An introduction to searches for gravitational waves

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General Relativity

- Current understanding of gravity at macroscopic scales based on classical general relativity
- Spacetime metric g_{ab} satisfies Einstein's equation: $R_{ab} - \frac{1}{2}Rg_{ab} = 16\pi G T_{ab}$
- For small deviations from flat space: $g_{ab} = \eta_{ab} + h_{ab}$, we get propagation of gravitational waves

$$\left(-\frac{\partial^2}{\partial t^2} + \nabla^2\right)\bar{h}_{ab} = 16\pi T_{ab}$$

  where $\bar{h}_{ab} = h_{ab} - \frac{1}{2}h\eta_{ab}$, Lorentz gauge: $\partial_a \bar{h}^{ab} = 0$.
- Can make a further gauge transformation so that $h_{0i} = h_{00} = h = 0$: This the transverse traceless gauge

Generation of gravitational waves

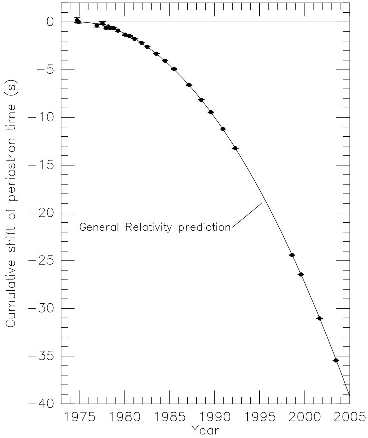
- As a first approximation, in the slow motion approximation, the generation of GWs is governed by the quadrupole formula

$$h_{ij}^{TT} = \frac{2G}{c^4 r}\ddot{\mathcal{I}}_{ij}^{TT}$$

- Here $\mathcal{I}_{ij}$ is the source's mass quadrupole moment
- $G/c^4 \sim 8 \times 10^{-45}\,\mathrm{s}^2\text{-}\mathrm{kg}^{-1}\text{-}\mathrm{m}^{-1}$ is a very small number
- We need astrophysical sources to produce detectable GWs
- Since GWs interact weakly with matter, they provide "clean" probes of the source if they can be detected – negligible absorption/dispersion by intervening matter

Generation of gravitational waves

- Indirect evidence of the existence of GWs - The Hulse-Taylor binary system



- Agreement with GR is within $(0.13 \pm 0.21)\%$
- This is still not a very relativistic system ($v/c \approx 0.15\%$)
- With GW observations, we can track systems with $v/c \sim \mathcal{O}(10^{-1})$

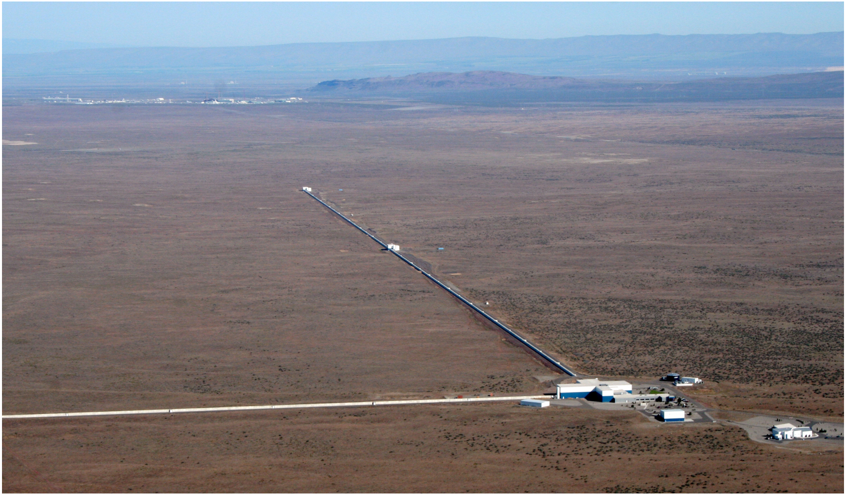
(Taylor& Weisberg (2004))

Interferometric detecors

Current and planned Earth based interferometric detectors:

- LIGO - Hanford and Livingston in the USA
- Virgo - Pisa, Italy
- GEO600 - Hannover, Germany
- KAGRA - Kamioka, Japan (Under construction)
- LIGO India - Site TBD

Detection of gravitational waves



(LIGO Hanford, USA)

Detection of gravitational waves

(LIGO Livingston, USA)

Detection of gravitational waves

(Virgo, Pisa, Italy)

Detection of gravitational waves

(GEO600, Hannover, Germany)

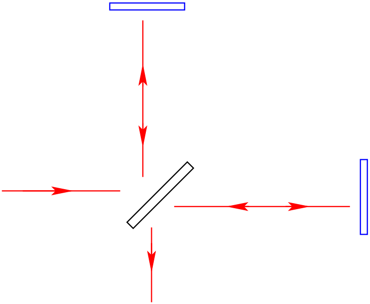
Detection of gravitational waves

- Wave traveling along z-axis, $+$ polarization:

$$ds^2 = -c^2 dt^2 + [1 + h_+]dx^2 + [1 - h_+]dy^2$$

Can use Michelson interferometer to detect GWs

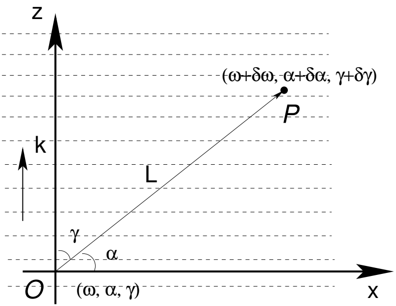
- x-arm changes by factor $1 + \frac{h(t)}{2}$
- y-arm changes by factor $1 - \frac{h(t)}{2}$
- This is actually not the right calculation!



Change in differential arm length $\delta L = h(t) \times L$

Detection of gravitational waves

- In practice one uses photons to “measure” the arm lengths
- Actual measurement is the phase-difference of the photon returning from the two arms
- The photon frequency is Doppler-shifted and its path is lensed by the gravitational wave



- Photon traveling from O to P
- GW traveling in z-direction

Detection of gravitational waves

- Change in frequency to linear order:

$$\frac{\delta\nu}{\nu} = \frac{1}{2}\frac{\alpha^2 - \beta^2}{1-\gamma}\Delta h_+ + \frac{\alpha\beta}{1-\gamma}\Delta h_\times = \frac{1}{2}\frac{\hat{x}^i\hat{x}^j}{1-\hat{k}\cdot\hat{x}}\Delta h_{ij}$$

- Depends only on h at O and P
- For ground based detectors (LIGO/Virgo etc.) consider long wavelength limit $\lambda_{GW} \gg L$
- Consider round trips around both arms of a Michelson interferometer with arms along $\hat{X}$ and $\hat{Y}$

$$\Delta\Phi = L(\hat{X}^i\hat{X}^j - \hat{Y}^i\hat{Y}^j)h_{ij}$$

The detector response

- Consider a wave traveling along the z-direction
- The GW is transverse, i.e. $h_{\mu\nu}$ is non-vanishing only in the x-y plane
- If we make a rotation in the $x - y$ plane by an angle ψ then

$$h_+ \to h_+ \cos 2\psi - h_\times \sin 2\psi$$

$$h_\times \to h_+ \sin 2\psi + h_\times \cos 2\psi$$

- For the general elliptically polarized case we can fix a frame such that

$$h_+ = A_+(t) \cos 2\Phi(t)$$

$$h_\times = A_\times(t) \sin 2\Phi(t)$$

$A_{+,\times}$ are slowly varying and $\Phi(t)$ is rapidly varying

The detector response

- Consider now a single detector with perpendicular arms
- Align the detector frame with these arms with the z-direction perpendicular to the detector plane
- Spherical coordinates define lines of latitude and longitude in the in the detector frame
- The orientation of the wave frame with respect to the lines of latitude and longitude defines the so-called *polarization angle* ψ

The detector response

- Go back to expression for the strain $h(t) = \delta L/L$:

$$h(t) = (\hat{X}^i \hat{X}^j - \hat{Y}^i \hat{Y}^j) h_{ij}$$

- $h(t)$ must depend linearly on $h_{+,\times}$
- Convince yourself that it must be of the form:

$$h(t) = F_+(\hat{n}, \psi) h_+(t) + F_\times(\hat{n}, \psi) h_\times(t)$$

- For a network of detectors, we use a fiducial frame fixed to Earth or to the SSB

Periodic gravitational waves from neutron stars

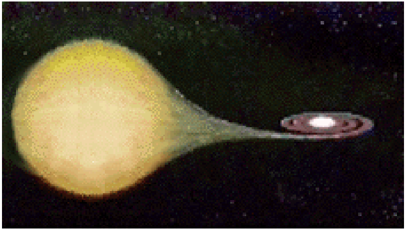
- The neutron star can emit GWs because of non-axisymmetries:

$$\epsilon = \frac{I_{xx} - I_{yy}}{I_{zz}}$$

- Commonly accepted reasons for emission:
  - Deformation of the crust
  - Oscillation modes in the neutron star fluid
- Regardless of mechanism, is there any reason to suspect that any neutron star is emitting detectable GWs? – accreting neutron stars

Periodic gravitational waves from neutron stars

- Observationally accreting NSs seem to spin slower than expected. Current record is 716 Hz and most spin much slower.
- Break-up frequency is expected to be at least 1kHz – Is this limit due to GW emission? (Bildsten, 1998)
- Spindown due to GW emission could be balancing spin-up torque due to accretion



(NASA HEASARC)

Signal from rotating neutron stars

In the rest frame of the star, the signal is a slowly varying sinusoid with a quadrupole pattern:

$$\begin{aligned} h_+(\tau) &= A_+ \cos\Phi(\tau) \qquad & h_\times(\tau) = A_\times \sin\Phi(\tau) \\ A_+ &= h_0 \frac{1+\cos^2\iota}{2} \qquad & A_\times = h_0 \cos\iota \\ h_0 &= \frac{16\pi^2 G}{c^4} \frac{I_{zz}\epsilon f_r^2}{d} & \end{aligned}$$

- ι: pulsar orientation w.r.t line of sight
- $\epsilon = (I_{xx} - I_{yy})/I_{zz}$: equatorial ellipticity
- f_r: rotation frequency
- d: distance to star
- value of h_0 (and ϵ) is model dependent

Signal from rotating neutron stars

The phase is very simple in the source frame:

$$\Phi(\tau) = \Phi_0 + 2\pi \left[f(\tau - \tau_0) + \frac{1}{2}\dot{f}(\tau - \tau_0)^2 + \dots \right]$$

Need to correct for the arrival times

- For an isolated pulsar:

$$\tau = t + \frac{\mathbf{r}_D \cdot \mathbf{n}}{c} + \text{relativistic corrections}$$

- For a pulsar in a binary system:

$$\tau = t + \frac{\mathbf{r}_D \cdot \mathbf{n}}{c} + \frac{\mathbf{r}_P \cdot \mathbf{n}}{c} + \text{relativistic corrections}$$

- $\mathbf{n}$: sky-position, $\mathbf{r}_D$: Detector in SSB frame, $\mathbf{r}_P$: Pulsar in binary frame

Signal from rotating neutron stars

- The signal as seen by the detector is Doppler shifted

$$f(t) = \hat{f}\left(1 + \frac{\mathbf{v} \cdot \mathbf{n}}{c}\right)$$

- Note that for orbital motion: $v/c \sim 10^{-4}$ and for Earth rotation, $v/c \sim 10^{-6}$
- Thus, $\delta f \sim f \times 10^{-4}$ – appreciable at the higher frequency end when $f \sim 1\,\mathrm{kHz}$
- Smaller variation due to Earth rotation over smaller timescale
- Earth rotation also causes amplitude modulation because relative orientation between detector and source changes

The spindown limit

- Simplest estimate of h_0 comes from energy conservation
- Energy carried away by GWs:

$$\dot{E}_{gw} = -\frac{c^3}{16\pi G} \oint_S \langle \dot{h}_+^2 + \dot{h}_\times^2 \rangle d^2 S$$

- Average over large sphere and over at least a few cycles
- This must be balanced by loss in rotational energy $\dot{E}_{rot} = \pi I f \dot{f}$
- In general $\dot{E}_{gw} \leq \dot{E}_{rot}$ then

$$h_0 \leq \frac{1}{d} \sqrt{\frac{5GI}{2c^3} \frac{|\dot{f}|}{f}}$$

The spindown limit

- If $\dot{E}_{gw} = x\dot{E}_{rot}$ then

$$h_0 \leq \frac{1}{d}\sqrt{\frac{5GxI}{2c^3}\frac{|\dot{f}|}{f}}$$

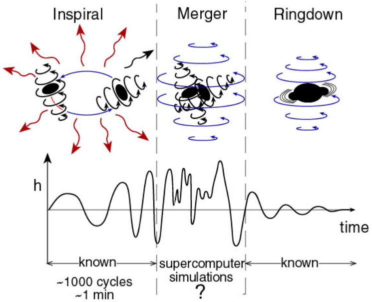
- At this amplitude, the equivalent ellipticity will be

$$\epsilon^{sd} = \sqrt{\frac{5c^5}{32\pi^4 G}\frac{x|\dot{f}|}{If^5}}$$

- This should be consistent with neutron star models
- Plausible Upper-limit:ϵ 10^{-5}

Signal from a coalescing binary

- The binary system has a time varying quadrupole moment, thus it will lose energy in GWs
- This means that the orbit shrinks and the orbital velocity increases

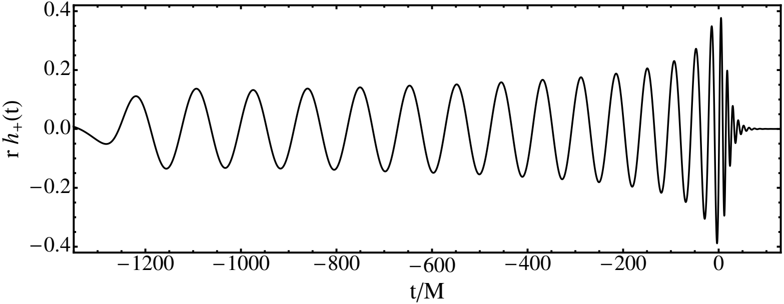


(K.Thorne)

- Inspiral and ringdown described by perturbative techniques
- Merger needs numerical simulations

Signal from a coalescing binary

- An example of a complete inspiral-merger-ringdown waveform (equal mass non-spinning BHs)
- Waveform obtained by stitching together PN and NR calculations



Signal from a coalescing binary

Some features of the signal

- The signal is a "chirp", it increases in pitch and loudness.
- At leading order, during the inspiral phase, the frequency evolves as
$$\frac{df}{dt} = \frac{96}{5}\pi^{8/3}\mathcal{M}^{5/3}f^{11/3}$$
- $\mathcal{M} = \mu^{3/5}M^{2/5}$ is called the chirp mass, $\mu = m_1 m_2/(m_1 + m_2)$ and $M = m_1 + m_2$
- For a $1.4\,M_\odot$-$1.4\,M_\odot$ system, starting at 40 Hz, it will coalesce in $\sim$ 30 sec
- For a starting frequency of about 10 Hz, this will be $\mathcal{O}(10^3)$ sec
- Coalescence will happen at $\sim$ 1.5 kHz

Signal from a coalescing binary

- We shall take the signal model to be

$$\Phi(t) = 2\phi_c + 2\phi(t - t_c; m_1, m_2)\,,$$

$$\eta(t) = \left(\frac{t_c - t}{5G\mathcal{M}/c^3}\right)^{-1/4}\,,$$

$$A_+ = -\frac{G\mathcal{M}}{c^2 D}\frac{1+\cos^2\iota}{2}\,, \quad A_+ = -\frac{G\mathcal{M}}{c^2 D}\cos\iota\,.$$

- Works for low-mass binaries but needs to be modified to deal with merger and spins if necessary
- ϕ_c, t_c, amlitudes, sky-position are extrinsic parameters
- m_1, m_2 and spins are intrinsic paremeters (determine phase evolution) (spins would also be intrinsic if they are considered)

Searching for CW signals

- Close analogy to much simpler problem of searching for monochromatic signals in noisy data
- If we had a signal $h(t) = A\sin(2\pi ft)$ in noise $n(t)$ so that data is $x(t) = h(t) + n(t)$
- Perform a Fourier transform: location of peak in spectrum yields f and height of peak yields A
- However now f and A are time dependent
- This time dependence depends in turn on sky-location, frequency and spindown parameters
- This leads to the most computationally challenging problem in GW data analysis

Searching for CW signals

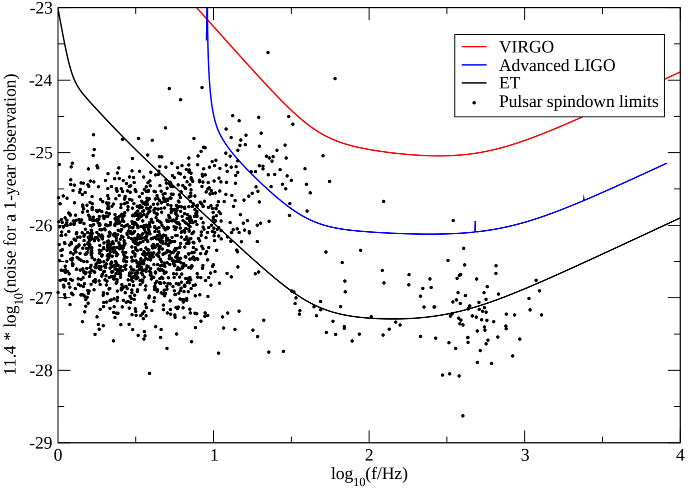
Three kinds of searches

- Targeted searches: Sky location, frequency and spindown all known (Pulsars studied in e.g. by radio telescopes)
- Directed searches: Sky location known but not frequency (e.g. galactic center, Cassiopea-A etc.)
- Blind searches: All sky surveys for unknown systems

Both Targeted and Blind searches are typically computationally limited

- $(h_0, \cos\iota, \phi_0, \psi)$ are “extrinsic” parameters – do not determine phase evolution
- We need a template grid over sky-position f and spindowns

The targeted searches



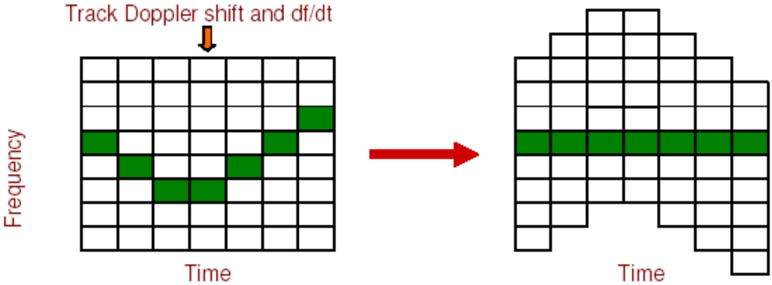
The targeted searches

- Spin-down limits for the Crab and Vela pulsars have been beaten
- The Crab-result: ApJ Lett. **683** (2008) 45 (see also ApJ **713** (2010) 671)
- The Vela result: ApJ **737** (2011) 93

Searching for CW signals

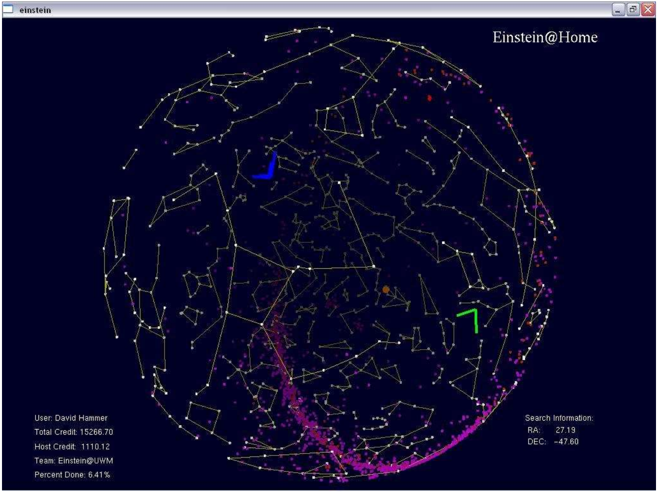
- Blind searches for CW signals over a wide parameter space are computationally limited
- Full matched filter searches are not feasible, and hierarchical techniques
- Break up full observation time into smaller segments
- Each segment is analysed coherently and they are later combined incoherently

Searching for CW signals



- This is still a very computationally intensive problem
- Recent work has involved Einstein@Home

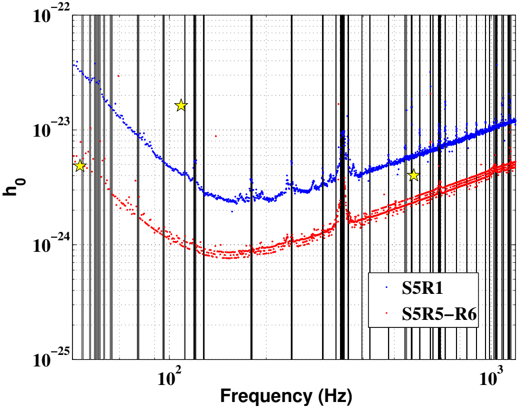
Blind searches for isolated pulsars



An example search result (PRD 87, 042001, 2013)

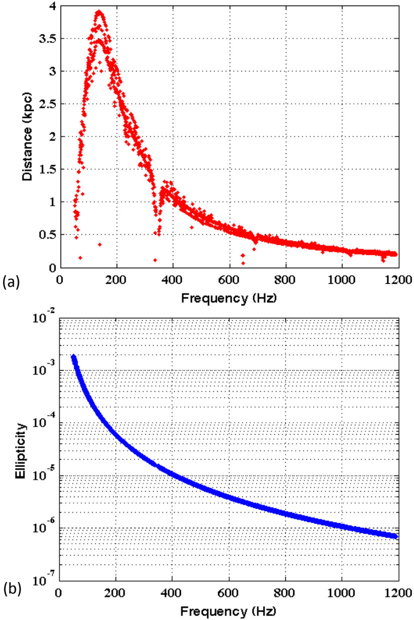
- Data taken between Nov 04,2005 - Sep 30, 2007 with the 4km LIGO instruments at Hanford and Livingston
- This was the "S5" run with the LIGO detectors at design sensitivity
- Search launched on Einstein@Home between January-October 2009
- 225,000 volunteers, 750,000 regiatered machines, 25,000 CPU years
- Data broken up into 25h segments (at least 40 hours of data in total in each segment)
- All sky search, $50\,\mathrm{Hz} \leq f \leq 1190\,\mathrm{Hz}$, $-2\mathrm{Hz/s} \leq \dot{f} \times 10^9 \leq 0.11\,\mathrm{Hz/s}$
- The incoherent combination of the different segments is performed by a Hough transform
- No signals were found

An example search result



- No limits set in marked bands
- Results better than previous Einstein@Home results by a factor of ~ 3

An example search result



- Maximum reach is ~ 4 kpc in optimal case

$$h_0^{sd} = \frac{8 \times 10^{-19}}{D_{kpc}} \frac{I}{I_{38}} \sqrt{\frac{|\dot{f}|}{f}}$$

Searching for low-mass systems

- Basic search is based the “FINDCHIRP” algorithm (Allen et al, `gr-qc/0509116`)
- Method is based on matched filtering, i.e. correlating the data with the expected signal with
- Additional tricks needed for dealing with real non-ideal data and coincidence between different detectors
- For a template $h(t)$ and data $x(t)$

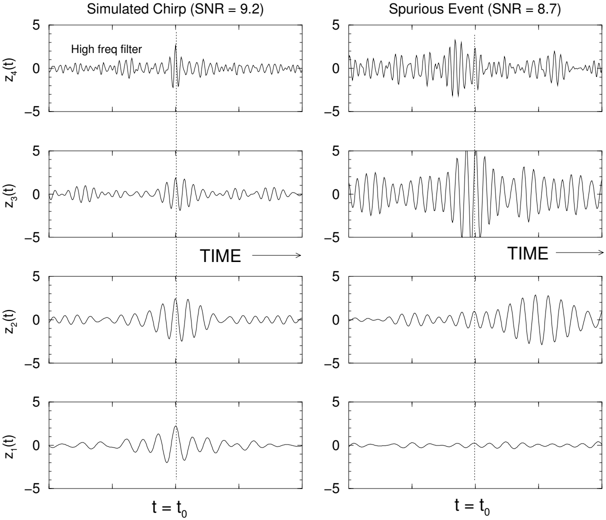
$$z(t) = 4\int_0^\infty \frac{\tilde{x}(f)\tilde{h}^\star(f)}{S_n(f)} e^{2\pi i f t} df$$

- ϕ_c searched by summing in quadrature, and arrival time by FFT
- Amplitude sets overall scale
- Template bank used in m_1, m_2 space

Searching for low-mass systems

- Matched filter as used here is close to optimal in Gaussian noise
- Data has frequent “glitches” which can ring-off a large number of templates
- A powerful mehod of discriminating between signals and glitches is the χ^2 test (Allen, `gr-qc/0405045`)
- Consider contribution from different frequency bins which are constructed to yield equal contributions for a real signal

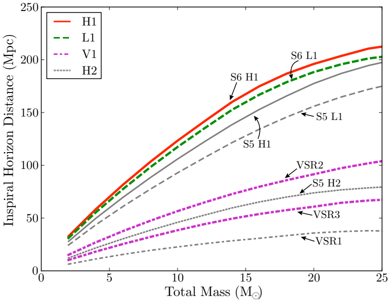
Searching for low-mass systems



(Allen, gr−qc/0405045)

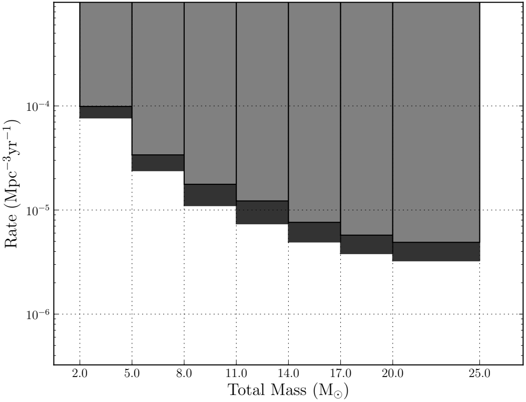
An example search result

- Published in `arXiv:1111.7314` (PRD **85** (2012) 082002)
- Search for systems with $2M_\odot \leq m_1 + m_2 \leq 25M_\odot$
- Data from LIGO and Virgo taken between July 7, 2009 - October 20, 2010 - No signals found
- The data contained a blind injection which was successfully recovered



- The "horizon distance" is a rough measure of the search sensitivity
- It is distance at which an optimally oriented/located binary would produce SNR of 8

An example search result



- Upper limit results are an improvement by a factor of 1.4 over previous LIGO-Virgo results
- Still 2-3 orders of magnitude away from astrophysical predictions

Conclusions

- GW searches are now becoming mature
- They can deal with large data volumes, real data and we are getting a handle on the computational problem
- Advanced LIGO is now collecting data!