

TMDs from MC evolution

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REF-Workshop: Resummation, Evolution, Factorization



Introduction: uPDFevolv code

- Sudakov formalism

- MC solution of the evolution equation

- uPDFevolv code

Results for integrated TMDs

- Initial k_T dependence

- Ordering dependence

Results for unintegrated TMDs

- Ordering dependence

Summary

Section 1

Introduction: uPDFevolv code

Sudakov formalism

Evolution equation for parton density

$$t \frac{\partial f(x,t)}{\partial t} = \frac{\alpha_s}{2\pi} \int \frac{dz}{z} P(z) f\left(\frac{x}{z}, t\right) - \frac{\alpha_s}{2\pi} f(x,t) \int dz P(z). \quad (1)$$

Introducing *Sudakov form factor*

$$\Delta_s(t, t_0) \equiv \Delta_s(t) = \exp\left(-\int_x^{z_{max}} dz \int_{t_0}^t \frac{\alpha_s}{2\pi} \frac{dt'}{t'} P(z)\right) \quad (2)$$

we can rewrite (1)

$$t \frac{\partial f(x,t)}{\partial t} = \frac{\alpha_s}{2\pi} \int \frac{dz}{z} P(z) f\left(\frac{x}{z}, t\right) + f(x,t) \frac{t}{\Delta_s(t)} \frac{\partial \Delta_s(t)}{\partial t}. \quad (3)$$

After integration

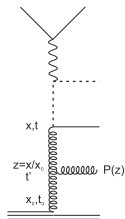
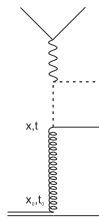
$$f(x,t) = f(x, t_0) \Delta_s(t) + \frac{\alpha_s}{2\pi} \int \frac{dt'}{t'} \frac{\Delta_s(t)}{\Delta_s(t')} \int \frac{dz}{z} P(z) f\left(\frac{x}{z}, t'\right). \quad (4)$$

Sudakov: probability of evolving from t_0 to t without any resolvable branching.

iterative solution:

$$f(x,t) = \lim_{n \rightarrow \infty} f_n(x,t) = \lim_{n \rightarrow \infty} \sum_n \frac{1}{n!} \log^n\left(\frac{t}{t_0}\right) A^n \otimes \Delta_s(t) f\left(\frac{x}{z}, t_0\right), \quad (5)$$

where $A = \frac{\alpha_s}{2\pi} \int \frac{dz}{z} P(z)$.



MC solution of the evolution equation

MC solution:

First branching: evolve from t_0 to t' obtained from $\Delta_s(t')$.

$$R_1 = \Delta_s(t'), \quad (6)$$

where R_1 is a random number in the interval $(0, 1)$.

If $t' > t$ evolution is stopped without any branching. If $t' < t$ branching is generated according to $P(z)$

$$\int_{z_{min}}^z dz' P(z') = R_2 \int_{z_{min}}^{z_{max}} dz' P(z') \quad (7)$$

and evolution continue.

Second branching: evolve from t' to t'' generated according to $\Delta_s(t'', t')$. If $t'' > t$ evolution is stopped only with one branching. If $t'' < t$ branching is generated according to $P(z)$ and evolution continue...etc.

Observation:

$$\frac{\partial}{\partial t'} \frac{\Delta_s(t)}{\Delta_s(t')} = \frac{\alpha_s}{2\pi} \frac{\Delta_s(t)}{\Delta_s(t')} \frac{1}{t'} \int_x^{z_{max}} dz P(z) \quad (8)$$

rewrite (4)

$$f(x, t) = f_0(x, t) + \int_x^1 \frac{dz'}{z'} \int_{t_0}^t d\Delta_s(t, t') P(z') f_0\left(\frac{x}{z'}, t'\right) \left(\int_x^{z_{max}} dz P(z)\right)^{-1} \quad (9)$$

uPDFevolv code

uPDFevolv is an evolution code based on SMALLX (G.Marchesini, B.Webber, 1991) for TMD parton densities.

▶ [Link to the TMDlib and TMDPlotter webpage](#)

Many options possible: DGLAP, CCFM, $f(x, t)$, $xf(x, t)$, 1 or 2 loop α_s , saturation, initial state branching ...

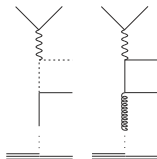
In this presentation: results from further developed updfevolv (including gluons , valence and sea quarks) for

- ▶ $xf(x, t)$,
- ▶ DGLAP evolution,
- ▶ LO in $P(z)$,
- ▶ 1-loop- α_s

compared to HERAPDF LO 1.5.

evolution in the code

We consider ep collisions in which we can measure different pdfs:



Final parton is not specified when the evolution begins.

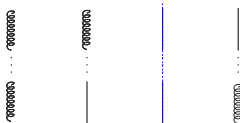
Two different evolution kernels are defined:

- ▶ initial quark (valence or sea) → quark grid,
- ▶ initial gluon → gluon grid.

Four different situations:

gluon at the beginning and at the end, gluon at the beginning and sea quark at the end, quark (valence or sea) at the beginning and gluon at the end and quark (valence or sea) at the beginning and quark (valence or sea) at the end. Valence quark at the end can come only from valence quark at the beginning.

Different kind of splittings can happen during the evolution process:

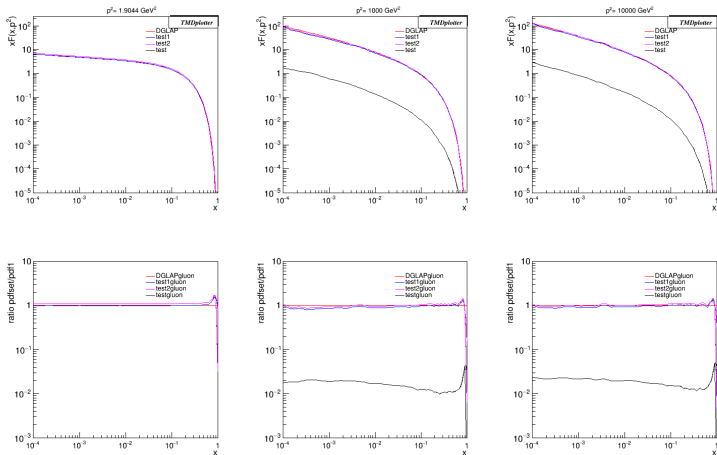


Kernels for evolution initiated by gluons and quarks are calculated separately and combined at the end.
 To get the final pdf: evolution kernel is folded with starting distribution

$$xf(x, t) = x \int dx_0 \int dz f_0(x_0) K(z, t) \delta(zx_0 - x)$$

Contribution from quark and gluon evolution

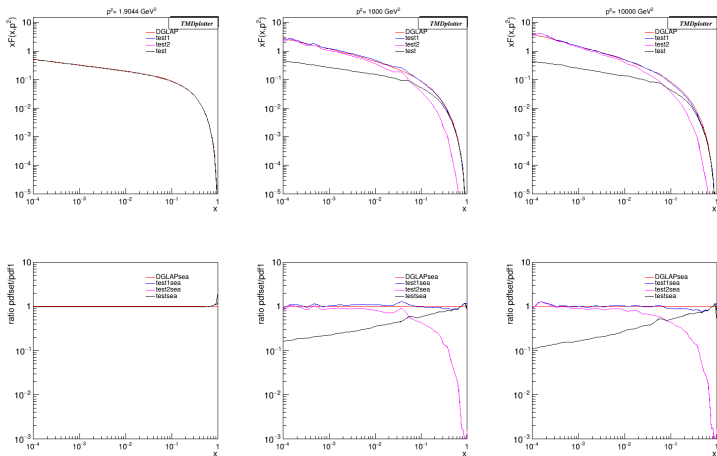
GLUON density: test1: merged quark and gluon kernel test2: gluon kernel test: quark kernel HERAPDF LO



Contribution from gluon kernel dominates, contribution from quark kernel (10^{-2}) times smaller.

Contribution from quark and gluon evolution

SEA density: test1: merged quark and gluon kernel test2: gluon kernel test: quark kernel HERAPDF LO



At small x main contribution to sea quark density from gluon kernel, for large x quark kernel dominates.

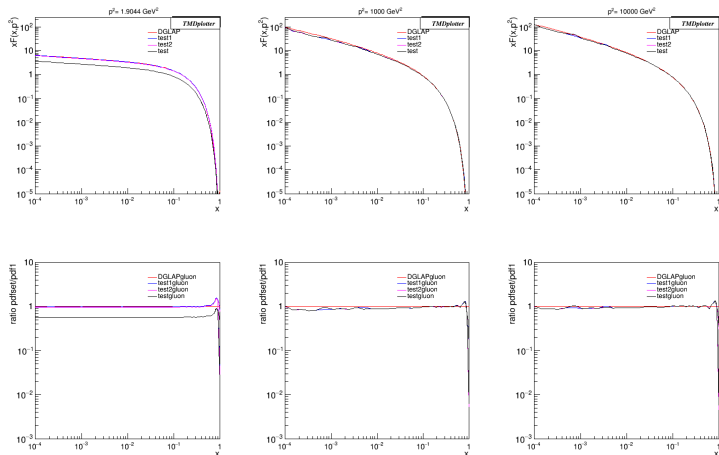
Section 2

Results for integrated TMDs

Initial k_T dependence: GLUON

Initial partons' k_T generated from gauss distribution with given σ :

test1: $\sigma = \frac{0.38}{\sqrt{2}}$ test2: $\sigma = \frac{0.75}{\sqrt{2}}$ test: $\sigma = \frac{1.5}{\sqrt{2}}$ HERAPDF LO

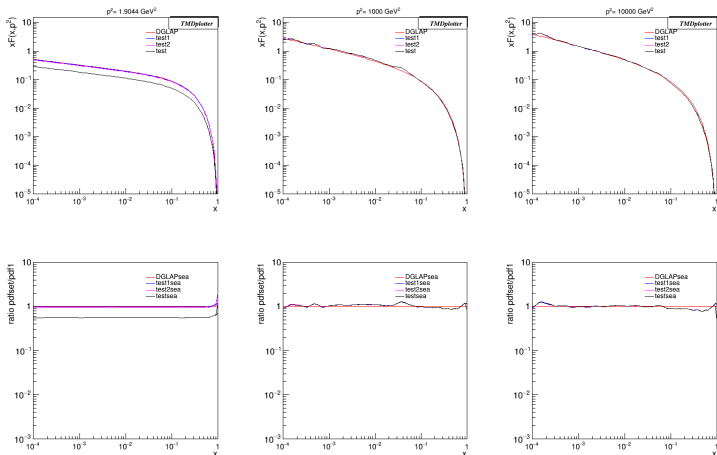


As long as the initial k_T is chosen to be $\ll p^2$ different choices of k_T give the same result.

Initial k_T dependence: SEA

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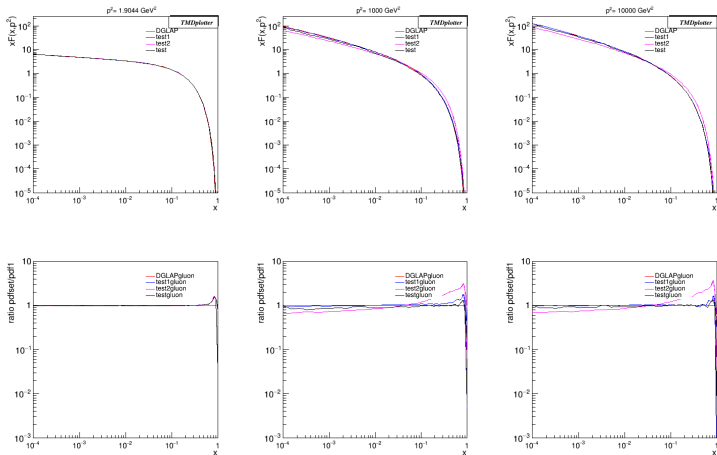


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Ordering dependence: GLUON

Possible choices of z_{max}

test1: angular ordering test2: Q ordering test: $z_{max} = 0.99$ HERAPDF LO

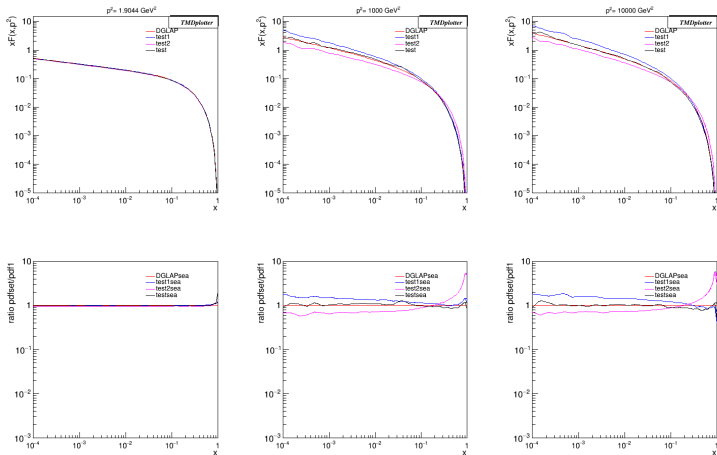


Different choices of z_{max} give different results

Ordering dependence: SEA

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Different choices of z_{max} give different results

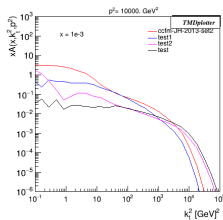
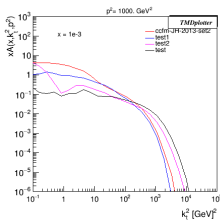
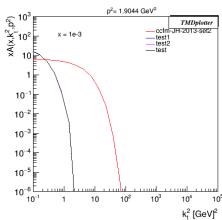
Section 3

Results for unintegrated TMDs

Ordering dependence: GLUON

Possible choices of z_{max}

test1: angular ordering test2: Q ordering test: $z_{max} = 0.99$ ccfm JH 2013 set2

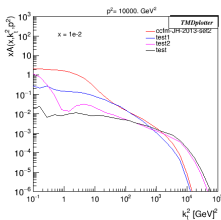
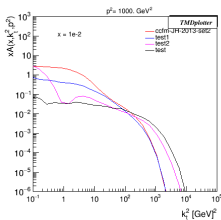
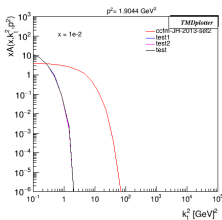


- ▶ Initial scale: intrinsic k_t distribution (different in ccfm set and uPDFevolv code).
- ▶ Drop for $k_T^2 \sim 2\text{GeV}^2$ - effect of matching intrinsic k_T with evolution.
- ▶ Different choices of z_{max} lead to different uTMDs, especially different large k_T tails .

Ordering dependence: GLUON

Possible choices of z_{max}

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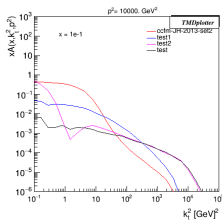
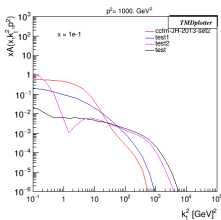
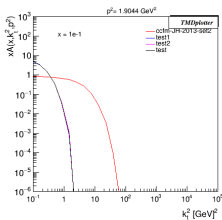


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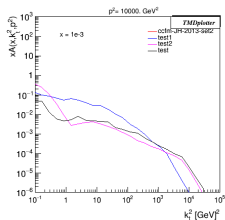
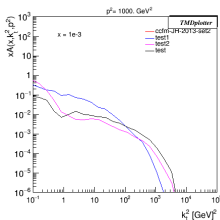
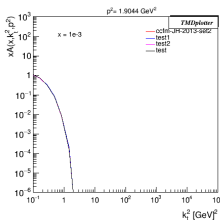


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Ordering dependence: SEA

Possible choices of z_{max}

test1: angular ordering test2: Q ordering test: $z_{max} = 0.99$

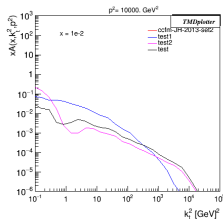
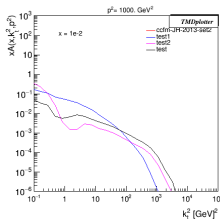
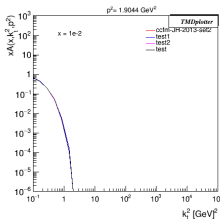


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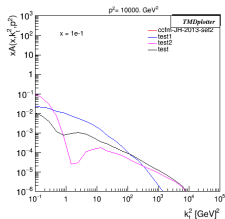
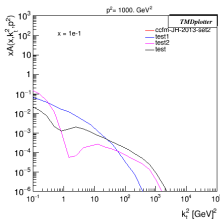
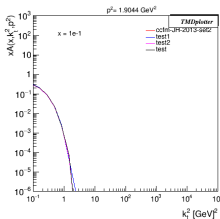


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Section 4

Summary

Summary:

Results for $xf(x, t)$ from uPDFevolve code based on DGLAP evolution and Sudakov form factor formalism were shown.

UPDFevolve code

- ▶ evolves TMD pdfs including all flavours,
- ▶ reproduce analytical solution (results consistent with HERAPDF LO).
- ▶ Effects on ordering observed (z_{max} choice).

Prospects:

- ▶ possibilities to study k_T effects from intrinsic k_T and from evolution,
- ▶ include NLO in $P(z)$,
- ▶ extension to full CCFM (small x),
- ▶ implementation into Herafitter.
- ▶ TMD MC in Cascade ?

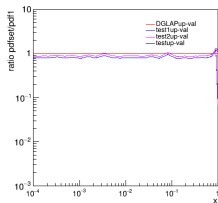
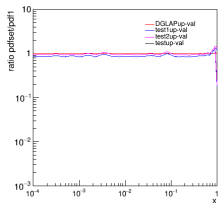
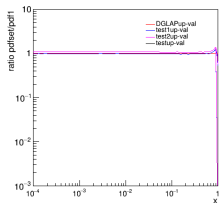
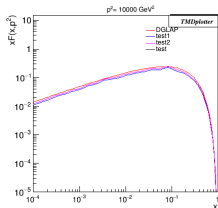
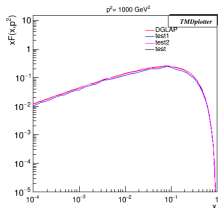
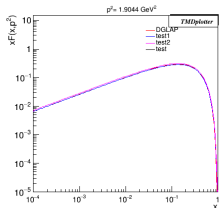
Thank you!

Section 5

Back up

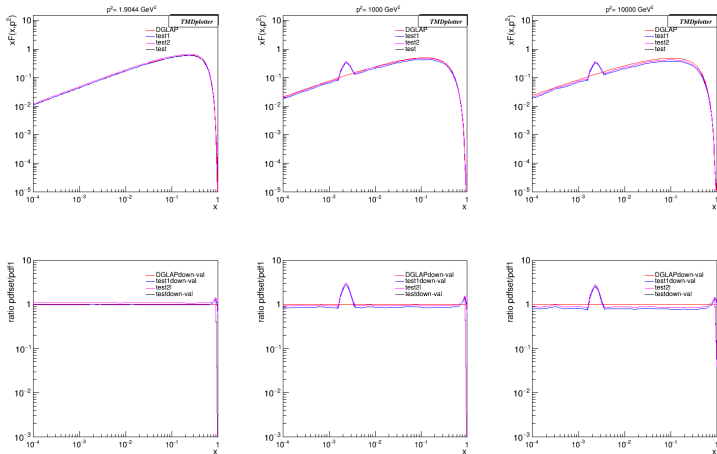
Contribution from quark and gluon evolution

UP-VAL density: test1: merged quark and gluon kernel test2: gluon kernel test: quark kernel HERAPDF LO

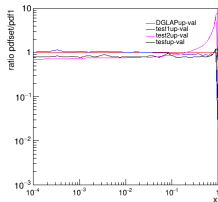
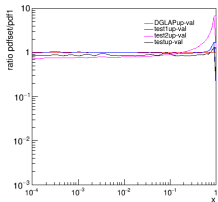
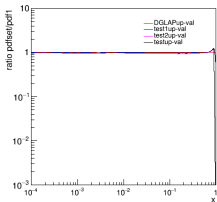
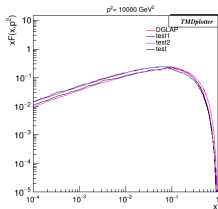
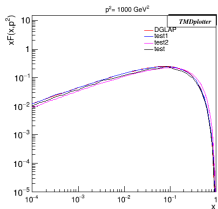
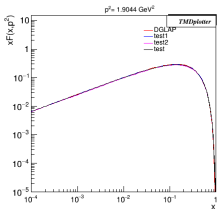


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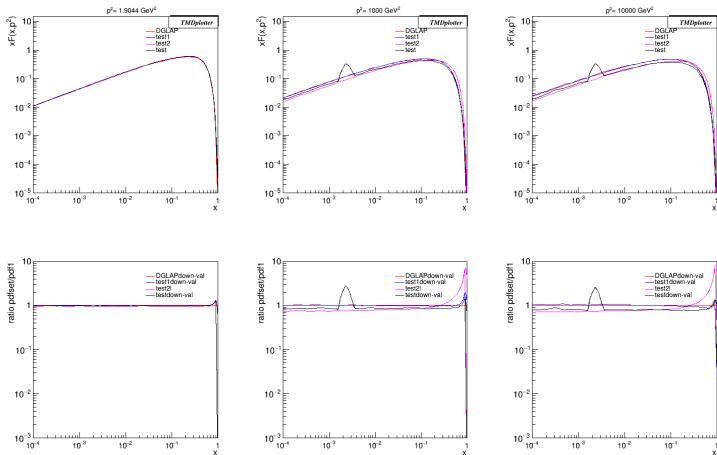
DOWN-VAL density: test1: merged quark and gluon kernel test2: gluon kernel test: quark kernel HERAPDF LO



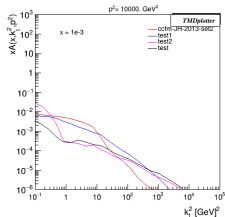
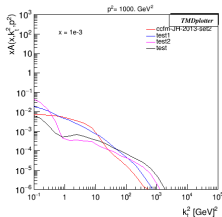
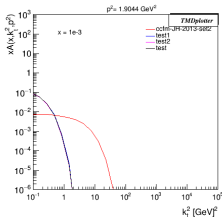
Ordering dependence: UP-VAL quarks

Possible choices of z_{max} test1: angular ordering test2: Q ordering test: $z_{max} = 0.99$ HERAPDF LO

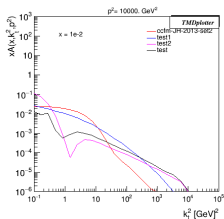
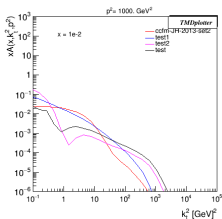
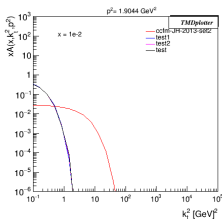
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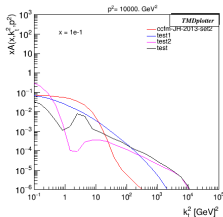
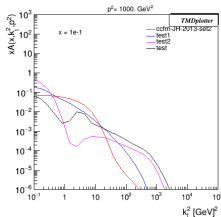
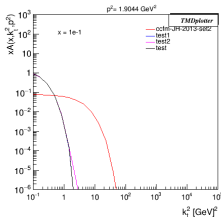
Ordering dependence: up-val

Possible choices of z_{max} test1: angular ordering test2: Q ordering test: $z_{max} = 0.99$ ccfm-JH-2013-set2

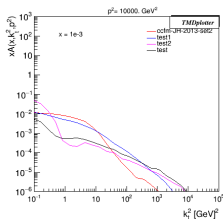
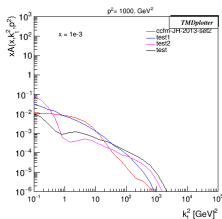
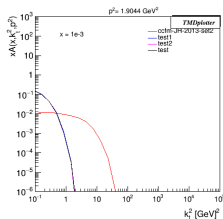
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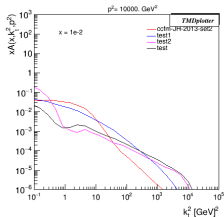
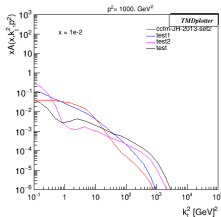
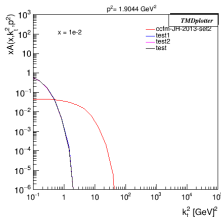
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Ordering dependence: down-val

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Ordering dependence: down-val

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