How do we calculate $\sigma(p\bar{p} \to t\bar{t})$?

Basic picture

Parton distribution function

$[x, x+dx]$

$F_{i/H}(x, \mu_F)$

$x_2 \quad F_{i/H_1}(x_1, \mu_F)\, F_{j/H_2}(x_2, \mu_F)$

ij

$d\sigma\big(i(x_1 P_1)\, j(x_2 P_2) \to t\bar{t}\big)$

$\frac{1}{2s}\, dLips\, |\mathcal{M}(ij \to t\bar{t})|^2$

$(2\pi)^4\, \delta(P - \sum p_i) \prod_k \frac{d^3 k_k}{(2\pi)^3\, 2E_k}$

picture

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$(2\pi)^4\, \delta\left(P - \sum p_i\right) \prod_i \frac{d^3 k_i}{(2\pi)^3\, 2E_k}$

It is the S-matrix ($S = \mathbb{1} + i\,T$) where the specific theory enters

Here QCD:

$$\mathcal{L}_{QCD} = \sum_i \bar{\psi}_i \left(i \not{D} - m_i \right) \psi_i - \frac{1}{4} G^a_{\mu\nu} G^{a\,\mu\nu}$$

$$\not{D} = \gamma^\mu D_\mu \,, \quad D_\mu = \partial_\mu - i g T_a A^a_\mu$$

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f_{abc} A^b_\mu A^c_\nu$$

From the QCD Lagrangian we obtain the Feynman rules:

$$\bar{\psi} (-i) g T^a A^a_\mu \gamma^\mu \psi \rightarrow$$

$$-i g T_a \gamma_\mu$$

$$-\delta^{ab} \frac{g^{\mu\nu}}{}$$

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$$-i g T_a \gamma_\mu$$

$$-\delta^{ab} \frac{g^{\mu\nu}}{p^2 + i\epsilon}$$

wave functions $u, \bar{u}, v, \bar{v}, \epsilon$

$$i\mathcal{M} = \bar{u}(k_1)(-i)g_s\gamma_\mu T^a v(k_2)\,\frac{-i g^{\mu\nu}\delta^{ab}}{(k_1+k_2)^2+i\epsilon}\,\bar{v}(p_2)(-i)g_s\gamma_\nu T^b u(p_1)$$

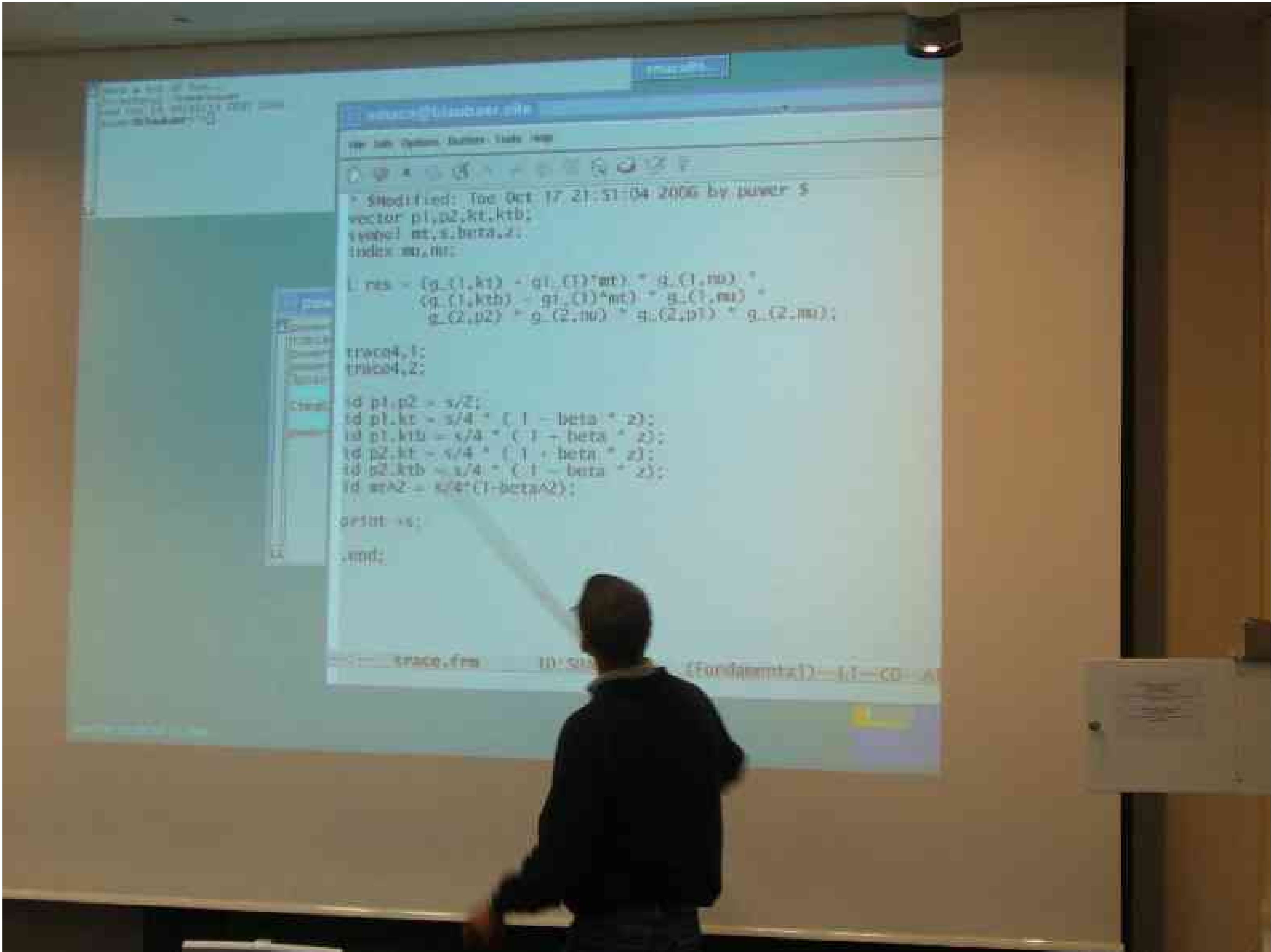
$$(k_1+k_2)^2 \equiv s$$

$$= i\frac{g_s^2}{s}\,\bar{u}(k_1)\gamma_\nu T^a v(k_2)\,\bar{v}(p_2)\gamma^\nu T^a u(p_1)$$

For the cross section we need $|\mathcal{M}|^2$

$$|\mathcal{M}|^2 = g_s^4\,\frac{1}{s^2}\,\bar{u}(k_1)\gamma_\nu T^a v(k_2)\,\bar{v}(p_2)\gamma^\nu T^a u(p_1)$$

$$\bar{v}(k_2)\gamma_\mu T^b u(k_1)\,\bar{u}(p_1)\gamma^\mu T^b v(p_2)$$



* $Modified: Tue Oct 17 21:51:04 2006 by puwer $
vector p1,p2,kt,ktb;
symbol mt,s,beta,z;
index mu,nu;
L res = (g_(1,kt) + gi_(1)*mt) * g_(1,mu) *
(g_(1,ktb) - gi_(1)*mt) * g_(1,nu) *
g_(2,p2) * g_(2,mu) * g_(2,p1) * g_(2,nu);
trace4,1;
trace4,2;
id p1.p2 = s/2;
id p1.kt = s/4 * (1 - beta * z);
id p1.ktb = s/4 * (1 + beta * z);
id p2.kt = s/4 * (1 + beta * z);
id p2.ktb = s/4 * (1 - beta * z);
id mt^2 = s/4*(1-beta^2);
print +s;
.end;
trace.frm

$$\frac{1}{3\cdot 3}\sum_{\substack{t\bar t\\ q\bar q}} (T^a)_{t\bar t}\,(T^b)_{\bar t t}\,(T^a)_{q\bar q}\,(T^b)_{\bar q q}$$

$$= \frac{1}{9}\,\mathrm{Tr}[T^a T^b]\,\mathrm{Tr}[T^a T^b]$$

$$= \frac{1}{9}\,\frac{1}{2}\,\delta^{ab}\,\delta^{ab}\,\frac{1}{2} = \frac{1}{9}\,\frac{1}{4}\,\delta^{aa} = \frac{1}{9}\,\frac{1}{4}\,8$$

$$\underset{SU(N)}{=} \frac{1}{N^2}\,\frac{1}{4}\,(N^2-1)$$

use $\sum_{s} u_\alpha(k_t, s)\,\bar u_\beta(k_t, s) = (\not k_t + m_t)_{\alpha\beta}$

$$\sum_{s} v_\alpha(k_{\bar t}, s)\,\bar v_\beta(k_{\bar t}, s) = (\not k_{\bar t} - m_t)_{\alpha\beta}$$

$$\sum |\mathcal{M}|^2 = C\,g_s^4\,\frac{1}{s^2}\,\mathrm{Tr}[(\not k_t + m)\gamma_\nu(\not k_{\bar t} - m)\gamma_\mu]\;\mathrm{Tr}[\not p_2\,\gamma^\nu\,\not p_1\,\gamma^\mu]$$

$$\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2\,g^{\mu\nu}$$

$SU(N)$ $\quad N^2 - 1$

use $\displaystyle\sum_s u_\alpha(k_t, s)\,\bar{u}_\beta(k_t, s) = (\not k_t + m_t)_{\alpha\beta}$

$$\sum_s v_\alpha(k_{\bar t}, s)\,\bar{v}_\beta(k_{\bar t}, s) = (\not k_{\bar t} - m_t)_{\alpha\beta}$$

$$\sum |M|^2 = C \; g_s^4 \; \frac{1}{s^2} \; \mathrm{Tr}\left[(\not k_t + m)\gamma_\nu (\not k_{\bar t} - m)\gamma_\mu \right]$$

$$\mathrm{Tr}\left[\not p_2 \, \gamma^\nu \, \not p_1 \, \gamma^\mu \right]$$

$$\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu = 2 g^{\mu\nu}$$

Hadronic cross section

$$d\sigma_{Had} = \int dx_1 dx_2 \Big\{ F_{q/p}(x_1,\mu_F)\, F_{\bar q/\bar p}(x_2,\mu_F) + F_{\bar q/p}(x_1,\mu_F)\, F_{q/\bar p}(x_2,\mu_F) \Big\}\, d\sigma(s_{hadron}\, x_1\, x_2, z)$$

$$s = 2p_1p_2 = 2\, x_1 P_1\, x_2 P_2 = x_1 x_2\, S_{hadron}$$

α_s and $F_{i/H}$ depend on a scale μ

in massless QCD the classical scale invariant

$\Rightarrow \alpha_s\ F_i$ become scale dependent

theoretical predictions cannot depend on these scales

$= x_1 p_1 \, x_2 p_2$

$= x_1 x_2 S$ frame

α_s and $F_{i/H}$ depend on a scale μ

in quantum QCD the classical scale invariance

$\Rightarrow$ α_s, F_i become scale dependent

theoretical predictions cannot depend on these scales

Consequences:

- F_H, α_S are not direct observable
- if the prediction is independent on scale μ we can use any value we like

that is what pure theory tells us...

How do we chose a good value for μ so that perturbation theory converges well.

$\ln\left(\frac{\mu^2}{s_{ij}}\right) \longrightarrow$ try to chose μ of the order of the typical energy scale of

How do we chose a good value for μ so that perturbation theory converges well

$\ln\left(\frac{\mu^2}{s_{ij}}\right) \Rightarrow$ try to chose μ of the order of the typical energy scale of the problem

$\mu_F = \mu_R = 2 m_t$

$\alpha_s = \alpha_s(2 m_t) \approx 0.1$

$F_{q/p}(x, \mu_F) = F_{q/p}(x, 2 m_t)$

CTEQ, MRST

pdflib, LHApdf

$q\bar{q} \to t\bar{t}$

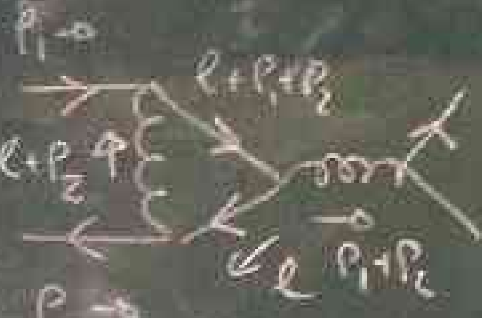
Additional rules:

- (-1) for closed fermion loops.
- additional symm. factors

$\frac{1}{2}$

- integration over unconstrained loop mom.

$$\int \frac{d^4 \ell}{(2\pi)^4}$$



$$i\mathcal{J} = \bar{u}(k_t)(-i)g_s\gamma_\mu v(k_{\bar{t}})$$

$$(-i)\frac{g^{\mu\nu}}{s}\int\frac{d^4\ell}{(2\pi)^4}\,\bar{v}(p_2)(-i)g_s\gamma^\alpha\, i\frac{\ell\!\!/}{\ell^2+i\epsilon}$$

$$\frac{-i}{(\ell+p_2)^2+i\epsilon}$$

$$= g_s^4 \, \frac{1}{s} \, \bar{u}(k_2)\gamma_\mu v(k_1)$$

$$\int \frac{d^4 l}{(2\pi)^4} \frac{1}{l^2 (l+p_2)^2 (l+p_1+p_2)^2} \{ \bar{v}(p_2)\gamma^\alpha \not{l} \gamma^\mu \not{l} \gamma_\alpha u(p_1)$$

$$+ \bar{v}(p_2)\gamma^\alpha \not{l}\gamma^\mu (\not{p}_1+\not{p}_2)\gamma_\alpha u(p_1)\}$$

$$\int \frac{l_\mu}{l^2 (l+p_2)^2 (l+p_1+p_2)^2} = C_{11} p_{2\mu} + C_{12} p_{1\mu}$$

$$\int \frac{l_\mu l_\nu}{l^2 (l+p_2)^2 (l+p_1+p_2)} = C_{21} p_2^\mu p_2^\nu + C_{22} p_1^\mu p_1^\nu + C_{23} \{p_1^\mu p_2^\nu + p_2^\mu p_1^\nu\}$$

$$+ C_{24} \, g^{\mu\nu}$$

$$\int \frac{l_\mu}{l^2 (l+p)^2} = B_1 \, p_\mu \quad | \; p^\mu$$

$$\int \frac{l \cdot p}{l^2 (l+p^2)} = p^2 B_1$$

$$\frac{1}{2} \int \frac{l^2 + 2lp + \cancel{p^2} - l^2 - p^2}{l^2 (l+p)^2} = \frac{1}{2} \int \frac{1}{l^2} - \frac{1}{2} \int \frac{1}{(l+p)^2} = p^2 B_1$$

$$- \frac{p^2}{2} \int \frac{1}{\quad}$$

$$= g_s^4 \frac{1}{s} \bar{u}(k_1) \gamma_\mu v(k_2)$$

$$\int \frac{d^4 l}{(2\pi)^4} \frac{1}{l^2 (l+p_2)^2 (l+p_1+p_2)^2} \{ \bar{v}(p_2) \gamma^\alpha \not{l} \gamma^\mu \not{k} \gamma_\alpha u(p_1)$$

$$+ \bar{v}(p_2) \gamma^\alpha \not{l} \gamma^\mu (\not{p}_1 + \not{p}_2) \gamma_\alpha u(p_1) \}$$

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$$- \frac{p^2}{2} \int \frac{1}{\quad}$$

$$\int \frac{\ldots}{\ell^2(\ell-p_2)^2(\ell+p_1+p_2)} = C_{21}\, p_2^\mu p_2^\nu + C_{22}\, p_1^\mu p_1^\nu + C_{23} \{ p_1^\mu p_2^\nu + p_2^\mu p_1^\nu \ldots$$

$$+ C_{24}\, g^{\mu\nu}$$

$$\int \frac{\ell_\mu}{\ell^2(\ell+p)^2} = B_1 \cdot p_\mu \quad \Big|\, p^\mu$$

$$\int \frac{\ell \cdot p}{\ell^2(\ell+p^2)} = p^2 \cdot B_1$$

$$\frac{1}{2} \int \frac{\ell^2 + 2\ell p + p^2 - \ell^2 - p^2}{\ell^2(\ell+p)^2} = \frac{1}{2}\int \frac{1}{\ell^2} - \frac{1}{2}\int\frac{1}{(\ell+p)^2} = p^2 B_1$$

$$- \frac{p^2}{2} \int \frac{1}{}$$

$$\mathcal{M}_1 = -C_0(1,2,3) + \underbrace{C_0(1,\cancel{2},3)}_{B_0(1,3)}$$

$$C_0(1,2,3) = \int \frac{d^4 l}{(2\pi)^4} \frac{1}{(l^2+i\epsilon)((l+p_2)^2+i\epsilon)((l+p_1+p_2)^2+i\epsilon)}$$

$$B_0(1,3) = \int \frac{d^4 l}{(2\pi)^4} \frac{1}{(l^2+i\epsilon)((l+p_1+p_2)^2+i\epsilon)}$$

Both integrals are divergent

B_0 diverges for $l \to \infty$ $\Rightarrow$ UV singularity

C_0 diverges due to singularities for example $l \to 0$

- soft, IR-singularities

B_0 diverges for $\ell \to \infty$ $\Rightarrow$ UV singularity

C_0 diverges due to singularities for example $\ell \to 0$

- soft, IR-singularities
- collinear sing., mass singularities

$$= \frac{1}{\varepsilon} - \ln\left(\frac{-s}{\mu^2}\right)^{+i\varepsilon} + O(\varepsilon) \qquad s=(p_1+p_2)^2$$

$$g_s^2 \to g_s^2(\mu)\,\mu^{2\varepsilon}$$

$$\langle \sigma \rangle \overset{?}{=} f_0(g_s^2)$$

$$\overset{?}{=} f_0(g_s^2) + f_1(g_s^2)$$

QED

$$\mathcal{L}_{QCD}(A^0, \psi^0, g_s^0, m^0)$$

$$= \mathcal{L}_{QCD}(Z_3^{1/2} A_R, Z_2^{1/2} \psi_R, Z_g g_R), Z_m m_R)$$

$$= \mathcal{L}_{QCD}(A_R, \psi_R, g_R, m_R)$$

$$+ \mathcal{L}_{CT}(A_R, \psi_R, g, \ldots, Z_2, Z_3, Z_g, Z_m)$$

$$\equiv \mathcal{L}_{QCD}(Z_3^{1/2} A_R, \ldots) - \mathcal{L}_{QCD}(A_R, \psi_R, \ldots)$$

$$(Z_2 - 1)\bar{\psi}_R(i\gamma_\mu \partial^\mu - m_R)\psi_R$$

$$(Z_2 - 1)\,\bar{\psi}_R\,(i\gamma_\mu\partial^\mu - m_R)\,\psi_R$$

$(Z_3$

$$\frac{1}{\varepsilon} \ldots$$

$$Z_g^{\overline{MS}} = 1 - \frac{1}{2}\frac{g_s^2(\mu)}{16\pi^2}\left(\frac{1}{\varepsilon} - \gamma + \ln(4\pi)\right)\frac{11N - 2n_f}{3}$$

$$\beta_\varepsilon = \mu\frac{d\, g_R(\mu)}{d\mu} = \mu\frac{d}{d\mu}\frac{\mu^{-\varepsilon} g_0}{Z_g(\mu)}$$

$$= -\varepsilon\,\mu\,\frac{\mu^{-\varepsilon-1}}{Z_g(\mu)}\, g_0 - \mu\frac{\mu^{-\varepsilon}}{Z_g^2(\mu)}\frac{d}{d\mu}(Z_g)\, g_0$$

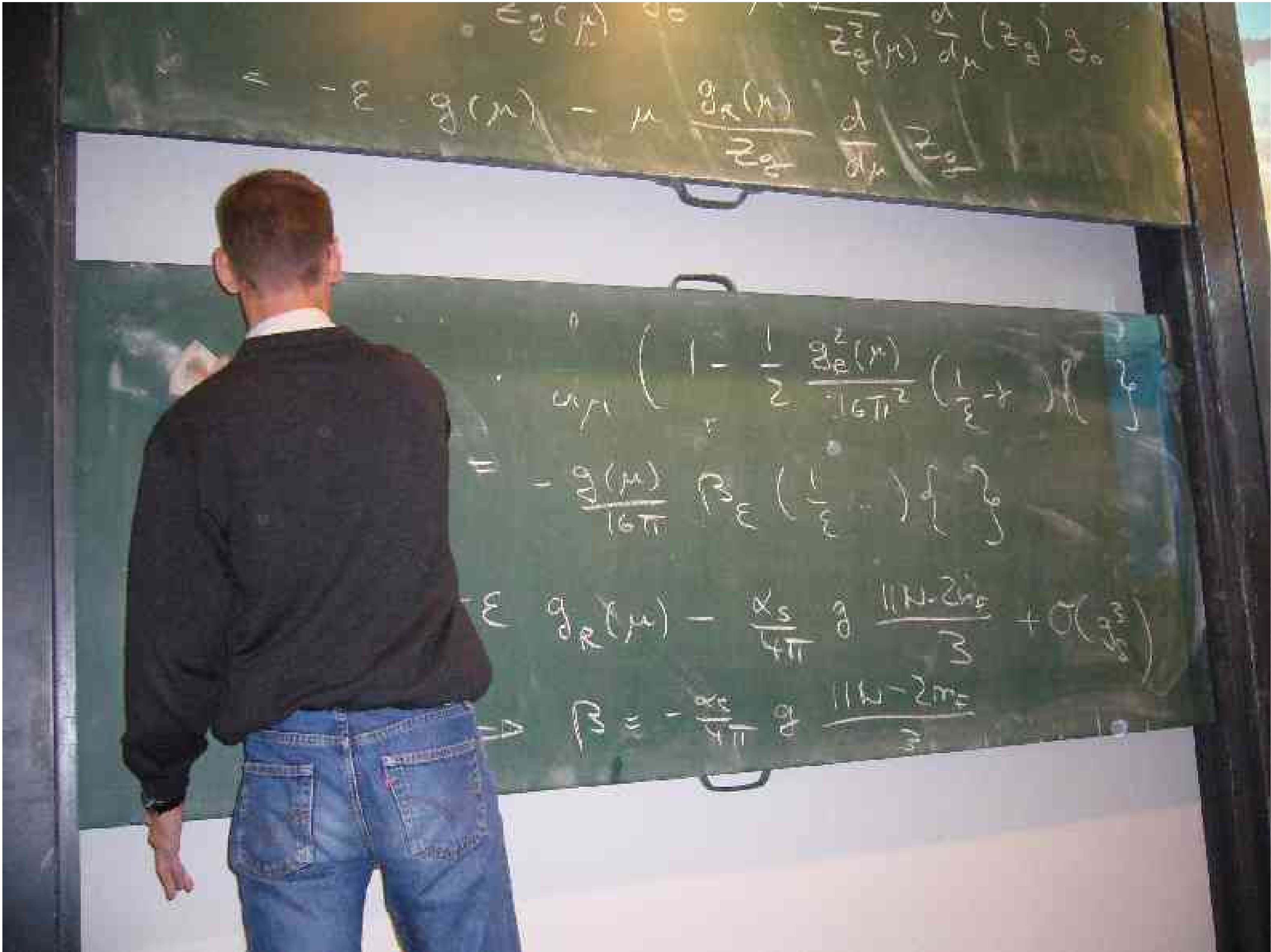
$$= -\varepsilon\, g(\mu) - \mu\frac{g_R(\mu)}{Z_g}\frac{d}{d\mu} Z_g$$

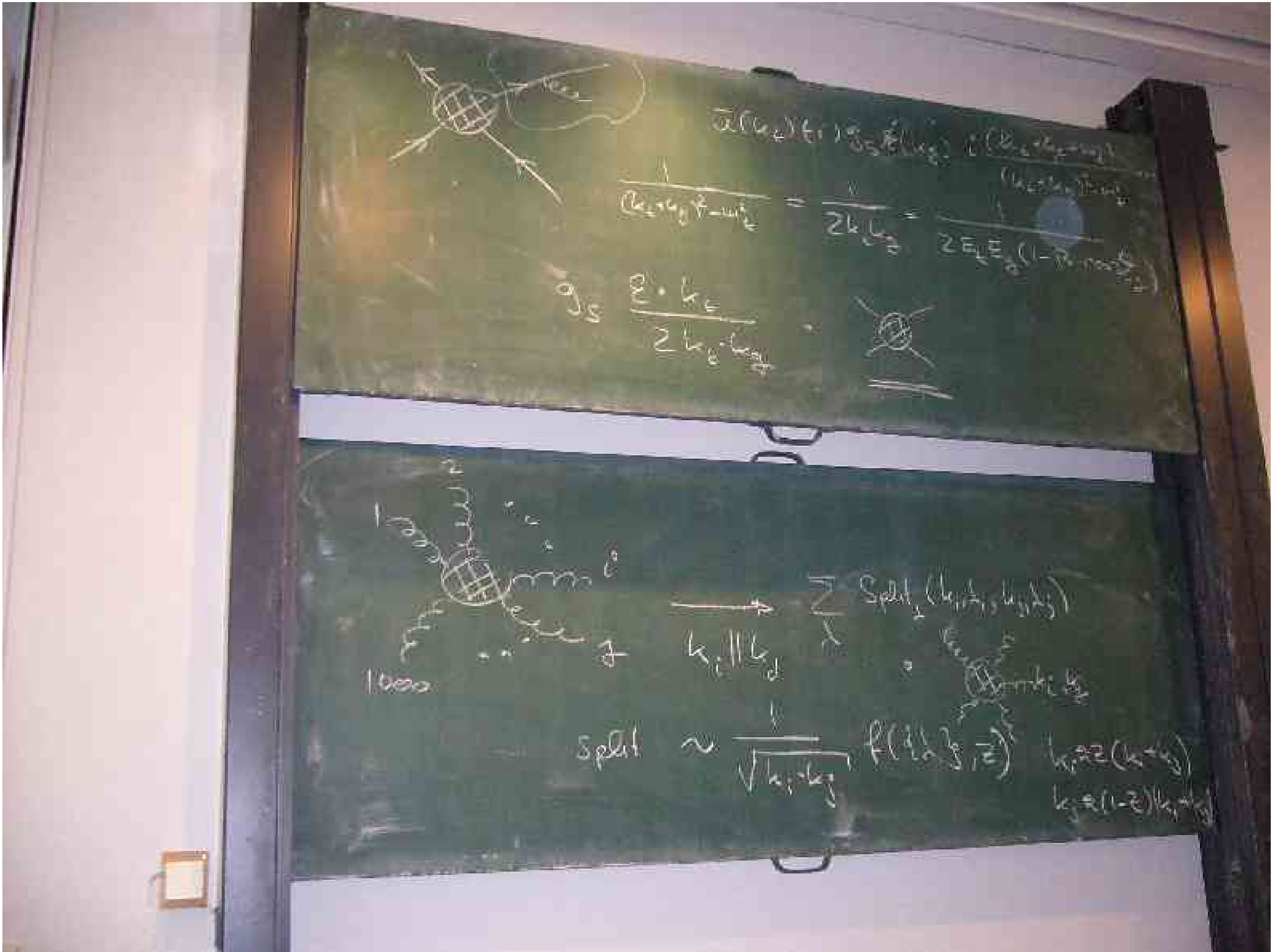
$$\mu\frac{d}{d\mu} Z_g = \mu\frac{d}{d\mu}\left(1 - \frac{1}{2}\frac{g_R^2(\mu)}{16\pi^2}\left(\frac{1}{\varepsilon} - \gamma\right)\{\ \}\right)$$

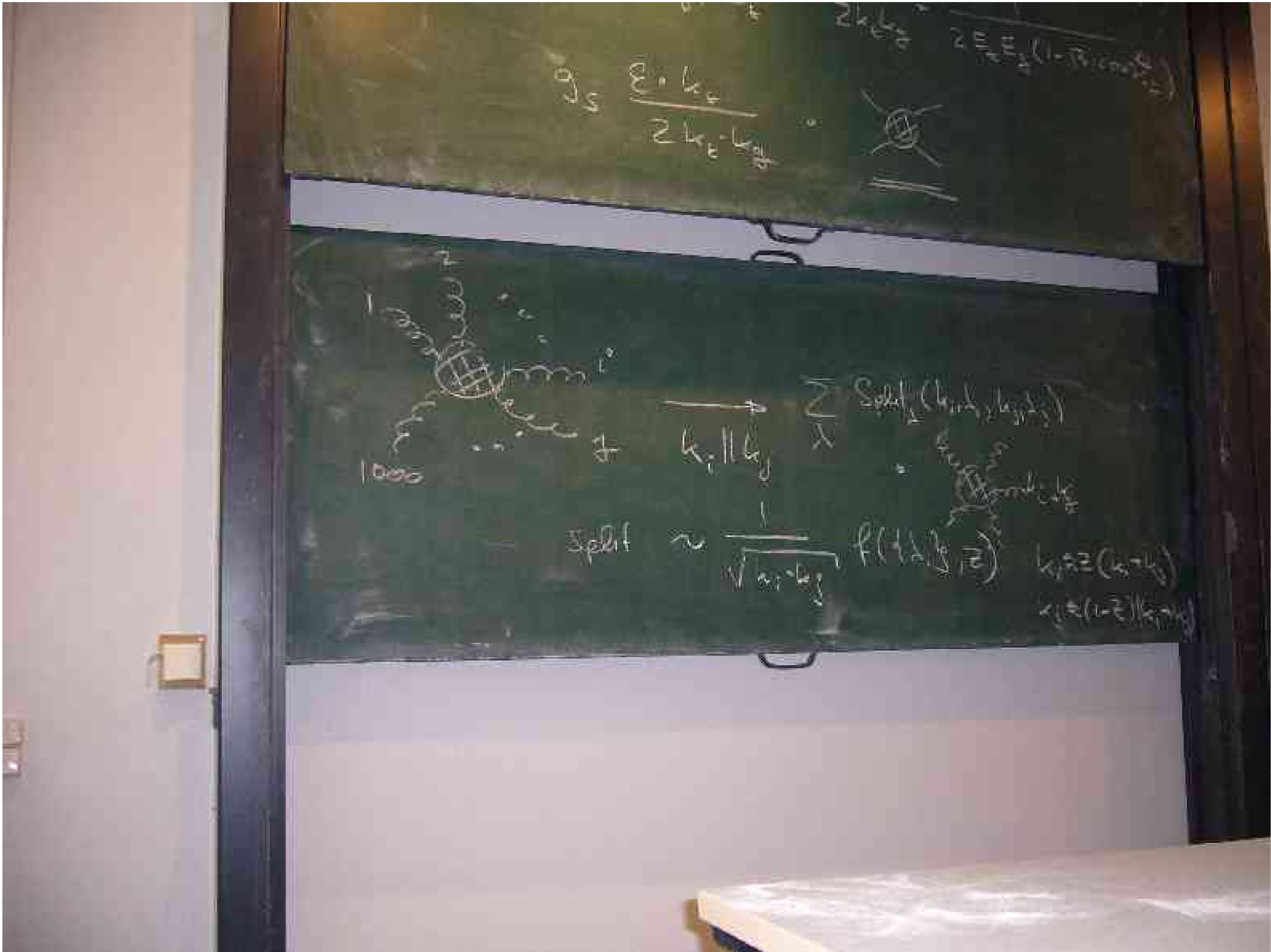
$$= -\frac{g(\mu)}{16\pi}\,\beta_\varepsilon\left(\frac{1}{\varepsilon}\ldots\right)\{\ \}$$

$$\beta_\varepsilon = -\varepsilon\, g_R(\mu) - \frac{\alpha_s}{4\pi}\, g\, \frac{11N - 2n_F}{3} + O(g_s^3)$$

$$\varepsilon \to 0 \Rightarrow \beta = -\frac{\alpha_s}{4\pi}\, g\, \frac{11N - 2n_F}{3}$$







g_s ε·k_t / 2k_t·k_g
1
2
10000
k_i || k_j
Σ_λ Split_λ(k_i,λ_i, k_j,λ_j)
Split ~ 1/√(k_i·k_j) f(λ_i,λ_j,z)

$$O_{n+1}(k_1, \ldots k_j \ldots k_{n+1}) \xrightarrow[k_j \to \text{soft}]{} O_n(k_1 \ldots k_{j-1}, k_{j+1} \ldots k_n)$$

$$O_{n+1}(k_1 \ldots k_j, \ldots k_k \ldots k_n) \xrightarrow[k_j \| k_k]{} O_n(k_1 \ldots (k_j + k_k), \ldots k_n)$$

$$|\mathcal{M}(q(p_1)\,\bar{q}(p_2) \to t(k_t)\,\bar{t}(k_{\bar{t}})\,g(p_3))|^2$$

$$\xrightarrow{p_1 \| p_3} \frac{1}{s_{13}} \frac{1}{z} \frac{1 + z^2 - \epsilon(1-z)^2}{1-z} \alpha_s C_F$$

$$2\, p_1 \cdot p_3$$

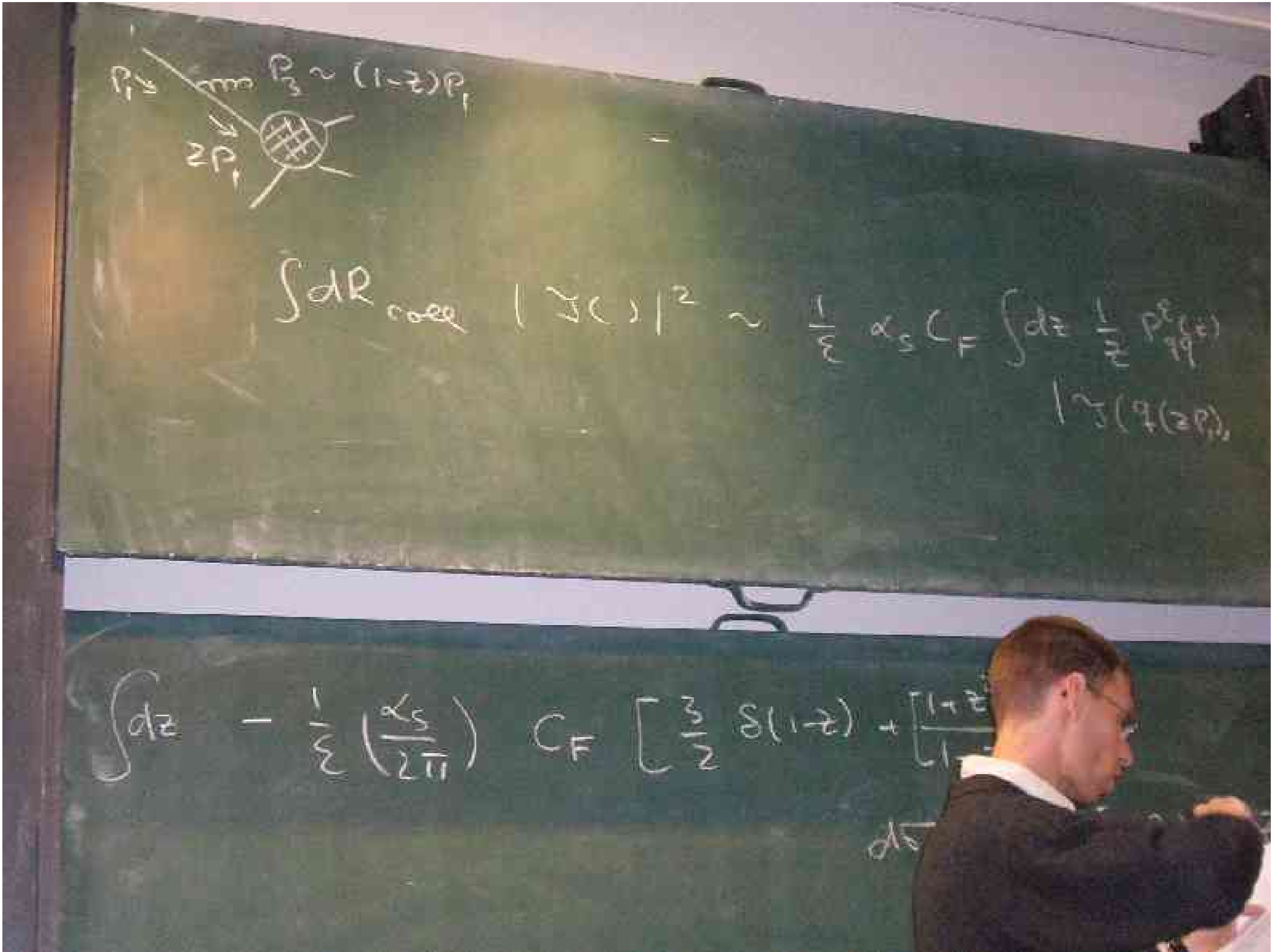
$$\times\, |\mathcal{M}(q(z p_1)\,\bar{q}(p_2) \to t(k_t) \ldots)|^2$$

$$|\mathcal{M}(q(p_1)\,\bar{q}(p_2) \to t(k_t)\,\bar{t}(k_{\bar{t}})\,g(p_3))|^2$$

$$\xrightarrow{p_1 \| p_3} \frac{1}{s}\,\frac{1}{z}\,\frac{1+z^2-\epsilon(1-z)^2}{1-z}\,\alpha_s C_F$$

$2\,p_1\cdot p_3$

$$\times\,|\mathcal{M}(q(zp_1)\,\bar{q}(p_2) \to t(k_t)\dots)|^2$$



p1
p3 ~ (1-z)p1
zp1
∫dR coll |J( )|² ~ 1/ε αs CF ∫dz 1/z Pε qq(z)
|J(q(zp1),
∫dz − 1/ε (αs/2π) CF [3/2 δ(1−z) +

$$\int dz \quad - \frac{1}{\epsilon} \left(\frac{\alpha_s}{2\pi}\right) \; C_F \left[\frac{3}{2}\, \delta(1-z) + \left[\frac{1+z^2}{1-z}\right]_+ \right]$$

$$d\hat{\sigma}\left(q(zP_1)\, \bar{q}(P_2) \rightarrow \ell \bar{\ell} \right)$$

$$\int dx \; F(x) \left(d\sigma_{LO} + d\sigma_{NLO} \right)$$

$$= \int dx \; F^{B}(x) \Big(d\sigma_{LO} + d\sigma_{NLO}$$

$$+ c_\varepsilon \left(\frac{\mu^2}{\mu_F^2}\right)^{\varepsilon} \frac{1}{\varepsilon} \frac{\alpha_s}{2\pi} \int dz \, P(z) \, d\sigma_{LO}(z \cdot x \cdot \ldots)$$

$$- c_\varepsilon \left(\frac{\mu^2}{\mu_F^2}\right)^{\varepsilon} \;\; ''$$

$$F_q^{R}(y, \mu_F) = F_q^{B}(y) + \delta F_q^{B}(y)$$

$$= F_q^{B}(y) - c_\varepsilon \frac{1}{\varepsilon} \left(\frac{\mu^2}{\mu_F^2}\right)^{-\varepsilon} \frac{\alpha_s}{2\pi} \int_y^1 dx \, \frac{1}{x} P_{qq}\left(\frac{y}{x}\right) F_q^{B}(\ldots$$

$$\Rightarrow \mu_F^2 \frac{\partial F_q^R}{\partial \mu_F^2} = \left(\frac{\alpha_s}{2\pi}\right) \int_y^1 dx \, \frac{1}{x} P_{qq}\left(\frac{y}{x}\right) F_q^R(x\ldots$$

$$F_q^R(y,\mu_F) = F_q^B(y) + \delta F_q^B(y)$$

$$= F_q^B(y) - C_\varepsilon \frac{1}{\varepsilon}\left(\frac{\mu^2}{\mu_F^2}\right)^{-\varepsilon} \frac{\alpha_S}{2\pi}\int_y^1 dx\, \frac{1}{x}\, P_{qq}\left(\frac{y}{x}\right) \cdot F_q^B(x)$$

$$\Rightarrow \mu_F^2 \frac{\partial F_q^R}{\partial \mu_F^2} = \left(\frac{\alpha_S}{2\pi}\right)\int_y^1 dx\, \frac{1}{x}\, P_{qq}\left(\frac{y}{x}\right) F_q^R(x)$$