Lorentz invariance relations for twist-3 functions and frame-independence of twist-3 cross sections

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See, for the detail,
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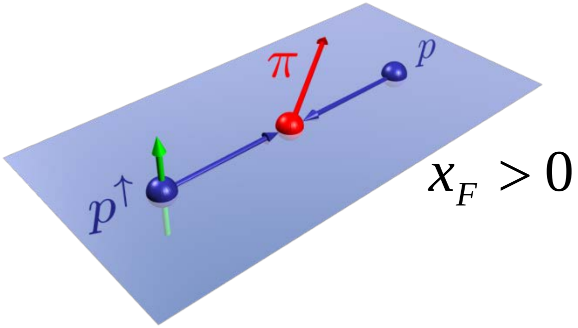
Contents:

⋆ Single (Transverse) Spin Asymmetry (SSA)

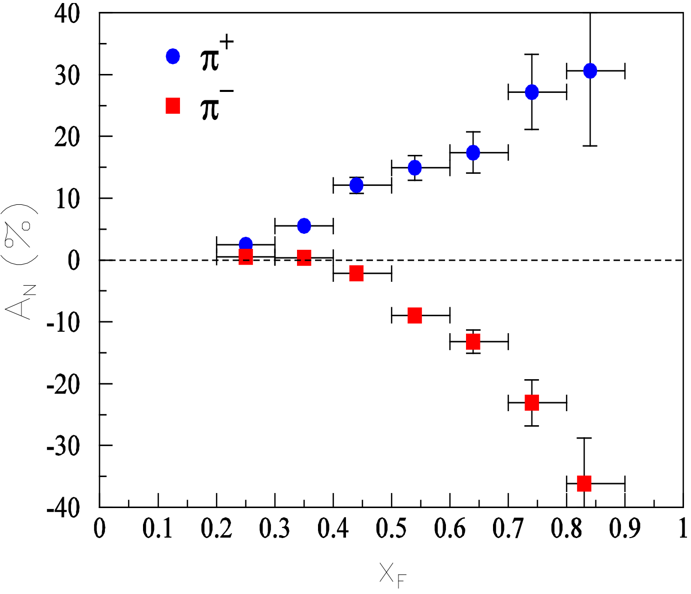
- $p^{\uparrow} p \to \pi X$ $\qquad A_N = \dfrac{\sigma^{\uparrow} - \sigma^{\downarrow}}{\sigma^{\uparrow} + \sigma^{\downarrow}}$



- FNAL-E704('91)($\sqrt{s} = 20$ GeV), RHIC($\sqrt{s} = 200$, 62 GeV): $A_N \sim 0.3$ at large $x_F = 2p_{\parallel}/\sqrt{s}$.

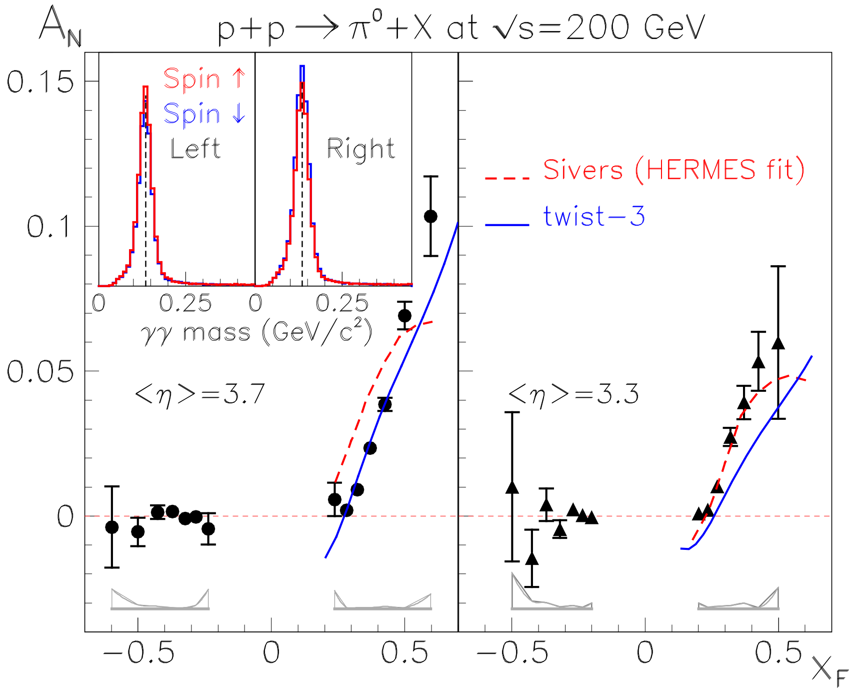
FNAL-E704

$\sqrt{s} = 20$ GeV



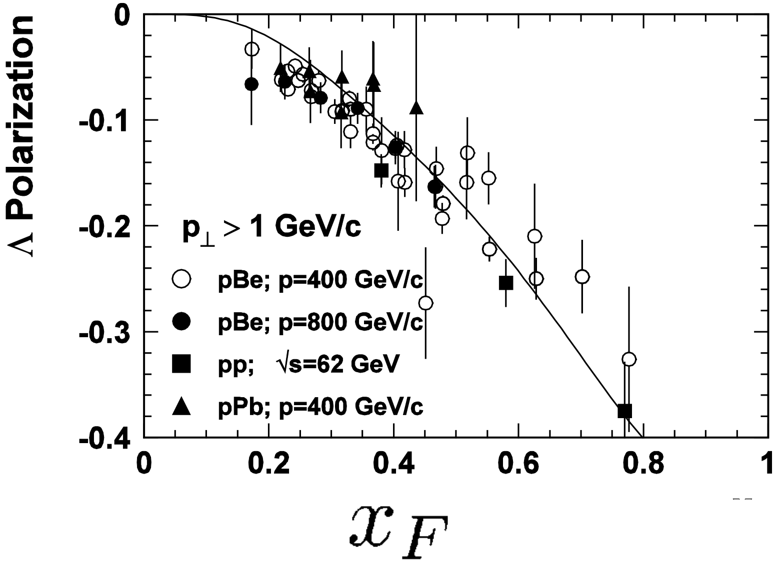
P.L. B264 ('91) 462
P.L. B261 ('91) 201

RHIC-STAR, PRL92('04)
hep-ex/0801.2990, PRL 101(2008)
$\sqrt{s} = 200$ GeV



⋆ Hyperon polarizations in unpolarized pp collisions

$$pp \to \Lambda^{\uparrow} X \text{ (in 80's and 90's)}$$



Also for other Hyperons.

- SSAs: naively "T-odd" twist-3 observables.
- A_{LT}: naively "T-even" twist-3 observables.
- Technique of calculating both types of twist-3 cross sections associated with twist-3 distribution/fragmentation functions is well developed in LO QCD.

⋆ We derive the complete set of the relations among all kinds of twist-3 distribution/fragmentation functions, and show that they are crucial to gurantee the Lorentz-invariance of the twist-3 cross sections.

2. Twist-3 distribution and fragmentation functions

2.1 Classifications

-Three types of twist-3 distribution/fragmentation functions:

Intrinsic, dynamical and kinematical twist-3 functions

- Twist-2 and 3- (collinear) quark distribution functions for the nucleon

$$\Phi_{ij}(x) = \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS|\bar{\psi}_j(0)[0,\lambda n]\psi_i(\lambda n)|PS\rangle \qquad [0,\lambda n] = P\exp\left\{ ig\int_\lambda^0 dt\, n_\mu A^\mu(tn) \right\}$$

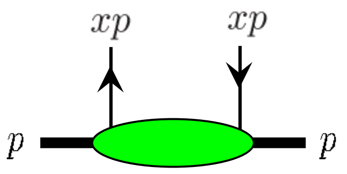
$p^2 = n^2 = 0,\ p\cdot n = 1$ Support: $|x| < 1$

Quark spin

		Ave.	$S_\parallel$	$S_\perp$
Nucleon spin	Ave.	$f_1(x)$, $e(x)$		
	$S_\parallel$		$g_1(x)$	$h_L(x)$
	$S_\perp$		$g_T(x)$	$h_1(x)$

Twist-2

Twist-3



Refered to "*intrinsic* twist-3" distributions

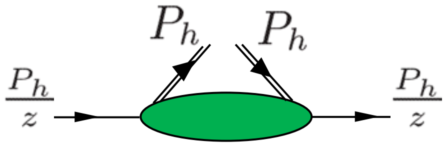
- Twist-2 and 3- (collinear) quark fragmentation functions

$$\Delta_{ij}(z) = \frac{1}{N}\sum_X \int \frac{d\lambda}{2\pi} e^{-i\lambda/z} \langle 0|[\pm\infty w, 0]\psi_i(0)|P_h S_h; X\rangle\langle P_h S_h; X|\bar{\psi}_j(\lambda w)[\lambda w, \pm\infty w]|0\rangle$$

$p_h^2 = w^2 = 0,\ p_h\cdot w = 1$ Support: $0 < z < 1$

Quark spin

		Ave.	$S_\parallel$	$S_\perp$
Hadron spin	Ave.	$D_1(z)$, $E(z)$, $H(z)$		
	$S_\parallel$	$E_L(z)$	$G_1(z)$	$H_L(z)$
	$S_\perp$	$D_T(z)$	$G_T(z)$	$H_1(z)$



Twist-2

Twist-3, "T-even"

Twist-3, "T-odd"

Refered to "*intrinsic* twist-3" fragmentation functions

· No constraint from T-invariance.

- *Dynamical* twist-3 distributions ("*F*-type" distribution)

$$\Phi_{F\,ij}(x_1,x_2)=\int\frac{d\lambda}{2\pi}\int\frac{d\mu}{2\pi}e^{i\lambda x_1+i\mu(x_2-x_1)}\langle PS|\bar{\psi}_j(0)gF^{\rho\tau}(\mu n)n_\tau\psi_i(\lambda n)|PS\rangle$$

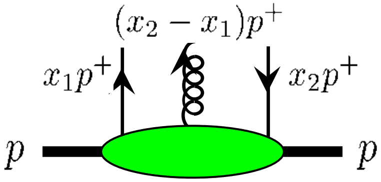
(gauge-link suppressed)

Nucleon spin

Ave.	$H_{FU}(x_1,x_2)$
$S_\parallel$	$H_{FL}(x_1,x_2)$
$S_\perp$	$F_{FT}(x_1,x_2)$, $G_{FT}(x_1,x_2)$

chiral-even

chiral-odd



(Other notations: $E_{FU}\sim E_F$, $F_{FT}\sim G_F$, $G_{FT}\sim\widetilde{G}_F$)

· These 4 functions are real.

· Support: $|x_1|<1$, $|x_2|<1$, $|x_2-x_1|<1$.

· H_{FU}, F_{FT} are symmetric, and H_{FL}, G_{FT} are anti-symmetric under $x_1\leftrightarrow x_2$ by P and T-invariance.

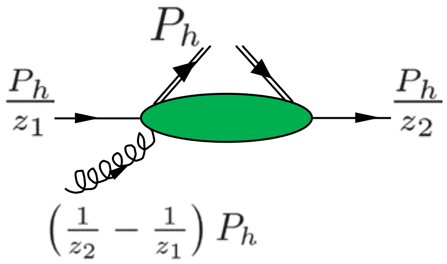
· $gF^{\rho\tau}(\mu n)n_\tau\to D_\perp^\rho(\mu n)$ defines another set of twist-3 distributions ("*D*-type" functions), which are related to the above "*F*-type" functions.

- *Dynamical* twist-3 fragmentation functions (“*F*-type” fragmentation function)

$$\Delta_{F\,ij}(z_1,z_2)=\frac{1}{N}\sum_X\int\frac{d\lambda}{2\pi}\int\frac{d\mu}{2\pi}\,e^{-i\frac{\lambda}{z_1}-i\mu\left(\frac{1}{z_2}-\frac{1}{z_1}\right)}\langle 0|[\pm\infty w,0]\psi_i(0)|P_hS_h,X\rangle$$
$$\times\langle P_hS_h,X|\bar{\psi}_j(\lambda w)[\lambda w,\mu w]gF^{\rho\tau}(\mu w)w_\tau[\mu w,\pm\infty w]|0\rangle$$

Hadron spin	Ave.	$\widehat{H}_{FU}(z_1,z_2)$
	$S_\parallel$	$\widehat{H}_{FL}(z_1,z_2)$
	$S_\perp$	$\widehat{D}_{FT}(z_1,z_2)$, $\widehat{G}_{FT}(z_1,z_2)$

chiral-even
chiral-odd



· These 4 functions are complex, and have no definite symmetry properties, since *T*-invariance gives no such constraint. (↔ Twist-3 distributions.)

· Support: $1 > z_2 > 0$ and $\infty > z_1 > z_2$.

· $gF^{\rho\tau}(\mu n)n_\tau \to D_\perp^\rho(\mu n)$ defines another set of twist-3 fragmentation functions (”*D*-type” functions), which are related to the above ”*F*-type” functions.

- *Kinematical* twist-3 distribution: $k_\perp$-moment of twist-2 TMD distributions

Quark spin

Nucleon spin

	Ave.	$S_\parallel$	$S_\perp$
Ave.			$h_1^{\perp(1)}(x)$
$S_\parallel$			$h_{1L}^{\perp(1)}(x)$
$S_\perp$	$f_{1T}^{\perp(1)}(x)$	$g_{1T}^{(1)}(x)$	

"T-even"
"T-odd"

Identities:
$f_{1T}^{\perp(1)}(x) = \pi F_{FT}(x,x)$
$h_1^{\perp(1)}(x) = \pi H_{FU}(x,x)$
(SGP function)

$$\Phi^\rho_\partial(x) = \int d^2k_T k_T^\rho \Phi(x,k_T)$$

$$\Phi_{ij}(x,k_T) = \int \frac{d\lambda}{2\pi}\int \frac{d^2\xi_T}{(2\pi)^2} e^{i\lambda x + i\xi_T\cdot k_T}\langle PS|\bar\psi_j(0)[0,\lambda n+\xi_T]\psi_i(\lambda n+\xi_T)|PS\rangle$$

- *Kinematical* twist-3 fragmentation functions: $k_\perp$-moment of twist-2 TMD fragmentation functions

Quark spin

Hadron spin

	Ave.	$S_\parallel$	$S_\perp$
Ave.			$H_1^{\perp(1)}(z)$
$S_\parallel$			$H_{1L}^{\perp(1)}(z)$
$S_\perp$	$D_{1T}^{\perp(1)}(z)$	$G_{1T}^{(1)}(z)$	

"T-even"
"T-odd"

Other notation: $H_1^{\perp(1)}(z) \sim \hat{H}(z)$

$$\Delta^\rho_\partial(x) = \int d^2k_T k_T^\rho \Delta(x,k_T)$$

$$\Delta_{ij}(z,k_T) = \frac{1}{N}\sum_X \int\frac{d\lambda}{2\pi}\int\frac{d^2\xi_T}{(2\pi)^2} e^{-i\lambda/z - ik_T\cdot\xi_T}\langle 0|[\pm\infty w,0]\psi_i(0)|P_hS_h;X\rangle$$
$$\times\langle P_hS_h;X|\bar\psi_j(\lambda w+\xi_T)[\lambda w+\xi_T,\pm\infty w+\xi_T]|0\rangle$$

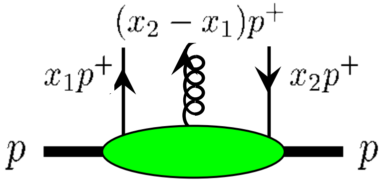
2.2 EOM relations and Lorentz invariance relations among twist-3 distribution/fragmentation functions

⋆ Example: twist-3 distributions in transversely polarized nucleon.

- Quark-gluon correlation functions

"*F*-type" (dynamical)

$$\int \frac{d\lambda}{2\pi}\int \frac{d\mu}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle PS|\bar{\psi}_j(0)\, gF^{\alpha\beta}(\mu n) n_\beta \psi_i(\lambda n)|PS\rangle$$
$$= \frac{M_N}{2}(\not{p})_{ij}\, \epsilon^{\alpha p n S_\perp} F_{FT}(x_1,x_2) - i\frac{M_N}{2}(\gamma_5 \not{p})_{ij}\, S_\perp^\alpha G_{FT}(x_1,x_2) + \cdots$$



"*D*-type"

$$\int \frac{d\lambda}{2\pi}\int \frac{d\mu}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle PS|\bar{\psi}_j(0) D_\perp^\alpha(\mu n) \psi_i(\lambda n)|PS\rangle$$
$$= \frac{M_N}{2}(\not{p})_{ij}\, \epsilon^{\alpha p n S_\perp} F_{DT}(x_1,x_2) - i\frac{M_N}{2}(\gamma_5 \not{p})_{ij}\, S_\perp^\alpha G_{DT}(x_1,x_2) + \cdots$$

M_N: Nucleon mass.

$$p^2 = n^2 = 0,\ p\cdot n = 1$$

- Relation between *D*-type and *F*-type from operator identity:

(Eguchi, YK, Tanaka, NPB752('06)1)

$$F_{DT}(x_1,x_2) = P\frac{1}{x_1-x_2}F_{FT}(x_1,x_2),$$
$$G_{DT}(x_1,x_2) = \delta(x_1-x_2)g_{1T}^{(1)}(x_1) + P\frac{1}{x_1-x_2}G_{FT}(x_1,x_2)$$

- EOM relation:

$$g_T(x) = -\frac{1}{x}\int dx' \left[F_{DT}(x,x') - G_{DT}(x,x')\right]$$
$$= \frac{1}{x}\left[g_{1T}^{(1)}(x) - P\int_{-1}^{1} dx' \frac{F_{FT}(x,x')-G_{FT}(x,x')}{x-x'}\right]$$

- Relations from identities for the nonlocal operators *not* on the lightcone.

(Eguchi, YK, Tanaka, NPB752('06)1)

· Operator identity for $z^2 \neq 0$:

$$z_\mu \left(\frac{\partial}{\partial z_\mu} \bar{\psi}(0)\gamma^\alpha \gamma_5 [0,z]\psi(z) - \frac{\partial}{\partial z_\alpha} \bar{\psi}(0)\gamma^\mu \gamma_5 [0,z]\psi(z) \right)$$

Balitsky, Braun, NPB311 (1988/89) 541.

$$= \int_0^1 dt\, \bar{\psi}(0)[0,tz]\not{z} \left\{ i\gamma_5 \left(t - \frac{1}{2} \right) gF^{\alpha\rho}(tz) z_\rho - \frac{1}{2} g\widetilde{F}^{\alpha\rho} z_\rho \right\} [tz,z]\psi(z)$$

$$+ \left[\bar{\psi}(0)\gamma_5 \sigma^{\alpha\rho} z_\rho i\not{D}\psi(z) - \bar{\psi}(0) i\overleftarrow{\not{D}}\gamma_5 \sigma^{\alpha\rho} z_\rho \psi(z) \right] + \text{(total translation)}$$

(Exact up to twist-4 ($O(z^2)$) correction.)

→ Fully incorporate all the constraints from Lorentz invariance.

(→ "Lorentz invariance relation (LIR)")

· Take the matrix element w.r.t. the nucleon state and let $z^2 \to 0$, one obtains

$$g_1^q(x) + x\frac{dg_T^q(x)}{dx} = -\mathcal{P}\int_{-1}^{1} dx' \frac{1}{x-x'} \left[\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial x'} \right) F_{FT}^q(x,x') - \left(\frac{\partial}{\partial x} - \frac{\partial}{\partial x'} \right) G_{FT}^q(x,x') \right].$$

· Combination with the EOM relation (previous slide) gives LIR:

$$g_T^q(x) = g_1^q(x) + \frac{d}{dx} g_{1T}^{(1),q}(x) - 2\,\mathcal{P}\int_{-1}^{1} dx' \frac{G_{FT}^q(x,x')}{(x-x')^2}.$$

⋆ Intrinsic and kinematical twist-3 distributions $g_T(x)$ and $g_{1T}^{(1)}(x)$ can be written in terms of the twist-2 distribution $g_1(x)$ and dynamical twist-3 distribution functions.

-Extension to twist-3 fragmentation functions

⋆ EOM relations for twist-3 fragmentation functions

"T-even"

"T-odd"

Unpolarized

real part

$$-\frac{E^q(z)}{2z} = \int_z^\infty \frac{dz'}{z'^2} \frac{\Re[\hat{H}^q_{FU}(z,z')]}{\frac{1}{z}-\frac{1}{z'}} - \frac{m_q}{2M_h} D_1^q(z) .$$

imaginary part

$$H_1^{\perp(1),q}(z) = -\frac{H^q(z)}{2z} + \int_z^\infty \frac{dz'}{z'^2} \frac{\Im[\hat{H}^q_{FU}(z,z')]}{\frac{1}{z}-\frac{1}{z'}},$$

Transversely polarized

$$G_{1T}^{(1),q}(z) = \frac{G_T^q(z)}{z} - \frac{m_q}{M_h} H_1^q(z) + \int_z^\infty \frac{dz'}{z'^2} \frac{\Re[\hat{D}^q_{FT}(z,z')] - \Re[\hat{G}^q_{FT}(z,z')]}{\frac{1}{z}-\frac{1}{z'}}$$

$$D_{1T}^{\perp(1),q}(z) = -\frac{D_T^q(z)}{z} + \int_z^\infty \frac{dz'}{z'^2} \frac{\Im[\hat{D}^q_{FT}(z,z')] - \Im[\hat{G}^q_{FT}(z,z')]}{\frac{1}{z}-\frac{1}{z'}},$$

Longitudinally polarized

$$H_{1L}^{\perp(1),q}(z) = -\frac{H_L^q(z)}{2z} + \frac{m_q}{2M_h} G_1^q(z) - \int_z^\infty \frac{dz'}{z'^2} \frac{\Re[\hat{H}^q_{FL}(z,z')]}{\frac{1}{z}-\frac{1}{z'}} .$$

$$\frac{E_L^q(z)}{2z} = -\int_z^\infty \frac{dz'}{z'^2} \frac{\Im[\hat{H}^q_{FL}(z,z')]}{\frac{1}{z}-\frac{1}{z'}},$$

⋆ Lorentz invariance relations (LIR) for twist-3 fragmentation functions

- "T-odd" fragmentation functions

· Transversely polarized spin=1/2 baryon:

$$\frac{-1}{z}\frac{d}{d(1/z)}\left(\frac{D_T^q(z)}{z}\right)=\int d\left(\frac{1}{z_1}\right)\mathcal{P}\left(\frac{1}{1/z-1/z_1}\right)\times\left\{\left(\frac{\partial}{\partial(1/z)}+\frac{\partial}{\partial(1/z_1)}\right)\frac{\Im[\hat{G}_{FT}^q(z,z_1)]}{z}-\left(\frac{\partial}{\partial(1/z)}-\frac{\partial}{\partial(1/z_1)}\right)\frac{\Im[\hat{D}_{FT}^q(z,z_1)]}{z}\right\}.$$

$$\longrightarrow \text{LIR:}\quad \frac{D_T^q(z)}{z}=-\left(1-z\frac{d}{dz}\right)D_{1T}^{\perp(1),q}(z)-\frac{2}{z}\int_z^\infty\frac{dz'}{z'^2}\frac{\Im[\hat{D}_{FT}^q(z,z')]}{(1/z-1/z')^2}.$$

· Unpolarized hadron

$$\frac{1}{z^2}\frac{dH^q(z)}{d(1/z)}=2\mathcal{P}\int d\left(\frac{1}{z_1}\right)\frac{1}{1/z_1-1/z}\left(\frac{\partial}{\partial\,(1/z_1)}-\frac{\partial}{\partial\,(1/z)}\right)\frac{\Im[\hat{H}_{FU}^q(z,z_1)]}{z}.$$

$$\longrightarrow \text{LIR:}\quad \frac{H^q(z)}{z}=-\left(1-z\frac{d}{dz}\right)H_1^{\perp(1),q}(z)-\frac{2}{z}\int_z^\infty\frac{dz'}{z'^2}\frac{\Im[\hat{H}_{FU}^q(z,z')]}{(1/z-1/z')^2}.$$

- "T-even" fragmentation functions

-LIRs can be obtained in parallel with the twist-3 distributions.

$$\frac{G_T^q(z)}{z}=\frac{G_1^q(z)}{z}+\left(1-z\frac{d}{dz}\right)G_{1T}^{(1),q}(z)-\frac{2}{z}\int_z^\infty\frac{dz'}{z'^2}\frac{\Re[\hat{G}_{FT}^q(z,z')]}{(1/z-1/z')^2},$$

$$\frac{H_L^q(z)}{z}=\frac{H_1^q(z)}{z}-\left(1-z\frac{d}{dz}\right)H_{1L}^{\perp(1),q}(z)+\frac{2}{z}\int_z^\infty\frac{dz'}{z'^2}\frac{\Re[\hat{H}_{FL}^q(z,z')]}{(1/z-1/z')^2},$$

- Several remarks

· All *intrinsic* and *kinematical* twist-3 distribution/fragmentation functions can be expressed in terms of the twist-2 functions and the *dynamical* twist-3 functions.

· All twist-3 cross sections can be expressed in terms of the twist-2 parton distribution/fragmentation and the dynamical twist-3 distribution/fragmentation functions.

· The dynamical twist-3 functions constitute a complete set of the basis for renormalization, and their LO evolution equations have been derived.

Balitsky, Braun, (1989); YK, Tanaka, (1995); YK, Nishiyama, (1997), Belitsky, Mueller, (1997); Belitsky, (1997); Braun, Manashov, Pirney, (2009);...

3. Lorentz-invariance properties of twist-3 cross sections.

- Distribution functions are defined from the lightcone correlation functions like

$$\Phi_{ij}(x) = \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS|\bar{\psi}_j(0)[0,\lambda n]\psi_i(\lambda n)|PS\rangle .$$

For $P^\mu = (E, \vec{P})$, we introduce two lightlike vectors, $p^\mu = \frac{|\vec{P}|}{\sqrt{2}}\left(1, \frac{\vec{P}}{|\vec{P}|}\right)$ and $n^\mu = \frac{1}{\sqrt{2}|\vec{P}|}\left(1, -\frac{\vec{P}}{|\vec{P}|}\right)$, which satisfy $P^\mu = p^\mu + \frac{M_N^2}{2} n^\mu$ and $p \cdot n = 1$.

- Likewise for the fragmentation functions

$$\Delta_{ij}(z) = \frac{1}{N}\sum_X \int \frac{d\lambda}{2\pi} e^{-i\lambda/z} \langle 0|[\pm\infty w, 0]\psi_i(0)|P_h S_h; X\rangle\langle P_h S_h; X|\bar{\psi}_j(\lambda w)[\lambda w, \pm\infty w]|0\rangle .$$

For $P_h^\mu = (E_h, \vec{P}_h)$, we have $p_h^\mu = \frac{|\vec{P}_h|}{\sqrt{2}}\left(1, \frac{\vec{P}_h}{|\vec{P}_h|}\right)$ and $w^\mu = \frac{1}{\sqrt{2}|\vec{P}_h|}\left(1, -\frac{\vec{P}_h}{|\vec{P}_h|}\right)$, which satisfy $P_h^\mu = p_h^\mu + \frac{M_N^2}{2} w^\mu$ and $p_h \cdot w = 1$.

⋆ In general, calculated twist-3 cross sections depend on n^μ and w^μ. But the physical cross sections should be Lorentz invariant functions of the physical momenta appearing in each process.

→ n^μ and w^μ dependence should be eliminated.

⋆ Example: $e(l) + N(P) \to h(P_h) + X$

3 Mandelstam variables: $S = (l + P)^2$, $T = (P - P_h)^2$, $U = (l - P_h)^2$

· In the twist-3 accuracy, one can regard $P^2 = P_h^2 = l^2 = 0$.

· n^μ and w^μ can be expanded as

$$\begin{aligned} n^\mu &= \chi\, P_h^\mu + \frac{2+\chi T}{S}\, l^\mu + \left[\frac{\chi U(2+\chi T)}{2S} + \frac{STU}{8}\chi_\epsilon^2\right] P^\mu + \chi_\epsilon\, \epsilon^{\mu l P P_h}, \\ w^\mu &= \eta\, P^\mu - \frac{2+\eta T}{U}\, l^\mu + \left[\frac{\eta S(2+\eta T)}{2U} + \frac{STU}{8}\eta_\epsilon^2\right] P_h^\mu + \eta_\epsilon \epsilon^{\mu l P P_h}. \end{aligned}$$

· These satisfy $n^2 = w^2 = 0$ and $p \cdot n = 1$, $p_h \cdot w = 1$.

· Different choices of χ, χ_ϵ, η and η_ϵ correspond to different frames.

- eN c.m. frame $\to n^\mu = \frac{2}{S} l^\mu$ (i.e., $\chi = \chi_\epsilon = 0$).

- Nh c.m. frame

  $\to n^\mu = \frac{-2P_h^\mu}{T}$ (i.e., $\chi = \frac{-2}{T}$, $\chi_\epsilon = 0$) and $w^\mu = \frac{-2P^\mu}{T}$ (i.e., $\eta = \frac{-2}{T}$, $\eta_\epsilon = 0$).

The cross sections should be independent of χ, χ_ϵ, η and η_ϵ.

-By using EOM relations and LIR, one can obtain the twist-3 cross sections in a Lorentz-invariant form which does not have n^μ and w^μ dependence.

i.e., Dependence on χ, χ_ϵ, η. η_ϵ vanishes owing to LIRs and EOM-relations.

⋆ SSA A_{UTU}: $e(l)+N^\uparrow(P,S_N)\to h(P_h)+X$

$$E_h\frac{d\sigma_{\rm LO}(S_N)}{d^3\vec{P}_h}=\frac{8\alpha_{\rm em}^2}{S}\sum_q e_q^2\int_0^1 dx\int_0^1\frac{dz}{z^3}\,\delta(s+t+u)$$
$$\times\epsilon^{lPP_hS_N}\Bigg[\pi M\left(1-x\frac{\rm d}{{\rm d}x}\right)F_{FT}^q(x,x)\,D_1^q(z)\left(\frac{s^2+u^2}{t^3u}\right)$$
$$+M_h\,h_1^q(x)\left(\frac{H^q(z)}{z}\left(\frac{2(u-s)}{t^3}\right)+\left(1-z\frac{\rm d}{{\rm d}z}\right)H_1^{\perp(1),q}(z)\left(\frac{2u}{t^3}\right)\right)\Bigg].$$

⋆ DSA A_{LTU}: $\vec{e}(l)+N^\uparrow(P,S_N)\to h(P_h)+X$

$$\begin{aligned}E_h\frac{d\Delta\sigma_{\rm LO}(S_N)}{d^3\vec{P}_h}&\equiv\frac{1}{2}\left(E_h\frac{d\sigma_{\rm LO}^{\lambda_e=+1}(S_N)}{d^3\vec{P}_h}-E_h\frac{d\sigma_{\rm LO}^{\lambda_e=-1}(S_N)}{d^3\vec{P}_h}\right)\\&=\frac{8\alpha_{\rm em}^2}{S}\sum_q e_q^2\int_0^1\frac{dx}{x}\int_0^1\frac{dz}{z^3}\delta(s+t+u)\Bigg[\left(\frac{z^2M(l\cdot S_N)}{-2T}\right)g_1^q(x)D_1^q(z)\left(\frac{s-u}{s}\right)\\&\quad+\left(\frac{T}{S}(l\cdot S_N)+(P_h\cdot S_N)\right)\Bigg[M\left(xg_1^q(x)\left(\frac{s-u}{2t^2}\right)+xg_T^q(x)\left(\frac{-s}{t^2}\right)+\left(1-x\frac{\rm d}{{\rm d}x}\right)g_{1T}^{(1),q}(x)\left(\frac{s(s-u)}{2t^2u}\right)\right)D_1^q(z)\\&\quad+M_hh_1^q(x)\frac{E^q(z)}{z}\left(\frac{-s}{t^2}\right)\Bigg]\Bigg].\end{aligned}\qquad(68)$$

Likewise for · SSA A_{UUT}: $e(l)+N(P)\to\Lambda^\uparrow(P_h,S_h)+X$.

· DSA A_{LUT}: $\vec{e}(l)+N(P)\to\Lambda^\uparrow(P_h,S_h)+X$.

4. Summary

· We have derived complete set of the relations among twist-3 distribution/fragmentation functions necessary to describe twist-3 cross sections by using QCD EOM and identities for the nonlocal operators. Results for the twist-3 fragmentation functions are wholly new.

· All intrinsic and kinematical twist-3 functions can be expressed in terms of twist-2 functions and dynamical twist-3 functions.

· Use of EOM relations and LIRs leads to Lorentz invariant form of the twist-3 cross sections.