Summing Threshold Logs with Parton Shower

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DESY-HH

In collaboration with Dave Soper

Factorization

- Let us consider an infrared safe observable and it has a typical resolution scale μ_J^2. This means every radiation under this scale is unresolvable and not visible by the observable.
- The all order cross section can be written in a factorized form. The soft and hard part is separated by the factorization (or shower) scale μ^2.

$$\sigma[O] = \underbrace{\big(1\big|\mathcal{O}(\mu_J^2)\mathcal{D}_{\rm NP}(\mu^2)}_{\text{"Soft part"}}\underbrace{\big[\mathcal{F}_{\rm F.S.}(\mu^2)\circ\mathcal{X}_{\rm S.S.}(\mu^2)\big]\big|\rho_H(\mu^2)\big)}_{\text{"Hard part"}}$$

This is the soft part and every radiation with $k_\perp^2, q^2, \dots < \mu^2$ are considered here.

This is the hard part and every radiation with $k_\perp^2, q^2, \dots > \mu^2$ are considered here.

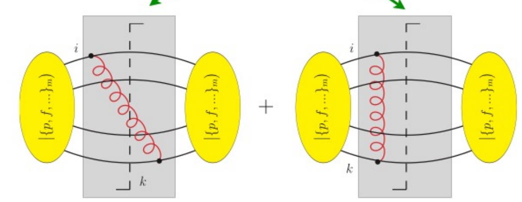
- It is important that we factorize out the parton emissions (real and virtual) instead of some kind of jet, soft and hard function.
- We work with states and operators in the statistical space.
- We don't have an all order proof for this factorization, yet. (But we are working on it…)
- We know the QCD amplitudes factorize in the singular limits and that what we use here.

Singular Operator

$$\mathcal{D}_{\rm NP}(\mu^2) = \big[\underbrace{\mathcal{F}_{\rm F.S.}(\mu^2)}_{\text{Renormalized PDF}}\circ\underbrace{\mathcal{K}_{\rm F.S.}(\mu^2)}_{\text{Defines the factorization scheme}}\circ\underbrace{\mathcal{Z}_F(\mu^2)}_{\text{Collinear counter-terms with explicit poles}}\big]\,\underbrace{\mathcal{D}(\mu^2)}_{\text{Partonic splitting operator with explicit and implicit singularities}}\,\underbrace{\big[\mathcal{F}_{\rm F.S.}(\mu^2)\circ\mathcal{X}_{\rm S.S.}\big]^{-1}}_{\text{Defines the shower scheme}}$$

The inverse operator of D is analogous to that is usually called to NLO subtraction term

$$\mathcal{D}(\mu^2) = 1 + \frac{\alpha_s(\mu^2)}{2\pi}\big(\mathcal{H}_{\rm R}(\mu^2) + \mathcal{H}_{\rm V}(\mu^2)\big) + \mathcal{O}(\alpha_s^2)$$



Visible Logs

When the factorization scale is small then the hard part suffers on large logarithms

$$\mathcal{F}(\mu^2)|\rho_{\rm H}(\mu^2)) = \left[1 + \frac{\alpha_s(\mu^2)}{2\pi}\mathcal{O}\left(\log^2\frac{\mu^2}{M^2}\right)\right]\underbrace{\mathcal{F}(M^2)|\rho_{\rm H}^{(0)}(M^2))}_{\text{Born level hard part}}$$

➡ This demands large factorization scale otherwise the hard part will be completely unreliable. It is clear that the 1GeV scale choice would be a disaster here.

$$\sigma[O] = \big(1\big|\mathcal{O}(\mu_J^2)\mathcal{D}_{\rm NP}(\mu_{\rm f}^2)\ \ \text{The gap between the hard and soft parts has to be bridged by partons shower.}\ \ \big[\mathcal{F}_{\rm F.S.}(M^2)\circ\mathcal{X}_{\rm S.S.}(M^2)\big]\big|\rho_H(M^2)\big)$$

Renormalization Group Equation

$$\sigma[O] = \underbrace{\big(1\big|\mathcal{O}(\mu_J^2)\mathcal{D}_{\rm NP}(\mu_{\rm f}^2)}_{\text{"Soft part"}}\underbrace{\mathbb{T}\exp\left\{\int_{\mu_{\rm f}^2}^{M^2}\mathcal{D}_{\rm NP}^{-1}(\mu^2)d\mathcal{D}_{\rm NP}(\mu^2)\right\}}_{\mathcal{U}(\mu_{\rm f}^2, M^2)}\underbrace{\big[\mathcal{F}_{\rm F.S.}(M^2)\circ\mathcal{X}_{\rm S.S.}(M^2)\big]\big|\rho_H(M^2)\big)}_{\text{"Hard part"}}$$

Standard Shower

As a first approximation: If we define the NLO (NNLO,…) subtraction scheme then we can have a reasonable parton shower algorithm. According to the previous slides we have to make sure that

$$\big(1\big|\mathcal{D}_{\rm NS}(\mu_{\rm f}^2) = \big(1\big|\left[1 + \frac{\alpha_s(\mu^2)}{2\pi}\mathcal{O}(1) + \cdots\right]$$

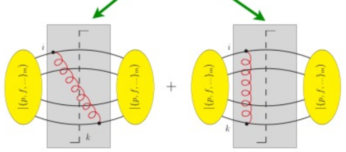
This can be easily done… ➡ *Unitarity!*

Unitarity condition: Do NOT define subtraction term directly for the 1-loop graphs, use the inclusive version of the real subtraction term via

$$-\big(1\big|[\mathcal{F}(\mu^2)\circ\mathcal{H}_{\rm V}(\mu^2)] = \big(1\big|[\mathcal{F}(\mu^2)\circ\mathcal{V}(\mu^2)] \equiv \big(1\big|\mathcal{F}(\mu^2)\mathcal{H}_{\rm R}(\mu^2)$$

This is the definitions of the inclusive splitting operator.

$$\mathcal{D}(\mu^2) = 1 + \frac{\alpha_s(\mu^2)}{2\pi}\big(\mathcal{H}_{\rm R}(\mu^2) + \mathcal{H}_{\rm V}(\mu^2)\big) + \mathcal{O}(\alpha_s^2)$$



- ✓ This certainly fulfils our requirement $\big(1\big|\mathcal{D}_{\rm NS}(\mu^2) = \big(1\big|$ for every μ^2
- ✓ This leads to a good NLO subtraction scheme.
- ✓ The meaning of the factorization scale is still debatable (kT, virtuality, angle or something else). See Bryan's talk!
- ✗ Is that all? Unfortunately not!

Inclusive Splitting Operator

After performing the y integral, we have

$$[\lambda_{{\rm a}k}(\mu^2;z)]_{\hat{a}a} = \underbrace{-\frac{1}{C_a}D_{a\hat{a}}(\mu^2/Q^2,z)}_{\text{Soft}\times\text{Collinear}+\text{Collinear}} - \delta_{a\hat{a}}\overbrace{C(\xi_{{\rm a}k},\mu^2/Q^2,z)}^{\text{Wide angle soft}}$$

Threshold logs with the right behaviour. They disappear in the $\mu^2 \to 0$ limit, thus they are summed up in the shower evolution.

$$\begin{aligned}
C(\xi, y_{\rm s}, z) = &\boxed{\left[\frac{2z}{1-z}\theta\big((1-z)^{\beta+1} < z\,y_{\rm s}\,r_\perp(z)\big)\right]_+\log\xi - \left[\frac{2z}{1-z}\theta\big((1-z)^{\beta+1} > z\,y_{\rm s}\,r_\perp(z)\big)\log\frac{\xi + \sqrt{\xi^2 + (1-\xi)\frac{4z^2y_{\rm s}^2r_\perp(z)^2}{(1-z)^{2\beta+2}}}}{2\xi}\right]_+}\\
&-\delta(1-z)\left\{\frac{(4\pi\,y_{\rm s})^\epsilon}{\Gamma(1-\epsilon)}\left(\frac{1}{\epsilon}+2\right)\log\xi - \int_0^1\frac{dx}{x}\,[\log x + \log(1-x)]\,w(\xi,x)\right\}\\
&-\delta(1-z)\int_0^1 du\,\frac{2u}{1-u}\theta\big((1-u)^{\beta+1} > u\,y_{\rm s}\,r_\perp(u)\big)\\
&\quad\times\log\frac{\xi + \sqrt{\xi^2 + (1-\xi)\frac{4u^2y_{\rm s}^2r_\perp(u)^2}{(1-u)^{2\beta+2}}}}{2}\,.
\end{aligned}$$

Inclusive Splitting Operator

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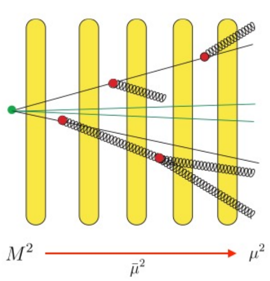
Threshold logs, they DON'T disappear in the $\mu^2 \to 0$ limit.

$$\begin{aligned}
D_{\hat{a}a}(y_{\rm s}, z) = &-\frac{1}{\epsilon}\frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}P_{a\hat{a}}(z)\\
&\boxed{+P_{a\hat{a}}^{(\epsilon)}(z) - P_{a\hat{a}}^{\rm reg}(z)\log\frac{(1-z)^{\beta-1}}{r_\perp(z)} - 2C_a\delta_{a\hat{a}}\left[\frac{1}{1-z}\log\frac{(1-z)^{\beta-1}}{r_\perp(z)}\right]_+}\\
&+2C_a\delta_{a\hat{a}}\left[\theta\big((1-z)^{\beta+1} < z\,y_{\rm s}\,r_\perp(z)\big)\frac{1}{1-z}\log\frac{(1-z)^{\beta+1}}{z\,y_{\rm s}\,r_\perp(z)}\right]_+\\
&+P_{a\hat{a}}^{\rm reg}(z)\,\theta\big((1-z)^{\beta+1} < z\,y_{\rm s}\,r_\perp(z)\big)\log\frac{(1-z)^{\beta+1}}{z\,y_{\rm s}\,r_\perp(z)}\\
&+\delta(1-z)\,\delta_{a\hat{a}}\frac{(4\pi\,y_{\rm s})^\epsilon}{\Gamma(1-\epsilon)}\left\{\frac{1}{\epsilon^2}C_a + \frac{1}{\epsilon}\gamma_a + \frac{\pi^2}{6}C_a\right\}\\
&-\delta(1-z)\,\delta_{a\hat{a}}\,2C_a\int_0^1\frac{du}{1-u}\log\frac{(1-u)^{\beta+1}}{u\,y_{\rm s}\,r_\perp(u)}\,\theta\big((1-u)^{\beta+1} > uy_{\rm s}\,r_\perp(u)\big)
\end{aligned}$$

Shower Evolution

Now the shower evolution operator is

$$\exp\left\{\int_{\mu_2^2}^{\mu_1^2}\frac{d\bar{\mu}^2}{\bar{\mu}^2}\Big(\underbrace{\mathcal{S}_{\rm uni}(\bar{\mu}^2)}_{\text{Unitary part}} + \overbrace{\big[\mathcal{F}_{\rm F.S.}(\bar{\mu}^2)\circ\mathcal{S}_{\rm thr}(\bar{\mu}^2)\big]\mathcal{F}_{\rm F.S.}^{-1}(\bar{\mu}^2)}^{\text{Threshold contribution}}\Big)\right\}$$



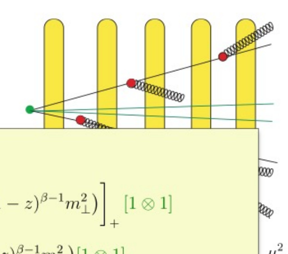
- ✓ This leads to a non-unitary shower.
- ✓ The threshold splitting operator doesn't change the number of the partons and their momenta. It operates in the colour and flavour space only.
- ✓ In LC+ approximation it leads to an extra factor that we have to insert after every step of the shower evolution.

$$\exp\left\{\int_{\mu_2^2}^{\mu_1^2}\frac{d\bar{\mu}^2}{\bar{\mu}^2}\big[\mathcal{F}_{\rm F.S.}(\mu^2)\circ\mathcal{S}_{\rm thr}^{\rm LC+}(\bar{\mu}^2)\big]\mathcal{F}_{\rm F.S.}^{-1}(\mu^2)\right\}$$

Shower Evolution

Now the shower evolution operator is

Threshold contribution



Direct term with diagonal color

$$\begin{aligned}
\mathcal{S}_{\rm thr}(\mu^2) = &-\frac{\alpha_s(\mu^2)}{2\pi}\left[\delta_{a\hat{a}}\frac{2Ca}{1-z}\theta\big((1-z)^{\beta+1} < z\,y_{\rm s}\,r_\perp(z)\big)\,\theta\big(\mu^2 > (1-z)^{\beta-1}m_\perp^2\big)\right]_+[1\otimes 1]\\
&-\frac{\alpha_s(\mu^2)}{2\pi}P_{a\hat{a}}^{\rm reg}(z)\,\theta\big((1-z)^{\beta+1} < z\,y_{\rm s}\,r_\perp(z)\big)\,\theta\big(\mu^2 > (1-z)^{\beta-1}m_\perp^2\big)[1\otimes 1]\\
&-\frac{\alpha_s(\mu^2)}{2\pi}\sum_{k\neq{\rm a}}\left[w\left(\xi_{{\rm a}k}, \frac{y_{\rm s}r_\perp(z)}{(1-z)^{\beta+1}}\right)\theta\big((1-z)^{\beta+1} > z\,y_{\rm s}\,r_\perp(z)\big)\,\theta\big(\mu^2 > (1-z)^{\beta-1}m_\perp^2\big)\right]_+\\
&\qquad\times\frac{1}{2}\big[(\boldsymbol{T}_{\rm a}\cdot\boldsymbol{T}_k)\otimes 1 + 1\otimes(\boldsymbol{T}_{\rm a}\cdot\boldsymbol{T}_k)\big] + \cdots
\end{aligned}$$

Colour interference part of the threshold logs

$$\exp\left\{\int_{\mu_2^2}^{\mu_1^2}\frac{d\bar{\mu}^2}{\bar{\mu}^2}\big[\mathcal{F}_{\rm F.S.}(\mu^2)\circ\mathcal{S}_{\rm thr}^{\rm LC+}(\bar{\mu}^2)\big]\mathcal{F}_{\rm F.S.}^{-1}(\mu^2)\right\}$$

Where is the Code?

- DEDUCTOR is designed to do a better job with color, spin and resummation of large logarithms compared

```
/*   Defining the ordering dependent functions for the INITIAL state splittings  */
template<bool _Is_msbar>
struct __ordering_traits<ini, ordering::lambda, _Is_msbar>
{
  /*  calculates the limits on the variable v    */
  static void vlimits(double&, double&, const __hard_params<ini> *);

  /*  calculates the z limits  */
  static void zlimits(double&, double&, const __hard_params<ini> *, int, int, double);

  /*  pdf scale  */
  static double pdf_scale(const __hard_params<ini> *pars, double x, double y) {…}

  /*  kT2/(v*Q2) \approx (1-z)^alpha   */
  static constexpr unsigned int kT_alpha = 1u;

  /*  mapping the indipendent splitting variables v and z to the normalized virtuality, y   */
  static double mapping_to_y(const __hard_params<ini> *, int, int, double v, double) {…}

  /*  mapping the indipendent splitting variables v and z to the normalized virtuality, y
   *  It also returns the jacobian of  v --> y mapping.
   */
  static void mapping_to_y(double& y, double& yjac, const __hard_params<ini> *, int, int, double v, double) {…}

  /*  helps to define the shower time: exp(-t) = v/vnull   */
  static double vnull(double Q2, const lorentzvector<double>& qnull, const lorentzvector<double>& pa) {…}


  /*   Ordering dependent properties for the threshold resummation    */
  struct threshold
  {…};
};
```

Deductor: A new parton Shower Project

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In collaboration with Dave Soper

DESY-HH, September 14, 2015

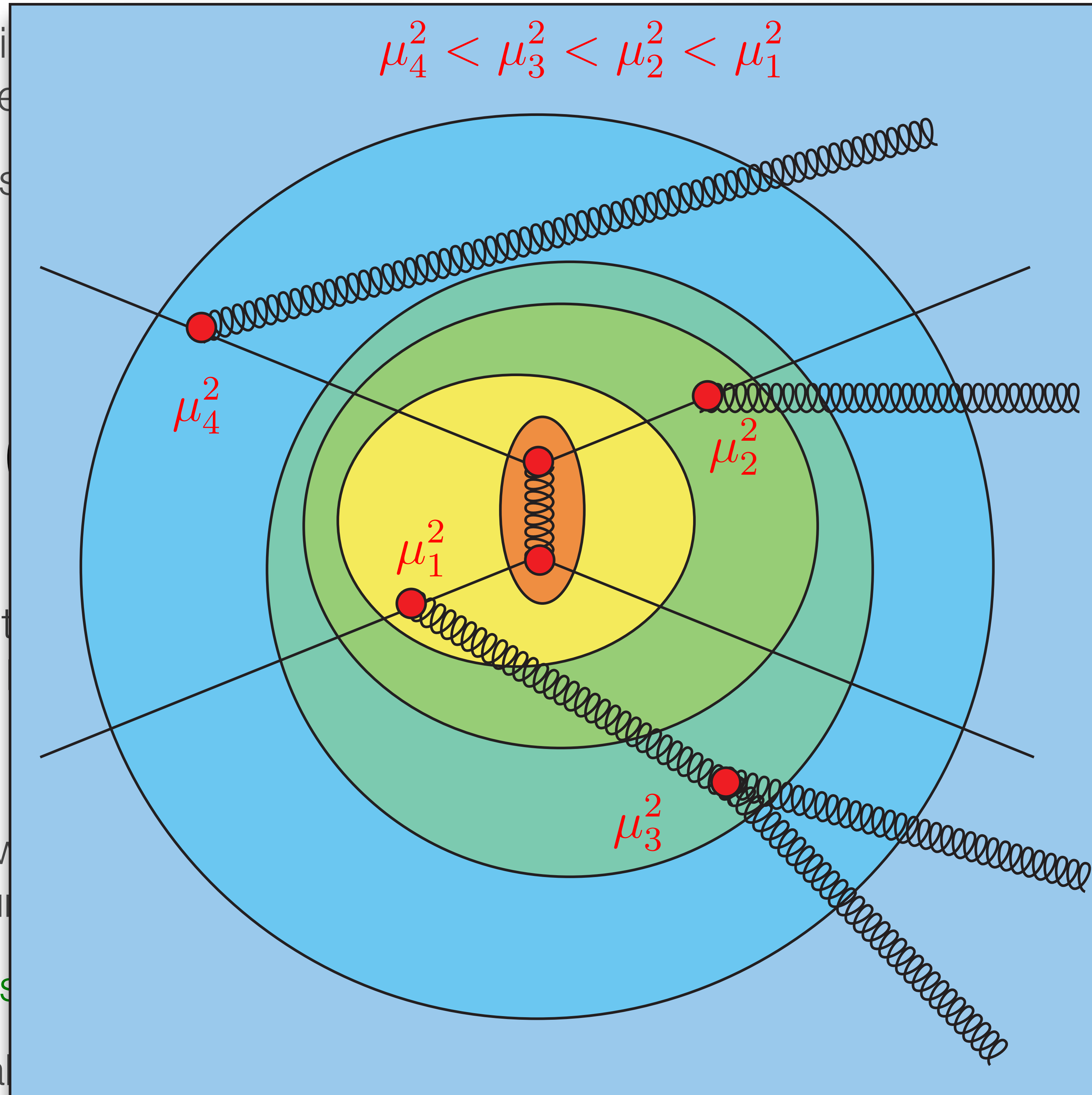
Introduction

- DEDUCTOR is NOT a project for ”just-to-describe-the-data”
    - At the moment we don’t hadronization
    - Only a few hard process is implemented (Drell-Yan, jet production, e+e-)
- The focus is to do theoretical studies and use parton shower as theory prediction
    - Providing a pQCD all order theory definition
        - Genuine higher order effects in the shower evolution
        - Higher order effect in the hard part (this is usually called matching)
    - Taking care of quantum effect
        - Color interferences
        - Spin correlations
    - Understand the large logarithms and their summation in parton shower
        - Visible logarithms (like Drell-Yan transverse momentum)
        - Invisible logarithms (like threshold effects)
        - Exotic logarithms (“factorization breaking” effects, super-leading logs,…)
    - Understanding the relation to BFKL physics

Definition of Parton Shower

Definition of Parton Shower

- Let us consider an i … μ_J^2. This means every radiation unde … e.
- The all order cross s … d part is separated by the factorization



$$\sigma[O] = (\ldots _H(\mu^2))$$

This is the … radiation with radiation wit … here. considered

- It is important that v … d of some kind of jet, soft and hard fu
- We work with states
- We don't have an a … on it…)
- We know the QCD amplitudes factorize in the singular limits and that what we use here.

Definition of Parton Shower

- Let us consider an infrared safe observable and it has a typical resolution scale μ_J^2. This means every radiation under this scale is unresolvable and not visible by the observable.
- The all order cross section can be written in a factorized form. The soft and hard part is separated by the factorization (or shower) scale μ^2.

"Soft part" *"Hard part"*

$$\sigma[O] = \big(1\big|\mathcal{O}(\mu_J^2)\mathcal{D}_{\rm NP}(\mu^2)\big[\mathcal{F}_{\rm F.S.}(\mu^2)\circ\mathcal{X}_{\rm S.S.}(\mu^2)\big]\big|\rho_H(\mu^2)\big)$$

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Definition of Parton Shower

The usual parton shower algorithms are theory motivated designs, phenomenological model of pQCD. It make some sense at LO level. As far as I know there is no formal definition even at leading order level.

$$\sigma[O_J] = \sum_{m=2}^{\infty} \big(\rho_m \big| \mathcal{O}_J \big| 1\big) = \sum_m \big[d\{p,f\}_m\big] \mathrm{Tr}\{\rho(\{p,f\}_m) O_J(\{p,f\}_m)\}$$

To proof this we need a kind of generalized factorization.

$$\begin{aligned}\sigma[O_J] = \big(1\big|\mathcal{O}_J &\Big[\mathcal{W}^{LO}(\mu_\mathrm{f}^2) + \mathcal{W}^{NLO}(\mu_\mathrm{f}^2) + \cdots\Big] && \textit{Finite corrections}\\ &\mathbb{T}\exp\left\{\int_{\mu_\mathrm{f}^2}^{\mu_0^2} \frac{d\mu^2}{\mu^2}\Big[\mathcal{S}^{LO}(\mu^2) + \mathcal{S}^{NLO}(\mu^2) + \cdots\Big]\right\} && \textit{Parton shower}\\ &\Big[\big|\rho^{LO}(\mu_0^2)\big) + \big|\rho^{NLO}(\mu_0^2)\big) + \big|\rho^{NNLO}(\mu_0^2)\big) + \cdots\Big] && \textit{Hard state}\end{aligned}$$

In the $\mu_\mathrm{f}^2 \to \mu_0^2$ limit it collapses to the corresponding NLO, NNLO fixed order expression.

Definition of Parton Shower

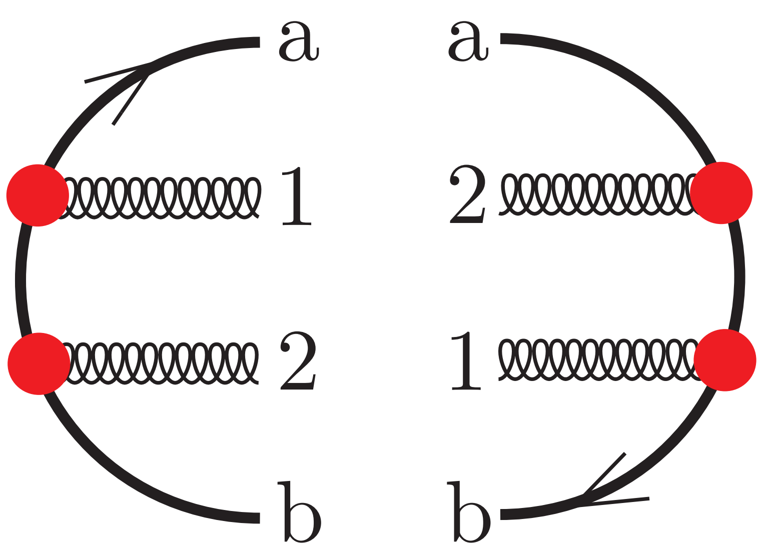
- Parton shower is defined based on a generalized factorization.
    - This has to be proven at all order level. Need a “constructive” proof.
    - The general scheme has been checked and works at LO level. This scheme is implemented (partly) in DEDUCTOR.
- The so called NLO, NNLO,.. matching is part of shower definition.
    - DEDUCTOR is matched to NLO computation in the HELAC framework. Unfortunately this implementation follows the POWHEG scheme.
- No enforced unitary. Loop graphs are explicitly considered.
    - Even in the LO order shower the events are weighted and not necessary positive, in fact they are complex number.
    - Negative weights come from the color structure and complex weight from the explicit loop graphs.
    - Able to predict not just the shape but the absolute normalization.

Quantum Effects: Color

Color

The fundamental object is quantum density matrix, with basis vectors $|\{c\}_m\rangle\langle\{c'\}_m|$

$$\rho = \sum_m \sum_{\{c,c'\}_m} \left|\{c'\}_m\right\rangle \rho(\{c,c'\}_m) \left\langle\{c\}_m\right|$$

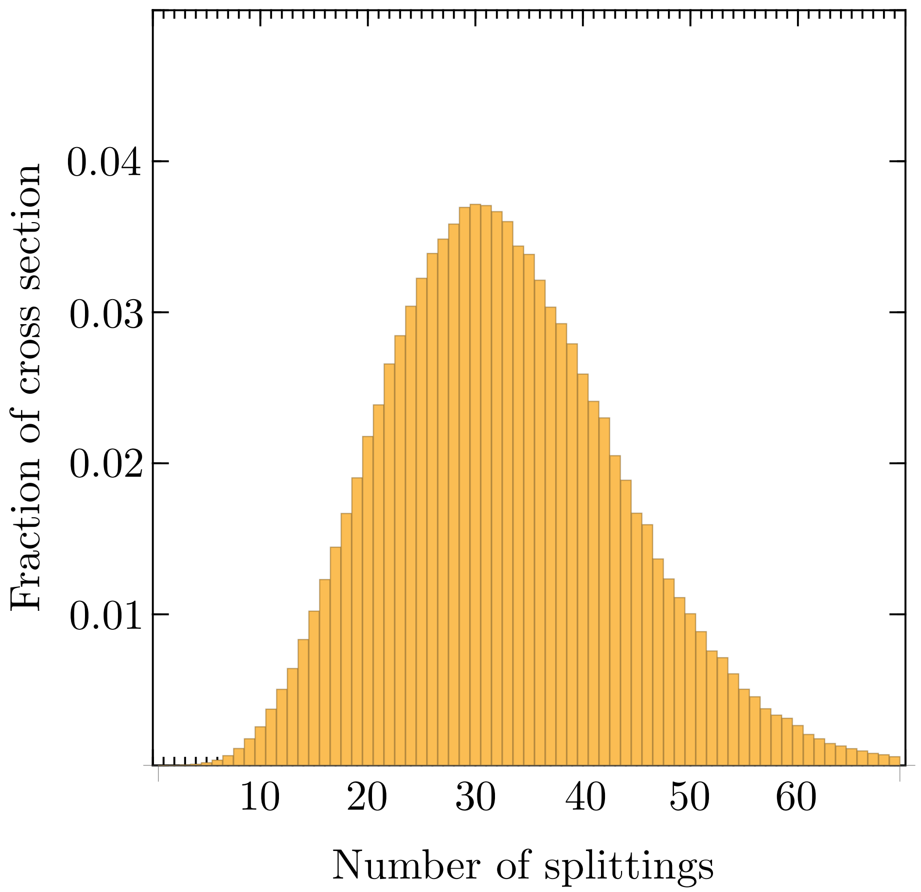


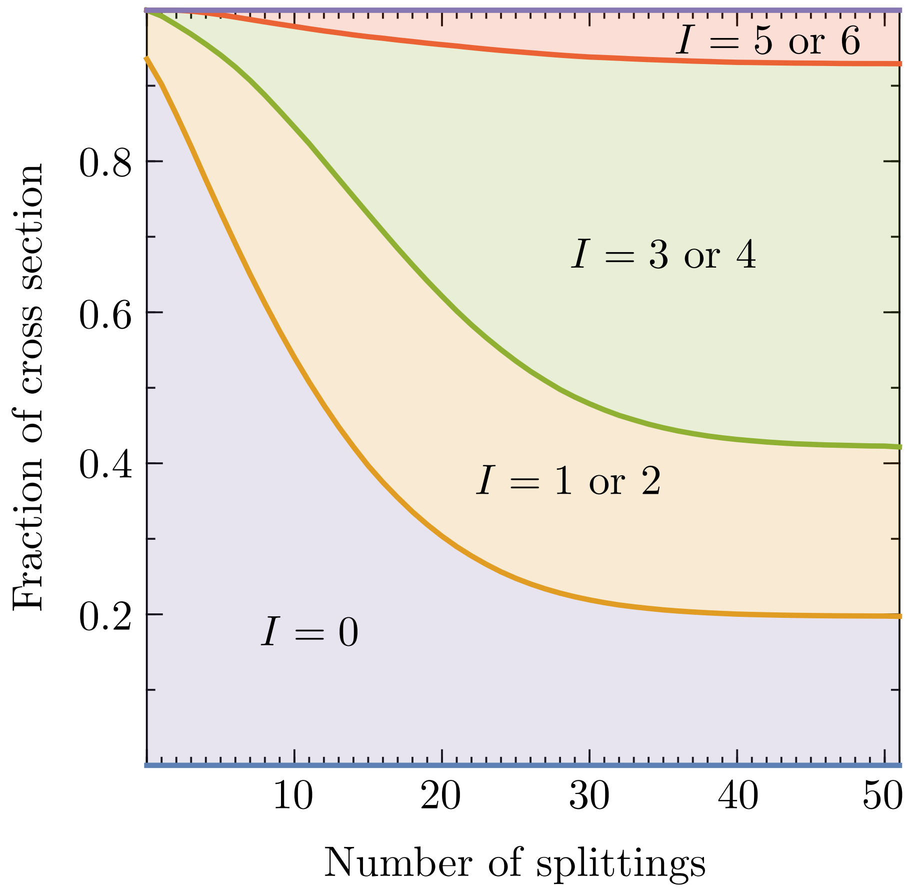
$\left|\{c\}_m\right\rangle$ $\left\langle\{c'\}_m\right|$

- Usually the dipole showers use leading color approximation (LC), considering only the diagonal color configurations before and after the splitting. (Approximation at matrix element square level.)
- Full color evolution is impossible in parton shower. Just impossible to exponentiate, say a 30! x 30! matrix…
- The LC approximation is not a systematical approximation, we cannot improve it perturbatively by adding back subleading correction in a controlled way.
- Solution is LC+ approximation. It can handle color interferences and the approximation is only in the splitting kernel. Perturbative improvable. (Approximation at amplitude level.)

Color

At LHC (8TeV) the average number of the splitting in an event is about 25-30 and most of them subleading color contribution

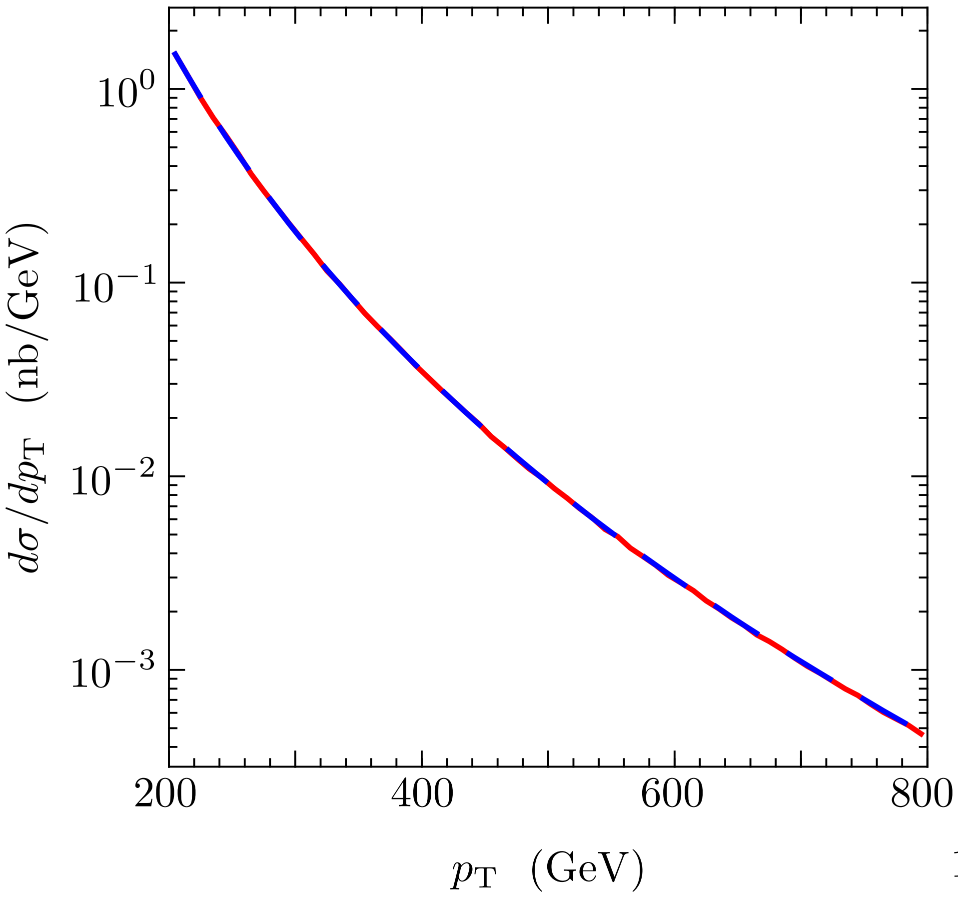




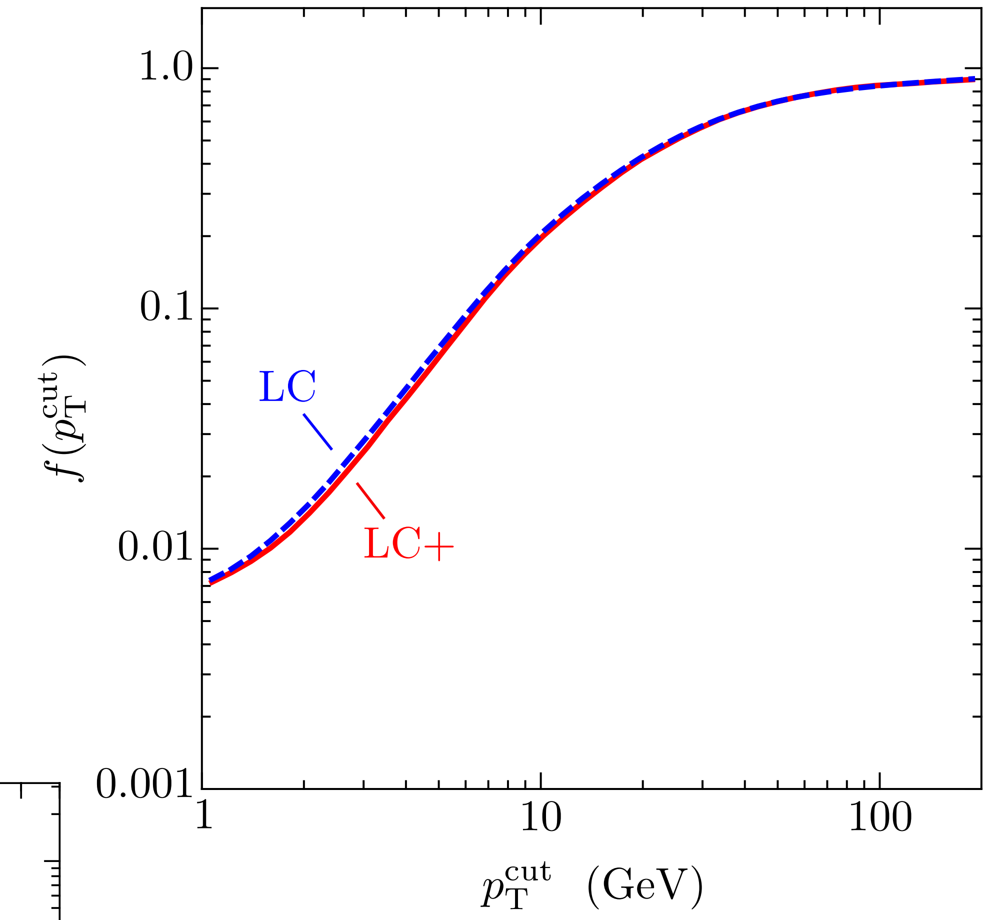
Does it have any visible effect on the observable compared to the LC result?

Color

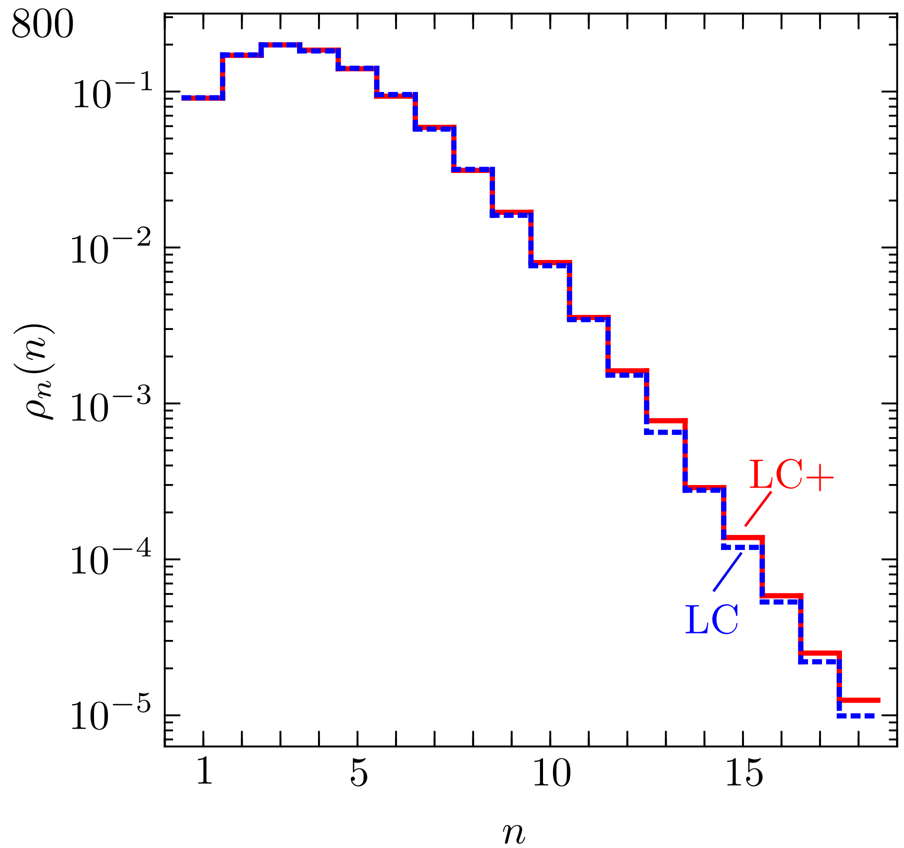
Inclusive jet cross section



Gap survival probability

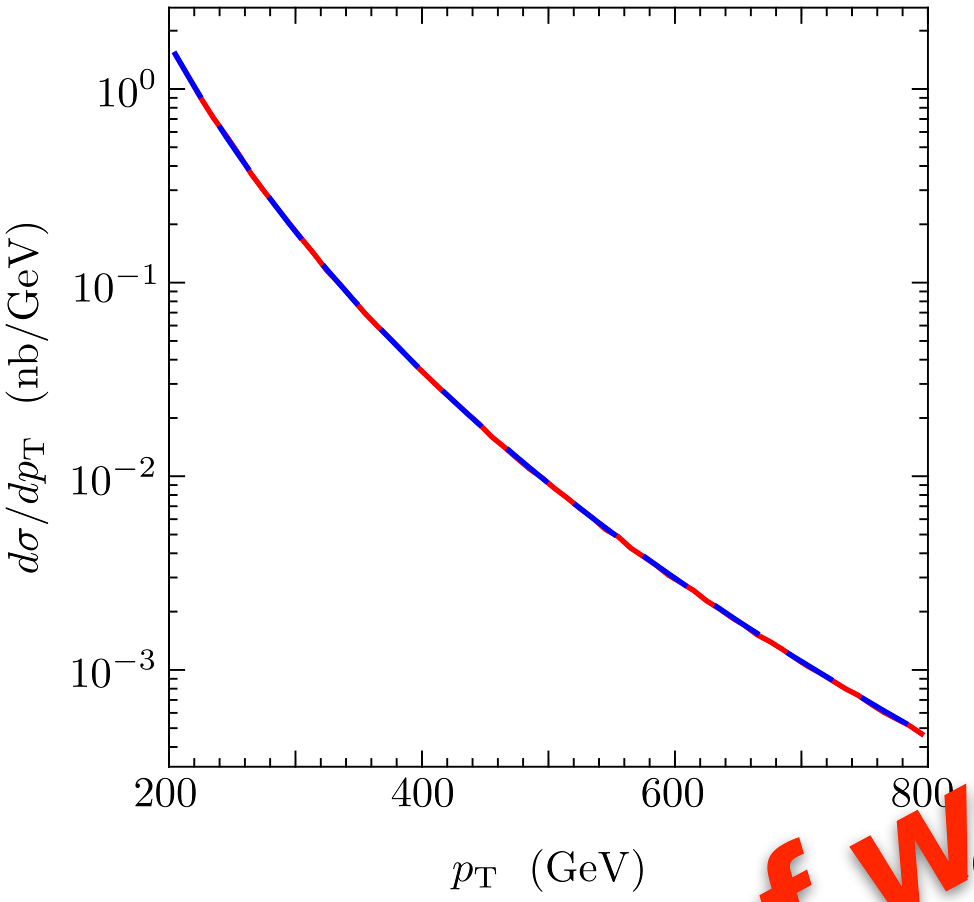


Number of parton in a jet



Color

Inclusive jet cross section



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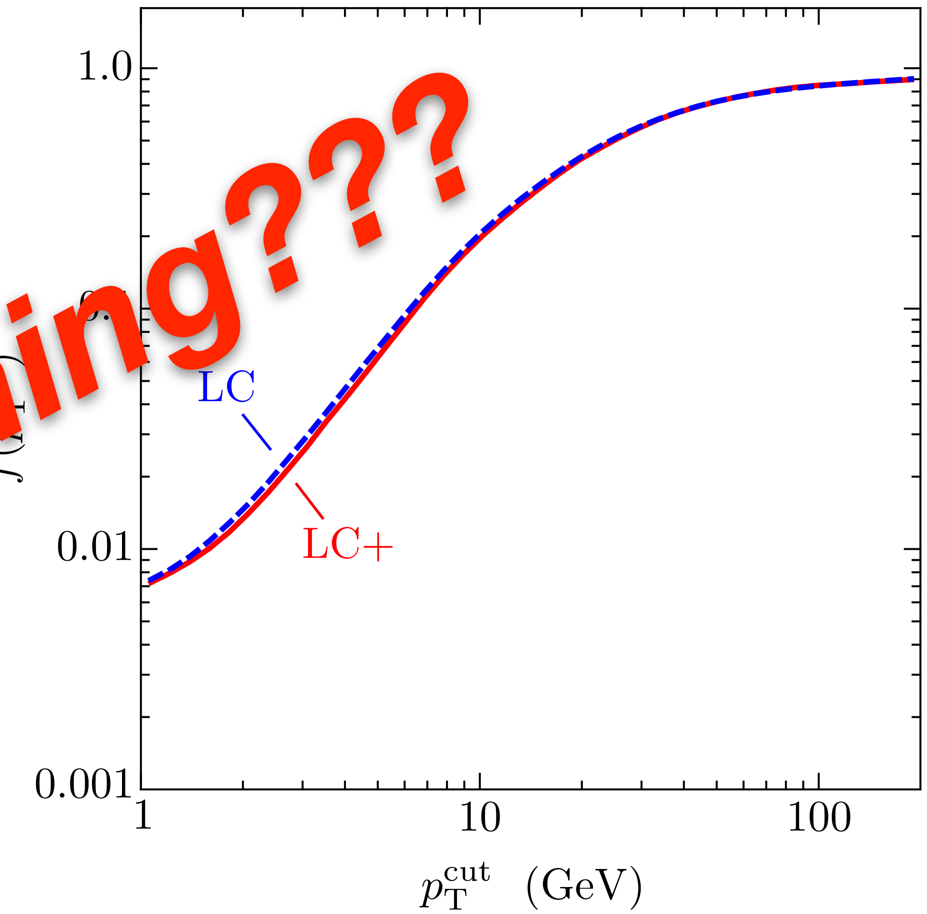
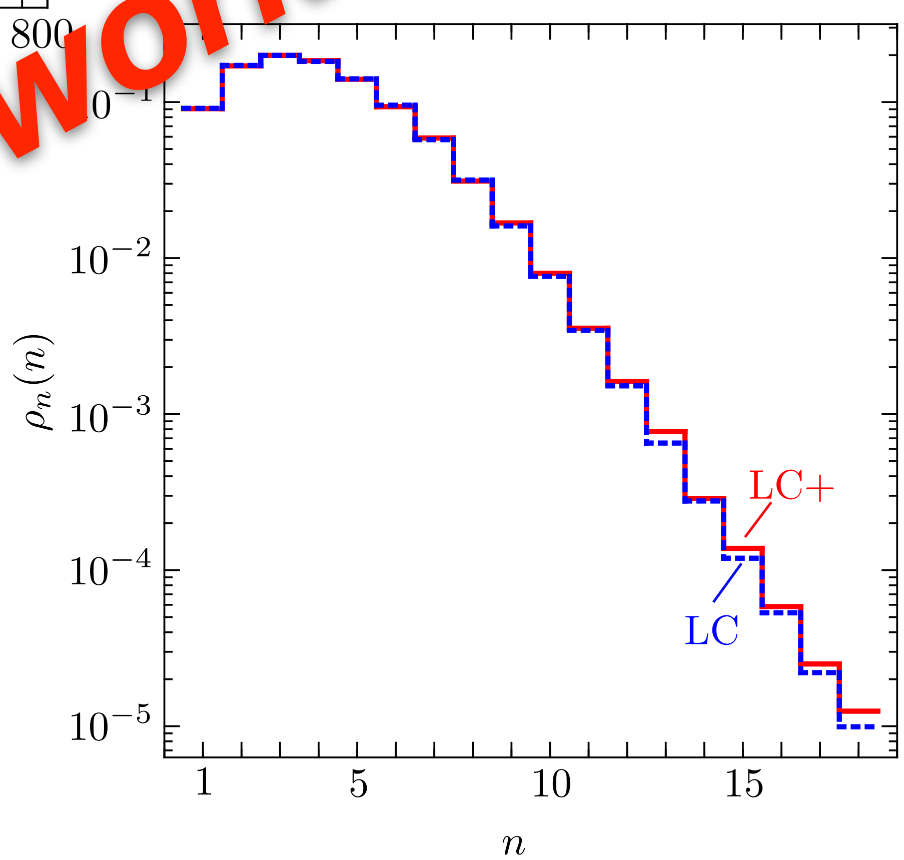


Lots of work for nothing???

Number of parton in a jet



Color

- As we expected the (very) inclusive observables are insensitive to color.

- The LC+ approximation is systematic but it considers only the coherent part of the soft gluon radiation. Most of the wide angle radiation are dropped.

- LC+ is improvable pertubatively

$$
\begin{aligned}
\mathcal{U}(t_\mathrm{f}, t_0) = &\, \mathcal{U}^{\mathrm{LC+}}(t_\mathrm{f}, t_0) \\
& + \int_{t_0}^{t_\mathrm{f}} d\tau\, \mathcal{U}^{\mathrm{LC+}}(t, \tau_1) \left[\Delta\mathcal{H}_I(\tau_1) - \Delta\mathcal{V}(\tau_1)\right] \mathcal{U}^{\mathrm{LC+}}(\tau_1, t_0) \\
& + \int_{t_0}^{t_\mathrm{f}} d\tau_2 \int_{t_0}^{\tau_2} d\tau_1\, \mathcal{U}^{\mathrm{LC+}}(t_\mathrm{f}, \tau_2) \left[\Delta\mathcal{H}_I(\tau_2) - \Delta\mathcal{V}(\tau_2)\right] \mathcal{U}^{\mathrm{LC+}}(\tau_2, \tau_1) \\
& \qquad \times \left[\Delta\mathcal{H}_I(\tau_1) - \Delta\mathcal{V}(\tau_1)\right] \mathcal{U}^{\mathrm{LC+}}(\tau_1, t_0) \\
& + \cdots \; .
\end{aligned}
$$

- Coulomb gluon effect can be considered in the LC+ approximation. This can lead to supper leading logs. This effects come from genuine loop contribution. (This is another reason to give up the concept of unweighted events…)

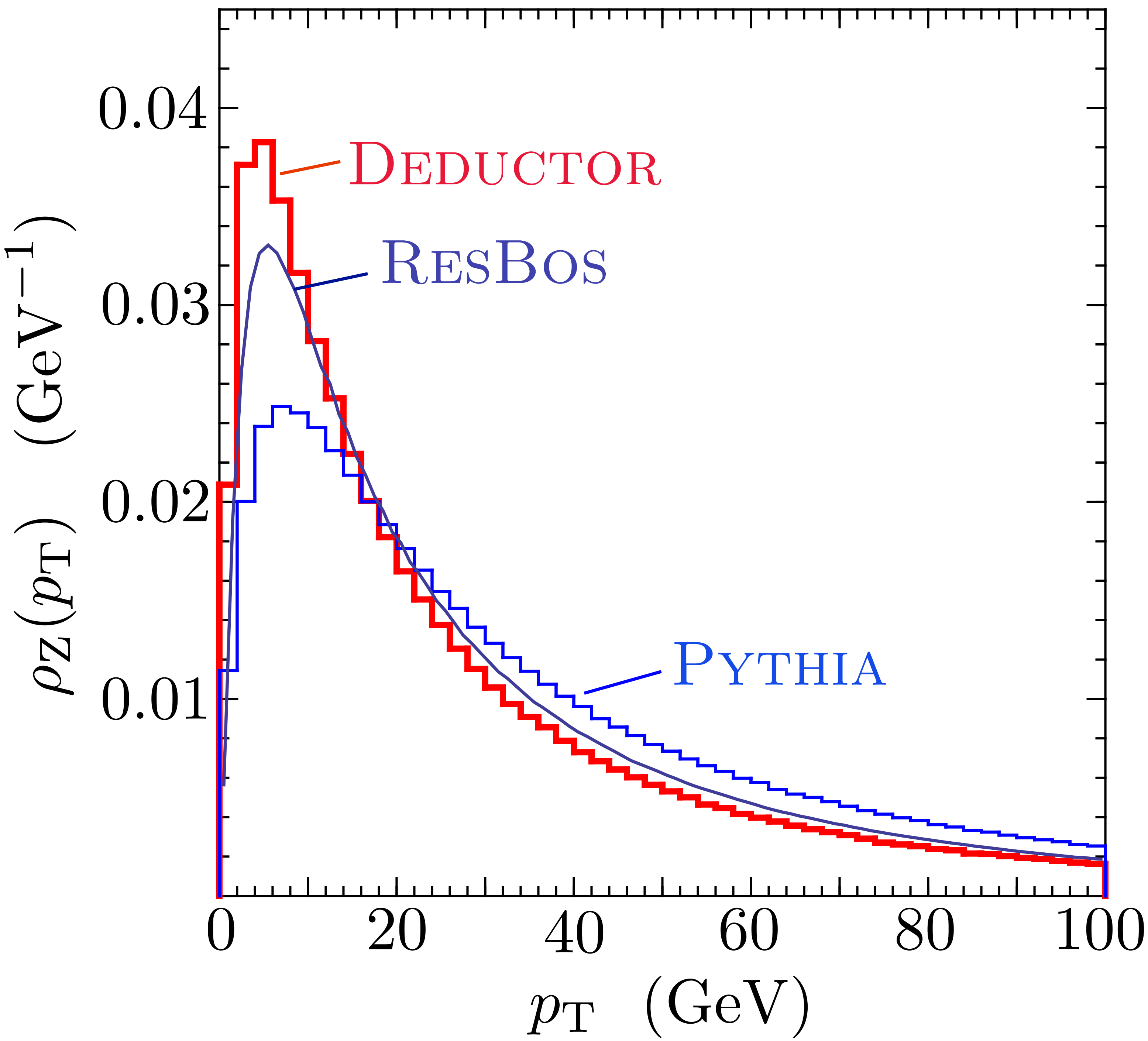
$$
\Delta\mathcal{V}(\tau)\big|\{p, f, ...\}_m\big) = \cdots + \mathrm{i}\pi \frac{\alpha_\mathrm{s}}{2\pi} \Big\{ [\boldsymbol{T}_l \cdot \boldsymbol{T}_k] \otimes 1 - 1 \otimes [\boldsymbol{T}_l \cdot \boldsymbol{T}_k] \Big\} \big|\{p, f, ...\}_m\big) + \cdots
$$

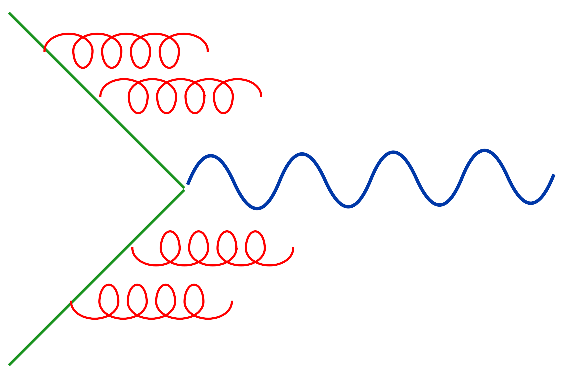
Summing Up Large Logarithms

Visible Logs

"Visible logs", like the Drell-Yan transverse momentum

$$\log(k_T^2/M^2)$$





Z boson gets recoil from soft gluon emission

Visible Logs

QCD result is

$$\frac{d\sigma}{d\boldsymbol{p}_\perp dY} \approx \int \frac{d^2\boldsymbol{b}}{(2\pi)^2}\, e^{\mathrm{i}\boldsymbol{b}\cdot\boldsymbol{p}_\perp}$$
$$\times \sum_{a,b} \int_{x_\mathrm{a}}^{1} \frac{d\eta_\mathrm{a}}{\eta_\mathrm{a}} \int_{x_\mathrm{b}}^{1} \frac{d\eta_\mathrm{b}}{\eta_\mathrm{b}}\; f_{a/A}(\eta_\mathrm{a}, C^2/\boldsymbol{b}^2)\; f_{b/B}(\eta_\mathrm{b}, C^2/\boldsymbol{b}^2)$$
$$\times \exp\left(-\int_{C^2/\boldsymbol{b}^2}^{M^2} \frac{d\boldsymbol{k}_\perp^2}{\boldsymbol{k}_\perp^2}\left[A(\alpha_\mathrm{s}(\boldsymbol{k}_\perp^2))\log\left(\frac{M^2}{\boldsymbol{k}_\perp^2}\right) + B(\alpha_\mathrm{s}(\boldsymbol{k}_\perp^2))\right]\right)$$
$$\times \sum_{a',b'} H^{(0)}_{a'b'}\; C_{a'a}\left(\frac{x_\mathrm{a}}{\eta_\mathrm{a}}, \alpha_\mathrm{s}\left(\frac{C^2}{\boldsymbol{b}^2}\right)\right) C_{b'b}\left(\frac{x_\mathrm{b}}{\eta_\mathrm{b}}, \alpha_\mathrm{s}\left(\frac{C^2}{\boldsymbol{b}^2}\right)\right) .$$

$$A(\alpha_\mathrm{s}) = 2\,C_\mathrm{F}\,\frac{\alpha_\mathrm{s}}{2\pi} + 2\,C_\mathrm{F}\left\{C_\mathrm{A}\left[\frac{67}{18} - \frac{\pi^2}{6}\right] - \frac{5\,n_\mathrm{f}}{9}\right\}\left(\frac{\alpha_\mathrm{s}}{2\pi}\right)^2 + \cdots \;,$$

$$B(\alpha_\mathrm{s}) = -4\,\frac{\alpha_\mathrm{s}}{2\pi} + \left[-\frac{197}{3} + \frac{34 n_\mathrm{f}}{9} + \frac{20\pi^2}{3} - \frac{8 n_\mathrm{f}\pi^2}{27} + \frac{8\zeta(3)}{3}\right]\left(\frac{\alpha_\mathrm{s}}{2\pi}\right)^2 + \cdots \;,$$

$$C_{a'a}(z,\alpha_\mathrm{s}) = \delta_{a'a}\delta(1-z) + \frac{\alpha_\mathrm{s}}{2\pi}\left[\delta_{a'a}\left\{\frac{4}{3}(1-z) + \frac{2}{3}\delta(1-z)\left(\pi^2 - 8\right)\right\} + \delta_{ag}\, z(1-z)\right]$$

$$x_\mathrm{A} = \sqrt{\frac{M^2}{s}}e^{Y} \qquad x_\mathrm{B} = \sqrt{\frac{M^2}{s}}e^{-Y} \qquad C = 2e^{-\gamma_E}$$

Visible Logs

Analytic result from DEDUCTOR

$$
\begin{aligned}
\frac{d\sigma}{d\boldsymbol{p}_\perp dY} \approx & \int \frac{d^2\boldsymbol{b}}{(2\pi)^2}\, e^{\mathrm{i}\boldsymbol{b}\cdot\boldsymbol{p}_\perp} \\
& \times \sum_{a,b} \int_{x_\mathrm{a}}^1 \frac{d\eta_\mathrm{a}}{\eta_\mathrm{a}} \int_{x_\mathrm{b}}^1 \frac{d\eta_\mathrm{b}}{\eta_\mathrm{b}}\; f_{a/A}\big(\eta_\mathrm{a}, C^2/\boldsymbol{b}^2\big)\; f_{b/B}\big(\eta_\mathrm{b}, C^2/\boldsymbol{b}^2\big) \\
& \times \exp\left(-\int_{C^2/\boldsymbol{b}^2}^{M^2} \frac{d\boldsymbol{k}_\perp^2}{\boldsymbol{k}_\perp^2}\left[A(\alpha_\mathrm{s}(\boldsymbol{k}_\perp^2))\log\left(\frac{M^2}{\boldsymbol{k}_\perp^2}\right) + B(\alpha_\mathrm{s}(\boldsymbol{k}_\perp^2))\right]\right) \\
& \times \sum_{a',b'} H_{a'b'}^{(0)}\; C_{a'a}\left(\frac{x_\mathrm{a}}{\eta_\mathrm{a}}, \alpha_\mathrm{s}\left(\frac{C^2}{\boldsymbol{b}^2}\right)\right) C_{b'b}\left(\frac{x_\mathrm{b}}{\eta_\mathrm{b}}, \alpha_\mathrm{s}\left(\frac{C^2}{\boldsymbol{b}^2}\right)\right) \quad .
\end{aligned}
$$

$$
A(\alpha_\mathrm{s}) = 2\,C_\mathrm{F}\,\frac{\alpha_\mathrm{s}}{2\pi} + 2\,C_\mathrm{F}\left\{C_\mathrm{A}\left[\frac{67}{18} - \frac{\pi^2}{6}\right] - \frac{5\,n_\mathrm{f}}{9}\right\}\left(\frac{\alpha_\mathrm{s}}{2\pi}\right)^2 + \cdots \quad ,
$$

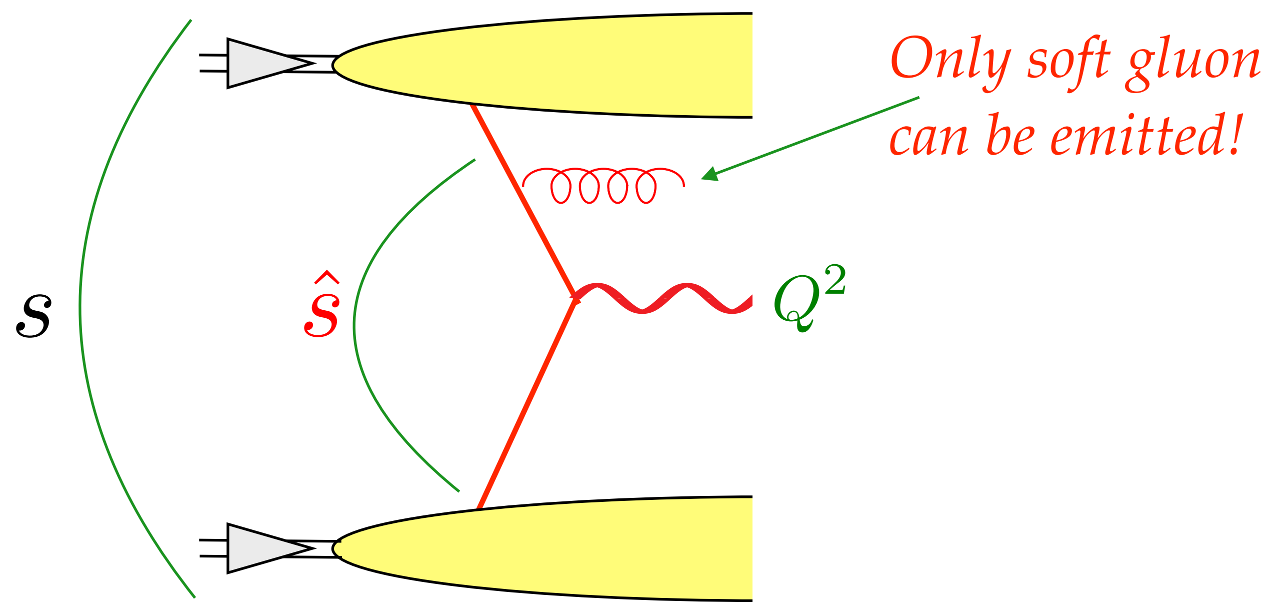
$$
B(\alpha_\mathrm{s}) = -4\,\frac{\alpha_\mathrm{s}}{2\pi} + \cancel{\left[-\frac{197}{3} + \frac{34 n_\mathrm{f}}{9} + \frac{20\pi^2}{3} - \frac{8 n_\mathrm{f}\pi^2}{27} + \frac{8\zeta(3)}{3}\right]\left(\frac{\alpha_\mathrm{s}}{2\pi}\right)^2} + \cdots \quad ,
$$

$$
C_{a'a}(z,\alpha_\mathrm{s}) = \delta_{a'a}\delta(1-z) + \cancel{\frac{\alpha_\mathrm{s}}{2\pi}\left[\delta_{a'a}\left\{\frac{4}{3}(1-z) + \frac{2}{3}\delta(1-z)(\pi^2-8)\right\} + \delta_{ag}\, z(1-z)\right]}
$$

$$
x_\mathrm{A} = \sqrt{\frac{M^2}{s}}e^{Y} \qquad x_\mathrm{B} = \sqrt{\frac{M^2}{s}}e^{-Y} \qquad C = 2e^{-\gamma_E}
$$

Invisible Logs

"Invisible logs", live under the integral. They are the so called threshold logs.



$$s \approx Q^2 \quad \Longrightarrow \quad \frac{Q^2}{\hat{s}} \approx 1 \quad \Longrightarrow \quad L = \log\left(1 - \frac{Q^2}{\hat{s}}\right)$$

- Simplest observable would be the total cross section.
- Standard parton showers cannot say anything about these logs. They are normalized to the Born level total cross section.
- The unitarity condition in shower is not God given, it is a technical choice to make the implementation simpler.
- To be able to sum up threshold logs we have to rid off the unitarity condition and consider genuine loop contributions in the splitting kernel.

Source of Threshold Logs

Let us start with the singular operator. This operator also defines the subtraction terms.

Collinear counter-terms with explicit poles

Renormalized PDF

$$\mathcal{D}_{\rm NP}(\mu^2) = \left[\mathcal{F}_{\rm F.S.}(\mu^2) \circ \mathcal{K}_{\rm F.S.}(\mu^2) \circ \mathcal{Z}_F(\mu^2)\right] \mathcal{D}(\mu^2) \left[\mathcal{F}_{\rm F.S.}(\mu^2) \circ \mathcal{X}_{\rm S.S.}\right]^{-1}$$

Defines the factorization scheme

Partonic splitting operator with explicit and implicit singularities

Source of Threshold Logs

Let us start with the singular operator. This operator also defines the subtraction terms.

Real subtraction term
(implicit singularities)

$$\mathcal{D}_{\rm NP}(\mu^2) = 1 + \frac{\alpha_{\rm s}(\mu^2)}{2\pi}\Big\{\mathcal{F}_{\rm F.S.}(\mu^2)\mathcal{H}_{\rm R}(\mu^2)\mathcal{F}^{-1}_{\rm F.S.}(\mu^2)$$

Virtual subtraction term
(explicit 1/ϵ singularities)

$$+ \mathcal{H}_{\rm V}(\mu^2) + \mathcal{R}^{(1)}_{\rm LSZ}(\mu^2)$$

$$+ \Big[\mathcal{F}_{\rm F.S.}(\mu^2) \circ \Big(\mathcal{K}^{(1)}_{\rm F.S.}(\mu^2) + \mathcal{Z}^{(1)}_F(\mu^2) - \mathcal{X}^{(1)}_{\rm S.S.}(\mu^2)\Big)\Big]\,\mathcal{F}^{-1}_{\rm F.S.}(\mu^2)\Big\} + \mathcal{O}(\alpha_{\rm s}^2)\quad .$$

Collinear counter-term
(explicit 1/ϵ singularities)

Source of Threshold Logs

Let us start with the singular operator. This operator also defines the subtraction terms.

$$\begin{aligned}\mathcal{D}_{\rm NP}(\mu^2) = 1 + \frac{\alpha_{\rm s}(\mu^2)}{2\pi}\Big\{ & \mathcal{F}_{\rm F.S.}(\mu^2)\mathcal{H}_{\rm R}(\mu^2)\mathcal{F}^{-1}_{\rm F.S.}(\mu^2) \\ & + \mathcal{H}_{\rm V}(\mu^2) + \mathcal{R}^{(1)}_{\rm LSZ}(\mu^2) \\ & + \Big[\mathcal{F}_{\rm F.S.}(\mu^2)\circ\Big(\mathcal{K}^{(1)}_{\rm F.S.}(\mu^2) + \mathcal{Z}^{(1)}_{F}(\mu^2) - \mathcal{X}^{(1)}_{\rm S.S.}(\mu^2)\Big)\Big]\,\mathcal{F}^{-1}_{\rm F.S.}(\mu^2)\Big\} + \mathcal{O}(\alpha_{\rm s}^2) \quad .\end{aligned}$$

Real subtraction term
(implicit singularities)

Virtual subtraction term
(explicit 1/∊ singularities)

Collinear counter-term
(explicit 1/∊ singularities)

It is still useful to introduce the inclusive splitting operator and its approximation as

$$\big(1\big|\big[\mathcal{F}(\mu^2)\circ\mathcal{V}(\mu^2)\big] = \big(1\big|\Big[\mathcal{F}(\mu^2)\circ\Big(\widetilde{\mathcal{V}}(\mu^2) + \mathcal{X}_{\rm S.S}(\mu^2)\Big)\Big] = \big(1\big|\mathcal{F}(\mu^2)\mathcal{H}_{\rm R}(\mu^2)$$

Defines the shower scheme
(only power suppressed terms)

Source of Threshold Logs

Let us start with the singular operator. This operator also defines the subtraction terms.

Real subtraction term
(implicit singularities)

Virtual subtraction term
(explicit 1/∊ singularities)

$$\mathcal{D}_{\rm NP}(\mu^2) = 1 + \frac{\alpha_{\rm s}(\mu^2)}{2\pi}\Big\{\mathcal{F}_{\rm F.S.}(\mu^2)\mathcal{H}_{\rm R}(\mu^2)\mathcal{F}^{-1}_{\rm F.S.}(\mu^2) + \mathcal{H}_{\rm V}(\mu^2) + \mathcal{R}^{(1)}_{\rm LSZ}(\mu^2) + \Big[\mathcal{F}_{\rm F.S.}(\mu^2)\circ\Big(\mathcal{K}^{(1)}_{\rm F.S.}(\mu^2) + \mathcal{Z}^{(1)}_{F}(\mu^2) - \mathcal{X}^{(1)}_{\rm S.S.}(\mu^2)\Big)\Big]\mathcal{F}^{-1}_{\rm F.S.}(\mu^2)\Big\} + \mathcal{O}(\alpha_{\rm s}^2)\ .$$

Collinear counter-term
(explicit 1/∊ singularities)

It is still useful to introduce the inclusive splitting operator and its approximation as

$$\big(1\big|\big[\mathcal{F}(\mu^2)\circ\mathcal{V}(\mu^2)\big] = \big(1\big|\Big[\mathcal{F}(\mu^2)\circ\Big(\widetilde{\mathcal{V}}(\mu^2) + \mathcal{X}_{\rm S.S}(\mu^2)\Big)\Big] = \big(1\big|\mathcal{F}(\mu^2)\mathcal{H}_{\rm R}(\mu^2)$$

Defines the shower scheme
(only power suppressed terms)

Unitary part, this leads to the standard shower

$$\mathcal{D}_{\rm NP}(\mu^2) = 1 + \frac{\alpha_{\rm s}(\mu^2)}{2\pi}\Big\{\Big(\mathcal{F}_{\rm F.S.}(\mu^2)\mathcal{H}_{\rm R}(\mu^2) - \big[\mathcal{F}_{\rm F.S.}(\mu^2)\circ\mathcal{V}(\mu^2)\big]\Big)\mathcal{F}^{-1}_{\rm F.S.}(\mu^2)$$
$$\boxed{+\,\mathcal{H}_{\rm V}(\mu^2) + \mathcal{R}^{(1)}_{\rm LSZ}(\mu^2) + \Big[\mathcal{F}_{\rm F.S.}(\mu^2)\circ\Big(\mathcal{K}^{(1)}_{\rm F.S.}(\mu^2) + \mathcal{Z}^{(1)}_{F}(\mu^2) + \widetilde{\mathcal{V}}(\mu^2)\Big)\Big]\mathcal{F}^{-1}_{\rm F.S.}(\mu^2)}\Big\} + \mathcal{O}(\alpha_{\rm s}^2)\ .$$

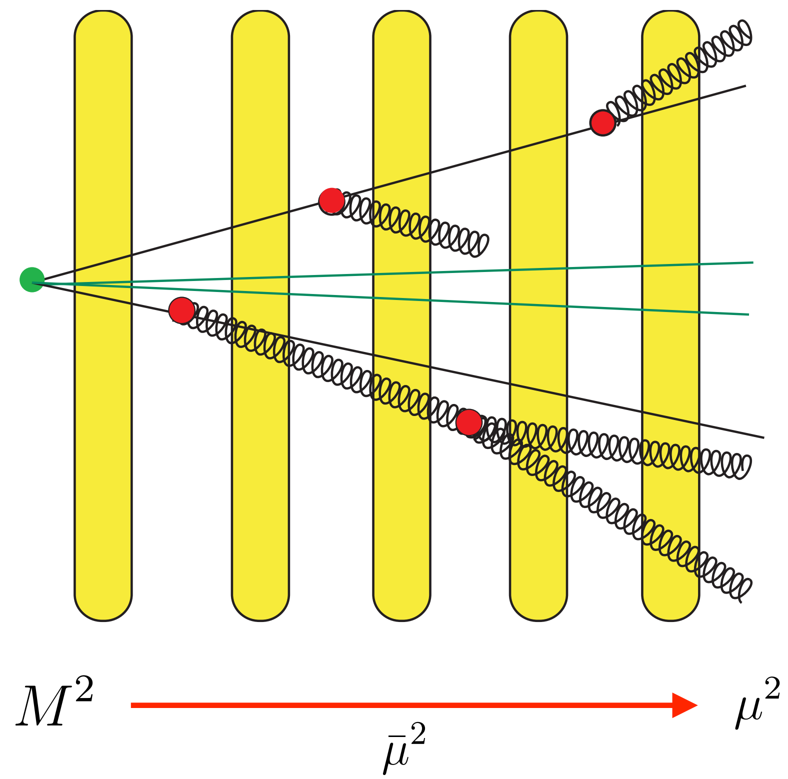
Finite in d=4 dimension

Completely neglected in the standard showers. The threshold contributions are here.

Shower Evolution

Now the shower evolution operator is

$$\exp\left\{\int_{\mu_2^2}^{\mu_1^2}\frac{d\bar{\mu}^2}{\bar{\mu}^2}\left(\underbrace{\mathcal{S}_{\rm uni}(\bar{\mu}^2)}_{\textit{Unitary part}}+\overbrace{\left[\mathcal{F}_{\rm F.S.}(\bar{\mu}^2)\circ\mathcal{S}_{\rm thr}(\bar{\mu}^2)\right]\mathcal{F}^{-1}_{\rm F.S.}(\bar{\mu}^2)}^{\textit{Threshold contribution}}\right)\right\}$$

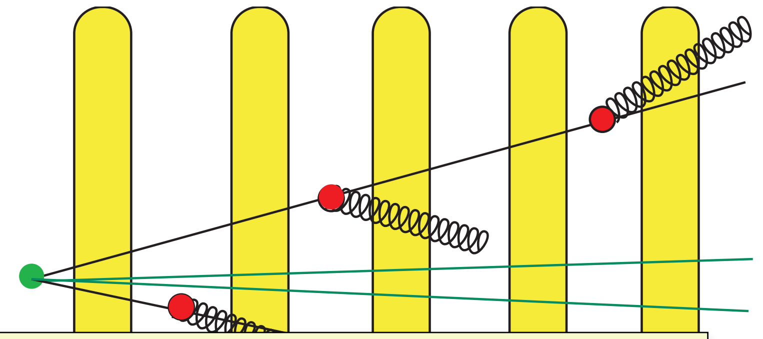


- ✓ This leads to a non-unitary shower.
- ✓ The threshold splitting operator doesn't change the number of the partons and their momenta. It operates in the colour and flavour space only.
- ✓ In LC+ approximation it leads to an extra factor that we have to insert after every step of the shower evolution.

$$\exp\left\{\int_{\mu_2^2}^{\mu_1^2}\frac{d\bar{\mu}^2}{\bar{\mu}^2}\left[\mathcal{F}_{\rm F.S.}(\mu^2)\circ\mathcal{S}^{\rm LC+}_{\rm thr}(\bar{\mu}^2)\right]\mathcal{F}^{-1}_{\rm F.S.}(\mu^2)\right\}$$

Shower Evolution

Now the shower evolution operator is



Threshold contribution

$$\exp\left\{\int^{\mu_1^2}\frac{d\bar\mu^2}{\cdots}\left(\mathcal{S}_{\cdots}(\bar\mu^2)+\overbrace{[\mathcal{F}_{\cdots}(\bar\mu^2)\circ\mathcal{S}_{\cdots}(\bar\mu^2)]\,\mathcal{F}^{-1}_{\cdots}(\bar\mu^2)}\right)\right\}$$

Direct term with diagonal color

$$\begin{aligned}\mathcal{S}_{\rm thr}(\mu^2) = &-\frac{\alpha_{\rm s}(\mu^2)}{2\pi}\left[\delta_{a\hat a}\frac{2Ca}{1-z}\,\theta\big((1-z)^{\beta+1}<z\,y_{\rm s}\,r_\perp(z)\big)\,\theta\big(\mu^2>(1-z)^{\beta-1}m_\perp^2\big)\right]_+[1\otimes 1]\\ &-\frac{\alpha_{\rm s}(\mu^2)}{2\pi}P^{\rm reg}_{a\hat a}(z)\,\theta\big((1-z)^{\beta+1}<z\,y_{\rm s}\,r_\perp(z)\big)\,\theta\big(\mu^2>(1-z)^{\beta-1}m_\perp^2\big)[1\otimes 1]\\ &-\frac{\alpha_{\rm s}(\mu^2)}{2\pi}\sum_{k\neq \rm a}\left[w\left(\xi_{\rm ak},\frac{y_{\rm s}r_\perp(z)}{(1-z)^{\beta+1}}\right)\theta\big((1-z)^{\beta+1}>z\,y_{\rm s}\,r_\perp(z)\big)\,\theta\big(\mu^2>(1-z)^{\beta-1}m_\perp^2\big)\right]_+\\ &\qquad\times\frac{1}{2}\left[(\boldsymbol{T}_{\rm a}\cdot\boldsymbol{T}_k)\otimes 1+1\otimes(\boldsymbol{T}_{\rm a}\cdot\boldsymbol{T}_k)\right]+\cdots\end{aligned}$$

Colour interference part of the threshold logs

✓ T

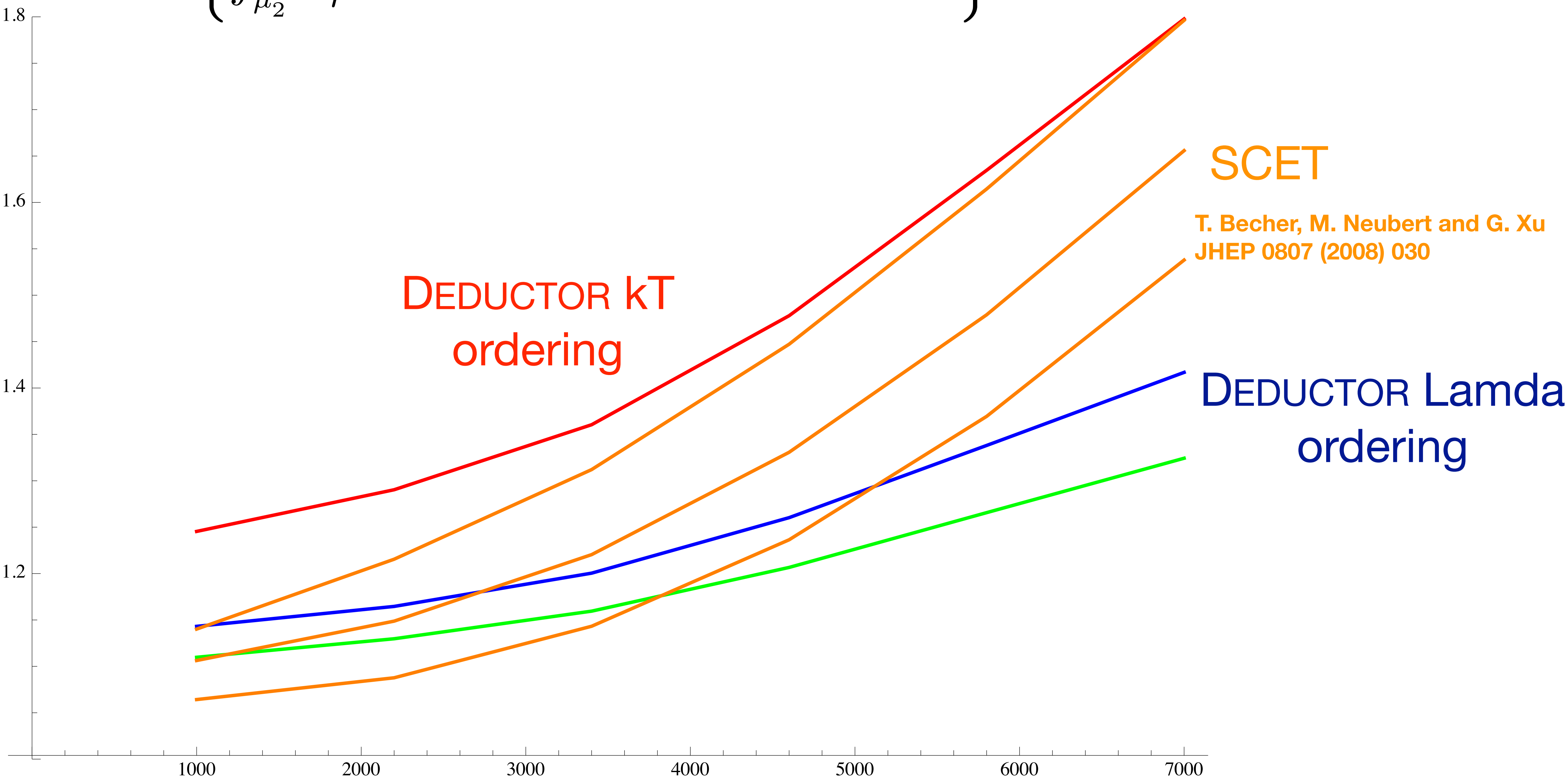
✓ T
t
f

✓ I
i

$$\exp\left\{\int_{\mu_2^2}^{\mu_1^2}\frac{d\bar\mu^2}{\bar\mu^2}\left[\mathcal{F}_{\rm F.S.}(\mu^2)\circ\mathcal{S}^{\rm LC+}_{\rm thr}(\bar\mu^2)\right]\mathcal{F}^{-1}_{\rm F.S.}(\mu^2)\right\}$$

Comparison to SCET

$$\exp\left\{\int_{\mu_2^2}^{\mu_1^2}\frac{d\bar{\mu}^2}{\bar{\mu}^2}\left[\mathcal{F}_{\rm F.S.}(\mu^2)\circ\mathcal{S}_{\rm thr}^{\rm LC+}(\bar{\mu}^2)\right]\mathcal{F}_{\rm F.S.}^{-1}(\mu^2)\right\}$$



PDF Factorization Scheme

Some of the threshold logarithms has to be summed up by the PDF functions by choosing factorization scheme appropriately. The first order kernel of the factorization scheme is

$$[K^{(1)}_{\text{F.S.}}(z,\mu^2)]_{a\hat{a}} = -P^{(\epsilon)}_{a\hat{a}}(z) + P^{\text{reg}}_{a\hat{a}}(z)\log\frac{(1-z)^{\beta-1}}{r_\perp(z)} + 2C_a\delta_{a\hat{a}}\left[\frac{1}{1-z}\log\frac{(1-z)^{\beta-1}}{r_\perp(z)}\right]_+$$

$$r_\perp(z) = \max\left\{1,(1-z)^{\beta-1}\frac{m_\perp^2}{\mu^2}\right\}$$

✓ Transverse momentum ordered shower $\beta = 1$

$$[K^{(1)}_{\text{F.S.}}(z,\mu^2)]_{a\hat{a}} = -P_{a\hat{a}}(z)\log\left(\max\{1, m_\perp^2/\mu^2\}\right) - P^{(\epsilon)}_{a\hat{a}}(z)$$

- For $m_\perp^2 < \mu^2$ we don't have to change the factorization scheme. MSbar PDF works perfectly.
- For $m_\perp^2 > \mu^2$ the PDFs get frozen.

✓ For other orderings (virtuality and angular) $\beta < 1$

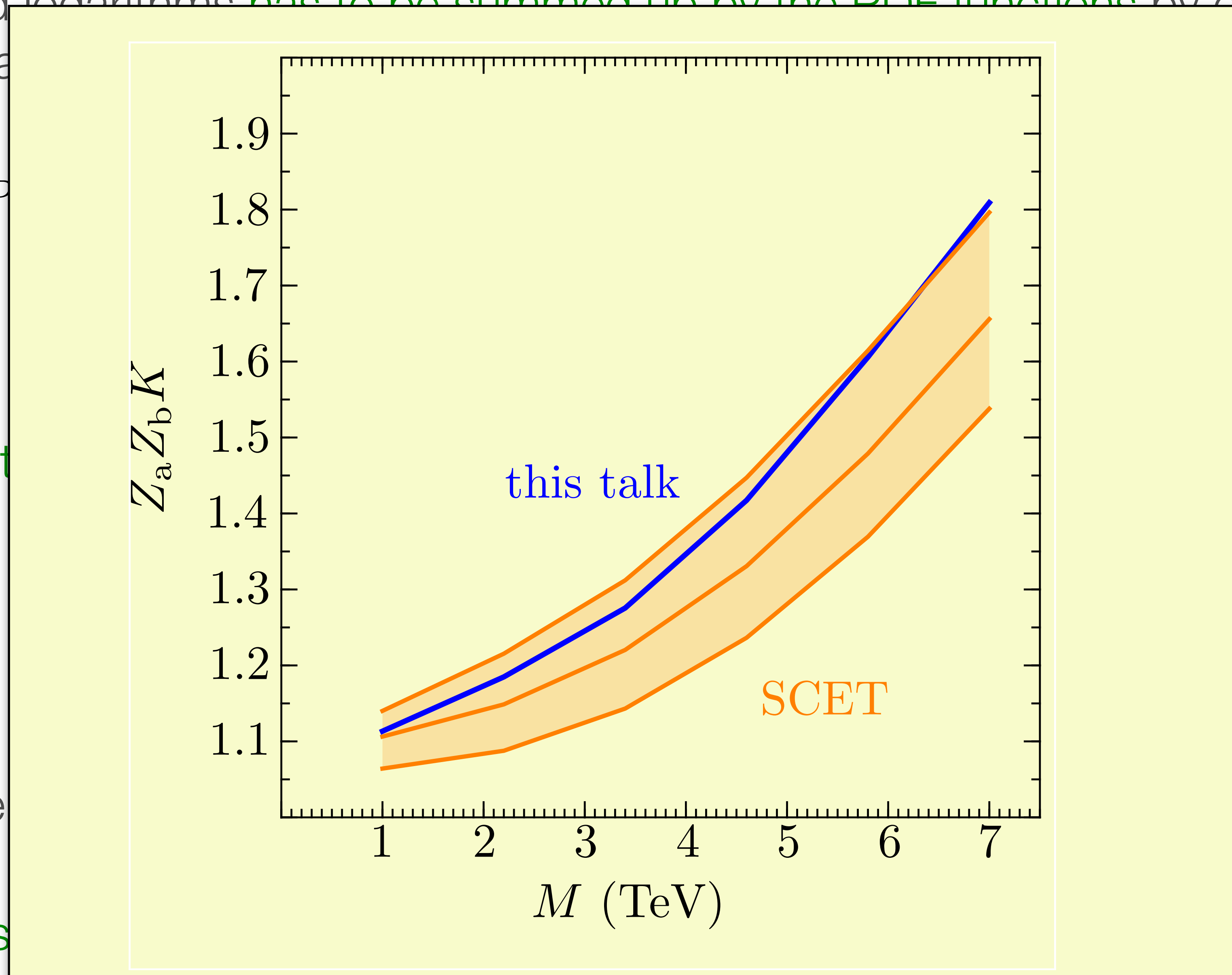
$$\begin{aligned}[K^{(1)}_{\text{F.S.}}(z,\mu^2)]_{a\hat{a}} = &- P^{(\epsilon)}_{a\hat{a}}(z) - P^{\text{reg}}_{a\hat{a}}(z)\log\left(\max\left\{(1-z)^{1-\beta},\frac{m_\perp^2}{\mu^2}\right\}\right)\\ &- 2C_a\delta_{a\hat{a}}\left[\frac{1}{1-z}\log\left(\max\left\{(1-z)^{1-\beta},\frac{m_\perp^2}{\mu^2}\right\}\right)\right]_+\end{aligned}$$

PDF Factorization Scheme

Some of the threshold logarithms has to be summed up by the PDF functions by choosing factorization scheme a … e is

$$[K^{(1)}_{\rm F.S.}(z,\mu^2)]_{a\hat{a}} = -P \;\ldots\; {}_{+}$$

$$\ldots\mathrm{x}\left\{1,\,(1-z)^{\beta-1}\,\frac{m_\perp^2}{\mu^2}\right\}$$



✓ Transverse moment …

- For $m_\perp^2 < \mu^2$ we … works perfectly.
- For $m_\perp^2 > \mu^2$ the …

✓ For other orderings …

$$[K^{(1)}_{\rm F.S.}(z,\mu^2)]_{a\hat{a}} = -\,P^{(\epsilon)}_{a\hat{a}}(z) - P^{\rm reg}_{a\hat{a}}(z)\log\left(\max\left\{(1-z)^{1-\beta},\,\frac{m_\perp^2}{\mu^2}\right\}\right)$$
$$-\,2C_a\delta_{a\hat{a}}\left[\frac{1}{1-z}\log\left(\max\left\{(1-z)^{1-\beta},\,\frac{m_\perp^2}{\mu^2}\right\}\right)\right]_+$$

Conclusions

✓ We have defined parton shower.

- ➠ We defined parton shower based on pQCD and factorization of QCD density matrices. The aim is the gain as much control as possible on the approximations (like unitarity condition)…
- ➠ We still need the all order proof of the factorization of the physical states. We want a constructive proof. Splitting operators (with many loops), momentum mapping, shower scale definition, …
- ➠ At higher order it is not possible to turn every subtraction scheme to parton shower.

✓ It works at NLO level.

- ➠ We recovered what is called "Standard Shower".
- ➠ We obtained threshold resummation basically for free. *Shower is not unitary!*
- ➠ If you want unitary shower, you need process dependent PDFs.
- ➠ Some threshold logs get resummed in the PDFs. MSbar PDFs only for transverse momentum ordered showers. In other shower schemes the PDF factorisation scheme has to be adjusted.
- ➠ There is a plan to implement the new factorization schemes in HERAFITTER.

✗ I didn't discuss in the talk.

- ➠ We obtained NLO matching for free, it is just part of the scheme.
- ➠ Genuine loop effects like $\mathrm{i}\pi/\epsilon$ terms.
- ➠ Final state heavy flavor threshold logs

Where is the Code?

- DEDUCTOR is designed to do a better job with color, spin and resummation of large logarithms compared to other shower generators.
    - Lambda ordering with and without initial state massive quarks
    - LC+ color treatment. It allows us to do color evolution at amplitude level
    - Spin correlations are not yet computed
- Next version is available soon…
    - The shower equation is implemented at very abstract level. It allows us to use other ordering variables like kT or angle (massless or massive initial state partons).
    - Initial state threshold log resummation.
    - Subleading (wide angle subleading colour, Coulomb gluon,…) contribution perturbative.
- It is available at

```
http://www.desy.de/~znagy/deductor
http://pages.uoregon.edu/soper/deductor
```

Where is the Code?

- DEDUCTOR is designed to do a better job with color, spin and resummation of large logarithms compared

```
/*   Defining the ordering dependent functions for the INITIAL state splittings  */
template<bool _Is_msbar>
struct __ordering_traits<ini, ordering::lambda, _Is_msbar>
{
  /*  calculates the limits on the variable v    */
  static void vlimits(double&, double&, const __hard_params<ini> *);

  /*  calculates the z limits  */
  static void zlimits(double&, double&, const __hard_params<ini> *, int, int, double);

  /*  pdf scale  */
  static double pdf_scale(const __hard_params<ini> *pars, double x, double y) {…}

  /*  kT2/(v*Q2) \approx (1-z)^alpha   */
  static constexpr unsigned int kT_alpha = 1u;

  /*  mapping the indipendent splitting variables v and z to the normalized virtuality, y   */
  static double mapping_to_y(const __hard_params<ini> *, int, int, double v, double) {…}

  /*  mapping the indipendent splitting variables v and z to the normalized virtuality, y
   *  It also returns the jacobian of  v --> y mapping.
   */
  static void mapping_to_y(double& y, double& yjac, const __hard_params<ini> *, int, int, double v, double) {…}

  /*  helps to define the shower time: exp(-t) = v/vnull   */
  static double vnull(double Q2, const lorentzvector<double>& qnull, const lorentzvector<double>& pa) {…}


  /*   Ordering dependent properties for the threshold resummation    */
  struct threshold
  {…};
};
```

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