close to the QCDNum results.
Effects on z_{max} observed: z_{max} values closer to 1 give better results:
for higher scale more splittings: effect of z_{max} being far away from 1 accumulated.
The differences between MC and QCDNum at x are an artefact of the histogram binning.

down quarks

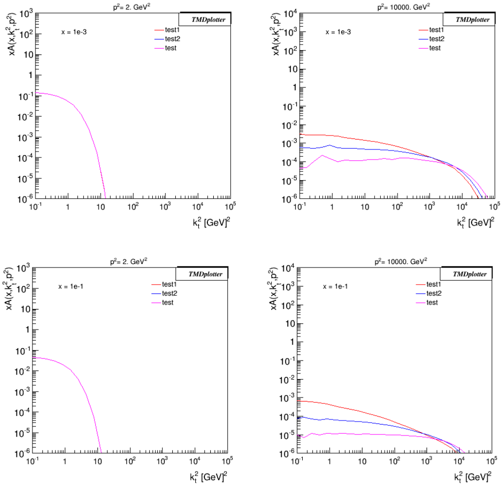
$z_{max} = 0.999$, $z_{max} = 0.99999$, $z_{max} = 0.999999999$



- Initial scale: intrinsic k_t distribution.
- Different choices of z_{max} lead to different uTMDs, especially different large k_T tails.

sea

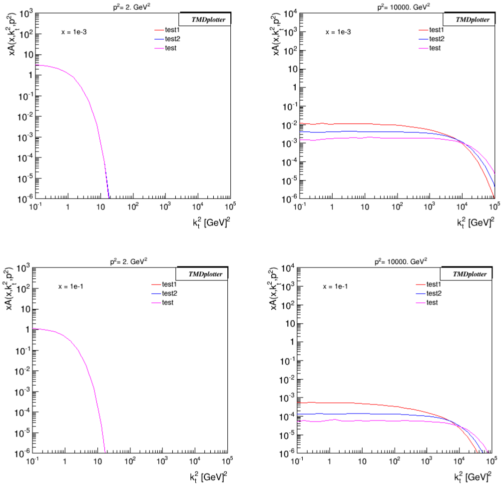
$z_{max} = 0.999$, $z_{max} = 0.99999$, $z_{max} = 0.999999999$



- Initial scale: intrinsic k_t distribution.
- Different choices of z_{max} lead to different uTMDs, especially different large k_T tails.

gluon

$z_{max} = 0.999$, $z_{max} = 0.99999$, $z_{max} = 0.999999999$



- Initial scale: intrinsic k_t distribution.
- Different choices of z_{max} lead to different uTMDs, especially different large k_T tails.