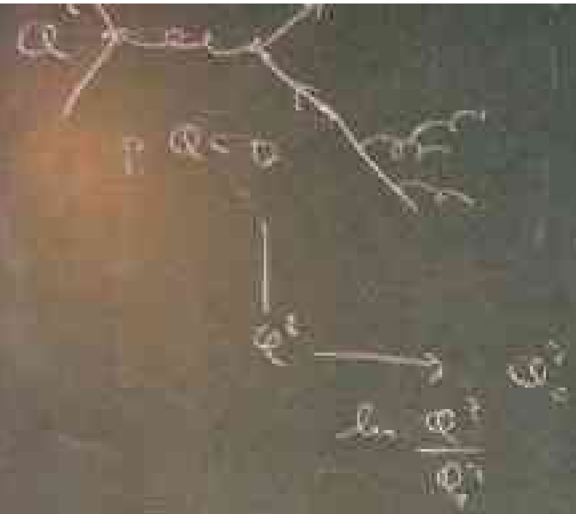


$$d\sigma_{n+1} = d\sigma_n \, \frac{\alpha_s}{2\pi} \, P(z)\, dz \, \frac{dk_\perp^2}{k_\perp^2}$$

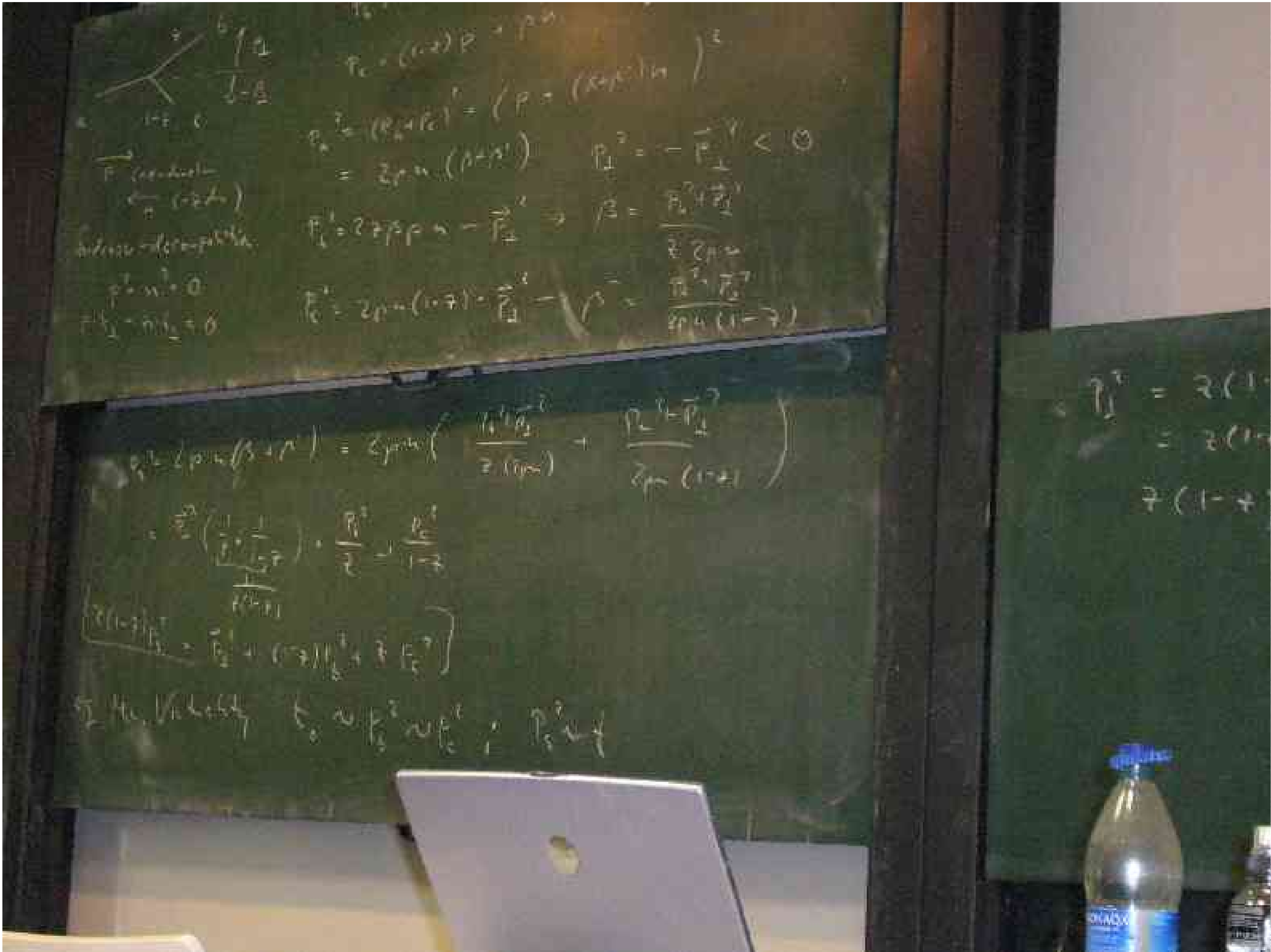
(Kollineare Faktorisation → Ell. lecture)

$$P(z) = C_F \, \frac{1+z^2}{1-z} \quad \xrightarrow{z \to 1} \quad \sim C_F \, \frac{1}{1-z} \qquad = P(z)$$

enhancement 1   long $\left(\alpha_s \ln \frac{Q^2}{Q_1^2}\right)^n$

$$t \frac{\partial}{\partial t} f_i(x,t) = \sum_j \int_x \frac{dz}{z} \, \frac{\alpha_s}{2\pi} \, P_{ij}(z) \, f_j\!\left(\frac{x}{z}, t\right)$$

Want to „undo" $\left.\frac{1+z^2}{1-z}\right|_+$ $|_+$-prescription and obtain a physical resolution → z-limit



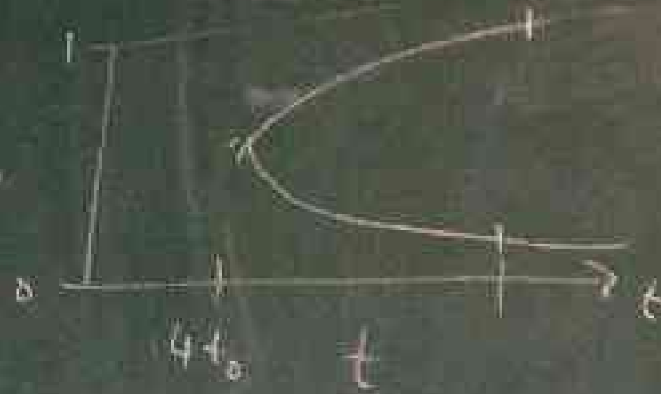
$$= z(1-z)t - (1-z)t_0 - z t_0$$

$$= z(1-z)t - t_0 \geq 0$$

$$z(1-z) \geq \frac{t_0}{t}$$

$\Rightarrow$ z-limits

$$= \frac{1}{2} \pm \frac{1}{2}\sqrt{1 - \frac{4t_0}{t}}$$



limits on z-integration $z_- < z < z_+$

$$\sqrt{t} \geq \sqrt{t_0} + \sqrt{t_0}$$

$$t\frac{\partial}{\partial t} D(x,t) = \int_x^1 \frac{dz}{z}\frac{\alpha_s}{2\pi} P(z)\, D\left(\frac{x}{z},t\right)$$

1 - prescription $f_+(z) = f(z) - \delta(1-z)\int_0^1 dy\, f(y)$

$P(z) \to \hat{P}(z)$ unregularized splitting function

$$\text{rhs} = \left(\int_{z_-}^{1} \frac{dz}{z}\frac{\alpha_s}{2\pi}\hat{P}(z)\, D\left(\frac{x}{z},t\right)\right) - \int dz\, \frac{\alpha_s}{2\pi}\hat{P}(z)\, D(x,t)$$

$$= \int_{z_-}^{z_+} dz\, \frac{\alpha_s}{2\pi}\hat{P}(z)\left(\frac{1}{z}D\left(\frac{x}{z},t\right) - D(x,t)\right)$$

$$\Delta(t) = \exp - \int_{t_0}^{t}\frac{dt'}{t'}\int_{z_-}^{z_+} dz\, \frac{\alpha_s(z,t)}{2\pi}\hat{P}(z)$$

usually $z_\pm = z_\pm(z,t)$

$$t\frac{\partial}{\partial t}\Delta(t) = -\cancel{t}\cdot\frac{1}{t}\left(\int_{z_-}^{z_+} dz\, \frac{\alpha_s}{2\pi}\hat{P}(z)\right)\Delta(t)$$

lhs of DGLAP $\left(t\frac{\partial}{\partial t}D(x,t) - \frac{1}{\Delta(t)}\, t\frac{\partial}{\partial t}\Delta(t)\, D(x,t)\right)\frac{1}{\Delta(t)}$

$$= \frac{1}{\Delta(t)}\int_{z_-}^{z_+} \frac{\alpha_s}{2\pi}\hat{P}(z)\,\frac{1}{z}D\left(\frac{x}{z},t\right)$$

$$t\left(\frac{1}{\Delta}\frac{\partial}{\partial t}D - \frac{1}{\Delta^2}\left(\frac{\partial}{\partial t}\Delta\right)D\right) = t\left(\frac{\partial}{\partial t}\frac{D}{\Delta}\right)$$

Alternative form of DGLAP equation

$$t\frac{\partial}{\partial t}\frac{D(x,t)}{\Delta(t)} = \int_{z_-}^{z_+}\frac{dz}{z}\frac{\alpha_s(z,t)}{2\pi}\hat{P}(z)\frac{D\left(\frac{x}{z},t\right)}{\Delta(t)}$$

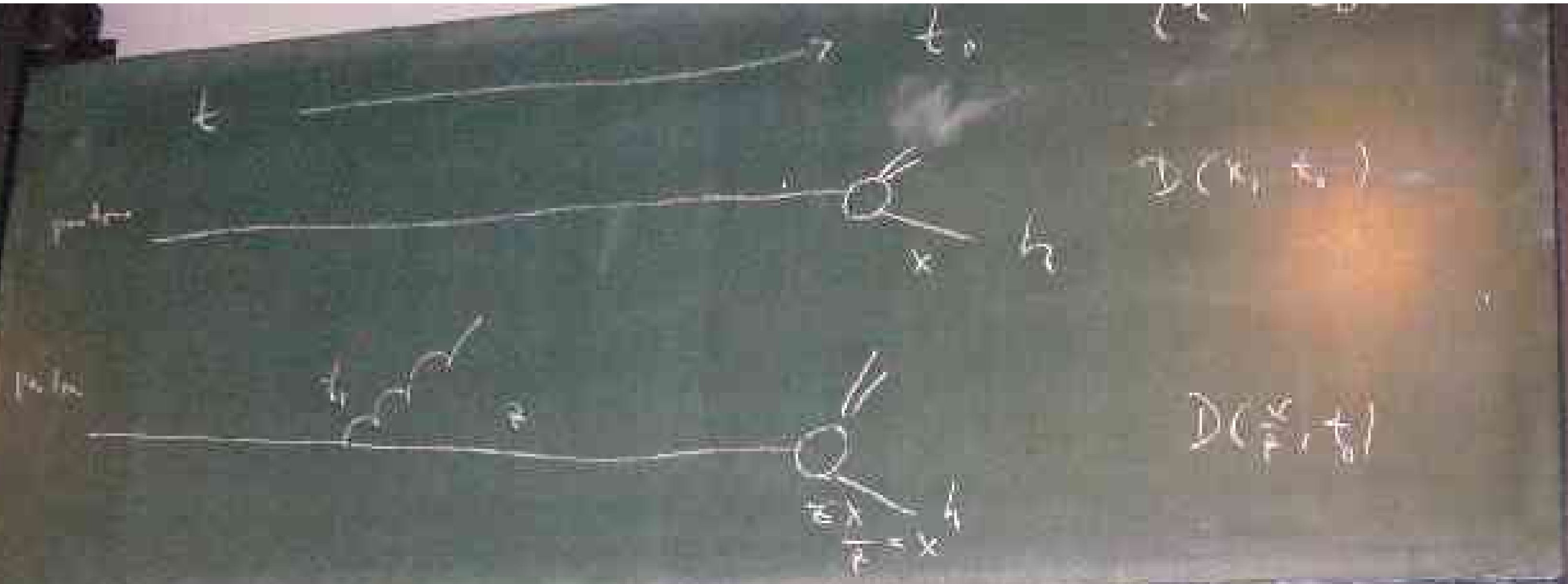
$t \to t'$, integrate over t', $t_0 < t' < t$

$$\frac{D(x,t)}{\Delta(t)} - \frac{D(x,t_0)}{\Delta(t_0)} = \int_{t_0}^{t}\frac{dt'}{t'}\int_{z_-}^{z_+}\frac{dz}{z}\frac{\alpha_s}{2\pi}\hat{P}(z)\frac{D\left(\frac{x}{z},t'\right)}{\Delta(t')}$$

$$D(x,t) = \frac{\Delta(t)}{\Delta(t_0)}D(x,t_0) + \int_{t_0}^{t}\frac{dt'}{t'}\int_{z_-}^{z_+}\frac{dz}{z}\frac{\alpha_s}{2\pi}\hat{P}(z)\frac{D\left(\frac{x}{z},t\right)}{\Delta(t)}$$

$$D_{(0)}(x,t) = \frac{\Delta(t)}{\Delta(t_0)}D(x,t_0)$$

$$D_{(1)}(x,t) = \frac{\Delta(t)}{\Delta(t_0)} + \int_{t_0}^{t}\frac{dt_1}{t_1}\int_{z_-}^{z_+}\frac{dz_1}{z_1}\frac{\Delta(t)}{\Delta(t_1)}\frac{\alpha_s}{2\pi}\hat{P}(z)\frac{\Delta(t_1)}{\Delta(t_0)}D\left(\frac{x}{z},t_0\right)$$



$\Rightarrow \dfrac{\Delta(t)}{\Delta(t_0)}$ = Sudakov Form Factor

= Probability (evolving NP obj D from $t > t_0 \to t_0$
without emission)

$$\frac{\Delta(t)}{\Delta(t_0)} = S(t) = P(\text{no branching from } t \to t_0)$$

$$1 - S(t) = P(\text{any \# of branchings while evolving from } t \to t_0)$$

$$\left(-\frac{dS(t)}{dt}\right) dt = P(\text{having next branching at } t)$$

Sample (t, z) (with random azimuth φ) $\to$ final splitting

$$\Delta(t) = \exp - \int \frac{dt'}{t'} \underbrace{\int_{z_0}^{z_1} dz \frac{\alpha_s}{2\pi} \hat{P}(z)}_{t\text{-density}}$$

Collinear factorization

$$d\sigma_{n+1} \sim d\sigma_n \, d\Phi_{\dots} P_{\dots}$$

$$|M_{n+1}|^2 d\Phi_{n+1} \to |M_n|^2 d\Phi_n \, d\Phi_1 P_{\dots}$$

Soft factorization

(Eik. limit)

$$\frac{\varepsilon \cdot q}{p \cdot q}$$

$$|M_{n+1}|^2 = \left| M_n \frac{\varepsilon \cdot q}{p \cdot q} \, d\Phi \right|^2$$

$\Rightarrow$ coherence :