

NLO impact factor for diffractive dijet production in DIS in the shockwave formalism

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RB, A.Grabovsky, L.Szymanowski, S.Wallon

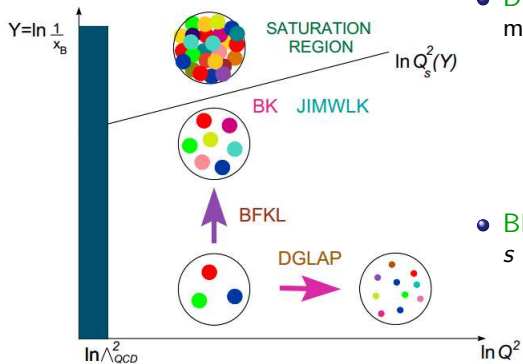
Paper in redaction. See [JHEP 409 \(2014\) 026](#) (arXiv:1405.7676)

[1512.00774\[hep-ph\]](#) [1511.02785\[hep-ph\]](#) and [1503.01782\[hep-ph\]](#) for previews

Overview

- 1 DGLAP regime vs BFKL regime
- 2 Diffractive DIS
 - Rapidity gap events at HERA
 - Collinear factorization approach
 - k_T -factorization approach : two exchanged gluons
 - Confrontation of the two approaches with HERA data
- 3 Diffractive production of jets : our approach
 - Leading Order Impact Factor
 - NLO Impact Factor : Virtual Corrections

Two regimes of perturbative QCD



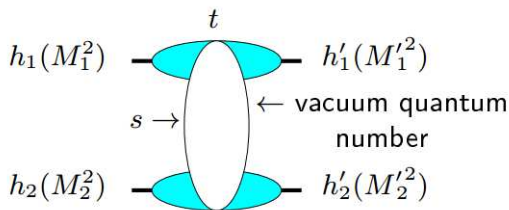
- **DGLAP** dynamics : $Q^2 \rightarrow \infty$
moderate x_B

- Governed by **collinear** dynamics
- Resummation of Q^2 logs :
 $(\alpha_s \ln Q^2)^n, \alpha_s (\alpha_s \ln Q^2)^n \dots$

- **BFKL** dynamics (Regge limit)
 $s \gg Q^2 \gg \Lambda_{QCD} \quad (x_B \ll 1)$

- Governed by **soft** dynamics
- Resummation of $\frac{1}{x_B} \sim s$ logs :
 $(\alpha_s \ln s)^n, \alpha_s (\alpha_s \ln s)^n \dots$

BFKL dynamics



$$\sigma = \frac{1}{s} \text{Im} \mathcal{A} \sim s^{\alpha(0)-1}$$

$$\alpha(0) - 1 = C \alpha_s$$

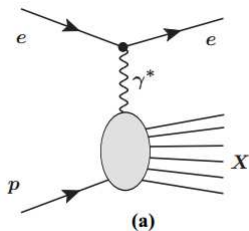
$$C > 0$$

\Rightarrow Violation of the Froissart bound

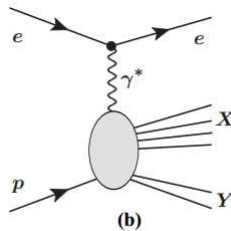
Diffractive DIS

Rapidity gap events at HERA

Experiments at HERA : about 10% of scattering events reveal a **rapidity gap**



DIS events

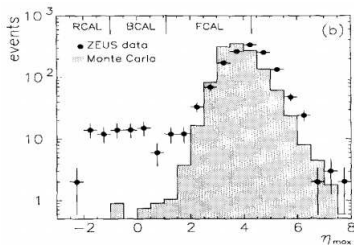


DDIS events

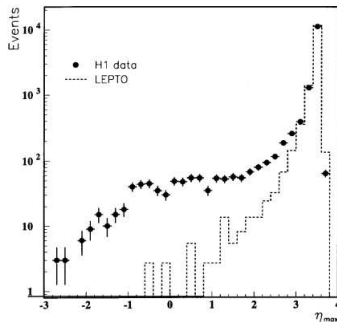
Diffractive DIS

Rapidity gap events at HERA

Experiments at HERA : about 10% of events reveal a rapidity gap



ZEUS, 1993



H1, 1994

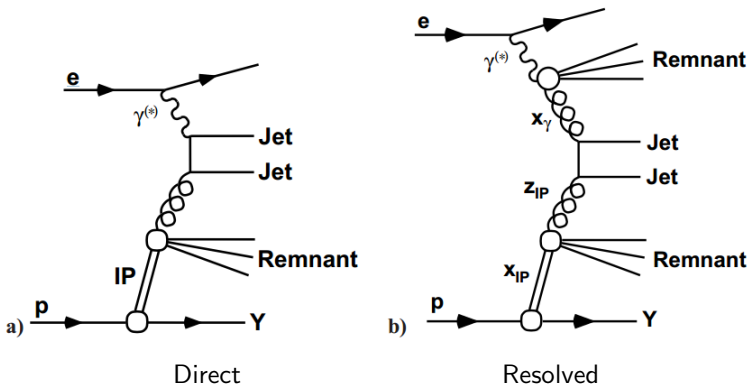
Diffractive DIS

Theoretical approaches for DDIS using pQCD

- **Collinear factorization** approach
 - Relies on QCD factorization theorem, using a hard scale such as the **virtuality Q^2** of the incoming photon
 - One needs to introduce a **diffractive distribution function** for partons *within a pomeron*
- **k_T factorization** approach for two exchanged gluons
 - low-x QCD approach : $s \gg Q^2 \gg \Lambda_{QCD}$
 - The pomeron is described as a **two-gluon color-singlet** state

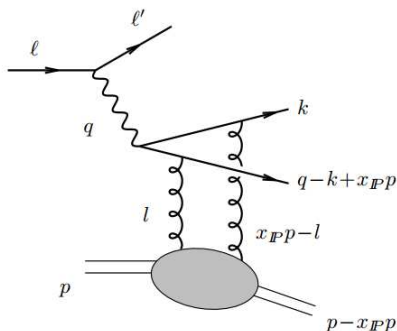
Theoretical approaches for DDIS using pQCD

Collinear factorization approach



Theoretical approaches for DDIS using pQCD

k_T -factorization approach : two gluon exchange

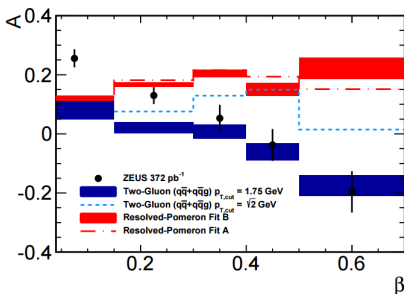


Bartels, Ivanov, Jung, Lotter, Wüsthoff

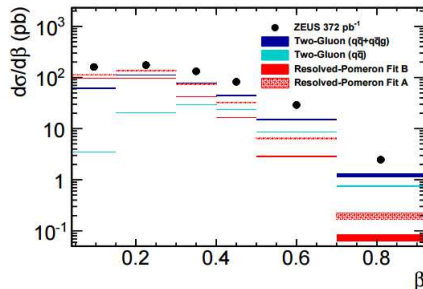
Braun and Ivanov developed a similar model in [collinear factorization](#)

Theoretical approaches for DDIS using pQCD

Confrontation of the two approaches with HERA data



ZEUS collaboration, 2015



ZEUS collaboration, 2015

Assumptions

- Regge limit : $s \gg Q^2 \gg \Lambda_{QCD}$
- **No approximation** for the outgoing gluon, contrary to e.g. :
 - **Collinear approximation** [Wüsthoff, 1995]
 - **Soft approximation** [Bartels, Jung, Wüsthoff, 1999]
- Lightcone coordinates (p^+, p^-, \vec{p}) and **lightcone gauge** $n_2 \cdot \mathcal{A} = 0$
- **Transverse dimensional regularization** $d = 2 + 2\epsilon$, **longitudinal cutoff**

$$p_g^+ < \alpha p_\gamma^+$$
- **Shockwave** (Wilson lines) approach [Balitsky, 1995]

The shockwave approach

One decomposes the gluon field \mathcal{A} into an **internal field** and an **external field** :

$$\mathcal{A}^\mu = A^\mu + b^\mu$$

The internal one contains the gluons with rapidity $p_g^+ > \alpha p_\gamma^+$ and the external one contains the gluons with rapidity $p_g^+ < \alpha p_\gamma^+$. One writes :

$$b^\mu(z) = \delta(z^+) B(\vec{z}) n_2^\mu$$

Intuitively, large boost λ along the + direction :

$$b^+(x^+, x^-, \vec{x}) \rightarrow \frac{1}{\lambda} b^+ \left(\lambda x^+, \frac{1}{\lambda} x^-, \vec{x} \right)$$

$$b^-(x^+, x^-, \vec{x}) \rightarrow \lambda b^- \left(\lambda x^+, \frac{1}{\lambda} x^-, \vec{x} \right)$$

$$b^i(x^+, x^-, \vec{x}) \rightarrow b^i \left(\lambda x^+, \frac{1}{\lambda} x^-, \vec{x} \right)$$

Propagator through a shockwave

$$G(z_2, z_0) = - \int d^4 z_1 \theta(z_2^+) \delta(z_1^+) \theta(-z_0^+) G(z_2 - z_1) \gamma^+ G(z_1 - z_0) U_1$$

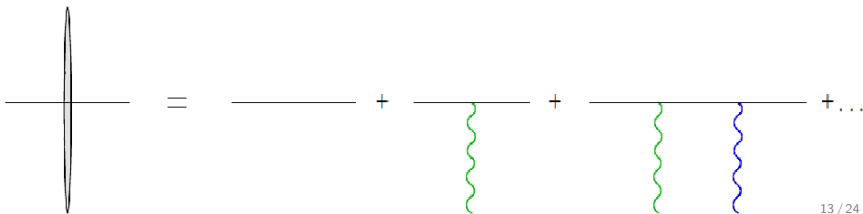
$$G(q, p) = (2\pi) \theta(p^+) \int d^D q_1 \delta(q + q_1 - p) G(q) \gamma^+ \tilde{U}_{\bar{q}_1} G(p)$$

Wilson lines :

$$U_i = U_{\bar{z}_i} = U(\bar{z}_i, \eta) = P \exp \left[ig \int_{-\infty}^{+\infty} b_{\eta}^-(z_i^+, \bar{z}_i) dz_i^+ \right]$$

$$U_i = 1 + ig \int_{-\infty}^{+\infty} b_{\eta}^-(z_i^+, \bar{z}_i) dz_i^+ + (ig)^2 \int_{-\infty}^{+\infty} b_{\eta}^-(z_i^+, \bar{z}_i) b_{\eta}^-(z_j^+, \bar{z}_j) \theta(z_{ji}^+) dz_i^+ dz_j^+$$

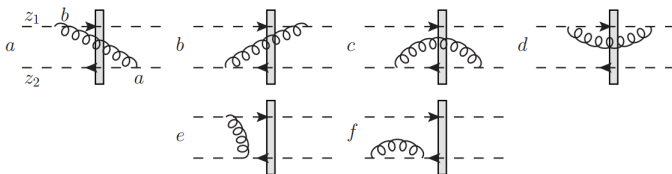
...



The BK equation

Dipole operator

$$\mathbf{U}_{12} = \frac{1}{N_c} \text{Tr} \left(U_1 U_2^\dagger \right) - 1$$

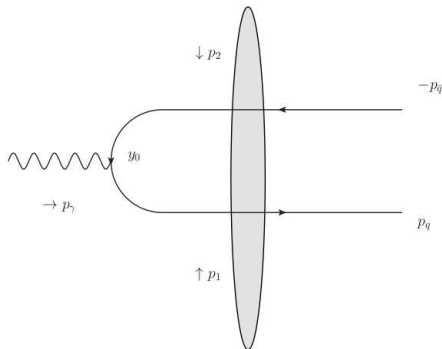


BK equation [Balitsky, 1995] [Kovchegov, 1999]

$$\frac{d\mathbf{U}_{12}}{d\ln\sigma} = \frac{\alpha_s N_c}{2\pi^2} \int d\vec{z}_3 \frac{\vec{z}_{12}^2}{\vec{z}_{13}^2 \vec{z}_{23}^2} [\mathbf{U}_{13} + \mathbf{U}_{32} - \mathbf{U}_{12} - \mathbf{U}_{13} \mathbf{U}_{32}]$$

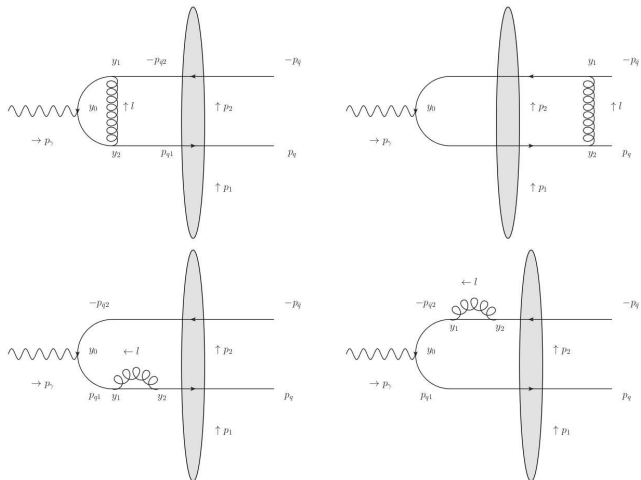
Non-linear term : **saturation**

Leading Order



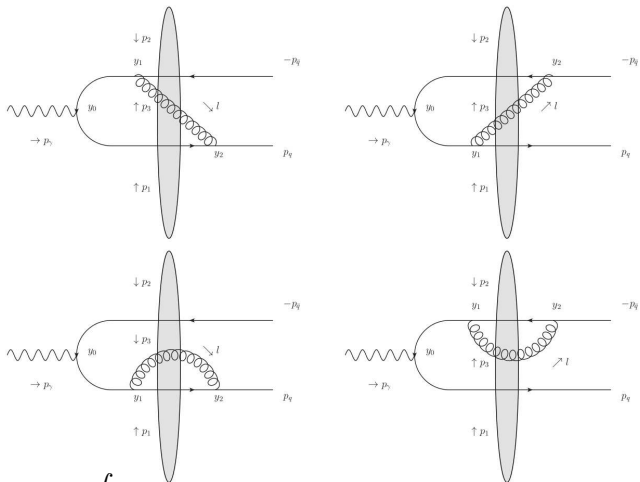
$$\mathcal{A}_0 = \varepsilon_\alpha N_c \int d^d \vec{p}_1 d^d \vec{p}_2 \Phi_0^\alpha(\vec{p}_1, \vec{p}_2) \delta(\vec{p}_{q1} + \vec{p}_{\bar{q}2}) \tilde{\mathbf{U}}_{12}$$

First kind of virtual corrections



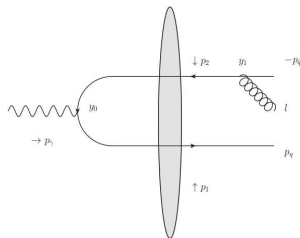
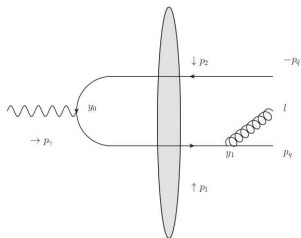
$$\mathcal{A}_{V1} \propto \varepsilon_\alpha N_c \int d^d \vec{p}_1 d^d \vec{p}_2 \Phi_{V1}^\alpha(\vec{p}_1, \vec{p}_2) \delta(\vec{p}_{q1} + \vec{p}_{\bar{q}2}) \left(\frac{N_c^2 - 1}{N_c} \right) \tilde{\mathbf{U}}_{12}$$

Second kind of virtual corrections



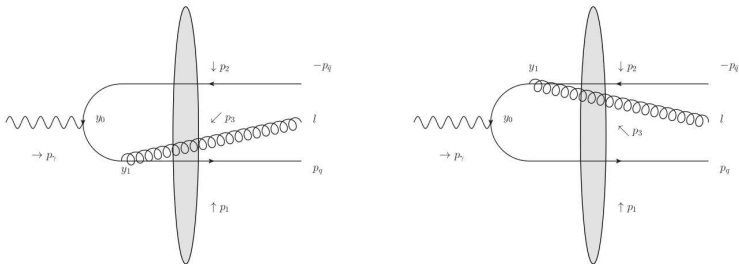
$$\begin{aligned}
 \mathcal{A}_{V2} \propto & \varepsilon_\alpha N_c \int d^d \vec{p}_1 d^d \vec{p}_2 d^d \vec{p}_3 \Phi_{V2}^\alpha(\vec{p}_1, \vec{p}_2, \vec{p}_3) \delta(\vec{p}_{q1} + \vec{p}_{q2} - \vec{p}_3) \\
 & \left[\delta(\vec{p}_3) \left(\frac{N_c^2 - 1}{N_c} \right) \tilde{\mathbf{U}}_{12} + N_c \left(\tilde{\mathbf{U}}_{13} \tilde{\mathbf{U}}_{32} + \tilde{\mathbf{U}}_{13} + \tilde{\mathbf{U}}_{32} - \tilde{\mathbf{U}}_{12} \right) \right]
 \end{aligned}$$

First kind of real corrections



$$\mathcal{A}_{R1} = \varepsilon_\alpha N_c \int d^d \vec{p}_1 d^d \vec{p}_2 \Phi_{R1}^\alpha(\vec{p}_1, \vec{p}_2) \delta(\vec{p}_{q1} + \vec{p}_{\bar{q}2} + \vec{p}_g) \left(\frac{N_c^2 - 1}{N_c} \right) \tilde{\mathbf{U}}_{12}$$

Second kind of real corrections



$$\begin{aligned}
 \mathcal{A}_{R2} = & \varepsilon_\alpha N_c \int d^d \vec{p}_1 d^d \vec{p}_2 d^d \vec{p}_3 \Phi_{R2}^\alpha(\vec{p}_1, \vec{p}_2, \vec{p}_3) \delta(\vec{p}_{q1} + \vec{p}_{q2} + \vec{p}_{g3}) \\
 & \left[\left(\frac{N_c^2 - 1}{N_c} \right) \tilde{\mathbf{U}}_{12} \delta(\vec{p}_3) + N_c \left(\tilde{\mathbf{U}}_{13} \tilde{\mathbf{U}}_{32} + \tilde{\mathbf{U}}_{13} + \tilde{\mathbf{U}}_{32} - \tilde{\mathbf{U}}_{12} \right) \right]
 \end{aligned}$$

Divergences

- UV divergence $\vec{p}_g^2 \rightarrow +\infty$

$$\Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*$$

- Soft divergence $p_g \rightarrow 0$

$$\Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*, \Phi_{R1}\Phi_{R1}^*$$

- Collinear divergence $p_g \propto p_q$ or $p_{\bar{q}}$

$$\Phi_{R1}\Phi_{R1}^*$$

- Soft and collinear divergence $p_g = \frac{p_g^+}{p_q^+} p_q$ or $\frac{p_g^+}{p_{\bar{q}}^+} p_{\bar{q}}$, $p_g^+ \rightarrow 0$

$$\Phi_{R1}\Phi_{R1}^*$$

- Rapidity divergence $p_g^+ \rightarrow 0$

$$\Phi_{V2}\Phi_0^* + \Phi_0\Phi_{V2}^*$$

Rapidity divergence

BK equation

$$\frac{\partial \tilde{\mathbf{U}}_{12}^\alpha}{\partial \log \alpha} = 2\alpha_s N_c \mu^{2-d} \int \frac{d^d \vec{k}_1 d^d \vec{k}_2 d^d \vec{k}_3}{(2\pi)^{2d}} \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 - \vec{p}_1 - \vec{p}_2) \left(\tilde{\mathbf{U}}_{13} \tilde{\mathbf{U}}_{32} + \tilde{\mathbf{U}}_{13} + \tilde{\mathbf{U}}_{32} - \tilde{\mathbf{U}}_{12} \right)$$

$$\left[2 \frac{(\vec{k}_1 - \vec{p}_1) \cdot (\vec{k}_2 - \vec{p}_2)}{(\vec{k}_1 - \vec{p}_1)^2 (\vec{k}_2 - \vec{p}_2)^2} + \frac{\pi^{\frac{d}{2}} \Gamma(1 - \frac{d}{2}) \Gamma^2(\frac{d}{2})}{\Gamma(d-1)} \left(\frac{\delta(\vec{k}_2 - \vec{p}_2)}{[(\vec{k}_1 - \vec{p}_1)^2]^{1-\frac{d}{2}}} + \frac{\delta(\vec{k}_1 - \vec{p}_1)}{[(\vec{k}_2 - \vec{p}_2)^2]^{1-\frac{d}{2}}} \right) \right]$$

η typical rapidity for the lower impact factor

$$\tilde{\mathbf{U}}_{12}^\alpha \Phi_0 \rightarrow \Phi_0 \tilde{\mathbf{U}}_{12}^\eta + \log\left(\frac{e^\eta}{\alpha}\right) \mathcal{K}_{BK} \Phi_0 \left(\tilde{\mathbf{U}}_{13} \tilde{\mathbf{U}}_{32} + \tilde{\mathbf{U}}_{13} + \tilde{\mathbf{U}}_{32} - \tilde{\mathbf{U}}_{12} \right)$$

Then $\tilde{\mathbf{U}}_{12}^\alpha \Phi_0 + \Phi_{V2}$ is **finite**.

Soft and collinear divergence

Jet cone algorithm

We define a **cone** width for each pair of particles with momenta p_i and p_k , rapidity difference ΔY_{ik} and relative azimuthal angle $\Delta\phi_{ik}$

$$(\Delta Y_{ik})^2 + (\Delta\phi_{ik})^2 = R_{ik}^2$$

If $R_{ik}^2 < R^2$, then the two particles together define a **single jet** of momentum $p_i + p_k$.

Applying this to our results in the small R^2 limit cancels the **soft and collinear** divergence.

Remaining divergence

Soft real emission

$$(\Phi_{R1}\Phi_{R1}^*)_{soft} \propto (\Phi_0\Phi_0^*) \int_{\text{outside the cones}} \left| \frac{p_q^\mu}{(p_q \cdot p_g)} - \frac{p_{\bar{q}}^\mu}{(p_{\bar{q}} \cdot p_g)} \right|^2 \frac{dp_g^+}{p_g^+} \frac{d^d p_g}{(2\pi)^d}$$

Collinear real emission

$$(\Phi_{R1}\Phi_{R1}^*)_{col} \propto (\Phi_0\Phi_0^*) (\mathcal{N}_q + \mathcal{N}_{\bar{q}})$$

Where \mathcal{N} is the number of jets in the quark or the antiquark

$$\mathcal{N}_k = \frac{(4\pi)^{\frac{d}{2}}}{\Gamma(2 - \frac{d}{2})} \int_{\alpha p_\gamma^+}^{p_{jet}^+} \frac{dp_g^+ dp_k^+}{2p_g^+ 2p_k^+} \int_{\text{in cone } k} \frac{d^d \vec{p}_g d^d \vec{p}_k}{(2\pi)^d \mu^{d-2}} \frac{\text{Tr}(\hat{p}_k \gamma^\mu \hat{p}_{jet} \gamma^\nu) d_{\mu\nu}(p_g)}{2p_{jet}^+ (p_k^- + p_g^- - p_{jet}^-)^2}$$

Those two contributions **cancel exactly the virtual divergences** (both UV and soft)

Conclusion and applications

- The paper is being written. The results will be used for [HERA phenomenology](#) and for [predictions for an Electron Ion Collider](#)
- Our results can also be used for [photoproduction](#) at [large \$t\$](#) or with a [large diffractive mass](#)
- They can be adapted to [ultraperipheral collisions](#) at the LHC
- They can be used to compute the impact factor for the production of a ρ meson
- In the future, we can study the massive quark case, and later on adapt it for the production of a J/ψ meson using the Color Evaporation Model [see tomorrow's talk]