Iterative Monte Carlo analysis of spin-dependent parton distributions

(arXiv:1601.07782)

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DIS Workshop 2016

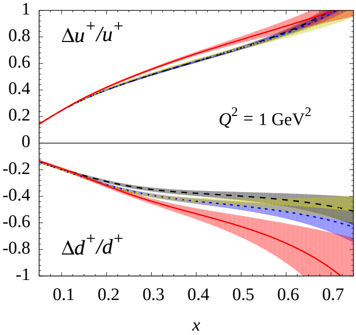
Motivations

Spin structure of nucleons

- spin carried by a quark of type q $\rightarrow$ $\frac{1}{2}\Delta q^{(1)} = \frac{1}{2}\int_0^1 dx\ \Delta q(x, Q^2)$
- spin sum rule $\rightarrow \frac{1}{2} = \frac{1}{2}\Delta\Sigma^{(1)} + \Delta g^{(1)} + \mathcal{L}$ $\rightarrow$ How big is $\mathcal{L}$?
- From existing global analysis $\rightarrow \Delta\Sigma^{(1)}_{[10^{-3},1]} \sim 0.3, \quad \Delta g^{(1)}_{[0.05,0.2]} \sim 0.1$

High x

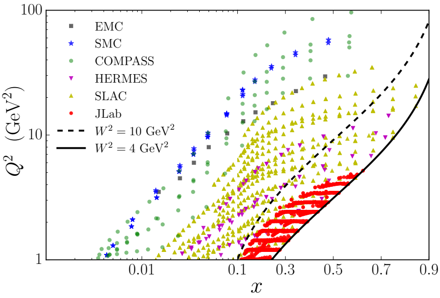
- $SU(6)$ spin-flavor symmetry:
  $\rightarrow \Delta u/u \rightarrow 2/3, \quad \Delta d/d \rightarrow -1/3$
- pQCD $\rightarrow \Delta q/q \rightarrow 1$



Higher twists

- d_2 matrix element
  $\rightarrow d_2 = 2g_1^{(3)}(Q^2) + 3g_2^{(3)}(Q^2)$
- Color forces experienced by struck quarks
  $\rightarrow \tilde{F}_E = 2d_2 + f_2, \quad \tilde{F}_B = 4d_2 - f_2$

Global Analysis



Data

- ✓ Polarized DIS $\rightarrow \Delta u^+, \Delta d^+$
- Polarized SIDIS: $\rightarrow \Delta\bar{d}, \Delta\bar{u}, \Delta s$
- Inclusive Jets/π^0 : $\rightarrow \Delta g$
- W production $\rightarrow \Delta\bar{d}, \Delta\bar{u}$

Theory

- ✓ Target mass corrections
- ✓ Twist-3 and twist-4 contributions in polarized structure functions
- ✓ Nuclear corrections for ^{3}He and deuteron targets
- Threshold resummation $\rightarrow (\alpha_S^m \log(1-x)^n)$

Tools

- ✓ Numerical codes developed within python framework
- ✓ Development of DGLAP evolution equations in Mellin space
- ✓ Fast calculation of observables $\rightarrow$ Mellin space techniques

Polarized DIS

Asymmetries

$$A_{||} = \frac{\sigma^{\uparrow\Downarrow} - \sigma^{\downarrow\Downarrow}}{\sigma^{\uparrow\Downarrow} + \sigma^{\downarrow\Downarrow}} = D(A_1 + \eta A_2)$$

$$A_{\perp} = \frac{\sigma^{\uparrow\Rightarrow} - \sigma^{\downarrow\Rightarrow}}{\sigma^{\uparrow\Rightarrow} + \sigma^{\downarrow\Rightarrow}} = d(A_2 - \xi A_1)$$

$$A_1 = \frac{(g_1 - \gamma^2 g_2)}{F_1} \qquad A_2 = \gamma \frac{(g_1 + g_2)}{F_1} \qquad \gamma^2 = \frac{4M^2 x^2}{Q^2}$$

Polarized structure functions

$$g_1(x, Q^2) = g_1^{\mathrm{LT+TMC}}(\Delta u^+, \Delta d^+, \Delta g, \ldots) + g_1^{\mathrm{T3+TMC}}(D_u, D_d) + g_1^{\mathrm{T4}}(H_{p,n})$$

$$g_2(x, Q^2) = g_2^{\mathrm{LT+TMC}}(\Delta u^+, \Delta d^+, \Delta g, \ldots) + g_2^{\mathrm{T3+TMC}}(D_u, D_d)$$

Fitting strategy

Parametrization

- $xf(x) = Nx^a(1-x)^b(1+c\sqrt{x}+dx)$
- LT quark distributions $\rightarrow \Delta u^+, \Delta d^+, \Delta s^+, \Delta g$
- T3 quark distributions $\rightarrow D_u, D_d$
- T4 structure functions $\rightarrow H_p, H_n$

Chi-squared minimization $\rightarrow$ with correlated systematic uncertainties

$$\chi^2 = \sum_i \left(\frac{D_i - T_i(1 - \sum_k r^k \beta_i^k / D_i)^{-1}}{\alpha_i} \right)^2 + \sum_k (r^k)^2$$

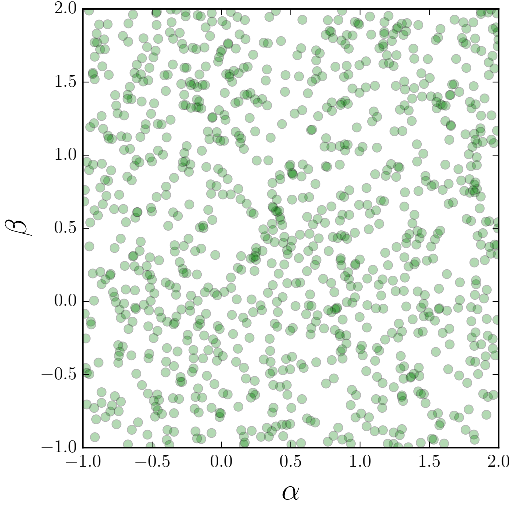
Issues

- Stability in the moments (e.g. $\Delta\Sigma^{(1)}$)
- Is the solution given by a single fit unique? $\rightarrow$ False minima
- Is over-fitting present in our fits?
- Which parameters should be fixed and at which value?
- Determination of uncertainty bands.

Solution $\rightarrow$ MC approach

Iterative Monte Carlo Analysis (IMC)

Toy example → fitting 2 model parameters α, β

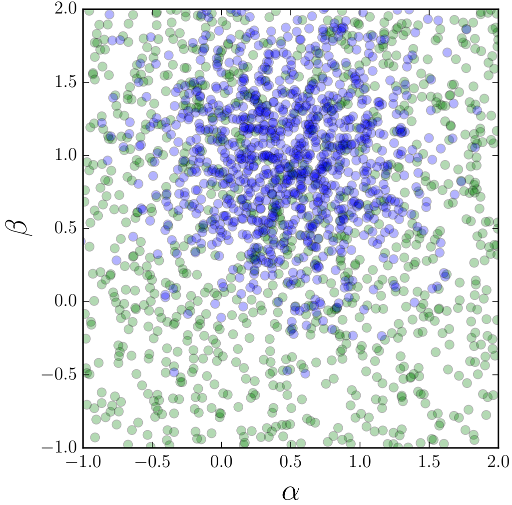


I. Flat sampling

Initial priors $\{(\alpha, \beta)\}$

Iterative Monte Carlo Analysis (IMC)

Toy example → fitting 2 model parameters α, β



I. Flat sampling

Initial priors $\{(\alpha, \beta)\}$

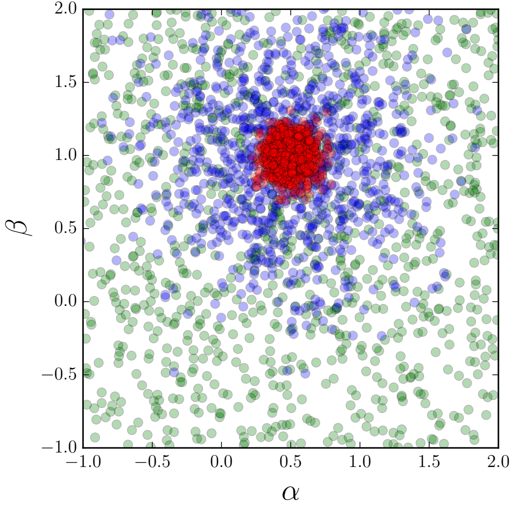
II. First iteration

priors $\{(\alpha, \beta)\}$

posteriors $\{(\alpha, \beta)\}$

Iterative Monte Carlo Analysis (IMC)

Toy example → fitting 2 model parameters α, β



I. Flat sampling

Initial priors $\{(\alpha, \beta)\}$

II. First iteration

priors $\{(\alpha, \beta)\}$

posteriors $\{(\alpha, \beta)\}$

III. Second iteration

priors $\{(\alpha, \beta)\}$

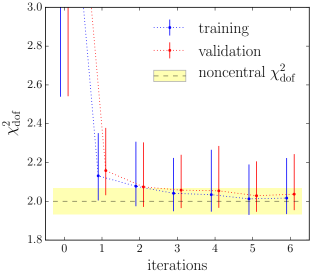
posteriors $\{(\alpha, \beta)\}$

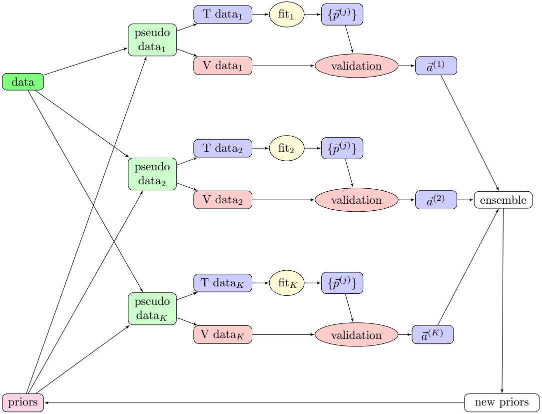
... until convergence

Each iteration

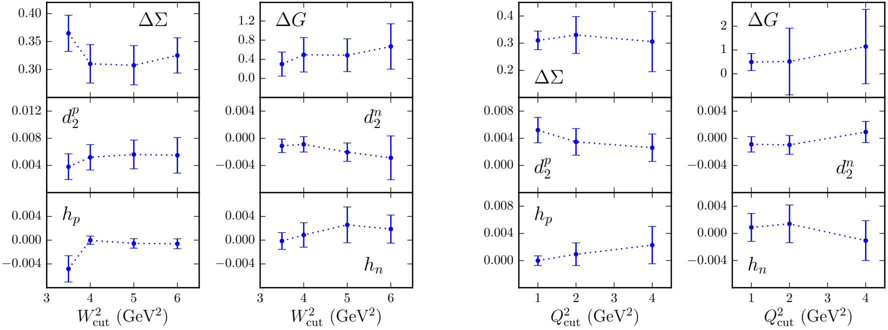
- Generate pseudo data sets via data resampling
- Random data partition → Training & Validation
- Fit the training set
- Validation

# of fits: 10000





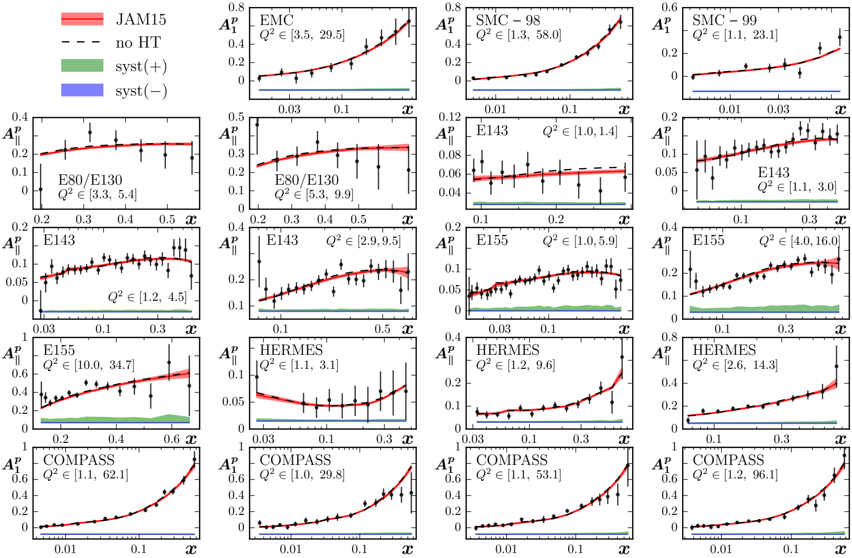
$W^2_{\rm cut}$ and $Q^2_{\rm cut}$



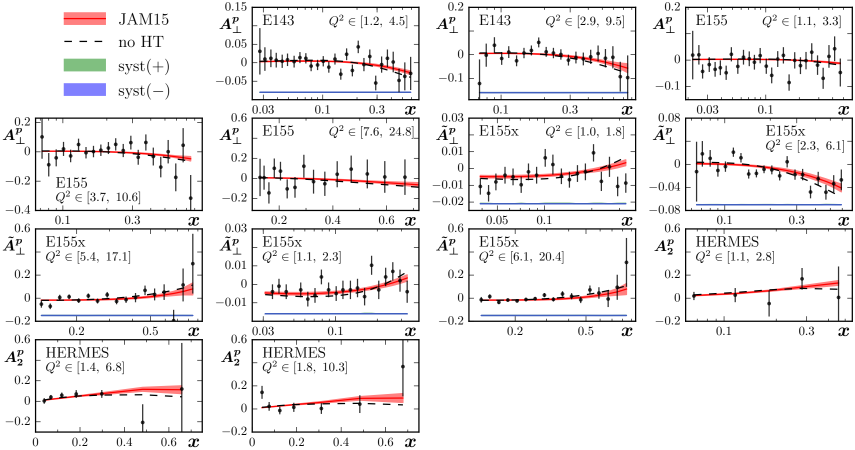
$W^2_{\rm cut}$ (GeV^2)	3.5	**4**	5	6	8	10
# points	2868	**2515**	1880	1427	943	854
$\chi^2_{\rm dof}$	1.20	**1.07**	1.03	1.02	0.99	0.97

$Q^2_{\rm cut}$ (GeV^2)	**1.0**	2.0	4.0
# points	**2515**	1421	611
$\chi^2_{\rm dof}$	**1.07**	1.08	0.95

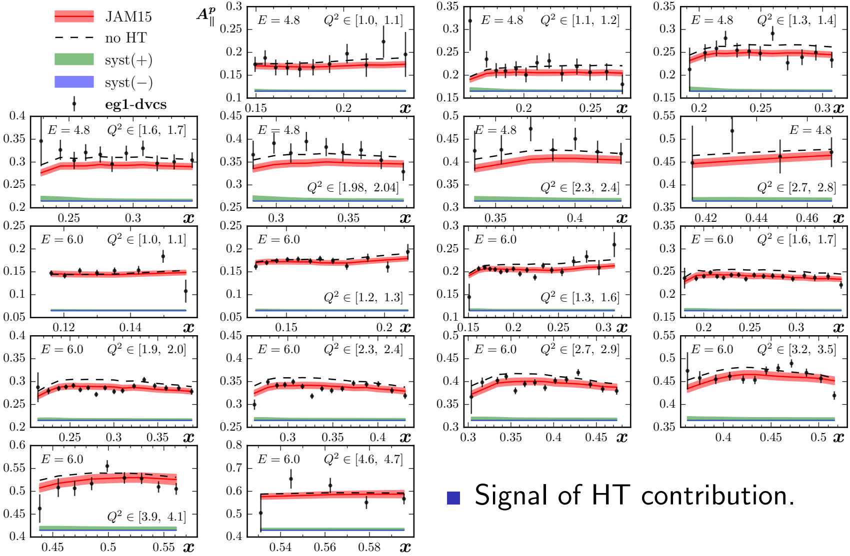
Data vs theory: proton



Data vs theory: proton

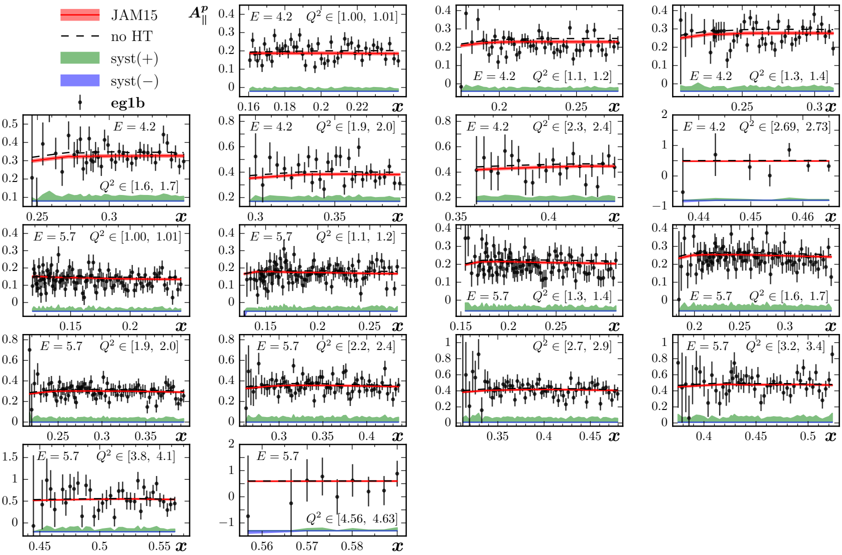


Data vs theory: proton JLab eg1b-dvcs

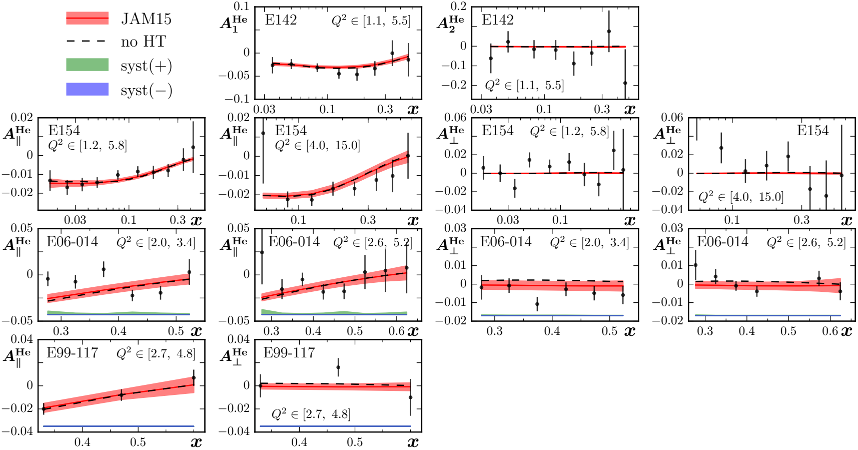


- Signal of HT contribution.

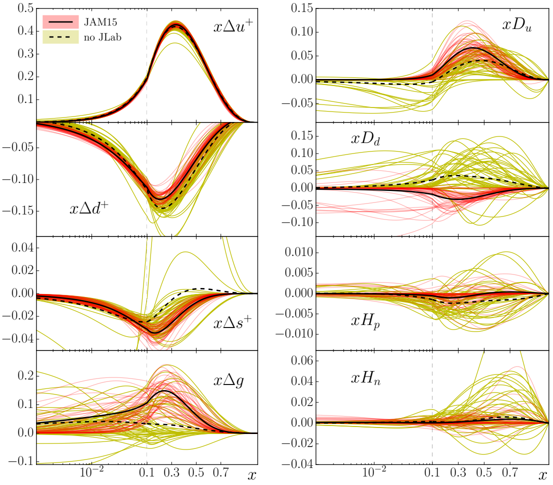
Data vs theory: proton JLab eg1b



Data vs theory: ^{3}He

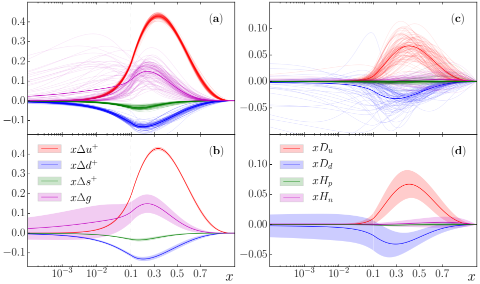


Impact of JLab data

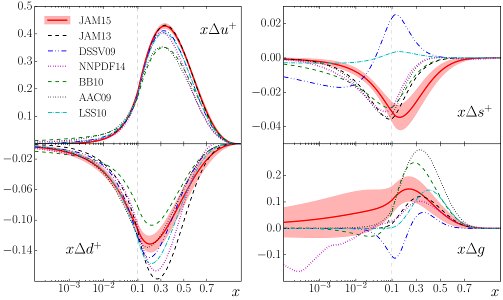


- JLab data $\rightarrow$ $0.1 < x < 0.7$
- Constraints on small x from large x $\rightarrow$ weak baryon decay constraints
- Large uncertainties in Δs^+, Δg removed by JLab data
- Non vanishing T3 quark distributions
- T4 distributions consistent with zero

Results

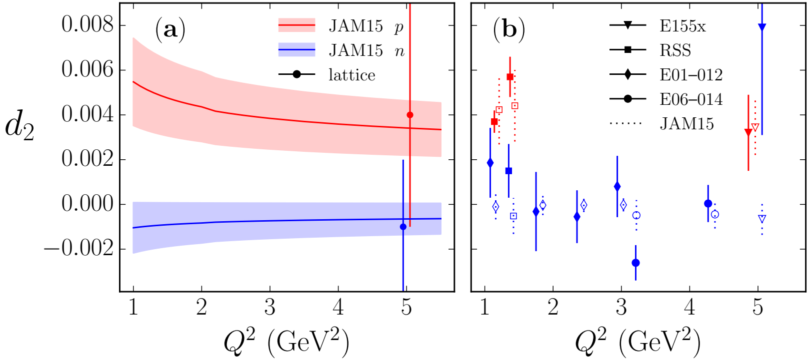


moment	truncated	full
Δu^+	0.82 ± 0.01	0.83 ± 0.01
Δd^+	-0.42 ± 0.01	-0.44 ± 0.01
Δs^+	-0.10 ± 0.01	-0.10 ± 0.01
$\Delta\Sigma$	0.31 ± 0.03	0.28 ± 0.04
ΔG	0.5 ± 0.4	1 ± 15
d_2^p	0.005 ± 0.002	0.005 ± 0.002
d_2^n	-0.001 ± 0.001	-0.001 ± 0.001
h_p	-0.000 ± 0.001	0.000 ± 0.001
h_n	0.001 ± 0.002	0.001 ± 0.003



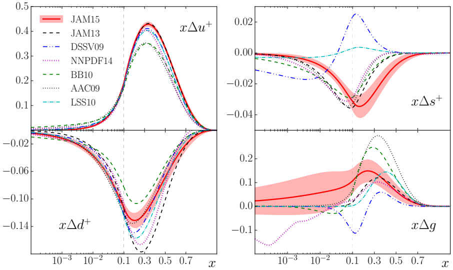
- Significant constraints on Δs^+ and Δg
- Non zero T3 quark distributions
- T4 contribution to g_1 consistent with zero
- **Negative** Δs^+
- JAM15 Δg compatible with recent DSSV fits.

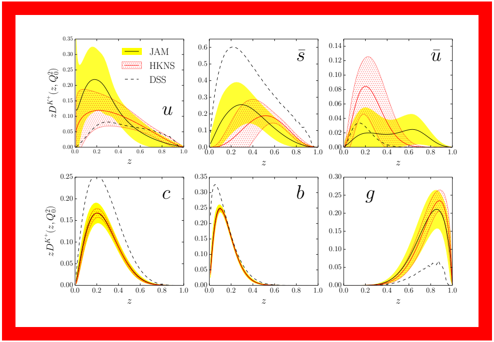
d_2 matrix element



- $d_2(Q^2) \equiv \int_0^1 dx x^2 \left[2g_1^{\mathrm{T3}}(x,Q^2) + 3g_2^{\mathrm{T3}}(x,Q^2)\right]$
- d_2 is related to "color polarizability" or the "transverse color force" acting on quarks.
- Existing measurements of d_2 are in the resonance region (contains TMC T4 and beyond.)
- Agreement with data indicates quark-hadron duality

The strange puzzle





- The sign of ΔS^+ from combined DIS and SIDIS depends on Kaon fragmentation: positive for DSS and negative for HKNS. (Leader, etal)

  NEW IMC FF analysis (to be published soon)

- Only SIA data is used : npts$=245$, $\chi^2 = 305.2$ (identical to HKNS)
- Size of Δs^+ similar to HKNS but smaller than DSS
- Combined DIS and SIDIS analysis unlikely to change Δs^+

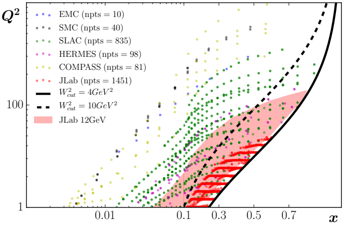
JAM: Outlook

JAM

- ✓ New JAM15 analysis to study impact of all JLab 6 GeV inclusive DIS data at low W and high x
- ✓ New extraction of LT & HT distributions
- Upcoming JAM16 analysis to study polarization of sea quarks & gluons.
  - SIDIS for flavor separation.
  - polarized pp cross sections (inclusive jet & π production) for Δg
  - W boson asymmetries
  - Threshold resummation impacts on large x
- Combined analysis of all inclusive (un)polarized DIS data
- Fits to helicity distributions

JLab 12

- Measurements at high-x $\rightarrow \Delta q/q$
- Wider coverage in $Q^2 \rightarrow \Delta g$
- Determination of pure twist-3 d_2 in DIS



Backup

Mellin trick

Curse of dimensionality $\rightarrow$ Mellin trick (Stratmann, Vogelsang)

$$
\begin{aligned}
I(x) &= \int_x^1 \frac{dy}{y} f(y) \int_y^1 \frac{dz}{z} g\left(\frac{x}{yz}\right) \qquad \leftarrow \qquad g(\xi) = \frac{1}{2\pi i} \int dN \xi^{-N} g_N \\
&= \frac{1}{2\pi i} \int dN g_N \left[\int_x^1 \frac{dy}{y} f(y) \int_y^1 \frac{dz}{z} \left(\frac{x}{yz}\right)^{-N} \right] \\
&= \frac{1}{2\pi i} \int dN g_N \mathcal{M}_N \\
&= \sum_{i,k} w_i^k j^k \mathrm{Im}\left(e^{i\phi} g_{N_j^k} \mathcal{M}_{N_j^k} \right) \qquad \leftarrow \qquad \text{Gaussian quadrature}
\end{aligned}
$$

- Time consuming part can be precalculated prior to the fit
- Extensible to higher dimensional integrals.
- A single fit that takes about 4 days $\rightarrow \approx$ **20 mins**

$\rightarrow \xi = \frac{2x}{1+(1+4\mu^2x^2)^{1/2}}$
$\rightarrow \mu^2 = M^2/Q^2$

Leading twist structure functions:

$$g_1^{\mathrm{LT+TMC}}(x,Q^2) = \frac{x}{\xi}\frac{g_1^{\mathrm{LT}}(\xi)}{(1+4\mu^2x^2)^{3/2}} + 4\mu^2x^2\frac{x+\xi}{\xi(1+4\mu^2x^2)^2}\int_\xi^1 \frac{dz}{z} g_1^{\mathrm{LT}}(z) - 4\mu^2x^2\frac{2-4\mu^2x^2}{2(1+4\mu^2x^2)^{5/2}}\int_\xi^1 \frac{dz}{z}\int_{z'}^1 \frac{dz'}{z'} g_1^{\mathrm{LT}}(z')$$

$$g_2^{\mathrm{LT+TMC}}(x,Q^2) = -\frac{x}{\xi}\frac{g_1^{\mathrm{LT}}(\xi)}{(1+4\mu^2x^2)^{3/2}} + \frac{x}{\xi}\frac{(1-4\mu^2x\xi)}{(1+4\mu^2x^2)^2}\int_\xi^1 \frac{dz}{z} g_1^{\mathrm{LT}}(z) + \frac{3}{2}\frac{4\mu^2x^2}{(1+4\mu^2x^2)^{5/2}}\int_\xi^1 \frac{dz}{z}\int_{z'}^1 \frac{dz'}{z'} g_1^{\mathrm{LT}}(z')$$

In the Bjorken limit ($Q^2 \to \infty$):

$$g_1^{\mathrm{LT+TMC}}(x,Q^2) \simeq g_1^{\mathrm{LT}}(x), \quad g_2^{\mathrm{LT+TMC}}(x,Q^2) \simeq -g_1^{\mathrm{LT}}(x) + \int_\xi^1 \frac{dz}{z} g_1^{\mathrm{LT}}(z)$$

Leading twist quark distributions:

$$g_1^{\mathrm{LT}}(x) = \frac{1}{2}\sum_q e_q^2 \left[\Delta C_{qq} \otimes \Delta q(x) + \Delta C_{qg} \otimes \Delta g(x)\right]$$

Polarized DIS

Twist-3 structure functions:

$$g_1^{\mathrm{T3+TMC}}(x,Q^2) = 4\mu^2x^2\frac{D(\xi)}{(1+4\mu^2x^2)^{3/2}} - 4\mu^2x^2\frac{3}{(1+4\mu^2x^2)^2}\int_\xi^1\frac{dz}{z}D(z) + 4\mu^2x^2\frac{2-4\mu^2x^2}{(1+4\mu^2x^2)^{5/2}}\int_\xi^1\frac{dz}{z}\int_{z'}^1\frac{dz'}{z'}D(z')$$

$$g_2^{\mathrm{T3+TMC}}(x,Q^2) = \frac{D(\xi)}{(1+4\mu^2x^2)^{3/2}} - \frac{1-8\mu^2x^2}{(1+4\mu^2x^2)^2}\int_\xi^1\frac{dz}{z}D(z) - \frac{12\mu^2x^2}{(1+4\mu^2x^2)^{5/2}}\int_\xi^1\frac{dz}{z}\int_{z'}^1\frac{dz'}{z'}D(z')$$

Bjorken limit($Q^2\to\infty$):

$$g_1^{\mathrm{T3+TMC}}(x,Q^2)\simeq 0 \qquad g_2^{\mathrm{T3+TMC}}(x,Q^2)\simeq D(x) - \int_\xi^1\frac{dz}{z}D(z)$$

Twist-3 quark distributions:

$$D(x,Q^2) = \frac{4}{9}D_u(x,Q^2) + \frac{1}{9}D_d(x,Q^2)$$

Polarized DIS

Twist-4 structure function (Nucleon d.o.f.):

$$g_1^{\mathrm{T4(p,n)}}(x, Q^2) = H^{(p,n)}(x)/Q^2$$

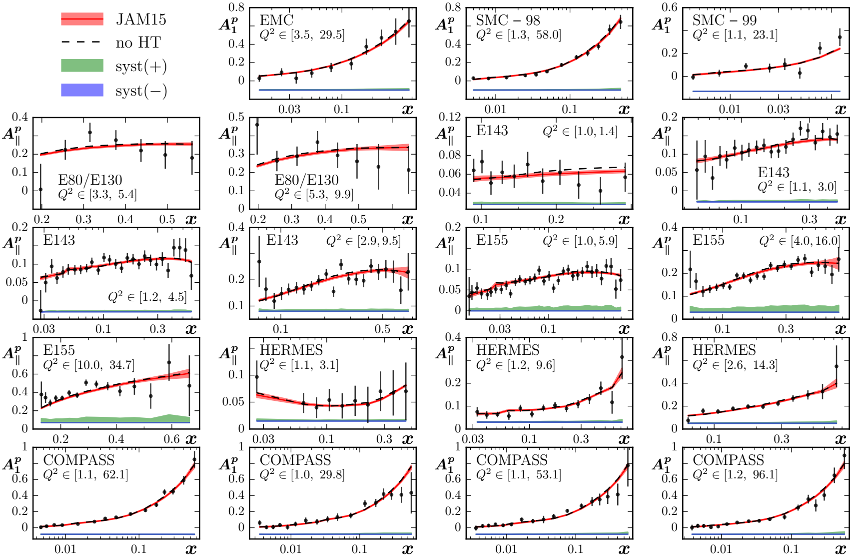
Nuclear corrections: $\rightarrow$ nuclear smearing functions

$$g_i^A(x, Q^2) = \sum_N \int \frac{dy}{y} f_{ij}^N(y, \gamma) g_j^N(x/y, Q^2)$$

Addition constraints: $\rightarrow$ weak neutron and hyperon decay constants

- $\Delta u^{+(1)} - \Delta d^{+(1)} = F + D = 1.269(3)$
- $\Delta u^{+(1)} + \Delta d^{+(1)} - 2\Delta s^{+(1)} = 3F - D = 0.586(31)$

Data vs theory: proton



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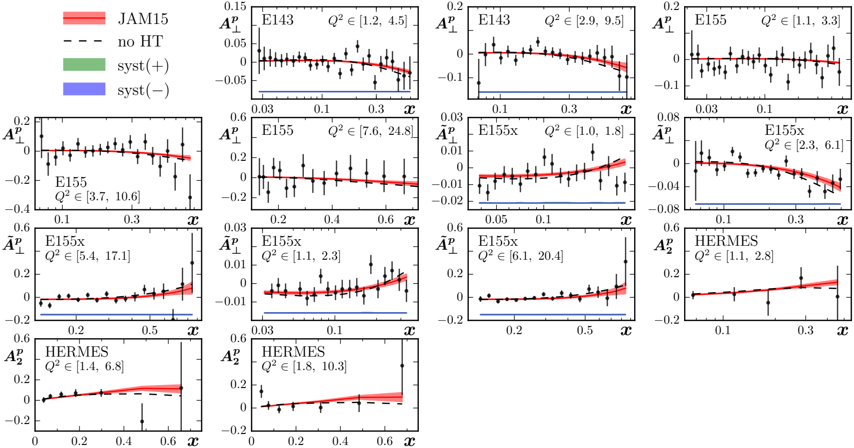
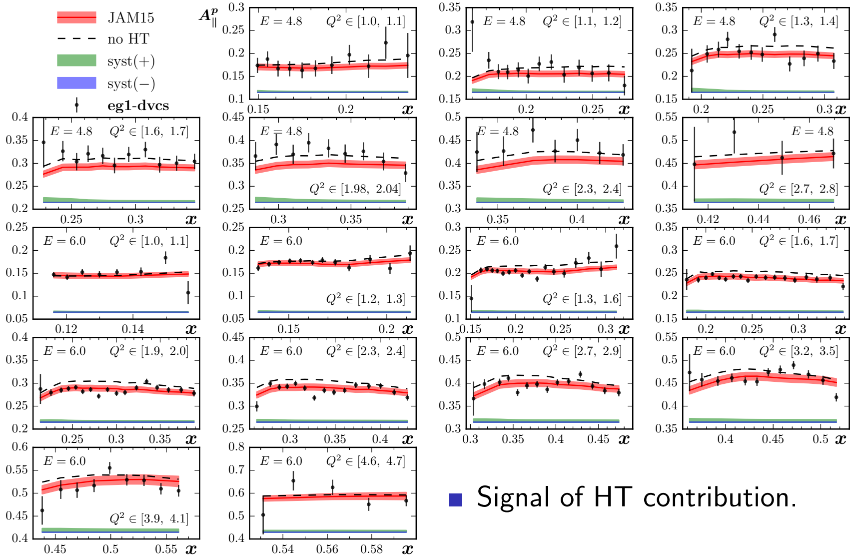
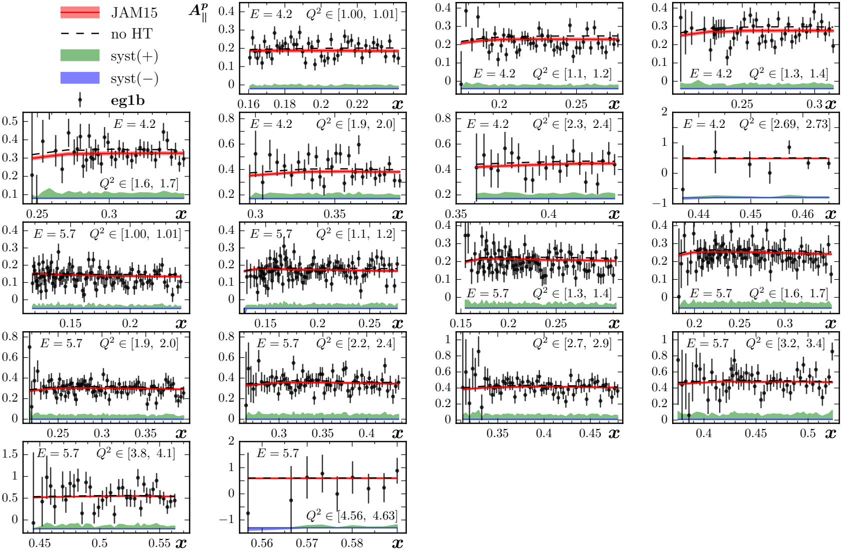


Data vs theory: proton JLab eg1b-dvcs

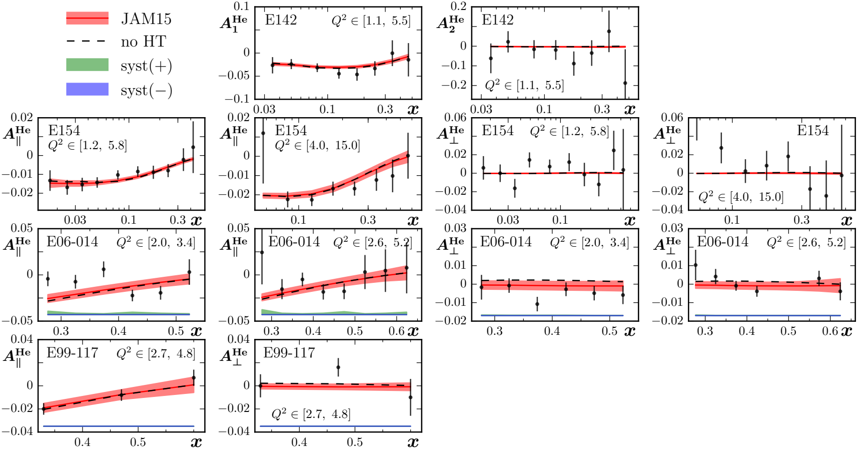


- Signal of HT contribution.

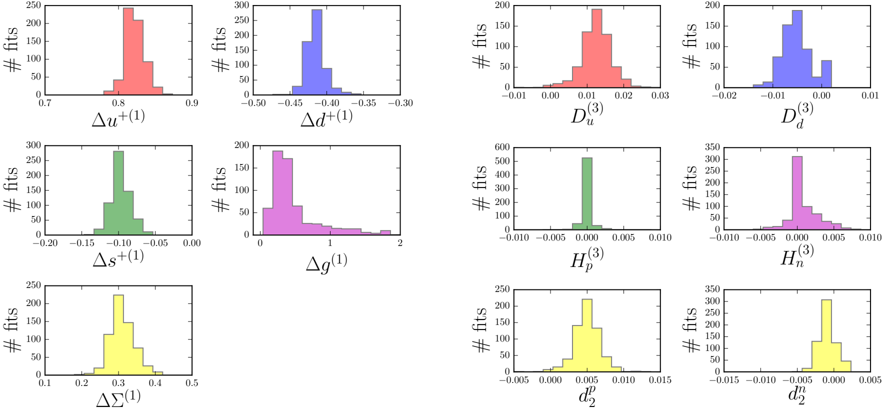
Data vs theory: proton JLab eg1b



Data vs theory: ^{3}He



JAM: moments



fits
$\Delta u^{+(1)}$
$\Delta d^{+(1)}$
$\Delta s^{+(1)}$
$\Delta g^{(1)}$
$\Delta\Sigma^{(1)}$
$D_u^{(3)}$
$D_d^{(3)}$
$H_p^{(3)}$
$H_n^{(3)}$
d_2^p
d_2^n