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#### Outline

#### Introduction to the Parton Reggeization Approach (PRA)

- $k_T$ -factorization, motivation for the PRA
- Reggeization of amplitudes in QCD
- $\bullet~{\rm Effective}~RR \to g~{\rm and}~RR \to c\bar{c}~{\rm vertices}$
- The effective action for high energy processes in QCD
- Factorization formula
- The Kimber-Martin-Ryskin unPDF
- Single D-meson production
  - Fragmentation approach. Subprocesses in the LO PRA
  - Numerical results
- $\bigcirc D\overline{D}$  pair production
  - Fragmentation approach. Subprocesses in the LO PRA

2/22

Numerical results

#### Motivation for $k_T$ -factorization and PRA

- Heavy final states (Higgs bosons,  $t\bar{t}, \ldots$ ) produced by large- $x \sim 10^{-1}$  initial partons  $\leftarrow$  soft and collinear gluons
- Light final states (small- $p_T$  quarkonia, single jets, prompt photons, ...) produced by small- $x \sim 10^{-3}$  initial partons  $\leftarrow$  additional hard jets
- To obtain the agreement with experimental data one needs to perform the pQCD calculations in NLO and higher  $\Rightarrow$  much time and computational resources are involved.
- Even that not all the observables are successfully described in the framework of collinear parton model (CPM), such as azimuthal angle distributions in pair production of particles.

At small x the logarithms  $\ln^n(1/x)$  can be resummed in all orders n by the BFKL-approach based on the property of gluon Reggeization.

#### Motivation for $k_T$ -factorization and PRA

• In the region of small  $x \sim \mu/\sqrt{S}$  most of the initial-state radiation is highly separated in rapidity from the central region, and can be factorized. In the small-*x* regime, initial-state partons carry the substantial transverse momentum (virtuality)  $|\mathbf{q}_T| \sim x\sqrt{S}$ , in contrast with the standard Collinear Parton Model (CPM), where  $|\mathbf{q}_T| \ll x\sqrt{S}$ , and can be neglected. This is the standard setup of the  $k_T$ -factorization [L. V. Gribov et. al. 1983; J. C. Collins et. al. 1991; S. Catani et. al. 1991].

The old  $k_T$ -factorization approach contains a prescription for a polarization vector of initial-state gluon with 4-momentum  $q = (q_0, \mathbf{q}_T, q_z)$ :

 $\epsilon^{\mu}(q) = \frac{q_T^{\mu}}{|\mathbf{q}_T|} \Rightarrow$  no gauge invariance for 3- and 4-gluon vertices; no generally accepted prescription for the treatment of off-shell initial-state quarks.

• We need special conditions for a gauge-invariant description of the processes with the off-shell initial state partons. The Reggeization of the amplitudes in QCD solves this problem.

In present time, two methods to generate the gauge-invariant amplitudes for the  $k_T\mbox{-}{\rm factorization}$  are proposed:

- The QCD in the Regge limit (see e. g. [B. Ioffe, V. S. Fadin, L. N. Lipatov, QCD Perturbative and Nonperturbative aspects] and [L. N. Lipatov, Nucl. Phys. B452 (1995) 369]).
- Methods based on the extraction of certain asymptotics of the amplitudes in the spinor-helicity representation (see e. g. [A. van Hameren *et. al.*, Phys.Lett. B727 226 (2013)]).

There is, in fact, a chain of succession between these methods.  $\bigcirc$ 

#### Reggeization of amplitudes in QCD.

PRA is based on the Reggeization of amplitudes in gauge theories (QED, QCD, Gravity). The *high energy asymptotics* of the  $2 \rightarrow 2 + n$  amplitude is dominated by the diagram with *t*-channel exchange of the effective (Reggeized) particle and Multi-Regge (MRK) or Quasi-Multi-Regge Kinematics (QMRK) of final state.



In the limit  $s \to \infty$ ,  $s_{1,2} \to \infty$ ,  $-t_1 \ll s_1$ ,  $-t_2 \ll s_2$  (Regge limit),  $2 \to 3$  amplitude has the form:

$$\mathcal{A}_{AB}^{A'B'C} = 2s\gamma_{A'A}^{R_1} \left(\frac{s_1}{s_0}\right)^{\omega(t_1)} \frac{1}{t_1} \times \\ \times \Gamma_{R_1R_2}^C(q_1, q_2) \times \frac{1}{t_2} \left(\frac{s_2}{s_0}\right)^{\omega(t_2)} \gamma_{B'B}^{R_2}$$

 $\Gamma^C_{R_1R_2}(q_1,q_2)$  - RRP effective production vertex,

 $\gamma^R_{A'A}$  - PPR effective scattering vertex,

 $\omega(t)$  - Regge trajectory.

Two approaches to obtain this asymptotics:

- BFKL-approach (Unitarity, renormalizability and gauge invariance), see [B. Ioffe, V. S. Fadin, L. N. Lipatov, QCD Perturbative and Nonperturbative aspects].
- Effective action approach [L. N. Lipatov, Nucl. Phys. B452 (1995) 369].

5/22

#### The field content of the effective theory.

To produce the amplitudes for the arbitrary QMRK processes, the effective-action approach is very useful. Light-cone coordinates and derivatives:

$$n^{+} = \frac{2P_2}{\sqrt{S}}, \ n^{-} = \frac{2P_1}{\sqrt{S}}, \ n^{+}n^{-} = 2$$
$$x^{\pm} = n^{\pm}x = x^0 \pm x^3, \ \partial_{\pm} = 2\frac{\partial}{\partial x^{\mp}}$$

Lagrangian of the effective theory  $L = L_{kin} + \sum_{y} (L_{QCD} + L_{ind}), v_{\mu} = v_{\mu}^{a} t^{a}$ ,

 $[t^a, t^b] = f^{abc}t^c$ . The rapidity space is sliced into the subintervals, corresponding to the groups of final-state particles, close in rapidity. Each subinterval in rapidity  $(1 \ll \eta \ll Y)$  has it's own set of QCD fields:

$$L_{QCD} = -\frac{1}{2} tr \left[ G_{\mu\nu}^2 \right], \ G_{\mu\nu} = \partial_{\mu} v_{\nu} - \partial_{\nu} v_{\mu} + g \left[ v_{\mu}, v_{\nu} \right].$$

Different rapidity intervals communicate via the gauge-invariant fields of Reggeized gluons  $(A_{\pm} = A_{\pm}^{a}t^{a})$  with the kinetic term:

$$L_{kin} = -\partial_{\mu}A^{a}_{+}\partial^{\mu}A^{a}_{-},$$

and the kinematical constraint:

#### The effective action for high energy processes in QCD.



Particles and Reggeons interact via induced interactions:

$$L_{ind} = - tr \left\{ \frac{1}{g} \partial_{+} \left[ P \exp\left( -\frac{g}{2} \int_{-\infty}^{x^{-}} dx'^{-} v_{+}(x') \right) \right] \cdot \partial_{\sigma} \partial^{\sigma} A_{-}(x) + \frac{1}{g} \partial_{-} \left[ P \exp\left( -\frac{g}{2} \int_{-\infty}^{x^{+}} dx'^{+} v_{-}(x') \right) \right] \cdot \partial_{\sigma} \partial^{\sigma} A_{+}(x) \right\}$$

Wilson lines lead to the infinite chain of the induced vertices:

$$\begin{split} L_{ind} &= tr \left\{ \begin{bmatrix} v_{+} - gv_{+}\partial_{+}^{-1}v_{+} + g^{2}v_{+}\partial_{+}^{-1}v_{+}\partial_{+}^{-1}v_{+} - \dots \end{bmatrix} \partial_{\sigma}\partial^{\sigma}A_{-} + \\ &+ \begin{bmatrix} v_{-} - gv_{-}\partial_{-}^{-1}v_{-} + g^{2}v_{-}\partial_{-}^{-1}v_{-}\partial_{-}^{-1}v_{-} - \dots \end{bmatrix} \partial_{\sigma}\partial^{\sigma}A_{+} \right\} \\ &+ \begin{bmatrix} v_{-} - gv_{-}\partial_{-}^{-1}v_{-} + g^{2}v_{-}\partial_{-}^{-1}v_{-}\partial_{-}^{-1}v_{-} - \dots \end{bmatrix} \partial_{\sigma}\partial^{\sigma}A_{+} \\ &+ \begin{bmatrix} v_{-} - gv_{-}\partial_{-}^{-1}v_{-} + g^{2}v_{-}\partial_{-}^{-1}v_{-}\partial_{-}^{-1}v_{-} - \dots \end{bmatrix} \partial_{\sigma}\partial^{\sigma}A_{+} \\ &+ \begin{bmatrix} v_{-} - gv_{-}\partial_{-}^{-1}v_{-} + g^{2}v_{-}\partial_{-}^{-1}v_{-} - \dots \end{bmatrix} \partial_{\sigma}\partial^{\sigma}A_{+} \\ &+ \begin{bmatrix} v_{-} - gv_{-}\partial_{-}^{-1}v_{-} + g^{2}v_{-}\partial_{-}^{-1}v_{-} - \dots \end{bmatrix} \partial_{\sigma}\partial^{\sigma}A_{+} \\ &+ \begin{bmatrix} v_{-} - gv_{-}\partial_{-}^{-1}v_{-} + g^{2}v_{-}\partial_{-}^{-1}v_{-} - \dots \end{bmatrix} \partial_{\sigma}\partial^{\sigma}A_{+} \\ &+ \begin{bmatrix} v_{-} - gv_{-}\partial_{-}^{-1}v_{-} + g^{2}v_{-}\partial_{-}^{-1}v_{-} - \dots \end{bmatrix} \partial_{\sigma}\partial^{\sigma}A_{+} \\ &+ \begin{bmatrix} v_{-} - gv_{-}\partial_{-}^{-1}v_{-} + g^{2}v_{-}\partial_{-}^{-1}v_{-} - \dots \end{bmatrix} \partial_{\sigma}\partial^{\sigma}A_{+} \\ &+ \begin{bmatrix} v_{-} - gv_{-}\partial_{-}^{-1}v_{-} + g^{2}v_{-}\partial_{-}^{-1}v_{-} - \dots \end{bmatrix} \partial_{\sigma}\partial^{\sigma}A_{+} \\ &+ \begin{bmatrix} v_{-} - gv_{-}\partial_{-}^{-1}v_{-} + g^{2}v_{-}\partial_{-}^{-1}v_{-} + g^{2}v_{-}\partial_{-}^{-1}v_{-} \\ &+ \begin{bmatrix} v_{-} - gv_{-}\partial_{-}^{-1}v_{-} + g^{2}v_{-}\partial_{-}^{-1}v_{-} + g^{2}v_{-}\partial_{-}^{-1}v_{-} \\ &+ \begin{bmatrix} v_{-} - gv_{-}\partial_{-}^{-1}v_{-} + g^{2}v_{-}\partial_{-}^{-1}v_{-} \\ &+ \begin{bmatrix} v_{-} - gv_{-}\partial_{-}^{-1}v_{-} + g^{2}v_{-}\partial_{-}^{-1}v_{-} \\ &+ \begin{bmatrix} v_{-} - gv_{-}\partial_{-}v_{-} + gv_{-} & gv_{-} \\ &+ \begin{bmatrix} v_{-} - gv_{-}\partial_{-}v_{-} & gv_{-} \\ &+ \begin{bmatrix} v_{-} - gv_{-} & gv_{-} & gv_{-} \\ &+ \begin{bmatrix} v_{-} - gv_{-} & gv_{-} & gv_{-} \\ &+ \begin{bmatrix} v_{-} - gv_{-} & gv_{-} & gv_{-} \\ &+ \begin{bmatrix} v_{-} - gv_{-} & gv_{-} & gv_{-} \\ &+ \begin{bmatrix} v_{-} - gv_{-} & gv_{-} & gv_{-} \\ &+ \begin{bmatrix} v_{-} & gv_{-} & gv_{-} & gv_{-} \\ &+ \begin{bmatrix} v_{-} & gv_{-} & gv_{-} & gv_{-} \\ &+ \begin{bmatrix} v_{-} & gv_{-} & gv_{-} & gv_{-} \\ &+ \begin{bmatrix} v_{-} & gv_{-} & gv_{-} & gv_{-} \\ &+ \begin{bmatrix} v_{-} & gv_{-} & gv_{-} & gv_{-} \\ &+ \begin{bmatrix} v_{-} & gv_{-} & gv_{-} & gv_{-} \\ &+ \begin{bmatrix} v_{-} & gv_{-} & gv_{-} & gv_{-} \\ &+ \begin{bmatrix} v_{-} & gv_{-} & gv_{-} & gv_{-} \\ &+ \begin{bmatrix} v_{-} & gv_{-} & gv_{-} & gv_{-} \\ &+ \begin{bmatrix} v_{-} & gv_{-} & gv_{-} & gv_{-} \\ &+ \begin{bmatrix} v_{-} & gv_{-} & gv_{-} & gv_{-} \\ &+ \begin{bmatrix} v_{-} & gv_{-} & gv_{-} & gv_{-} \\ &+ \begin{bmatrix} v$$

#### Effective Fadin-Kuraev-Lipatov vertex.

Using  $\mathbf{Rg}$  and  $\mathbf{Rgg}$  vertices, the Fadin-Kuraev-Lipatov  $RR \to g$  vertex can be constructed:

the effective vertex is gauge-invariant, even for the off-shell initial state partons  $(q_{1,2}^2 < 0, (q_1 + q_2)^2 = 0)$ :

$$(q_1 + q_2)_{\mu} \Gamma_{abc}^{-\mu+}(q_1, q_2) = 0.$$

It contributes to the  $RR \rightarrow c\bar{c}$  vertex:

#### Factorization of the cross section

Factorization:



Factorization formula:

$$d\sigma = \int \frac{d^2 \mathbf{q}_{T1}}{\pi} \int \frac{dx_1}{x_1} \Phi(x_1, t_1, \mu_F) \times \int \frac{d^2 \mathbf{q}_{T2}}{\pi} \int \frac{dx_2}{x_2} \Phi(x_2, t_2, \mu_F) d\hat{\sigma}_{PRA}$$

Where  $\Phi$  - Unintegrated PDFs.

Partonic cross section:

$$\begin{aligned} d\hat{\sigma}_{PRA} &= \frac{(2\pi)^4}{2x_1 x_2 S} \overline{|\mathcal{M}|^2}_{PRA} \delta^{(4)}(P_{[i]} - P_{[f]}) \times \\ &\times \prod_{j=[f]} \frac{d^3 \mathbf{p}_j}{(2\pi)^3 2 p_j^0}, \end{aligned}$$

Normalization of the unPDF:

$$\int^{\mu^2} dt \Phi(x,t,\mu^2) \approx x f(x,\mu^2),$$

where  $f(x, \mu^2)$  - collinear PDF, implies, that the *collinear limit* holds for the amplitude (at the *small x*):

$$\int \frac{d\phi_1 d\phi_2}{(2\pi)^2} \lim_{t_{1,2} \to 0} \overline{|\mathcal{M}|^2}_{PRA} \approx \overline{|\mathcal{M}|^2}_{CPM}$$

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#### The Kimber-Martin-Ryskin unPDF.

#### Kimber M. A., Martin A. D., Ryskin M. G., Phys. Rev. D **63**, 114027, (2001), [arXiv:hep-ph/0101348]

KMR prescription to obtain unintegrated PDF from collinear one is based on the mechanism of last step parton  $k_T$ -dependent radiation and the assumption of strong angular ordering:

$$\Phi_g(x, k_T^2, \mu^2) = T_g(k_T, \mu) \frac{\alpha_s(k_T^2)}{2\pi} \int_x^{1-\Delta} dz \int_x^{k_T^2} \frac{dq_T^2}{q_T^2} \times \left[ P_{gg}(z) f_g\left(\frac{x}{z}, q_T^2\right) + P_{gq}(z) f_q\left(\frac{x}{z}, q_T^2\right) \right].$$

Where  $P_{gg}(z)$ ,  $P_{gq}(z)$ - DGLAP splitting functions,  $T_g(k_T, \mu)$ - Sudakov formfactor:

$$T_{g}(k_{T},\mu) = exp\left\{-\int_{k_{T}^{2}}^{\mu^{2}} \frac{dq_{T}^{2}}{q_{T}^{2}} \frac{\alpha_{s}(q_{T}^{2})}{2\pi} \sum_{a'} \int_{\Delta}^{1-\Delta} P_{ga'}(z')dz'\right\}$$

where  $\Delta = \frac{k_T}{\mu + k_T}$  ensures the rapidity ordering of the last emission and particles produced in the hard subprocess.

10/22

#### Fragmentation approach. Subprocesses in the LO PRA

In the fragmentation approach [B. Mele, P. Nason, 1991], the cross section of the inclusive production of *D*-meson is related with the parton cross section as follows:

$$\frac{d\sigma}{dp_T dy} \left( p + p \to D_i(p) + X \right) = \sum_a \int_0^1 \frac{dz}{z} D_i(z, \mu^2) \frac{d\sigma}{dq_T dy} \left( p + p \to a(p/z) + X \right)$$

where  $D_i(z, \mu^2)$ -fragmentation function for the meson  $D_i$  (which depends on  $\mu$ -scale unlike the Peterson ansatz). In our calculations we use the LO set of FFs by [B. A. Kniehl, G. Kramer *et. al.*] fitted on the  $e^+e^-$  annihilation data. We take into account the following parton subprocesses:

$$R(q_1) + R(q_2) \quad \to \quad g(q_3) \left[ \to D(p) \right], \tag{1}$$

$$R(q_1) + R(q_2) \quad \to \quad c(q_3) \left[ \to D(p) \right] + \bar{c}(q_4), \tag{2}$$

where  $q_1^2 = -\mathbf{q}_{T1}^2 = -t_1$ ,  $q_2^2 = -\mathbf{q}_{T2}^2 = -t_2$ . Subprocess (2) contains the collinear divergence, which is regularized by the finite  $m_c$ .

D and  $D\bar{D}$  pair production at the LHCb in the Parton Reggeization Approach Numerical results for single D-meson production

### LHCb data, $2.0 < y < 4.5, \sqrt{S} = 7$ TeV.



Figure 1: Transverse momentum distributions of  $D^0$  and  $D^+$  mesons in *pp* scattering with  $\sqrt{S} = 7$  TeV and 2.0 < y < 4.5. Dashed line represents the contribution of gluon fragmentation, dash-dotted line – the *c*-quark-fragmentation contribution, solid line is their sum. The LHCb data at the LHC are from the [LHCb Collaboration, R. Aaij *et al.*, Nucl.Phys. **B871**, 1-20 (2013)].

D and  $D\bar{D}$  pair production at the LHCb in the Parton Reggeization Approach Numerical results for single D-meson production

### LHCb data, $2.0 < y < 4.5, \sqrt{S} = 7$ TeV.



Figure 2: Transverse momentum distributions of  $D^{\star+}$  and  $D_s^+$  mesons in pp scattering with  $\sqrt{S} = 7$  TeV and 2.0 < y < 4.5.

D and  $D\bar{D}$  pair production at the LHCb in the Parton Reggeization Approach Numerical results for single D-meson production

## ALICE data, |y| < 0.5, $\sqrt{S} = 7$ TeV.

Published result:

 Karpishkov A. V., Nefedov M. A., Saleev V. A., Shipilova A. V., Phys. Rev. D 91, 054009



Figure 3: Transverse momentum distributions of  $D^0$ ,  $D^+$ ,  $D^{*+}$ , and  $D_s^+$  mesons in pp scattering with  $\sqrt{S} = 7$  TeV and |y| < 0.5. The notations as in the Fig. 1. The ALICE data at the LHC are from the [ALICE Collaboration, B. Abelev *et al.*, JHEP **1207**, 191 (2012)].

D and  $D\bar{D}$  pair production at the LHCb in the Parton Reggeization Approach  $D\overline{D}$  pair production

## Fragmentation approach (pair production). Subprocesses in the LO PRA.

In case of pair D-meson production we can write down the cross section of the inclusive pair production of D-mesons in the following form:

$$\begin{aligned} \frac{d\sigma}{dp_{TD}dy_{D}dp_{T\overline{D}}dy_{\overline{D}}}\left(p+p\to D_{i}(p_{D})+\overline{D}_{j}(p_{\overline{D}})+X\right) = \\ = \sum_{ab} \int_{0}^{1} \frac{dz_{1}}{z_{1}} D_{i}(z_{1},\mu^{2}) \int_{0}^{1} \frac{dz_{2}}{z_{2}} D_{j}(z_{2},\mu^{2}) \frac{d\sigma}{dq_{3T}dy_{3}dq_{4T}dy_{4}} \left(p+p\to a(\frac{p_{D}}{z_{1}})+b(\frac{p_{\overline{D}}}{z_{2}})+X\right) d\sigma \end{aligned}$$

We take into account the following partonic subprocesses:

$$R(q_1) + R(q_2) \quad \to \quad g(q_3) \left[ \to D(p_D) \right] + g(q_4) \left[ \to \overline{D}(p_{\overline{D}}) \right], \tag{3}$$

$$R(q_1) + R(q_2) \quad \to \quad c(q_3) \left[ \to D(p_D) \right] + \bar{c}(q_4) \left[ \to \overline{D}(p_{\overline{D}}) \right], \tag{4}$$

where  $q_1^2 = -\mathbf{q}_{T1}^2 = -t_1$ ,  $q_2^2 = -\mathbf{q}_{T2}^2 = -t_2$ . Subprocesses (3) and (4) contains the collinear divergence, which is regularized by the finite  $m_c$ .

Numerical results for  $D\overline{D}$  pair production  $D^0\overline{D}^0$  spectra

## LHCb data, 2 < y < 4, $\sqrt{S} = 7$ TeV.



16/22

Collab. R. Aaii et al., JHEP **1206**, 141 (2012)].

D and  $D\overline{D}$  pair production at the LHCb in the Parton Reggeization Approach Numerical results for  $D\overline{D}$  pair production

Numerical results for DD pair production  $D^0 D^-$  spectra

## LHCb data, 2 < y < 4, $\sqrt{S} = 7$ TeV.



Figure 5:  $\Delta \phi$ ,  $p_T$ , Y, and  $M_{D\bar{D}}$  spectra for  $D^0 D^-$  pair. LHCb data from the LHCb  $O^{O^-}$ Collab. R. Aaij *et al.*, JHEP **1206**, 141 (2012)].

D and  $D\overline{D}$  pair production at the LHCb in the Parton Reggeization Approach Numerical results for  $D\overline{D}$  pair production

 $D^0 D_{\circ}^-$  spectra

## LHCb data, 2 < y < 4, $\sqrt{S} = 7$ TeV.



Figure 6:  $\Delta \phi$ ,  $p_T$ , Y, and  $M_{D\bar{D}}$  spectra for  $D^0 D_s^-$  pair. LHCb data from the [LHCb<sup>=</sup>  $O \bigcirc \bigcirc$ Collab. R. Aaij *et al.*, JHEP **1206**, 141 (2012)].

D and  $D\overline{D}$  pair production at the LHCb in the Parton Reggeization Approach Numerical results for  $D\overline{D}$  pair production

 $D^+D^-$  spectra

## LHCb data, 2 < y < 4, $\sqrt{S} = 7$ TeV.



Figure 7:  $\Delta \phi$ ,  $p_T$ , Y, and  $M_{D\bar{D}}$  spectra for  $D^+D^-$  pair. LHCb data from the LHCb O @ O O OCollab. R. Aaij *et al.*, JHEP **1206**, 141 (2012)].

Numerical results for  $D\overline{D}$  pair production

 $D^+D_{\circ}^-$  spectra

## LHCb data, 2 < y < 4, $\sqrt{S} = 7$ TeV.



Collab. R. Aaii et al., JHEP **1206**, 141 (2012)].

20 / 22

#### Conclusions

- In the single D-meson production the contribution of the  $RR \to g [\to D]$  subprocess has been found to be significant.
- We have described the inclusive pair production of  $D\overline{D}$  mesons in the Parton Reggeization Approach within uncertainties and without free parameters.
- In case of  $D\overline{D}$  production we can see that the  $RR \to c\overline{c}$  contribution lies upper than  $RR \to gg$ . This subprocesses have the same order of  $\alpha_S$  but the probability of fragmentation of *c*-quark into *D* meson is higher than the gluon one.

DD pair production at the LHCb will be discussed in the next talk by Prof. Szczurek.

## Thank you for your attention!