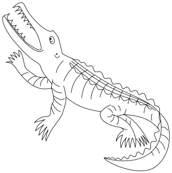
Quantifying bottom-quark mass effects in $b\bar{b}H$ production

DIS16
DESY, HAMBURG 11-15 APRIL 2016

UNIVERSITY OF CAMBRIDGE



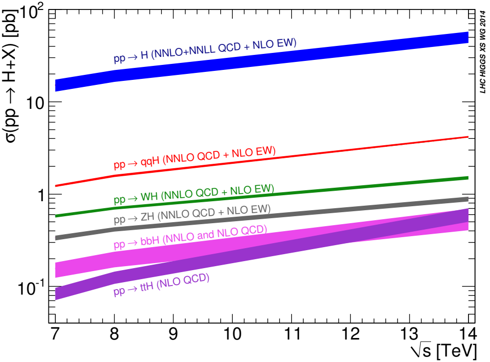
Andrew Papanastasiou

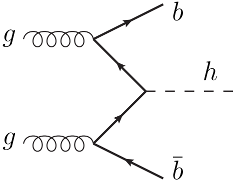
Cavendish Laboratory, University of Cambridge

In collaboration with **Marco Bonvini** (Oxford) and **Frank Tackmann** (DESY)

arXiv: 1508.03288

$b\bar{b}H$ @ LHC is small but non-negligible





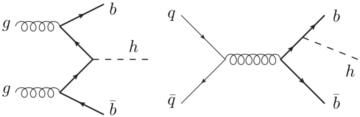
$b\bar{b}H$ is interesting

- access to the bottom-quark Yukawa, Y_b
- enhanced in BSM scenarios, e.g. large $\tan\beta$ 2HDM, SUSY, ...
- common features/issues with other processes involving "initial-state" heavy quarks (e.g. single-top, $Zb\bar{b}$, ...)

4F/5F schemes are valid in opposite limits

$m_b \sim m_H$

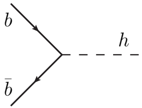
4-flavour scheme



- finite-m_b effects ✓
- collinear logs $\sim \log(m_b/m_H)$ may spoil perturbative behaviour of cross section ✗

$m_b \ll m_H$

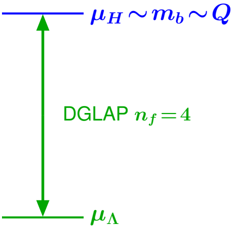
5-flavour scheme



- $\log(m_b/m_H)$-terms resummed via DGLAP evolution in effective b-quark PDF ✓
- no mass power-corrections ✗

Aside from power-corrections m_b^2/m_H^2, the two methods of computing the cross section agree at all orders.

Fixed order region: $m_b \sim Q \sim m_H$
Single matching step required



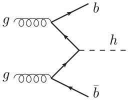
full QCD

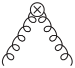
$\downarrow$

theory of collinear gluons & light quarks

(up to corrections of $\Lambda^2_{\rm QCD}/Q^2$)

[standard QCD factorization]



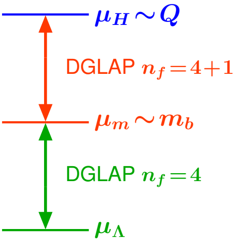


$$\sigma^{\rm FO} = \sum_{i,j=g,q,\bar{q}} \boldsymbol{D_{ij}(m_H, m_b, \mu_F)} f_i^{[4]}(\mu_F) f_j^{[4]}(\mu_F)$$

$$f_i^{[4]}(\mu_F) = \boldsymbol{U_{ik}^{[4]}(\mu_F, \mu_0)} f_k^{[4]}(\mu_0)$$

Resummation region: $m_b \ll Q \sim m_H$

Two matching steps required



full QCD

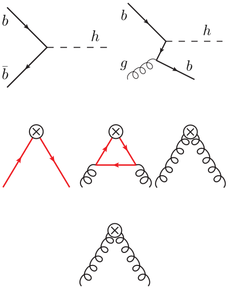
$\downarrow$

theory of collinear gluons, light quarks & b-quarks

(up to corrections of m_b^2/Q^2)

$\downarrow$

theory of collinear gluons & light quarks

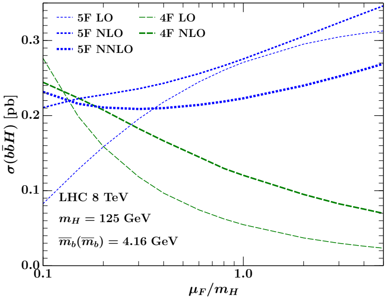


$$\sigma^{\text{Resum}} = \sum_{i,j=g,q,\bar{q},b,\bar{b}} \boldsymbol{C_{ij}(m_H, \mu_F)} f_i^{[5]}(m_b, \boldsymbol{\mu_m}, \mu_F) f_j^{[5]}(m_b, \boldsymbol{\mu_m}, \mu_F)$$

$$f_i^{[5]}(m_b, \boldsymbol{\mu_m}, \mu_F) = \sum_{\substack{k=g,q,\bar{q},b,\bar{b} \\ l,p=g,q,\bar{q}}} \boldsymbol{U_{jk}^{[5]}(\mu_F, \mu_m)} \mathcal{M}_{kl}(m_b, \underset{\uparrow}{\boldsymbol{\mu_m}}) \boldsymbol{U_{lp}^{[4]}(\mu_b, \mu_0)} f_p^{[4]}(\mu_0)$$

<u>standard</u> 5F-PDF set construction ($\mu_m = m_b$)

At fixed order results can differ wildly



- LO: huge μ_F dependence
- NLO: not much better

Until recently, a theoretically consistent approach for combining virtues of 4F and 5F schemes for $b\bar{b}H$ was missing.

- 'best' prediction was a weighted average of 4F & 5F: Santander Matching [Harlander,Krämer,Schumacher]
- recent activity: [Forte,Napoletano,Ubiali; Bonvini,AP,Tackmann] , LHC YR

Perturbative counting

b-PDF is perturbative and counts as $\mathcal{O}(\alpha_s)$

- counting in 4F and 5F schemes assigned only to $\boldsymbol{D_i}$ & $\boldsymbol{C_{i,b}}$
- in particular, 5FS counts $f_b^{[5]} \sim 1$

$$f_b^{[5]}(m_b, \mu_H) = \Big[\underbrace{\boldsymbol{U_{bg}^{[5]}(\mu_H, \mu_m)}}_{\sim 1} + \underbrace{\boldsymbol{U_{bb}^{[5]}(\mu_H, \mu_m)}\mathcal{M}_{bg}^{(1)}(m_b, \mu_m)}_{\sim \alpha_s} + \dots\Big]\boldsymbol{f_g^{[4]}(\mu_m)}$$

(naïvely)

Perturbative counting

b-PDF is perturbative and counts as $\mathcal{O}(\alpha_s)$

- counting in 4F and 5F schemes assigned only to $\boldsymbol{D_i}$ & $\boldsymbol{C_{i,b}}$
- in particular, 5FS counts $f_b^{[5]} \sim 1$

$$f_b^{[5]}(m_b, \mu_H) = \Big[\underbrace{\boldsymbol{U_{bg}^{[5]}(\mu_H, \mu_m)}}_{\sim 1} + \underbrace{\boldsymbol{U_{bb}^{[5]}(\mu_H, \mu_m)}\mathcal{M}_{bg}^{(1)}(m_b, \mu_m)}_{\sim \alpha_s} + \dots\Big]\boldsymbol{f_g^{[4]}(\mu_m)}$$

(naïvely)

- however, $\boldsymbol{U_{bg}^{[5]}}$ is off-diagonal evolution factor: $\sim \alpha_s \log(\mu_H/\mu_m)$
- $\boldsymbol{U_{bg}^{[5]}} \sim 1$ formally only for $\mu_H \gg \mu_m$, $\boldsymbol{U_{bg}^{[5]}} \sim \alpha_s$ a more appropriate counting for $\mu_m \sim m_b$ and typical LHC hard scales
- for LHC pheno: $\boldsymbol{f_b^{[5]}(m_b, \mu_H)}$ effectively counts as $\boldsymbol{\mathcal{O}(\alpha_s)}$

Combining fixed-order and resummation

Add pure m_b^2/Q^2 corrections to resummed result

Want to combine the results $d\sigma^{\text{FO}}$ and $d\sigma^{\text{resum}}$ valid different parametric regions to obtain a result valid for any value of m_b/Q

- terms missing in $d\sigma^{\text{resum}}$ are the $\mathcal{O}\left(\frac{m_b^2}{Q^2}\right)$ terms in $d\sigma^{\text{FO}}$

Write full cross section as

$$d\sigma = d\sigma^{\text{resum}} + \underbrace{\left(d\sigma^{\text{FO}} - \overbrace{d\sigma^{\text{resum}}\Big|_{\mu_m=\mu_H}}^{d\sigma^{\text{singular}}}\right)}_{d\sigma^{\text{nonsingular}}}$$

- taking perturbative counting we prescribe, then $d\sigma^{\text{resum}}\Big|_{\mu_m=\mu_H}$ exactly reproduces all singular contributions (terms that do not vanish in $m_b \to 0$ limit) in $d\sigma^{\text{FO}}$
- note: $d\sigma \to d\sigma^{\text{FO}}$ in limit $\mu_m \to \mu_H$

NLO+NLL matched result

$$\sigma^{\text{FO+Resum}} = \sum_{i,j=b,\bar{b}} \boldsymbol{C_{ij}(m_H,\mu_F)} \otimes f_i^{[5]}(m_b,\mu_F) \otimes f_j^{[5]}(m_b,\mu_F)$$
$$+ \sum_{\substack{i=b,\bar{b}\\ j=g,q,\bar{q}}} \left[\boldsymbol{C_{ij}(m_H,\mu_F)} \otimes f_i^{[5]}(m_b,\mu_F) \otimes f_j^{[5]}(m_b,\mu_F) + (i \leftrightarrow j)\right]$$
$$+ \sum_{i,j=g,q,\bar{q}} \boldsymbol{\bar{C}_{ij}(m_H,m_b,\mu_F)} \otimes f_i^{[5]}(m_b,\mu_F) \otimes f_j^{[5]}(m_b,\mu_F),$$

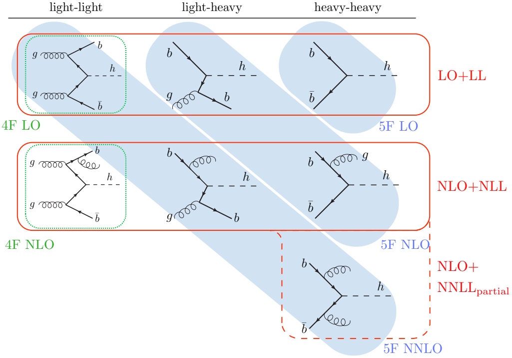
- fixed-order finite m_b-effects contained in $\boldsymbol{\bar{C}_{ij}}$ (up to NLO)

[similar construction to S-ACOT and FONLL coefficient functions]

Expanding (with perturbative counting as before):

LO+LL	$\sigma = \alpha_s^2 \boldsymbol{\bar{C}_{ij}^{(2)}} f_i f_j + \alpha_s 4\boldsymbol{C_{bg}^{(1)}} f_b f_g + 2\boldsymbol{C_{b\bar{b}}^{(0)}} f_b f_b$	$\sim \alpha_s^2$
NLO+NLL	$+ \alpha_s^3 \boldsymbol{\bar{C}_{ij}^{(3)}} f_i f_j + \alpha_s^2 4\boldsymbol{C_{bk}^{(2)}} f_b f_k + \alpha_s 2\boldsymbol{C_{b\bar{b}}^{(1)}} f_b f_b$	$\sim \alpha_s^3$

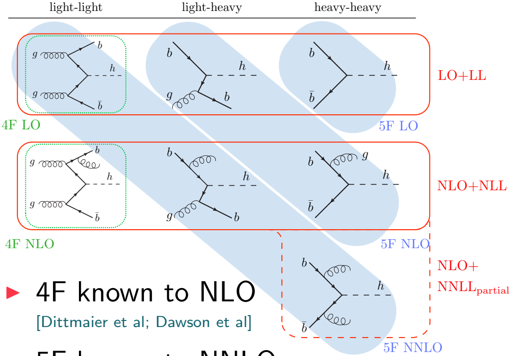
NLO+NLL matched result: ingredients



$$\text{LO+LL} \qquad \sigma = \alpha_s^2 \bar{C}_{ij}^{(2)} f_i f_j + \alpha_s 4 C_{bg}^{(1)} f_b f_g + \quad 2 C_{b\bar{b}}^{(0)} f_b f_b \qquad \sim \alpha_s^2$$

$$\text{NLO+NLL} \qquad + \alpha_s^3 \bar{C}_{ij}^{(3)} f_i f_j + \alpha_s^2 4 C_{bk}^{(2)} f_b f_k + \alpha_s 2 C_{b\bar{b}}^{(1)} f_b f_b \qquad \sim \alpha_s^3$$

NLO+NLL matched result: ingredients



- 4F known to NLO
  [Dittmaier et al; Dawson et al]
- 5F known to NNLO
  [Harlander, Kilgore; Bühler et al]

- we extract $D_i^{(3)}(m_H, m_b, \mu_H)$ using MG5_aMC@NLO [Alwall et al.]
- in-house implementation of $C_{b\bar{b}}^{(1,2)}$ $C_{bg}^{(1,2)}$, $C_{bq}^{(2)}$
- $\mathcal{M}_{ij}^{(2)}(m_b, \mu_m)$ known [Buza et al] : we implemented in APFEL [Bertone et al] for general μ_m
- construct $\bar{C}_{ij}^{(1,2)}$ from these known ingredients

$$\text{LO+LL} \qquad \sigma = \alpha_s^2 \bar{C}_{ij}^{(2)} f_i f_j + \alpha_s 4 C_{bg}^{(1)} f_b f_g + 2 C_{b\bar{b}}^{(0)} f_b f_b \qquad \sim \alpha_s^2$$

$$\text{NLO+NLL} \qquad + \alpha_s^3 \bar{C}_{ij}^{(3)} f_i f_j + \alpha_s^2 4 C_{bk}^{(2)} f_b f_k + \alpha_s 2 C_{b\bar{b}}^{(1)} f_b f_b \qquad \sim \alpha_s^3$$

Other approaches to initial-state HQs available (mainly DIS)

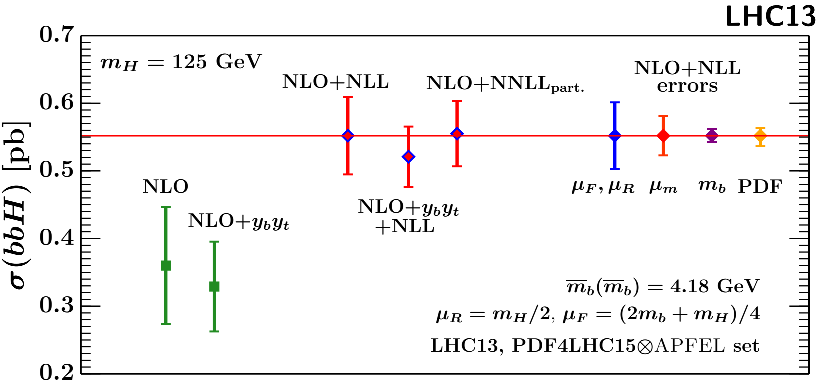
Alternative approaches to combining fixed-order mass effects with resummation of logarithms in DIS have been around for a while ...

- ACOT [Aivazis,Collins,Olness,Tung]
- TR [Thorne,Roberts]
- S-ACOT(χ) [Collins; Krämer,Olness,Soper; ...]
- FONLL [Cacciari,Greco,Nason; Forte,Laenen,Nason,Rojo]

These differ mainly in the way in which they choose to incorporate the m_b^2/Q^2 corrections. Compared to what we have outlined:

- construction of coefficient functions equivalent to S-ACOT and FONLL
- we consider $f_b(m_b, \mu_H)$ a perturbative $\mathcal{O}(\alpha_s)$ object (strictly expanded together with coefficient functions)
- we take the matching scale μ_m to be $\mathcal{O}(m_b)$, but <u>not</u> strictly equal to m_b

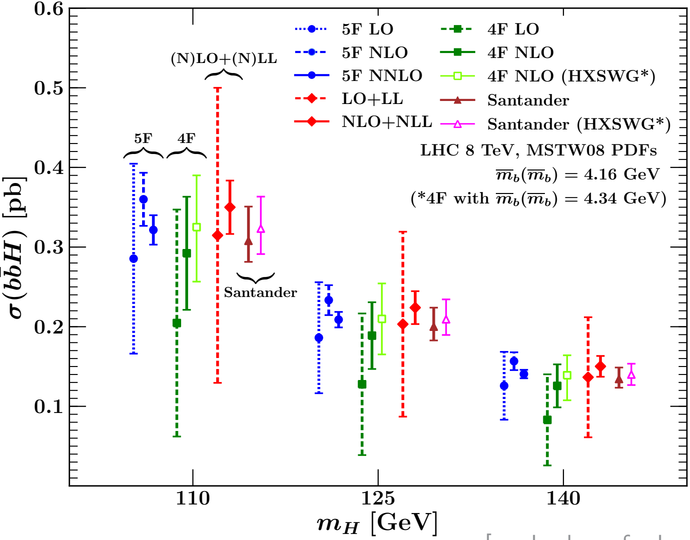
$\sigma^{\text{NLO+NLL}}$ and its uncertainties (vary all scales)



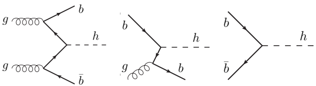
- $y_b y_t$: interferences of Born-level diagams with diagrams involving a top-quark loop
- NLO+NNLL$_{\text{part.}}$ contains pure 2-loop terms from 5F which are higher order in our apprach

$\sigma^{\mathrm{Santander}}$ systematically different from $\sigma^{\mathrm{NLO+NLL}}$

LHC8



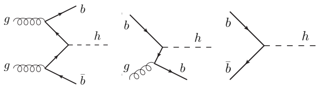
[see backups for larger m_H]



To take away

- theoretically consistent combination of m_b^2/Q^2-corrections with $\log(m_b/Q)$-resummation for $b\bar{b}H$ has been made
  - μ_m does not have to equal m_b! ($\Rightarrow$ additional uncertainty that is not accounted for in mainstream 5F PDF sets)
  - $f_b^{[5]}(m_b, \mu_H) \sim \alpha_s$ for phenomenology relevant to LHC
  - improvement to both 4F and 5F predictions
- robust error-estimate through variation of <u>all</u> matching scales (fixed-order + resummation uncertainties)
- Santander-matched cross section systematically different NLO+NLL (however, size of error bands seems reasonable)

Conclusions & Outlook



To take away

- theoretically consistent combination of m_b^2/Q^2-corrections with $\log(m_b/Q)$-resummation for $b\bar{b}H$ has been made
  - μ_m does not have to equal m_b! ($\Rightarrow$ additional uncertainty that is not accounted for in mainstream 5F PDF sets)
  - $f_b^{[5]}(m_b, \mu_H) \sim \alpha_s$ for phenomenology relevant to LHC
  - improvement to both 4F and 5F predictions
- robust error-estimate through variation of <u>all</u> matching scales (fixed-order + resummation uncertainties)
- Santander-matched cross section systematically different NLO+NLL (however, size of error bands seems reasonable)

Thank you for your attention!

Comparing approaches

A practical way to combine 4F and 5F predictions is given by:

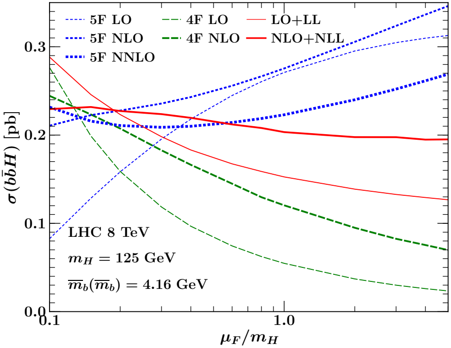
$$\boldsymbol{\sigma}^{\text{Santander}} = \frac{\boldsymbol{\sigma}^{\text{4FS,NLO}} + \omega \boldsymbol{\sigma}^{\text{5FS,NNLO}}}{1+\omega}, \quad \text{where} \quad \omega = \log\left(\frac{m_H}{m_b}\right) - 2,$$

and uncertainties obtained by applying formula to upper and lower 4F & 5F results.

Meaningful/fair comparison:

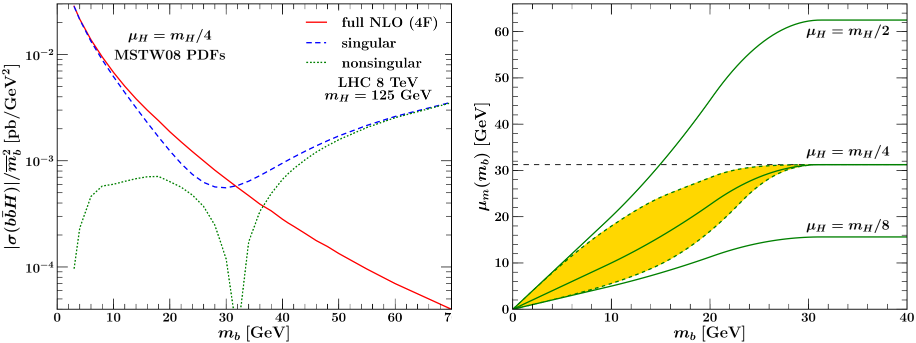
- PDFs: $\mathrm{MSTW08}$ (NLO/NNLO and `nf4`/`nf5` as appropriate)
- $m_b = 4.75$ GeV (as used in $\mathrm{MSTW08}$)
- $\overline{m_b}(\overline{m_b}) = 4.18$ GeV, same RGE-evolution for all results
- central scale: $\mu_H = (m_H + 2m_b)/4$ $\quad (\mu_H = \mu_F = \mu_R)$

(fixing $\mu_m = m_b$)



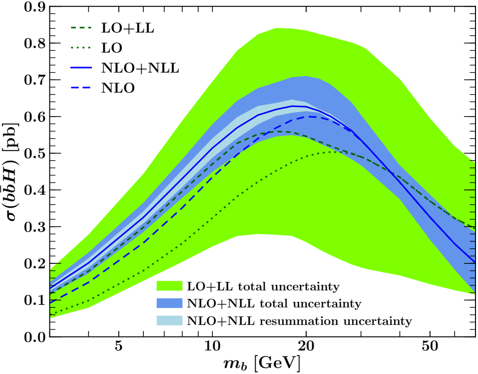
- scale dependence of LO+LL and NLO+NLL improved w.r.t LO and NLO 4F and 5F
- 4F stabilized: resum large logs
- 5F stabilized: put together channels that contribute at same perturbative order ($b\bar{b}$, bg, gg)

Structure of fixed-order cross section



- small m_b: singular terms dominate cross section $\Rightarrow$ resum logs
- large m_b: delicate balance between singular and nonsingular
  $\Rightarrow$ switch off resummation
- smooth transition between resummation & FO regions controlled by $\mu_m(m_b)$ ('profiles' standard in EFTs)

- checks smooth matching to FO
- ($\Rightarrow$ very stringent check of construction of coefficients $\bar{C}_i$ and implementation of $\mathcal{M}_{ij}$)
- check whether our uncertainty determination is useful



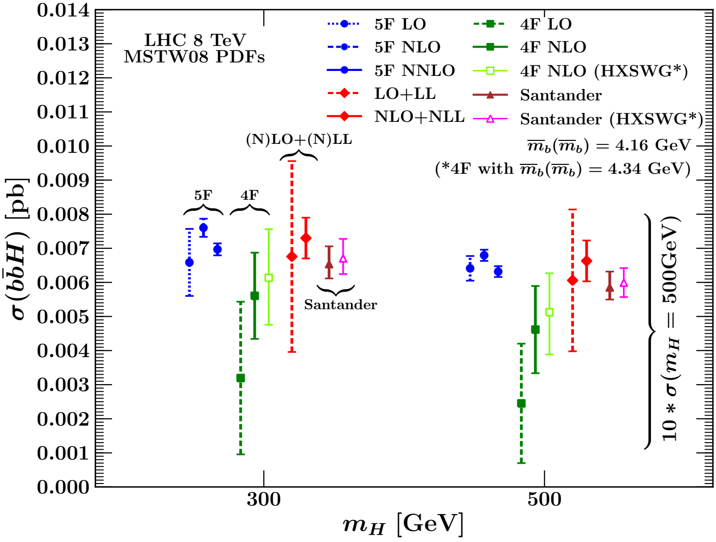
$m_H = 125$ GeV, $\mu_H = m_H/4$

✓ $d\sigma^{\mathrm{(N)LO+(N)LL}} \to \sigma^{\mathrm{(N)LO}}$ in limit $\mu_m \to \mu_H$ (large m_b)

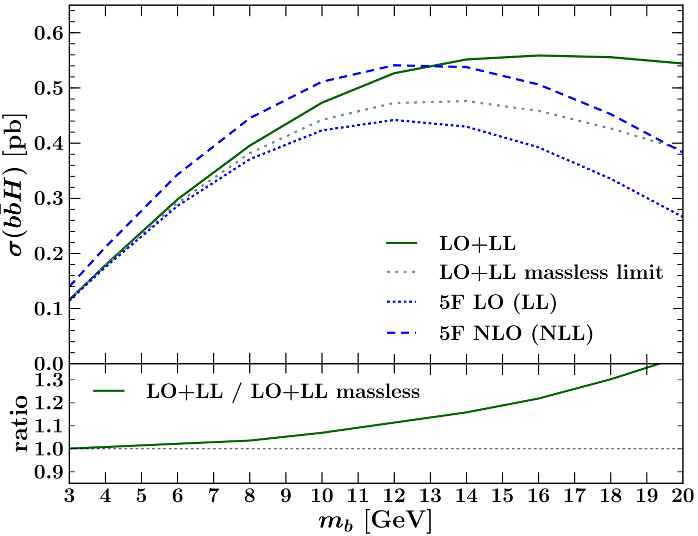
✓ NLO+NLL lies within LO+LL uncertainty band

✓ resummed result appears perturbatively more stable than FO result

$\sigma^{\text{Santander}}$ systematically lower than $\sigma^{\text{NLO+NLL}}$



Size of m_b^2/m_H^2 corrections in $b\bar{b}H$



Comparison to FONLL approach [Forte,Napoletano,Ubiali]

The $b\bar{b}H$ cross section has also recently been obtained in the FONLL approach [Forte,Napoletano,Ubiali] .

- this has a different perturbative counting to the one we adopt
- LO 4F is combined with NNLO 5F (i.e. fixed order m_b effects are included at LO)
  - includes $\boldsymbol{C^{(2)}_{b\bar{b}}}$, which is a higher-order term in our approach
  - does not include $\boldsymbol{\bar{C}^{(3)}_{gg}}$, $\boldsymbol{\bar{C}^{(3)}_{q\bar{q}}}$ and $\boldsymbol{\bar{C}^{(3)}_{qg}}$ (NLO 4F contributions)
- for $m_b = 4.75$ GeV, m_b^2/m_H^2 corrections are small, the FONLL result is very close to the 5F NNLO prediction
- work in progress to include 4F NLO (will be done for YR4)
- no estimate of resummation/matching uncertainty