# BFKL effects and central rapidity dependence in Mueller–Navelet jet production at 13 TeV LHC


Francesco Giovanni Celiberto

francescogiovanni.celiberto@fis.unical.it

Università della Calabria
Dipartimento di Fisica

Università della Calabria & INFN-Cosenza
Italy

INFN


based on

[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2016)]

to appear in **Eur. Phys. J.**

arXiv:1601.07847

*DIS 2016: 24$^{th}$ International Workshop on Deep Inelastic Scattering and Related Subjects*

April 11$^{st}$ - 15$^{th}$, 2016
DESY Hamburg

# Outline

# Outline

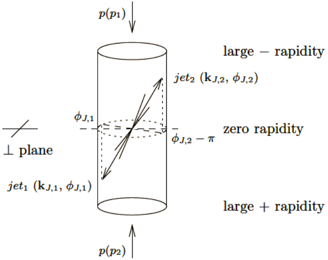
# Mueller-Navelet jets

Process: $\mathrm{proton}(p_1) + \mathrm{proton}(p_2) \to \mathrm{jet}_1(k_1) + \mathrm{jet}_2(k_2) + X$ *...LHC physics!*

$$\frac{d\sigma}{dx_{J_1}\,dx_{J_2}\,d^2k_{J_1}\,d^2k_{J_2}} = \sum_{i,j=q,\bar{q},g} \int_0^1 dx_1 \int_0^1 dx_2\, f_i(x_1,\mu) f_j(x_2,\mu) \frac{d\hat{\sigma}_{i,j}(x_1 x_2 s,\mu)}{dx_{J_1}\,dx_{J_2}\,d^2k_{J_1}\,d^2k_{J_2}}$$

- ◇ *large jet transverse momenta:* $\vec{k}_{J,1}^{\,2} \sim \vec{k}_{J,2}^{\,2} \gg \Lambda^2_{\mathrm{QCD}}$ ⇒ pQCD allowed
- ◇ *large rapidity gap between jets (high energies)* ⇒ $\Delta y = \ln \frac{x_{J,1} x_{J,2} s}{|\vec{k}_{J,1}||\vec{k}_{J,2}|}$

⇒ *BFKL resummation:* $\sum_n \left( a_n^{(0)}\ \alpha_s^n \ln^n s + a_n^{(1)}\ \alpha_s^n \ln^{n-1} s \right)$




*Mueller-Navelet jets at LO:*
*a back-to-back di-jet reaction*

*Picture from*
*[D. Colferai, F. Schwennsen, L. Szymanowski, S. Wallon (2010)]*

# Outline

1. Introduction
   - Mueller-Navelet jet production
2. Theoretical setup
   - BFKL resummation
   - Cross section and central rapidity range
   - BLM optimization procedure
3. Results
   - Numerical analysis
4. Conclusions

# The BFKL resummation

pQCD, **semi-hard processes**: $s \gg Q^2 \gg \Lambda^2_{\rm QCD}$

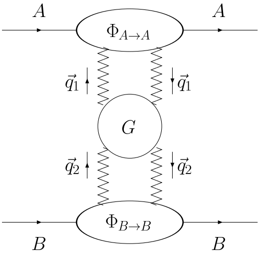
total cross section for $A + B \to X$: $\sigma_{AB}(s) = \frac{\mathcal{I}m_s\left(\mathcal{A}_{AB}^{AB}\right)}{s}$ $\Leftarrow$ optical theorem

- ◇ Pomeron channel: $t = 0$ + singlet colour representation in the $t$-channel
- ◇ Regge limit: $s \simeq -u \to \infty$, $t$ not growing with $s$

- **BFKL resummation**:

  **leading logarithmic approximation (LLA)**: $\alpha_s^n (\ln s)^n$

  **next-to-leading logarithmic approximation (NLA)**: $\alpha_s^{n+1} (\ln s)^n$




- $\mathrm{Im}_s\left(\mathcal{A}_{AB}^{AB}\right)$ factorization:

  convolution of the **Green's function** of two interacting Reggeized gluons with the **impact factors** of the colliding particles.

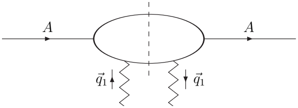
$$\mathrm{Im}_s\left(\mathcal{A}\right)=\frac{s}{(2\pi)^{D-2}}\int\frac{d^{D-2}q_1}{\vec{q}_1^{\,2}}\Phi_A(\vec{q}_1,s_0)\int\frac{d^{D-2}q_2}{\vec{q}_2^{\,2}}\Phi_B(-\vec{q}_2,s_0)\int_{\delta-i\infty}^{\delta+i\infty}\frac{d\omega}{2\pi i}\left(\frac{s}{s_0}\right)^{\omega}G_\omega(\vec{q}_1,\vec{q}_2)$$

- **Green's function** is **process-independent**

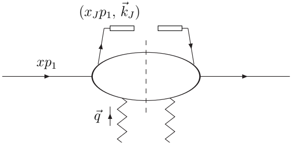
  $\longrightarrow$ determined through the **BFKL equation**
  [Ya.Ya. Balitsky, V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1975)]

- **Impact factors** are **process-dependent**

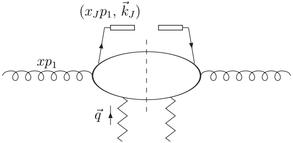
  $\longrightarrow$ known in the NLA just for few processes




* forward jet production




quark jet vertex



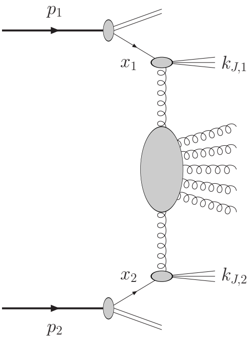

gluon jet vertex

[J. Bartels, D. Colferai, G.P. Vacca (2003)]
(small-cone approximation) [F. Caporale, D.Yu. Ivanov, B. Murdaca, A. Papa, A. Perri (2012)]
(small-cone approximation) [D.Yu. Ivanov, A. Papa (2012)]
(several jet algorithms discussed) [D. Colferai, A. Niccoli (2015)]

# Outline

# BFKL cross section...

$$\frac{d\sigma}{dx_{J_1}dx_{J_2}d^2k_{J_1}d^2k_{J_2}} = \sum_{i,j=q,\bar{q},g} \int_0^1 dx_1 \int_0^1 dx_2\, f_i(x_1,\mu) f_j(x_2,\mu) \frac{d\hat{\sigma}_{i,j}(x_1x_2s,\mu)}{dx_{J_1}dx_{J_2}d^2k_{J_1}d^2k_{J_2}}$$




- slight change of variable in the final state
- project onto the eigenfunctions of the LO BFKL kernel, i.e. transfer from the reggeized gluon momenta to the $(n,\nu)$-representation
- suitable definition of the **azimuthal coefficients**

$$\frac{d\sigma}{dx_{J_1}dx_{J_2}\,d|\vec{k}_{J_1}|\,d|\vec{k}_{J_2}|d\phi_{J_1}d\phi_{J_2}} = \frac{1}{(2\pi)^2}\left[\mathcal{C}_0 + \sum_{n=1}^{\infty} 2\cos(n\phi)\,\mathcal{C}_n\right]$$

with $\phi = \phi_{J_1} - \phi_{J_1} - \pi$

...useful definitions: $Y = \ln \frac{x_{J_1}x_{J_2}s}{|\vec{k}_{J_1}||\vec{k}_{J_2}|}$, $Y_0 = \ln \frac{s_0}{|\vec{k}_{J_1}||\vec{k}_{J_2}|}$

# ...and azimuthal coefficients

$$\mathcal{C}_n = \int_{-\infty}^{+\infty} d\nu \; e^{(Y-Y_0)\left[\bar\alpha_s(\mu_R)\chi(n,\nu)+\bar\alpha_s^2(\mu_R)K^{(1)}(n,\nu)\right]} \alpha_s^2(\mu_R)$$
$$\times c_1(n,\nu)\, c_2(n,\nu) \left[1 + \alpha_s(\mu_R)\left(\frac{c_1^{(1)}(n,\nu)}{c_1(n,\nu)} + \frac{c_2^{(1)}(n,\nu)}{c_2(n,\nu)}\right)\right]$$

where

$$\chi(n,\nu) = 2\psi(1) - \psi\left(\frac{n}{2}+\frac{1}{2}+i\nu\right) - \psi\left(\frac{n}{2}+\frac{1}{2}-i\nu\right)$$

$$K^{(1)}(n,\nu) = \bar\chi(n,\nu) + \frac{\beta_0}{8N_c}\chi(n,\nu)\left(-\chi(n,\nu) + \frac{10}{3} + \imath\frac{d}{d\nu}\ln\left(\frac{c_1(n,\nu)}{c_2(n,\nu)}\right) + 2\ln\left(\mu_R^2\right)\right)$$

$$c_1(n,\nu,|\vec k|,x) = 2\sqrt{\frac{C_F}{C_A}}(\vec k^{\,2})^{i\nu-1/2}\left(\frac{C_A}{C_F}f_g(x,\mu_F) + \sum_{a=q,\bar q} f_a(x,\mu_F)\right)$$

...several NLA-equivalent expressions can be adopted for $\mathcal{C}_n$!
$\longrightarrow$ ...we use the *exponentiated* one

[F. Caporale, D.Yu Ivanov, B. Murdaca, A. Papa, (2014)]

# Exclusion of central jet rapidities

## Motivation...

◇ At given $Y = y_{J_1} - y_{J_2}$ ...

→ $|y_{J_i}|$ could be so small ($\lesssim 2$), that the jet $i$ is actually produced in the central region, rather than in one of the two forward regions

→ longitudinal momentum fractions of the parent partons $x \sim 10^{-3}$

→ for $|y_{J_i}|$ and $|k_{J_i}| < 100$ GeV $\Rightarrow$ increase of $C_0$ by 25% due to NNLO PDF effects [J. Currie, A. Gehrmann-De Ridder, E. W. N. Glover, J. Pires (2014)]

! Our BFKL description of the process could be not so accurate...

## ...let's return to the original Mueller-Navelet idea!

◇ remove regions where jets are produced at central rapidities...

→ ...in order to reduce as much as possible thoeretical uncertainties

# Outline

# BLM method

NLO BFKL corrections to $C_0$ with opposite sign with respect to the leading order (LO) result and large in absolute value.

- ◇ ...call for some optimization procedure...
- ◇ ...choose scales to mimic the most relevant subleading terms

- **BLM** [ S.J. Brodsky, G.P. Lepage, P.B. Mackenzie (1983)]
  - ✓ preserve the conformal invariance of an observable...
  - ✓ ...by making vanish its $\beta_0$-dependent part

* "Exact" BLM:

suppress NLO IFs + NLO Kernel $\beta_0$-dependent factors

* Partial (approximated) BLM:

a) $\left(\mu_R^{BLM}\right)^2 = k_1 k_2 \ \exp\left[2\left(1+\frac{2}{3}I\right) - f\left(\nu\right) - \frac{5}{3}\right]$ ← NLO IFs $\beta_0$

b) $\left(\mu_R^{BLM}\right)^2 = k_1 k_2 \ \exp\left[2\left(1+\frac{2}{3}I\right) - 2\ f\left(\nu\right) - \frac{5}{3} + \frac{1}{2}\chi\left(\nu, n\right)\right]$ ← NLO Kernel $\beta_0$

$f\left(\nu\right) = 0$ for this process

[F. Caporale, D.Yu. Ivanov, B. Murdaca, A. Papa, (2015)]

# Outline

# Observables and kinematics

- **Observables:**

$\phi$-averaged cross section $\mathcal{C}_0$ , $\langle \cos\left[n\left(\phi_{J_1}-\phi_{J_2}-\pi\right)\right]\rangle \equiv \frac{\mathcal{C}_n}{\mathcal{C}_0}$ , with $n=1,2,3$

$$\frac{\langle \cos\left[2\left(\pi-\Delta\phi\right)\right]\rangle}{\langle \cos\left(\pi-\Delta\phi\right)\rangle}=\frac{\mathcal{C}_2}{\mathcal{C}_1}\,,\quad \frac{\langle \cos\left[3\left(\pi-\Delta\phi\right)\right]\rangle}{\langle \cos\left[2\left(\pi-\Delta\phi\right)\right]\rangle}=\frac{\mathcal{C}_3}{\mathcal{C}_2}\,,\ \text{with}\ \ \Delta\phi=\phi_{J_2}-\phi_{J_1}\ .$$

◇ *Integrated coefficients:*

$$C_n=\int_{y_{1,\mathrm{min}}}^{y_{1,\mathrm{max}}}dy_1\int_{y_{2,\mathrm{min}}}^{y_{2,\mathrm{max}}}dy_2\int_{k_{J_1,\mathrm{min}}}^{\infty}dk_{J_1}\int_{k_{J_2,\mathrm{min}}}^{\infty}dk_{J_2}$$
$$\delta\left(y_1-y_2-Y\right)\theta\left(|y_1|-y_{\mathrm{max}}^{\mathrm{C}}\right)\theta\left(|y_2|-y_{\mathrm{max}}^{\mathrm{C}}\right)\mathcal{C}_n\left(y_{J_1},y_{J_2},k_{J_1},k_{J_2}\right)$$

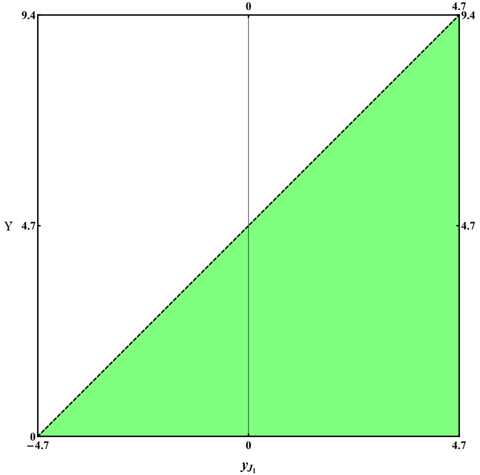
- **Kinematic settings:**
  - ◇ $R=0.5$ and $\sqrt{s}=13$ TeV
  - ◇ $y_{\mathrm{max}}^{\mathrm{C}}\leq|y_{J_{1,2}}|\leq 4.7$, with $y_{\mathrm{max}}^{\mathrm{C}}=0,\ 1.5\simeq 4.7/3,\ 2.5$
  - ◇ **1.** $k_{J_1}\geq 20,\ 35$ GeV; $k_{J_2}\geq 20,\ 35$ GeV; symmetric cuts, 2 choices
    **2.** $k_{J_1}\geq 20$ Gev; $k_{J_2}\geq 35,\ 40,\ 45$ GeV; asymmetric cuts, 3 choices

- **Numerical tools:** FORTRAN + NLO MSTW 2008 PDFs + CERNLIB

[A.D. Martin, W.J. Stirling, R.S. Thorne, G. Watt, (2009)]
http://cernlib.web.cern.ch/cernlib

# Rapidity range

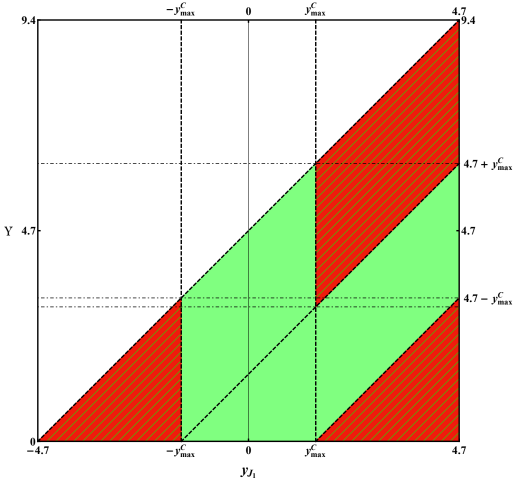
$$\int_{-4.7}^{4.7} dy_1 \int_{-4.7}^{4.7} dy_2 \; \delta(y_1 - y_2 - Y) \; \theta\left(|y_1| - y_{\max}^{C}\right) \theta\left(|y_2| - y_{\max}^{C}\right) \; \mathcal{C}_n\left(y_{J_1}, y_{J_2}, k_{J_1}, k_{J_2}\right)$$




$Y = y_{J_1} - y_{J_2}$

# Rapidity range

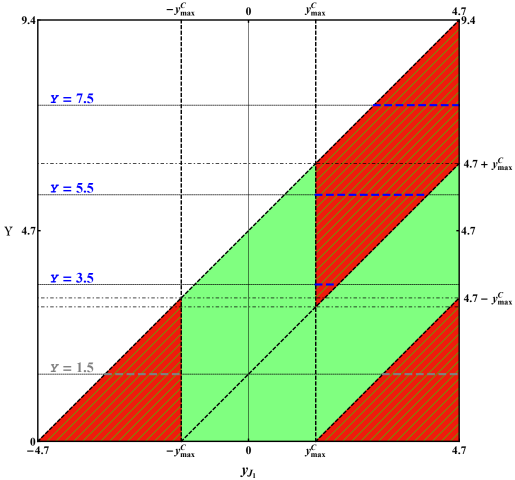
$$\int_{-4.7}^{4.7} dy_1 \int_{-4.7}^{4.7} dy_2 \; \delta(y_1 - y_2 - Y) \; \theta\left(|y_1| - y_{max}^C\right) \theta\left(|y_2| - y_{max}^C\right) \mathcal{C}_n \left(y_{J_1}, y_{J_2}, k_{J_1}, k_{J_2}\right)$$




$Y = y_{J_1} - y_{J_2}$

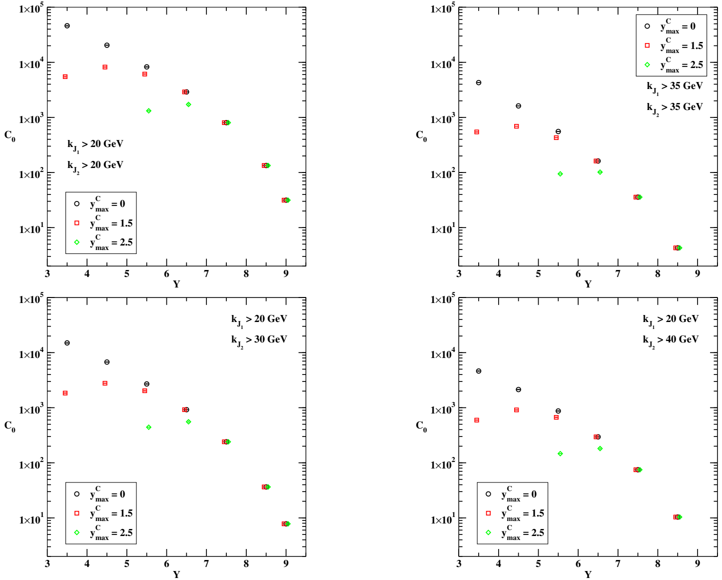
# Rapidity range

$$\int_{-4.7}^{4.7} \mathbf{dy_1} \int_{-4.7}^{4.7} \mathbf{dy_2}\ \delta(\mathbf{y_1} - \mathbf{y_2} - \mathbf{Y})\ \theta\left(|\mathbf{y_1}| - \mathbf{y}^{\rm C}_{\rm max}\right) \theta\left(|\mathbf{y_2}| - \mathbf{y}^{\rm C}_{\rm max}\right) \mathcal{C}_{\mathbf{n}}\left(\mathbf{y_{J_1}}, \mathbf{y_{J_2}}, \mathbf{k_{J_1}}, \mathbf{k_{J_2}}\right)$$

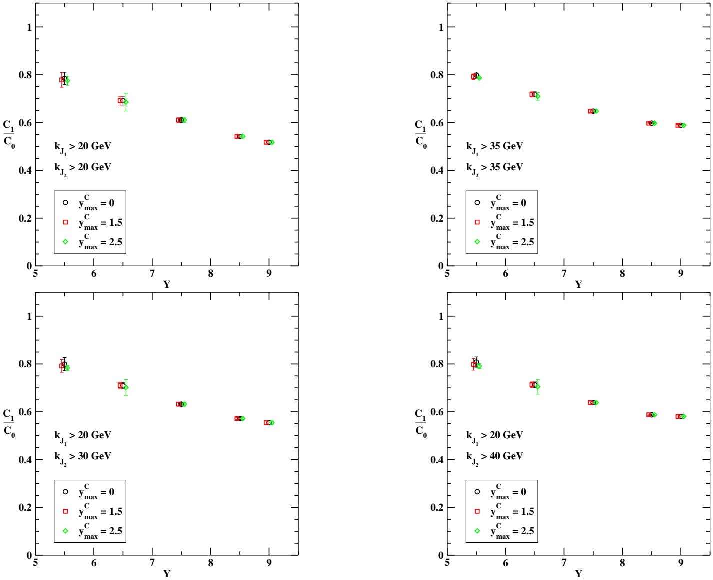



$$Y = y_{J_1} - y_{J_2}$$

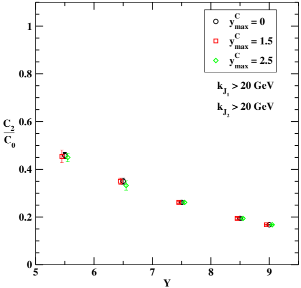
# $C_0$ vs $Y = y_{J_1} - y_{J_2}$ - "exact" BLM method

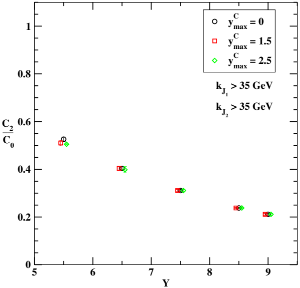


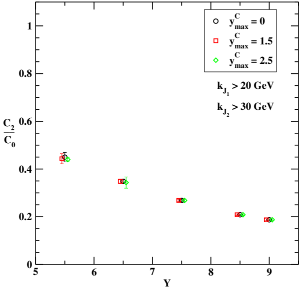
# $C_1/C_0$ vs $Y$ - "exact" BLM method

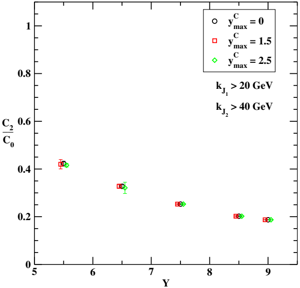


# $C_2/C_0$ vs $Y$ - "exact" BLM method

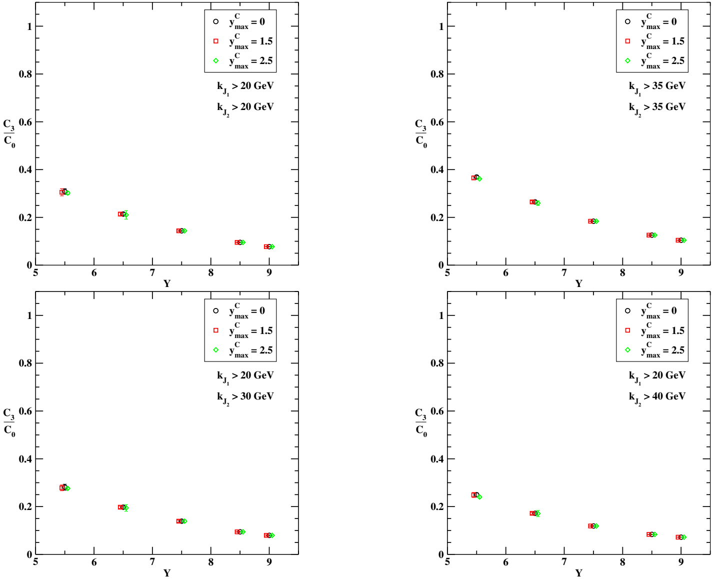


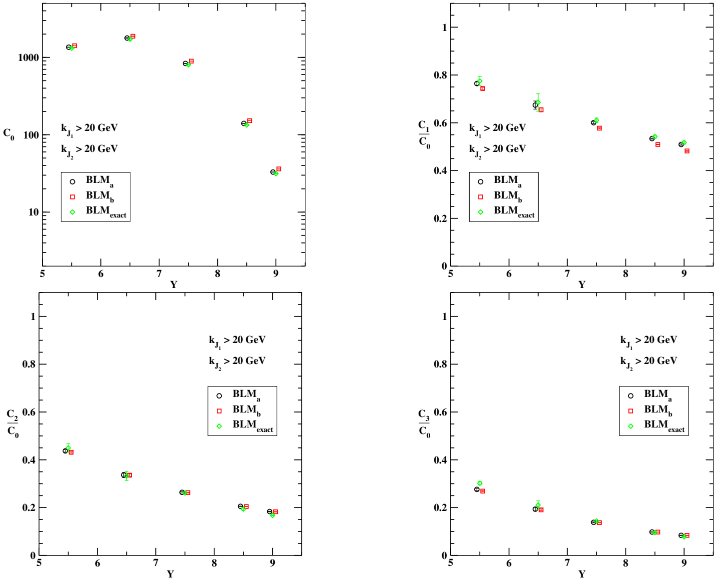



# $C_3/C_0$ vs $Y$ - "exact" BLM method



# BLM comparisons of $C_0$ and $R_{n0}$ vs $Y$ - $y^C_{\max} = 2.5$



# Conclusions

Comparison of predictions for $C_0$ several $R_{nm}$ ratios in full NLA BFKL approach

- Implementation of exact **BLM method** $\xrightarrow{\textit{compared to}}$ two different partial ones!
- Symmetric and asymmetric kinematics of detected jets

[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa, (2015)]

- Central rapidity range exclusion...

$\rightarrow$ in order to rule out from the final state a kinematics which could not be descripted by the BFKL approach

▶ effects on $R_{nm}$ negligible

$\Downarrow \quad \Downarrow \quad \Downarrow$

We strongly suggest experimentalist collaborations to measure $C_0$ by escluding central rapidity range...

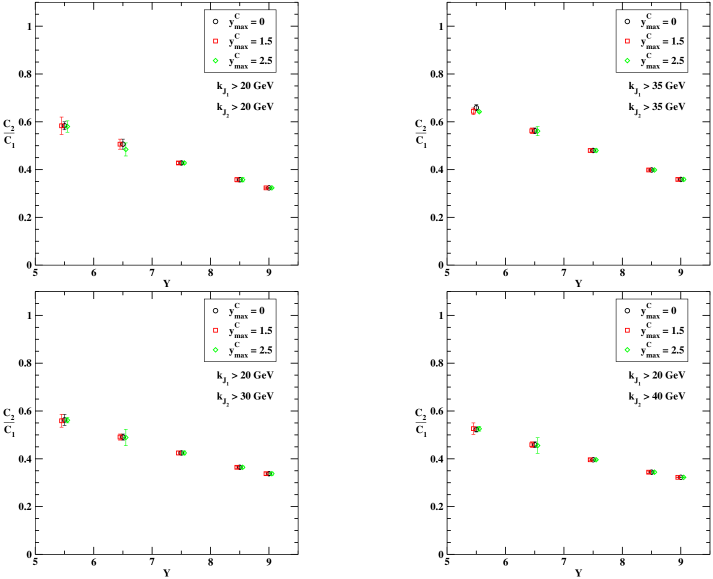
◇ ...to discriminate BFKL from different theoretical approaches (HEF, fixed order DGLAP, ...)

◇ ...to discriminate from different BFKL approaches ($C_n$ representation, scale optimization method, ...)

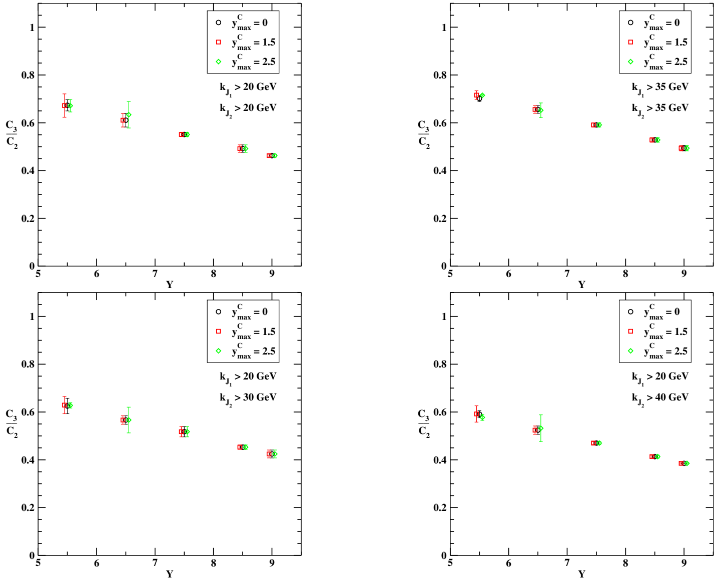
**Thanks for your attention!!**

# BACKUP slides

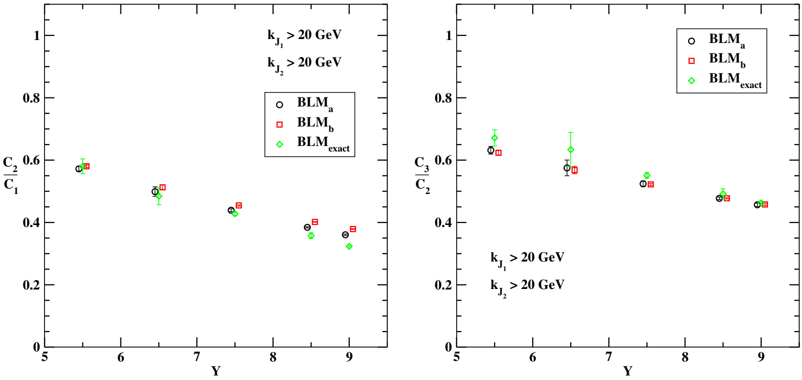
# $C_2/C_1$ vs $Y$ - "exact" BLM method



# $C_3/C_2$ vs $Y$ - "exact" BLM method



## BLM comparisons of $C_2/C_1$ and $C_3/C_2$ vs $Y$ - $y^C_{\max} = 2.5$



# The "exact" BLM cross section

$$\mathcal{C}_n^{\rm BLM} = \frac{x_{J_1} x_{J_2}}{|\vec{k}_{J_1}||\vec{k}_{J_2}|} \int_{-\infty}^{+\infty} d\nu \; e^{(Y-Y_0)\bar{\alpha}_s^{\rm MOM}(\mu_R^{\rm BLM})\left[\chi(n,\nu)+\bar{\alpha}_s^{\rm MOM}(\mu_R^{\rm BLM})\left(\bar{\chi}(n,\nu)+\frac{T^{\rm conf}}{N_c}\chi(n,\nu)\right)\right]}$$

$$\times (\alpha_s^{\rm MOM}(\mu_R^{\rm BLM}))^2 c_1(n,\nu,|\vec{k}_{J_1}|,x_{J_1}) c_2(n,\nu,|\vec{k}_{J_2}|,x_{J_2})$$

$$\times \left[1 + \alpha_s^{\rm MOM}(\mu_R^{\rm BLM}) \left\{ \frac{\bar{c}_1^{(1)}(n,\nu,|\vec{k}_{J_1}|,x_{J_1})}{c_1(n,\nu,|\vec{k}_{J_1}|,x_{J_1})} + \frac{\bar{c}_2^{(1)}(n,\nu,|\vec{k}_{J_2}|,x_{J_2})}{c_2(n,\nu,|\vec{k}_{J_2}|,x_{J_2})} + \frac{2\,T^{\rm conf}}{N_c} \right\}\right] ,$$

with the $\mu_R^{\rm BLM}$ scale chosen as the solution of the following integral equation...

$$\mathcal{C}_n^{\beta} \equiv \frac{x_{J_1} x_{J_2}}{|\vec{k}_{J_1}||\vec{k}_{J_2}|} \int_{-\infty}^{\infty} d\nu \left(\frac{s}{s_0}\right)^{\bar{\alpha}_s^{\rm MOM}(\mu_R^{\rm BLM})\chi(n,\nu)} \left(\alpha_s^{\rm MOM}(\mu_R^{\rm BLM})\right)^3$$

$$\times\, c_1(n,\nu) c_2(n,\nu) \frac{\beta_0}{2N_c} \left[\frac{5}{3} + \ln\frac{(\mu_R^{\rm BLM})^2}{Q_1 Q_2} - 2\left(1+\frac{2}{3}I\right)\right.$$

$$\left. + \bar{\alpha}_s^{\rm MOM}(\mu_R^{\rm BLM}) \ln\frac{s}{s_0}\, \frac{\chi(n,\nu)}{2} \left(-\frac{\chi(n,\nu)}{2} + \frac{5}{3} + \ln\frac{(\mu_R^{\rm BLM})^2}{Q_1 Q_2} - 2\left(1+\frac{2}{3}I\right)\right)\right] = 0$$

...which represents the condition that terms proportional to $\beta_0$ in $C_n$ disappear

$$\alpha^{\rm MOM} = -\frac{\pi}{2T}\left[1-\sqrt{1+4\alpha_s\left(\mu_R\right)\frac{T}{\pi}}\right] ,$$

with $T = T^\beta + T^{\rm conf}$,

$$T^\beta = -\frac{\beta_0}{2}\left(1+\frac{2}{3}I\right) ,$$

$$T^{\rm conf} = \frac{C_A}{8}\left[\frac{17}{2}I + \frac{3}{2}\left(I-1\right)\xi + \left(1-\frac{1}{3}I\right)\xi^2 - \frac{1}{6}\xi^3\right] ,$$

where $I = -2\int_0^1 dx \frac{\ln(x)}{x^2-x+1} \simeq 2.3439$ and $\xi$ is a gauge parameter.

# The BLM cases $(a)$ and $(b)$ cross section

a) $\left(\mu_R^{BLM}\right)^2 = k_1 k_2 \ \exp\left[2\left(1+\frac{2}{3}I\right) - f\left(\nu\right) - \frac{5}{3}\right] \sim 5^2 k_1 k_2$

b) $\left(\mu_R^{BLM}\right)^2 = k_1 k_2 \ \exp\left[2\left(1+\frac{2}{3}I\right) - 2\ f\left(\nu\right) - \frac{5}{3} + \frac{1}{2}\chi\left(\nu, n\right)\right] < (11.5)^2 k_1 k_2$

$$\mathcal{C}_n^{\rm BLM,a} = \frac{x_{J_1} x_{J_2}}{|\vec{k}_{J_1}||\vec{k}_{J_2}|} \int_{-\infty}^{+\infty} d\nu \ e^{(Y-Y_0)\left[\bar{\alpha}_s^{\rm MOM}(\mu_{R,a}^{\rm BLM})\chi(n,\nu) + (\bar{\alpha}_s^{\rm MOM}(\mu_{R,a}^{\rm BLM}))^2\left(\bar{\chi}(n,\nu) + \frac{T^{\rm conf}}{N_c}\chi(n,\nu) - \frac{\beta_0}{8N_c}\chi^2(n,\nu)\right)\right]}$$

$$\times\, (\alpha_s^{\rm MOM}(\mu_{R,a}^{\rm BLM}))^2 c_1(n,\nu,|\vec{k}_{J_1}|,x_{J_1}) c_2(n,\nu,|\vec{k}_{J_2}|,x_{J_2})$$

$$\times \left[1 + \alpha_s^{\rm MOM}(\mu_{R,a}^{\rm BLM}) \left\{ \frac{\bar{c}_1^{(1)}(n,\nu,|\vec{k}_{J_1}|,x_{J_1})}{c_1(n,\nu,|\vec{k}_{J_1}|,x_{J_1})} + \frac{\bar{c}_2^{(1)}(n,\nu,|\vec{k}_{J_2}|,x_{J_2})}{c_2(n,\nu,|\vec{k}_{J_2}|,x_{J_2})} + \frac{2T^{\rm conf}}{N_c} \right\}\right]$$

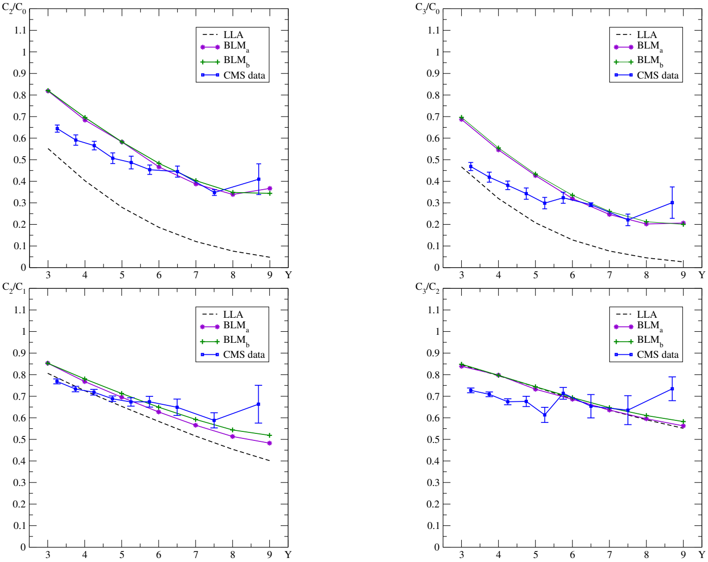
$$\mathcal{C}_n^{\rm BLM,b} = \frac{x_{J_1} x_{J_2}}{|\vec{k}_{J_1}||\vec{k}_{J_2}|} \int_{-\infty}^{+\infty} d\nu \ e^{(Y-Y_0)\left[\bar{\alpha}_s^{\rm MOM}(\mu_{R,b}^{\rm BLM})\chi(n,\nu) + (\bar{\alpha}_s^{\rm MOM}(\mu_{R,b}^{\rm BLM}))^2\left(\bar{\chi}(n,\nu) + \frac{T^{\rm conf}}{N_c}\chi(n,\nu)\right)\right]}$$

$$\times\, (\alpha_s^{\rm MOM}(\mu_{R,b}^{\rm BLM}))^2 c_1(n,\nu,|\vec{k}_{J_1}|,x_{J_1}) c_2(n,\nu,|\vec{k}_{J_2}|,x_{J_2})$$

$$\times \left[1 + \alpha_s^{\rm MOM}(\mu_{R,b}^{\rm BLM}) \left\{ \frac{\bar{c}_1^{(1)}(n,\nu,|\vec{k}_{J_1}|,x_{J_1})}{c_1(n,\nu,|\vec{k}_{J_1}|,x_{J_1})} + \frac{\bar{c}_2^{(1)}(n,\nu,|\vec{k}_{J_2}|,x_{J_2})}{c_2(n,\nu,|\vec{k}_{J_2}|,x_{J_2})} + \frac{2T^{\rm conf}}{N_c} + \frac{\beta_0}{4N_c}\chi(n,\nu) \right\}\right]$$

# BACKUP slides

## Why BLM? MN jets - symmetric kinematics




[F. Caporale, D.Yu. Ivanov, B. Murdaca, A. Papa, (2014)]