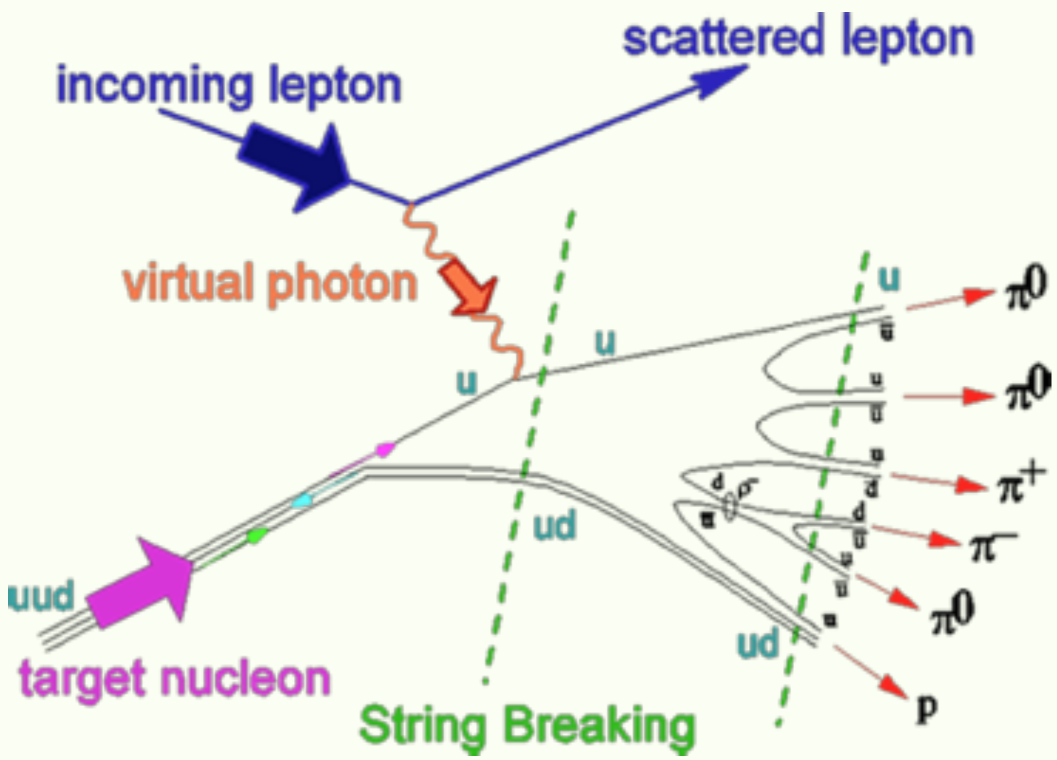
Study of TMD evolution in SIDIS at moderate *Q*



REF 2015 (Resummation, Evolution, Factorization)

Nov. 3rd, 2015

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Based on Phys. Rev. D 89 (2014) C. Aidala, . B. Field, LG, T. Rogers

Comments

- Collins-Soper evol. kernel has perturbative-short distance & non-perturbative (**NP**) large-distance content
- Non-pertb. large-distance is ***strongly universal*** -many interesting predictions
- Universal character can exploited in observables "Bessel Weighting" another time and place
  (Boer Gamberg, Musch Prokudin JHEP 2011, Aghasyan, Avakian, Gamberg, Prokudin, Rossi et al 2014)
- Global fits, based on larger Q Drell-Yan–data/processes find substantial contributions from nonperturbative regions in the Collins-Soper evolution kernel-e.g. BNLY PRD 67(2003) & Konychev Nadolsky PLB 2005
- Recent demonstrations that applying larger Q DY fits result in very rapid evolution for SIDIS data which are "HERMES/COMPASS/JLAB like"
- We investigate SIDIS measurements in the region of a few GeV, where sensitivity to *NP* transverse momentum dependence is more important or even dominate the evolution
- Performed a study that isolates/places bounds on it/we quantify it s.t. both high-energy DY fits as well respects the lower energy experiments

Outline

- Review of elements TMD factorization in QCD... in particular strong universal factor from *NP* content of CSS evolution kernel
- Study of evolution transverse momentum broadening SIDIS and role of universal role of *NP* content of evolution kernel
- Prediction of strong universality of TMD factorization
- Implications for TMDs & Nucleon Structure
- **Role of** *Y*- Term matching of low and high q_T behavior of cross section @ moderate *Q*--new work to appear
- Some conclusions ... work

TMD factorized cross section

$$\frac{d\sigma}{dP_T^2} \propto \sum_{jj'} \mathcal{H}_{jj',\,\text{SIDIS}}(\alpha_s(\mu),\mu/Q) \int d^2\boldsymbol{b}_T e^{i\boldsymbol{b}_T\cdot\boldsymbol{P}_T}\, \tilde{F}_{H_1}(x,b_*;\mu_b,\mu_b^2)\tilde{D}_{H_2}(z,b_*;\mu_b,\mu_b^2)$$

$$\exp\left\{ -g_{\text{PDF}}(x,b_T;b_{\max}) - g_{\text{FF}}(z,b_T;b_{\max}) - 2g_K(b_T;b_{\max})\ln\left(\frac{Q}{Q_0}\right)\right.$$

$$\left. + 2\ln\left(\frac{Q}{\mu_b}\right)\tilde{K}(b_*;\mu_b) + \int_{\mu_b}^{Q}\frac{d\mu'}{\mu'}\left[\gamma_{\text{PDF}}(\alpha_s(\mu');1) + \gamma_{\text{FF}}(\alpha_s(\mu');1) - 2\ln\left(\frac{Q}{\mu'}\right)\gamma_K(\alpha_s(\mu'))\right]\right\}$$

$$+\, Y_{\text{SIDIS}} \cdot \;+\; P.S.C$$

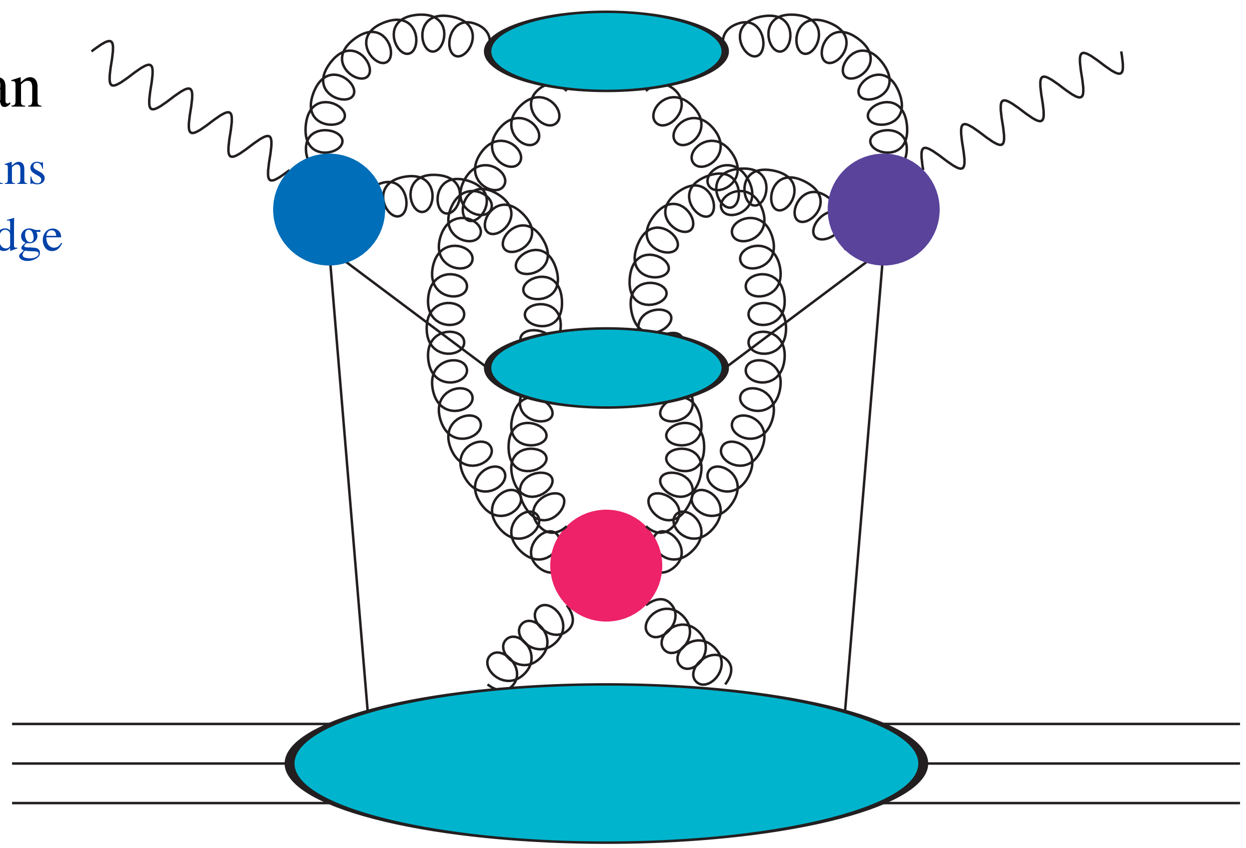
Collins 2011 (Cambridge Univ. Press)

Elements of TMD Fact. Cross section

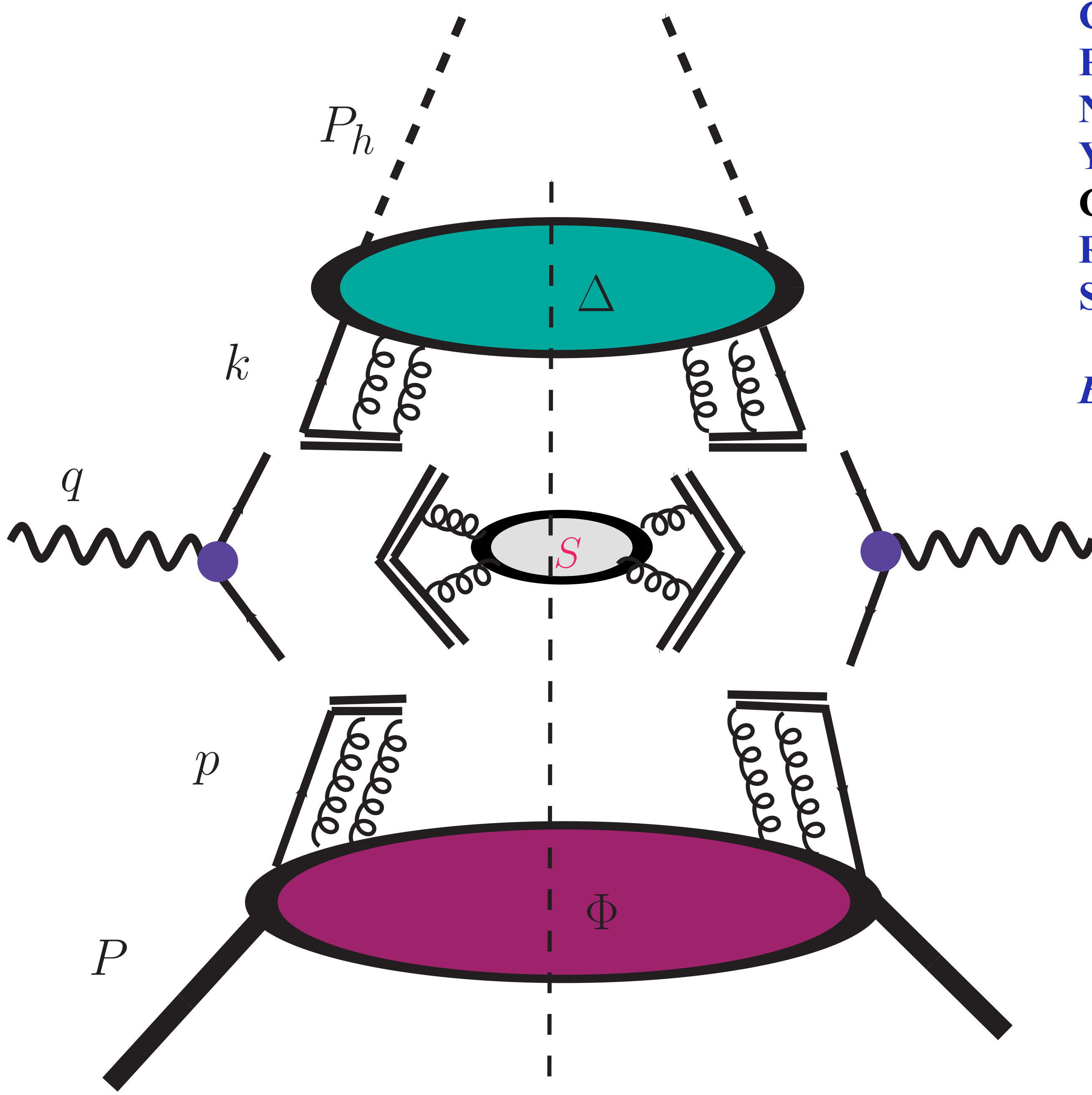
- H hard scattering part
- F & D are TMD PDFs dominates when $P_T \sim k_T << Q$
- Y term serves to correct expression for structure function when $P_T \sim Q$
- Exponent contains both perturbative and non-perturbative content arising from TMD factorization ⟷ evolution
- Where does this structure come from ... of course this is based upon earlier CS 81 & CSS 85 formalism but new treatment of soft factor and CSS equations effectively implements "resummation" of large logs.

TMD Factorization Procedure while maintaining a Parton Model picture: CSS + JC 2011 To study nucleon structure

- Leading Regions-power counting Libby Sterman PRD 1978 (see Collins PRD 1980 nongauge theories, Collins Soperp NPB& CSS formalism 1982-85... Collins 2011 Cambridge Univ. Press)
- "Reduced Diagrams"
- Apply Ward Identities get factorized form
  - Soft Factor w/ gauge links
  - TMDs w/ gauge links
  - Hard contribution



TMD factorization

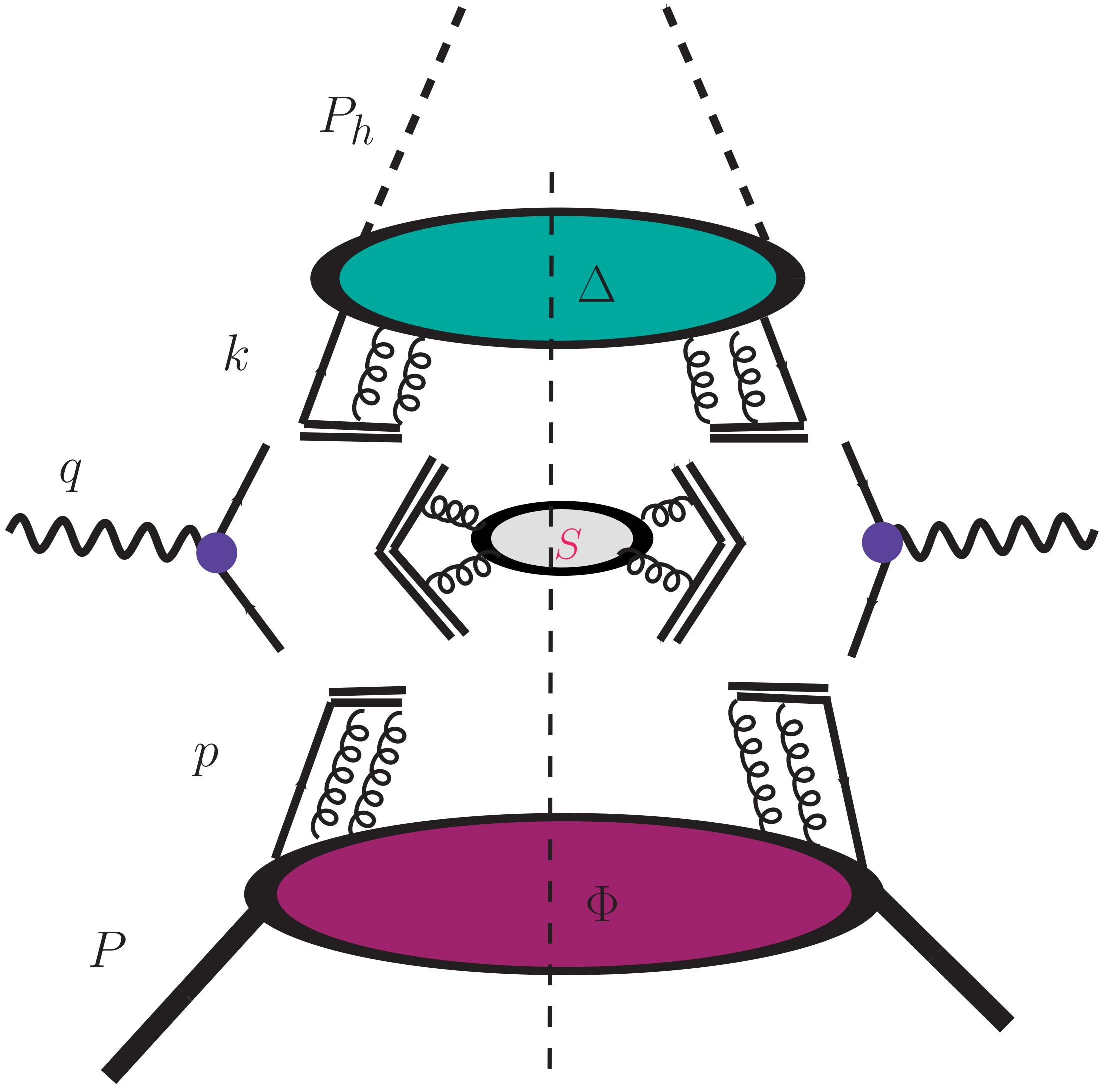


Collins Soper NPB 1981,1982, CSS NPB 1985, Collins, Hautman PLB 00, Collins Metz PRL 2004, Collins Oxford Press 2011, Boer NPB 2001, 2009,2013, Ji, Ma, Yuan PLB 2004, PRD 2005, Ibildi, Ji, Yuan PRD 2004, Cherednikov, Karanikas, Stefanis NPB 2010, Collins QFT 2011, Abyat, Rogers PRD 2011, Abyat, Collins, Qiu, Rogers PRD 2012, Collins Rogers 2013, Echevarria, Idilbi, Scimemi JHEP 2012

ETC ....

Elements of Factorization

- TMDs w/Gauge links: color invariant
- In addition Soft factor
- Hard factor



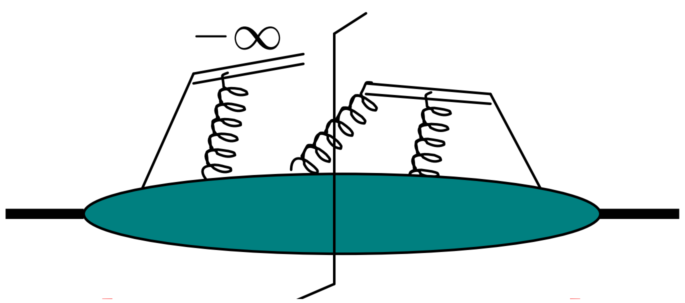
- TMD PDFs & Soft factor have rapidity/LC divergences
- Rapidity regulator introduced to regulate these divergences
- Treatment of LC/Rapidity divergences in TMD factorization

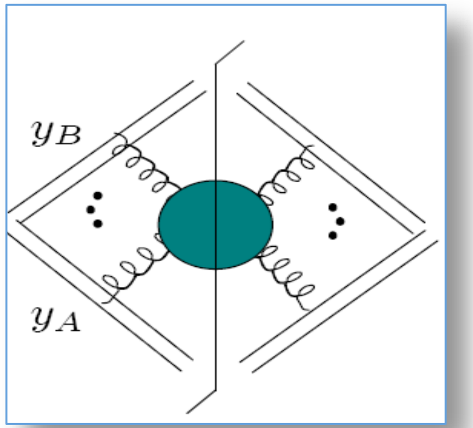
Collins Soper NPB 1981, Collins Hautmann 2000, Collins Metz PRL 2004,
Ji, Ma, Yuan PRD 2004, Collins 2011, Collins Rogers 2012

Further treatment achieve full factorization using Soft Factor in CSS

- Lightlike Wilson lines in TMDs $W(\infty, x; n) = P \exp\left[-ig_0 \int_0^\infty dsn \cdot A_0^a(x + sn)t^a\right].$
  - Infinite rapidity QCD radiation in the wrong direction.
  - In soft factor/fragmentation function too.

$$y_B = \frac{1}{2} \ln \left(\frac{n^+}{n^-} \right)$$





$$n^- = 0 \rightarrow$$

$$\lim y_B \rightarrow -\infty$$

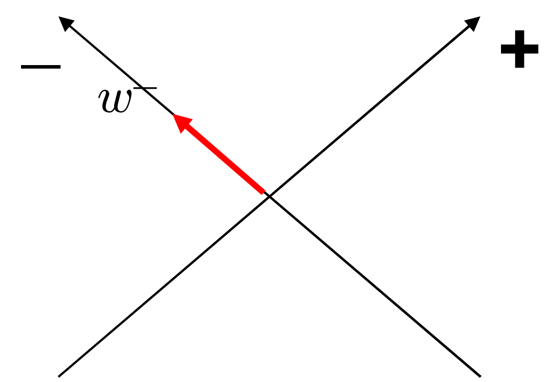
- Finite rapidity Wilson lines
  - Regulate rapidity of extra gluons.

$$n^- = (-e^{2y_B}, 1, \mathbf{0})$$

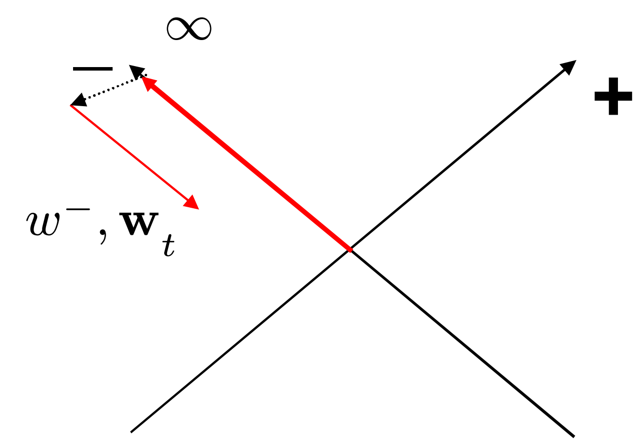
Introduces rapidity scale parameter

$$y = \frac{1}{2}\ln\left(\frac{n^+}{n^-}\right)$$

Paths of Wilson Lines in Coordinate Space



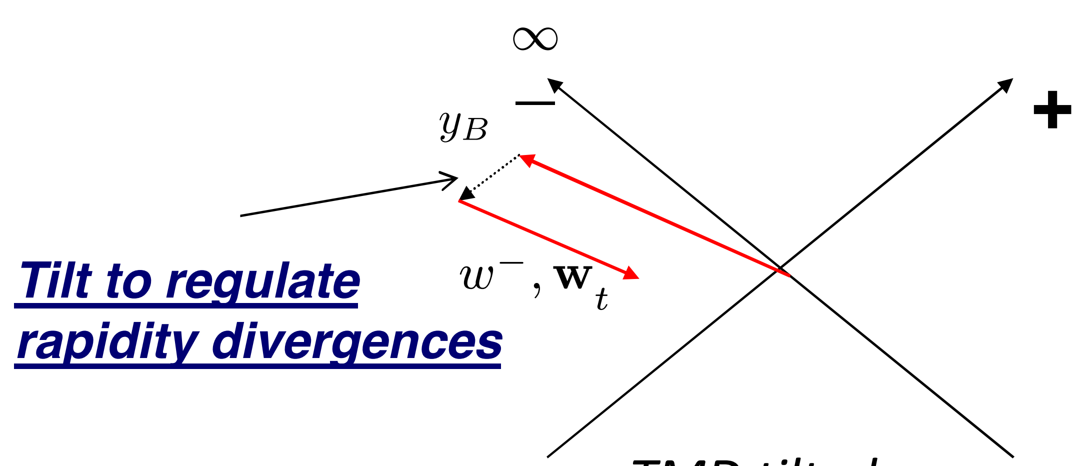
Standard Integrated



TMD first try

Regulate

$$n^- = (-e^{2y_B}, 1, \mathbf{0})$$



TMD tilted

$$\zeta_F = M_P^2 x^2 e^{2(y_P - y_s)} \quad \Longleftrightarrow \quad y_s$$

Emergence of Soft Factor in Cross section

$$d\sigma = |\mathcal{H}|^2 \frac{\tilde{F}_1^{\rm unsub}(y_1 - (-\infty)) \times \tilde{F}_2^{\rm unsub}(+\infty - y_2)}{\tilde{S}(+\infty, -\infty)}$$

Collins Act Pol. 2003
Ji Ma Yuan 2004, 2005

TMDs are still "entangled" not yet fully factorized

LC/rapidity divergences regulated

Use soft factor properties to fully factorize and perform evolution

Collins 2011 Cam. Univ. Press see also Aybat Rogers PRD 2011

Emergence of Soft Factor in TMDs

$$d\sigma = |\mathcal{H}|^2 \, \frac{\tilde{F}_1^{\rm unsub}(y_1 - (-\infty)) \times \tilde{F}_2^{\rm unsub}(+\infty - y_2)}{\tilde{S}(+\infty, -\infty)}$$

Soft factor further "repartitioned"
This is done to

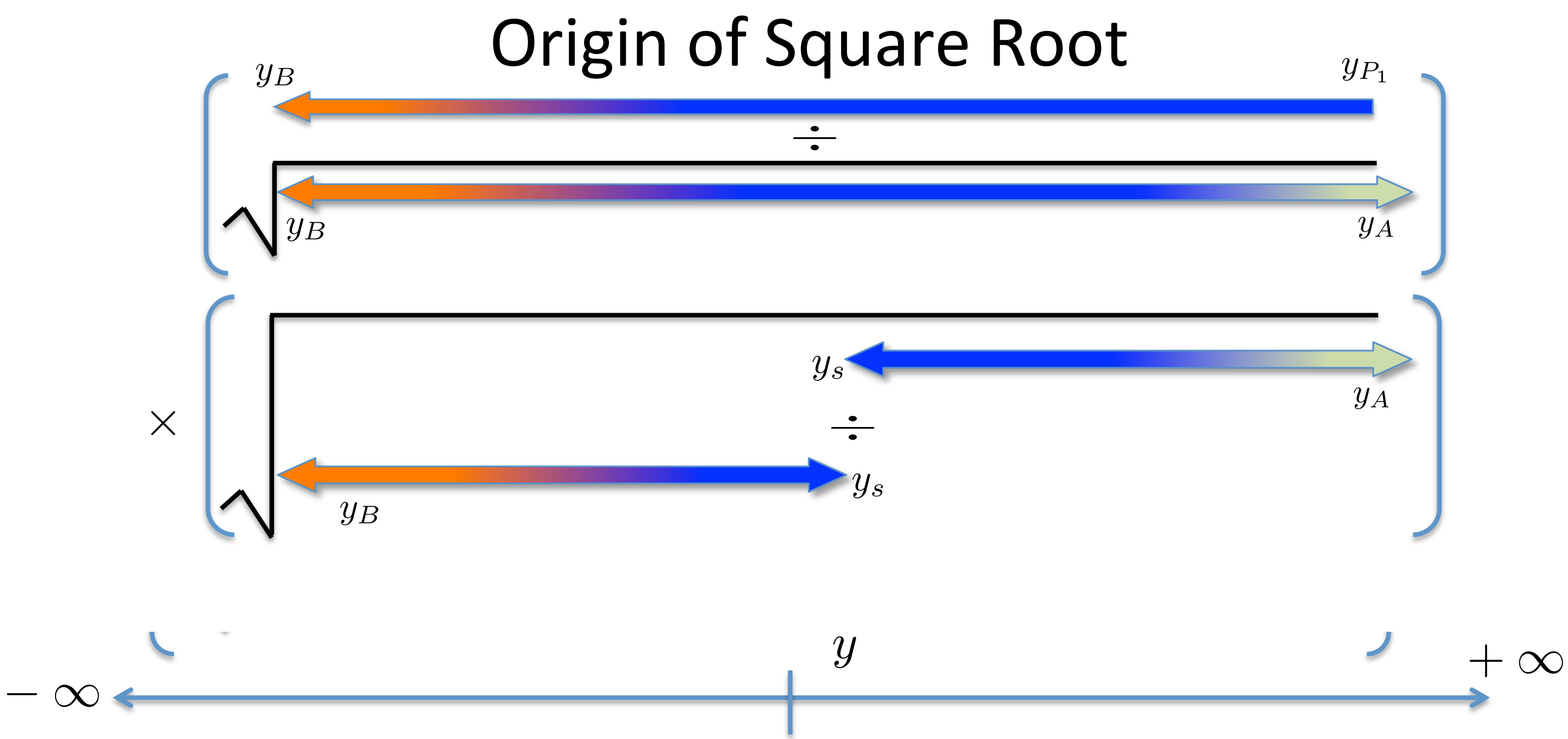
1) cancel LC divergences in "unsubtracted" TMDs
2) separate "right & left" movers i.e. full factorization
3) remove double counting of momentum regions

$$d\sigma = |\mathcal{H}|^2 \, \left\{ F_1^{\rm unsub}(y_1 - (-\infty)) \sqrt{\frac{\tilde{S}(+\infty, y_s)}{\tilde{S}(+\infty,-\infty)\tilde{S}(y_s,-\infty)}} \right\} \times \left\{ \tilde{F}_2^{\rm unsub}(+\infty - y_2) \sqrt{\frac{\tilde{S}(y_s,-\infty)}{\tilde{S}(+\infty,-\infty)\tilde{S}(+\infty,y_s)}} \right\}$$

Separately Well-defined

Each TMD is "factorized"

$$\tilde{F}_H^{\rm sub}(x, b_T; \mu, y_s) = \lim_{\substack{y_A \to \infty \\ y_B \to -\infty}} \tilde{F}_H^{\rm unsub}(x, b_T; \mu, y_P - y_B) \sqrt{\frac{\tilde{S}(b_T; y_A, y_s)}{\tilde{S}(b_T; y_A, y_B)\tilde{S}(b_T; y_s, y_B)}}$$



From Ted Rogers

N.B. here y_s is regulator

$$\frac{d\sigma}{dP_T^2} \propto \sum_{jj'} \mathcal{H}_{jj',\,\mathrm{SIDIS}}(\alpha_s(\mu), \mu/Q) \int d^2\boldsymbol{b}_T e^{i\boldsymbol{b}_T\cdot\boldsymbol{P}_T}\ \tilde{F}_{j/H_1}(x, b_T; \mu, \zeta_1)\, \tilde{D}_{H_2/j'}(z, b_T; \mu, \zeta_2) \ + \ Y_{\mathrm{SIDIS}}$$

$$\zeta_1 = M_P^2 x^2 e^{2(y_P - y_s)} \quad \Longleftrightarrow \quad y_s$$

In full QCD, the auxiliary parameters are exactly arbitrary and this is reflected in the the Collins-Soper (CS) equations for the TMD PDF, and the renormalization group (RG) equations

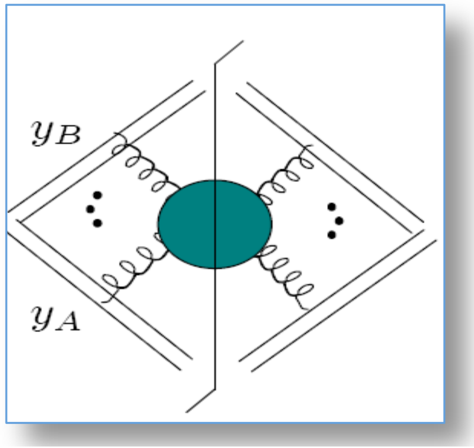
Collins arXiv: 1212.5974

Evolution follows from their independence of rapidity scale

$$\tilde{F}_H^{\rm sub}(x, b_T; \mu, y_n) = \lim_{\substack{y_A \to \infty \\ y_B \to -\infty}} \tilde{F}_H^{\rm unsub}(x, b_T; \mu, y_P - y_B) \sqrt{\frac{\tilde{S}(b_T; y_A, y_n)}{\tilde{S}(b_T; y_A, y_B)\tilde{S}(b_T; y_n, y_B)}}$$

From operator definition get

Collins-Soper Equation:



$$-\quad \frac{\partial \ln \tilde{F}(x, b_T, \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_T; \mu)$$

$$\tilde{K}(b_T; \mu) = \frac{1}{2} \frac{\partial}{\partial y_n} \ln \frac{\tilde{S}(b_T; y_n, -\infty)}{\tilde{S}(b_T; +\infty, y_n)}$$

Along with .... Renormalization group Equations

$$\frac{d\tilde{K}}{d\ln\mu} = -\gamma_K(g(\mu))$$

$$\frac{d\ln\tilde{F}(x, b_T; \mu, \zeta)}{d\ln\mu} = -\gamma_F(g(\mu); \zeta/\mu^2)$$

RGE:
get anomalous
for *F* & *K*

Solve Collins Soper & RGE eqs. to obtain "evolved TMDs"

Evolved TMDs

- Small b_T -Perturbative
- Large b_T -non-perturbative

Small b_T

Expansion for small b_T can be made in terms of the integrated PDFs - OPE
After Fourier transformation, this gives both the large- k_T behavior, and the normalization of the integral over the whole k_T region.

$$\tilde{F}_H(x, b_T; \mu, \zeta_F) = \sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}_{f/j}(x/\hat{x}, b_T; \zeta_F, \mu, g(\mu)) f_{j/P}(\hat{x}; \mu) + \mathcal{O}((\Lambda_{\mathrm{QCD}} b_T)^a).$$

Large b_T

$$\frac{d\sigma}{dP_T^2} \propto \sum_{jj'} \mathcal{H}_{jj',\,\mathrm{SIDIS}}(\alpha_s(\mu), \mu/Q) \int d^2\boldsymbol{b}_T e^{i\boldsymbol{b}_T\cdot\boldsymbol{P}_T}\ \tilde{F}_{j/H_1}(x, b_T; \mu, \zeta_1)\, \tilde{D}_{H_2/j'}(z, b_T; \mu, \zeta_2)$$

$$+\ Y_{\mathrm{SIDIS}}\ + \mathrm{P.S.C} \quad O(\Lambda/Q)^a$$

Practical issue: is that the ”TMD contribution” term is calculated in coordinate space and Fourier transformed back into momentum space

Calculations of *FT* term include non-perturbative behavior at large b_T

In the Fourier transforms that connect these calculations to cross sections, non-perturbative effects from large b_T can migrate to unexpectedly large P_T, and perturbative effects from small b_T can migrate to small P_T.

Must match these regions

Solve Collins Soper & RGE eqs. obtain Evolution kernal

Collins Soper Sterman NPB 85

- Prescription for matching large and small b_T
- Replace b_T in hard part with

$$\mathbf{b}_* = \frac{\mathbf{b}_T}{\sqrt{1 + b_T^2/b_{\max}^2}}, \qquad \mu_b = \frac{C_1}{b_*}.$$

- Maximizes the perturbative content while providing a TMD formalism that is applicable over the entire range of P_T

Nonperturbative part of $\tilde{K}(b_T, \mu)$

$$g_K(b_{\mathrm{T}}; b_{\max}) = -\tilde{K}(b_{\mathrm{T}}, \mu) + \tilde{K}(b_*, \mu)$$

Collins Soper Sterman NPB 85

Totally universal related to derivative of soft factor independent of *x* & hadron. Contains essential content on evolution of nucleon structure

Non-perturbative part of $\tilde{K}(b_T, \mu)$

$$g_K(b_{\rm T}; b_{\rm max}) = -\tilde{K}(b_{\rm T}, \mu) + \tilde{K}(b_*, \mu)$$

Collins Soper Sterman NPB 85

Solve RGE:

$$\tilde{K}(b_T; \mu) = \tilde{K}(b_*; \mu_b) - \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \gamma_K(g(\mu')) - g_K(b_T)$$

$$\mathbf{b}_* = \frac{\mathbf{b}_T}{\sqrt{1 + b_T^2/b_{\rm max}^2}}, \qquad \mu_b = \frac{C_1}{b_*}.$$

$b_{\rm max}$ chosen so that b_* doesn't go too far beyond the pertb. region maximize perturbative content in evolving TMDs and cross section

Non-perturbative part of TMD

$$
\begin{aligned}
\tilde{F}_H(x, b_T; \mu, \zeta_F) &= \tilde{F}_H(x, b_*; \mu, \zeta_F) \frac{\tilde{F}_H(x, b_T; \mu, \zeta_F)}{\tilde{F}_H(x, b_*; \mu, \zeta_F)} \\
&= \tilde{F}_H(x, b_*; \mu, \zeta_F) \frac{\tilde{F}_H(x, b_T; \mu_0, \zeta_0) e^{\int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \gamma_F\left(g(\mu'), \frac{\gamma_0}{\mu'^2}\right)} e^{\tilde{K}(b_T,\mu) \ln \sqrt{\frac{\zeta_F}{\zeta_0}}}}{\tilde{F}_H(x, b_*; \mu_0, \zeta_0) e^{\int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \gamma_F(g(\mu'), \frac{\gamma_0}{\mu'^2})} e^{\tilde{K}(b_*,\mu) \ln \sqrt{\frac{\zeta_F}{\zeta_0}}}} \\
&= \tilde{F}_H(x, b_*; \mu, \zeta_F) \frac{\tilde{F}_H(x, b_T; \mu_0, \zeta_0)}{\tilde{F}_H(x, b_*; \mu_0, \zeta_0)} e^{-g_K(b_*) \ln \sqrt{\frac{\zeta_F}{\zeta_0}}} \\
&= \tilde{F}_H(x, b_*; \mu, \zeta_F) e^{-g_1(x, b_T; b_{max}) - g_K(b_*) \ln \sqrt{\frac{\zeta_F}{\zeta_0}}}
\end{aligned}
$$

Evolved Structure Function & TMDs

$$\mathcal{F}_{UU}(x,z,b,Q^2)=\sum_a \tilde{F}^a_{H1}(x,b_T,\mu,\zeta_F)\tilde{D}^a_{H2}(z_h,b_T,\mu,\zeta_D)H_{UU}(Q^2,\mu^2)$$

Non-perturbative large b_T behavior

Totally universal related to derivative of soft factor independent of x & hadron

$$\tilde{F}_{H_1}(x,b_T;Q,Q^2)=\tilde{F}_{H_1}(x,b_*;\mu_b,\mu_b^2)\exp\left\{-g_1(x,b_T;b_{\max})-g_K(b_T;b_{\max})\ln\left(\frac{Q}{Q_0}\right)+\ln\left(\frac{Q}{\mu_b}\right)\tilde{K}(b_*;\mu_b)+\int_{\mu_b}^{Q}\frac{d\mu'}{\mu'}\left[\gamma_{\mathrm{PDF}}(\alpha_s(\mu');1)-\ln\left(\frac{Q}{\mu'}\right)\gamma_K(\alpha_s(\mu'))\right]\right\}$$

Perform OPE on

perturbative small b_T behavior

These functions have good perturbative behavior at entire range of b_T

Comments Factorization

- This strong form of universality is, therefore, an important basic test of the TMD factorization theorem. It is related to the soft factors—the vacuum expectation values of Wilson loops—that are needed in the TMD definitions for consistent factorization with a minimal number of arbitrary cutoffs
- Constraining the non-perturbative component of the evolution probes fundamental aspects of soft QCD

Studies that impact TMD Factorization

Fixed scale phenomenology- Stage 1+

A.V. Efremov, K. Goeke, S. Menzel, A. Metz, and P. Schweitzer, Phys. Lett. B 612, 233 (2005).
W. Vogelsang and F. Yuan, Phys. Rev. D 72, 054028 (2005).
M. Anselmino et al., Phys. Rev. D 71, 074006 (2005).
S. Arnold, A. Efremov, K. Goeke, M. Schlegel, P. Schweitzer, arXiv:0805.2137.
M. Anselmino et al., Eur. Phys. J. A 39, 89 (2009).
A. Bacchetta and M. Radici, Phys. Rev. Lett. 107, 212001 (2011).
A. Signori, Bacchetta, Radici,Schnell, JHEP 1311 (2013)
Anselmino, Boglione, O. Gonzalez, S. Melis , Prokudin JHEP 1404 (2014) ....

Stage 2 w/ evolution of various forms

D. Boer, Nucl. Phys. B603, 195 (2001); B806, 23 (2009); B874, 217 (2013).
Z.-B. Kang, B.-W. Xiao, and F. Yuan, Phys. Rev. Lett. 107, 152002 (2011).
S. M. Aybat and T. C. Rogers, Phys. Rev. D 83, 114042 (2011).
S. M. Aybat, J. C. Collins, J.-W. Qiu, and T. C. Rogers, Phys. Rev. D 85, 034043 (2012).
M. Aybat, A. Prokudin, T. Rogers, Phys.Rev.Lett. 108 (2012)
M. G. Echevarria, A. Idilbi, A. Schafer, and I. Scimemi, Eur. Phys. J. C 73, 2636 (2013).
Bacchetta & Prokuding PLB 2013
P. Sun and F. Yuan, Phys. Rev. D 88, 034016 (2013).
Aidala, Field, Gamberg, Rogers, PRD 89 (2014)
M. Echevarria, A. Idilbi, Z-B.Kang, I. Vitev Phys.Rev. D89 (2014) 074013
J. Collins, T. Rogers PRD91 (2015)

Comments on Stage 2 Fitting

It was recently illustrated that the rapid evolution given by extrapolating the non-perturbative extractions from Drell-Yan cross sections at large Q is too fast to adequately account for data in the region of Q of order a few GeV.

The current phenomenological situation is further complicated by the observation that parametrizations obtained by extrapolating large Q fits to small Q implies suspiciously rapid evolution in the region of a few GeV, a result very clearly demonstrated in the recent work of Sun Yuan PRD 2013, Boer NPB 2015....

More recently, Aidala,Field,LG,Rogers PRD 14 examed the HERMES and COMPASS data. We agree that there is indeed a discrepancy between these data and the predictions based on the earlier Drell–Yan data but argue that the low- and high-energy data are mostly probing different regions of transverse position; thus, the discrepancy concerns the extrapolation of a parametrization of non-perturbative physics outside the region where it was fitted.

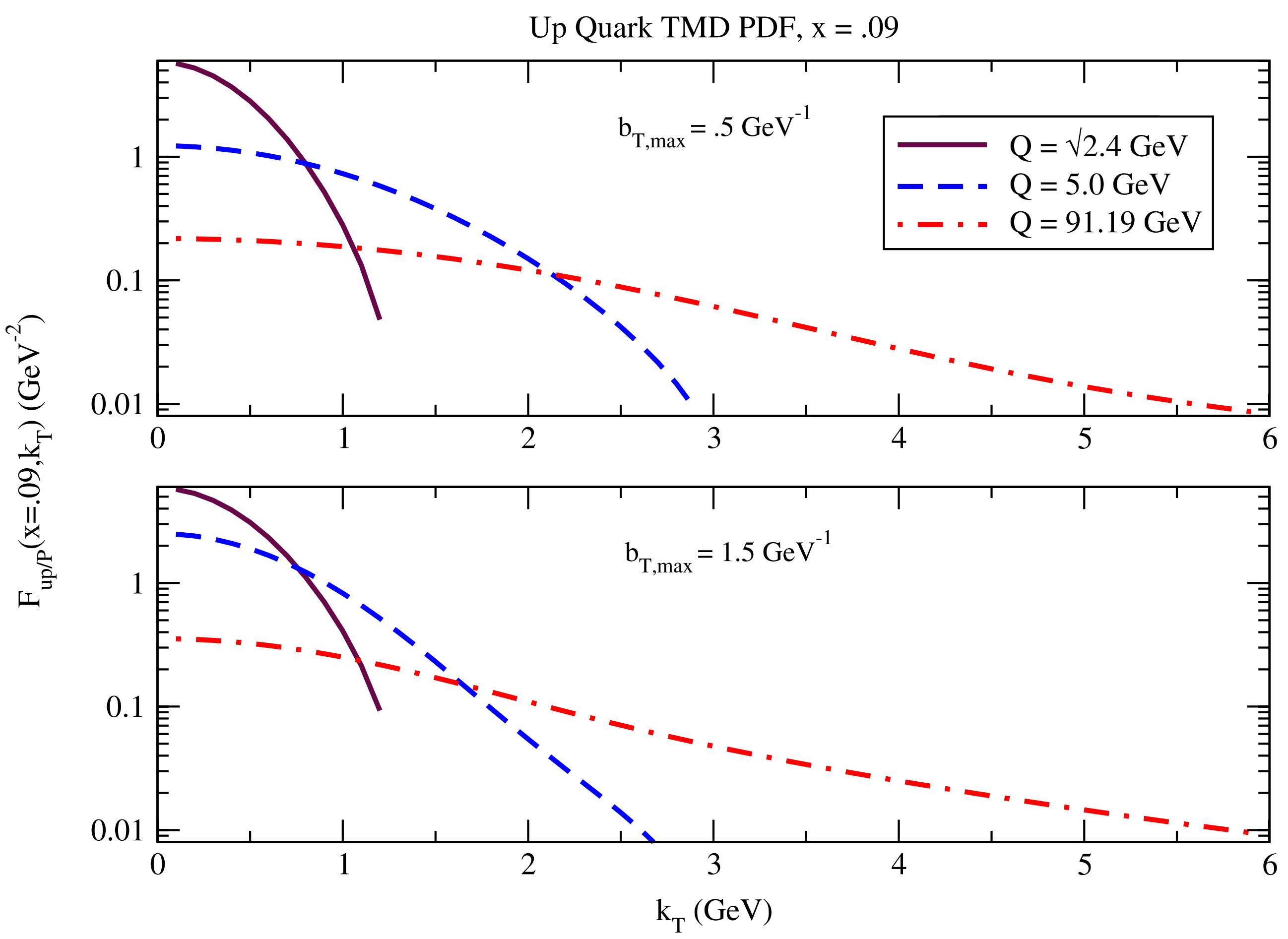
- Frequently used ansatz for *DY*

$$g_K(b_T; b_{\max}) = g_2(b_{\max}) \frac{1}{2} b_T^2,$$

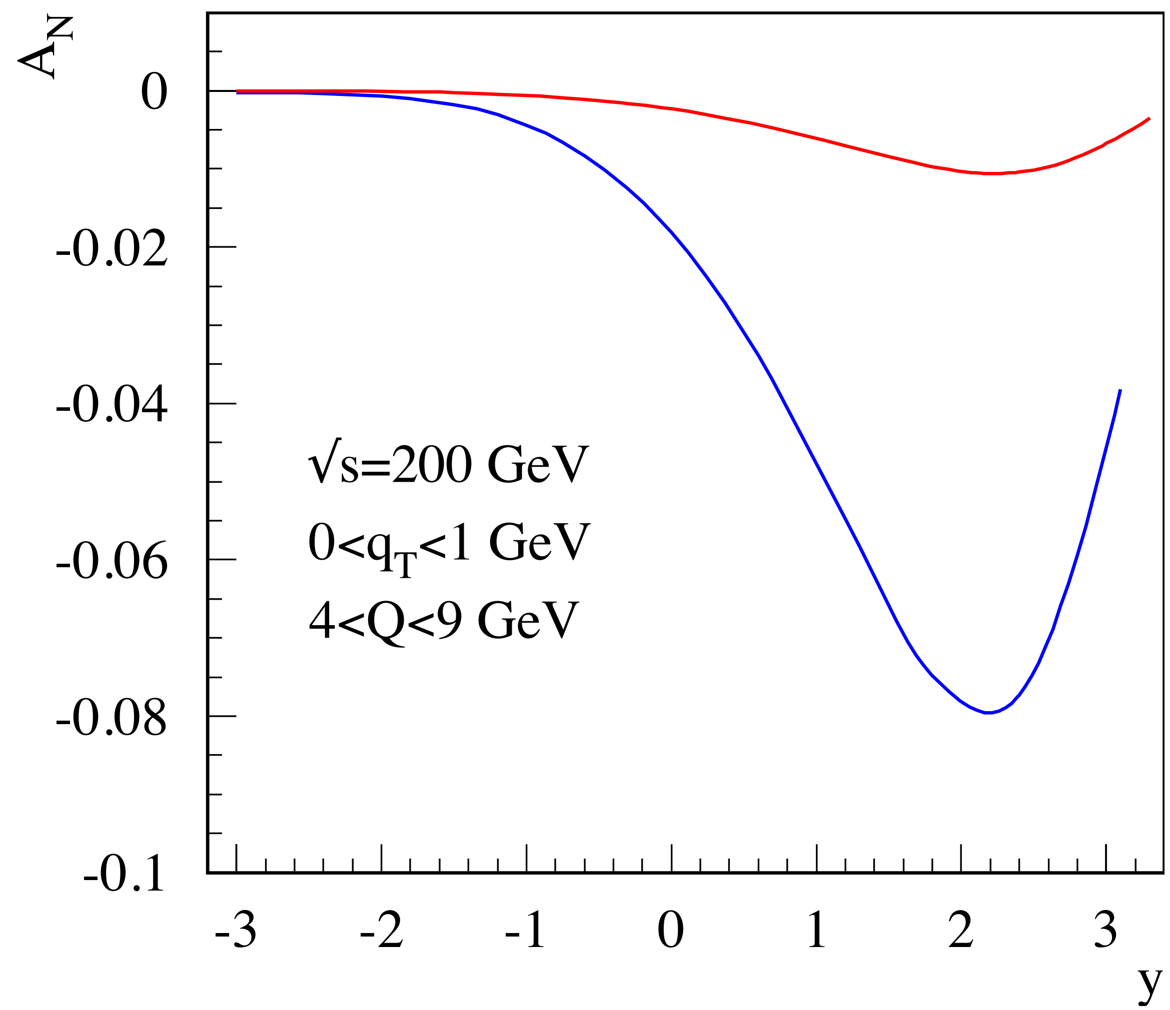
Rapid TMD Evolution ???

In the momentum-space TMD PDF, the evolution corresponds to rapid suppression at small k_T , of order $k_T \sim 1$ GeV, with increasing Q.
The effect can be observed in the small k_T region of the curves

PHYSICAL REVIEW D **83,** 114042 (2011)



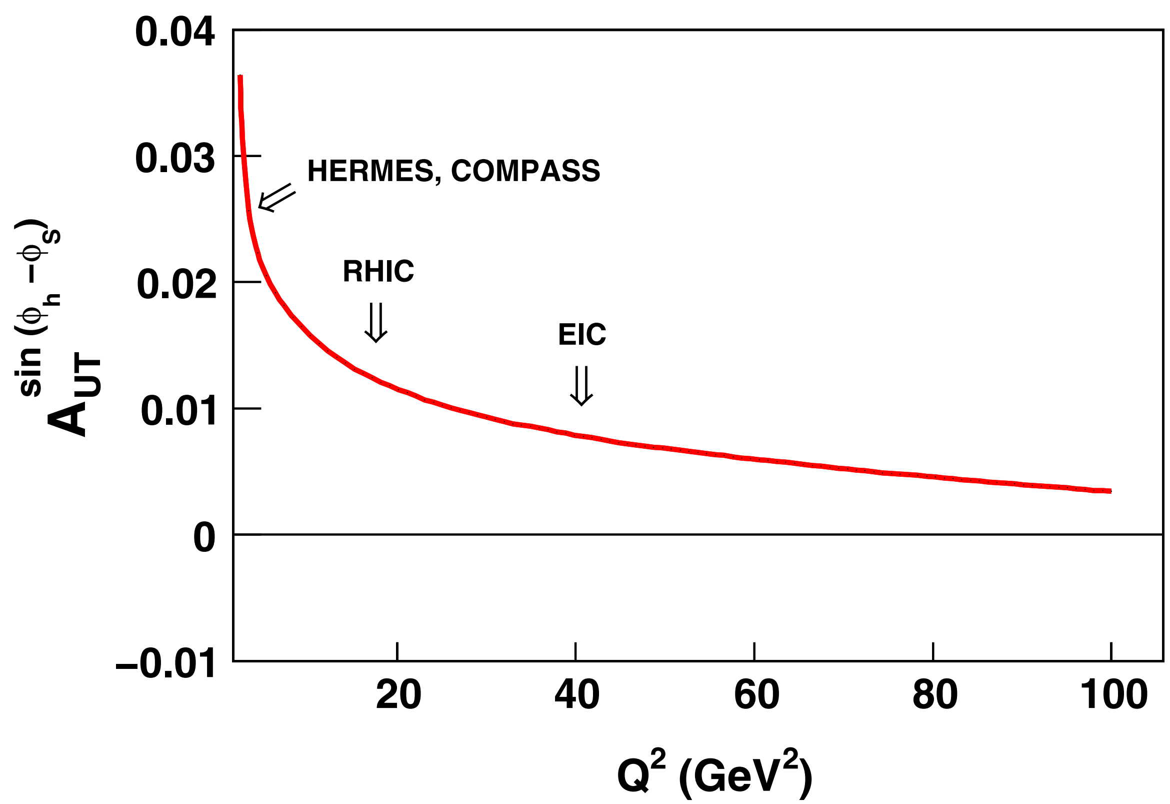
Kang QCD Evolution 2013



b_{max} = 0.5 and g_2 = 0.68 and start from Gaussian at HERMES

See also Sun & Yuan 2013 PRD, Boglione Prokudin Melis Anselmino et al ....

!! See also Elke's talk at REF 2015



Sivers evolution integrated over x
Aybat Prokudin Rogers PRL 2012

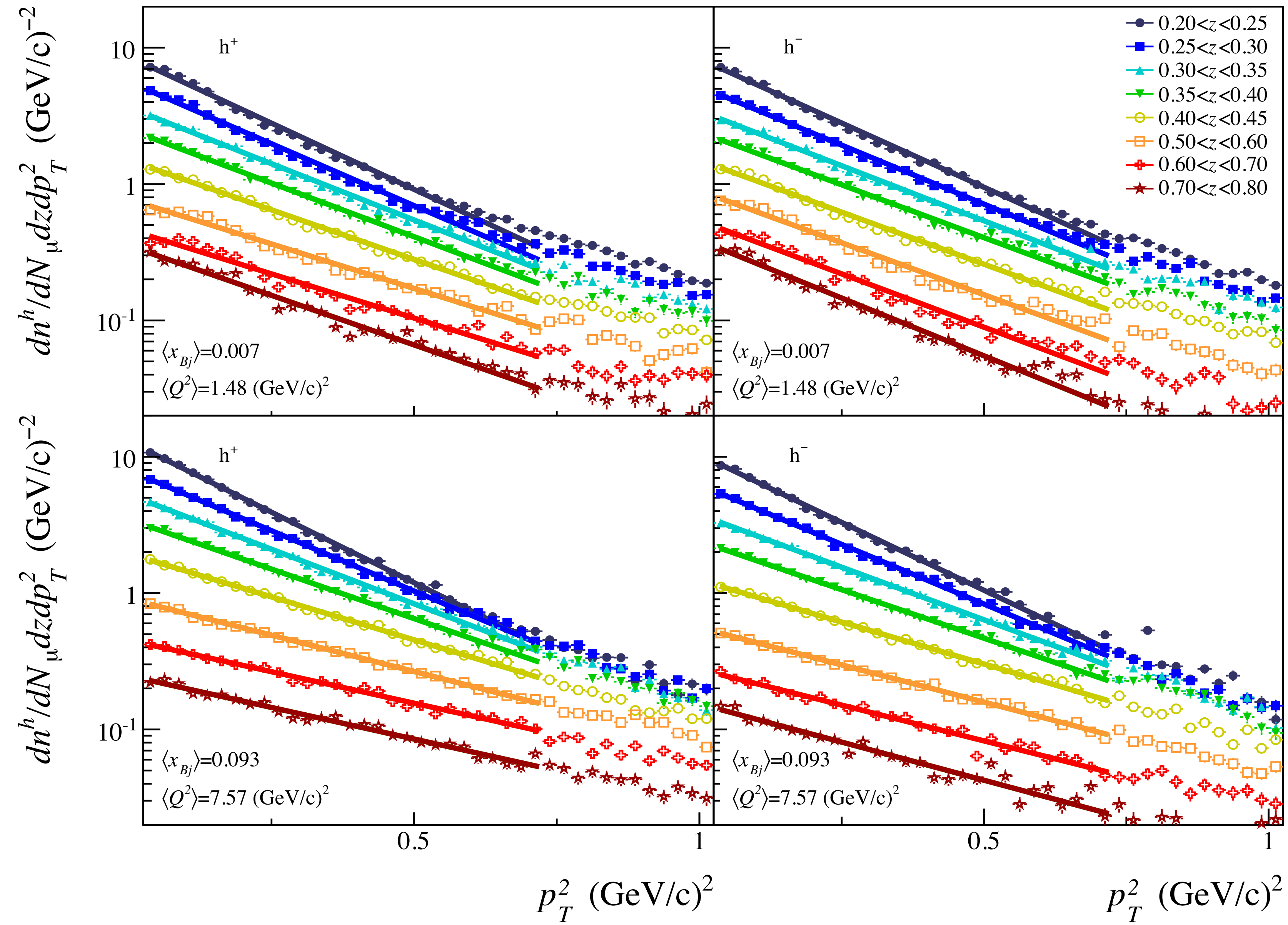
Comments on Stage 2 Fitting

To maintain consistency with the general aim of extracting properties intrinsic to specific hadrons we would ideally vary Q while holding x, z, and hadron species fixed.

In experiments, however, these variables are correlated, and practical fitting becomes challenging.

We appeal to the multi-differential COMPASS data to study the variation in the multiplicity distribution with small variations in Q and roughly fixed x and z bins within the same experiment.

From COMPASS, *C. Adolph et al., arXiv:1305.7317*



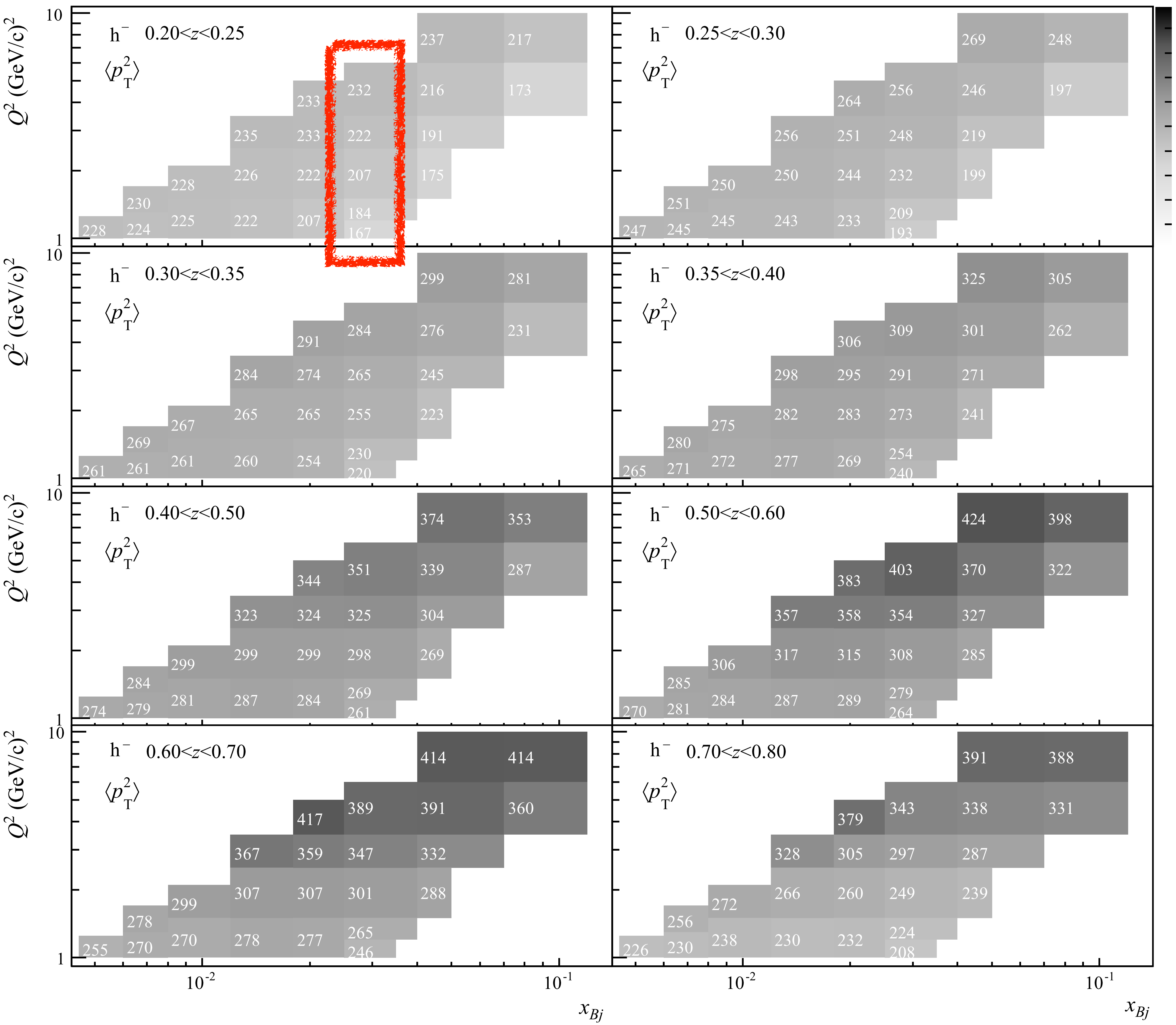
Approx. fixed x,z bins

Eur. Phys. J. C (2013) Adolph et al.

Bin	x_{bj}^{min}	x_{bj}^{max}	$\langle x_{bj}\rangle$	Q^2_{min}	Q^2_{max}	$\langle Q^2\rangle$
1	0.0045	0.0060	0.0052	1.0	1.25	1.11
2	0.0060	0.0080	0.0070	1.0	1.30	1.14
3	0.0060	0.0080	0.0070	1.3	1.70	1.48
4	0.0080	0.0120	0.0099	1.0	1.50	1.22
5	0.0080	0.0120	0.0099	1.5	2.10	1.76
6	0.0120	0.0180	0.0148	1.0	1.50	1.22
7	0.0120	0.0180	0.0148	1.5	2.50	1.92
8	0.0120	0.0180	0.0150	2.5	3.50	2.90
9	0.0180	0.0250	0.0213	1.0	1.50	1.23
10	0.0180	0.0250	0.0213	1.5	2.50	1.92
11	0.0180	0.0250	0.0213	2.5	3.50	2.94
12	0.0180	0.0250	0.0216	3.5	5.00	4.07
13	0.0250	0.0350	0.0295	1.0	1.20	1.10
14	0.0250	0.0400	0.0316	1.2	1.50	1.34
15	0.0250	0.0400	0.0318	1.5	2.50	1.92
16	0.0250	0.0400	0.0319	2.5	3.50	2.95
17	0.0250	0.0400	0.0323	3.5	6.00	4.47
18	0.0400	0.0500	0.0447	1.5	2.50	1.93
19	0.0400	0.0700	0.0533	2.5	3.50	2.95
20	0.0400	0.0700	0.0536	3.5	6.00	4.57
21	0.0400	0.0700	0.0550	6.0	10.0	7.36
22	0.0700	0.1200	0.0921	3.5	6.00	4.62
23	0.0700	0.1200	0.0932	6.0	10.0	7.57

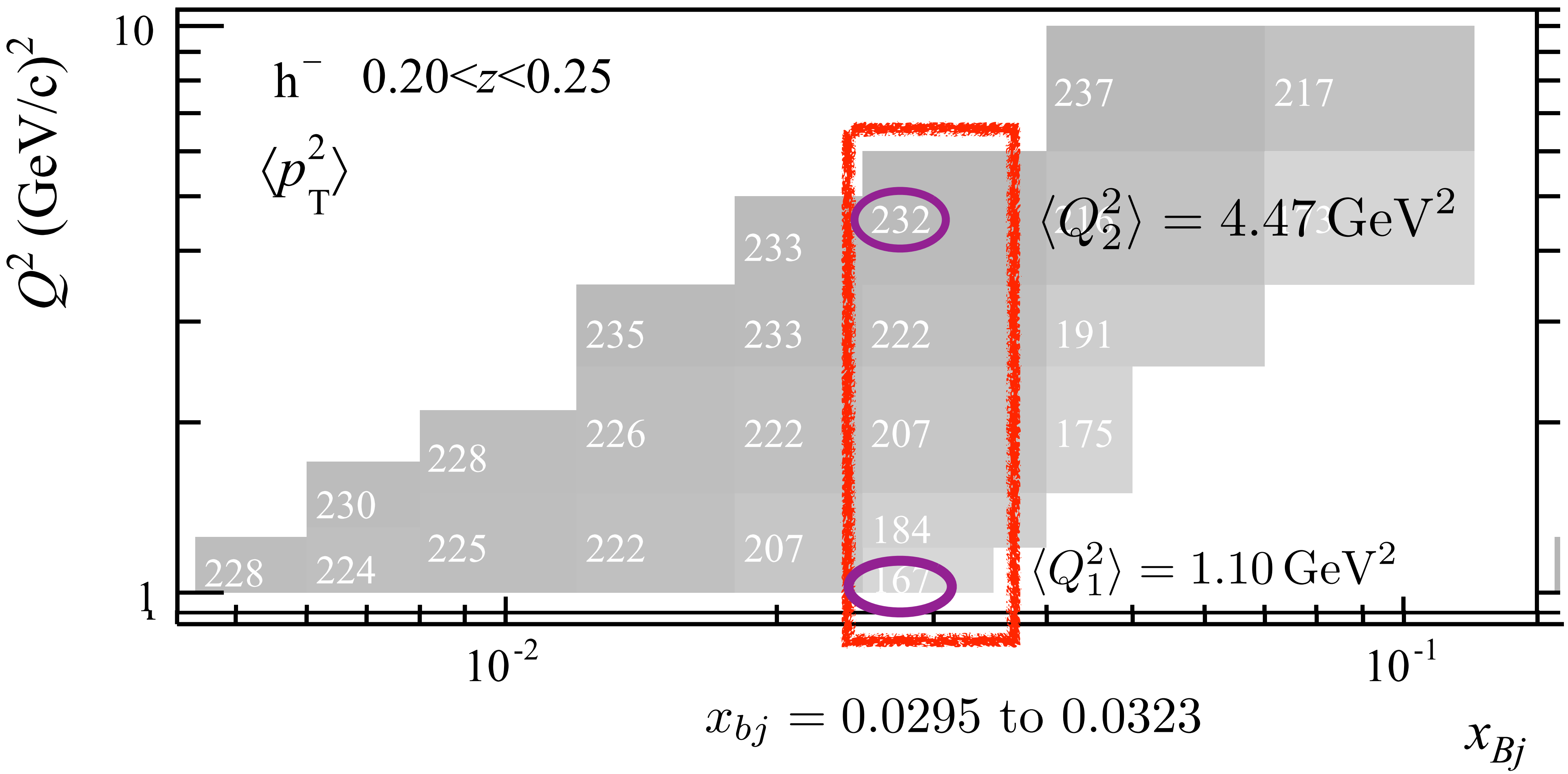
$\langle Q_1^2\rangle$

$\langle Q_2^2\rangle$



Window of fixed x and z

Panels are fixed z-bins & columns are fixed x bins for Q^2 vs. $\langle P_T^2 \rangle$



Quantifying the Evolution

COMPASS data for hadron multiplicities are fitted using a Gaussian form
We then quantified/bounded the P_{T} broadening

$$\tilde{\sigma}_{\text{TMD term}} \equiv \mathcal{H}(\alpha_s(Q))\tilde{F}_{H_1}(x, b_T; Q, Q^2)\tilde{D}_{H_2}(z, b_T; Q, Q^2)$$

$$\tilde{\sigma}_{\text{TMD term}} \approx \exp\left\{-\frac{b_T^2\langle P_T^2\rangle}{4}\right\}$$

$$g_{\text{PDF}}(x, b_T; b_{\max}) \propto g_{\text{FF}}(z, b_T; b_{\max}) \propto b_T^2$$

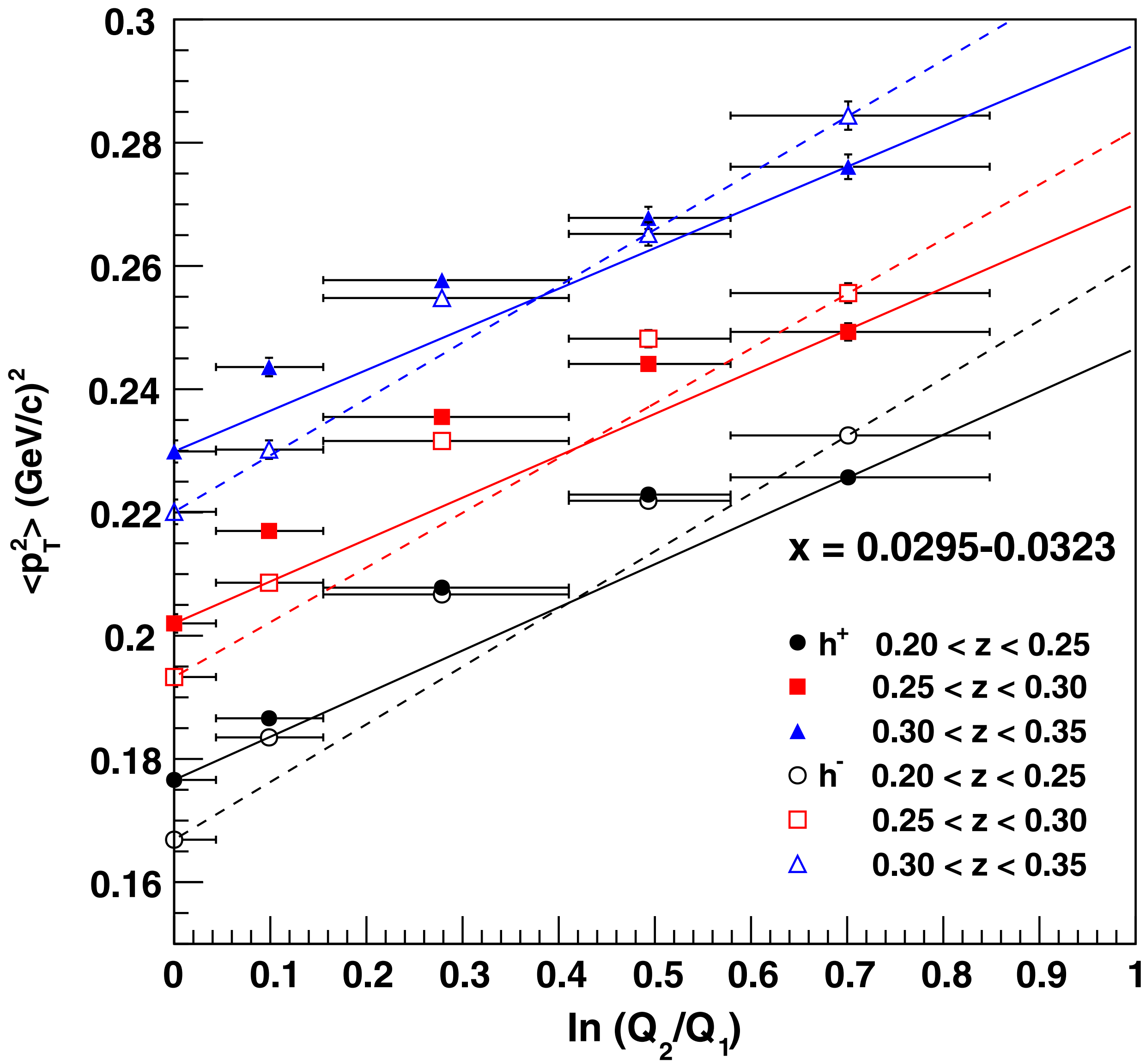
$$\left.\frac{d\ \ln\ \tilde{\sigma}_{\text{TMD term}}}{d\ \ln\ Q^2}\right|_{\mathrm{b_T dep}} = \tilde{K}(b_T; \mu_0)|_{\mathrm{b_T dep}}$$

$$\frac{d\sigma}{dP_T^2} \propto \text{F.T.} \exp\left\{-\frac{b_T^2}{4}\left(\langle P_T^2\rangle_0 + 4C_{\text{evol}}\ \ln\left(\frac{Q_2}{Q_1}\right)\right)\right\}$$

$$\Delta\langle P_T^2\rangle(Q_1, Q_2) \approx 4C_{\text{evol}}\ \ln\left(\frac{Q_2}{Q_1}\right)$$

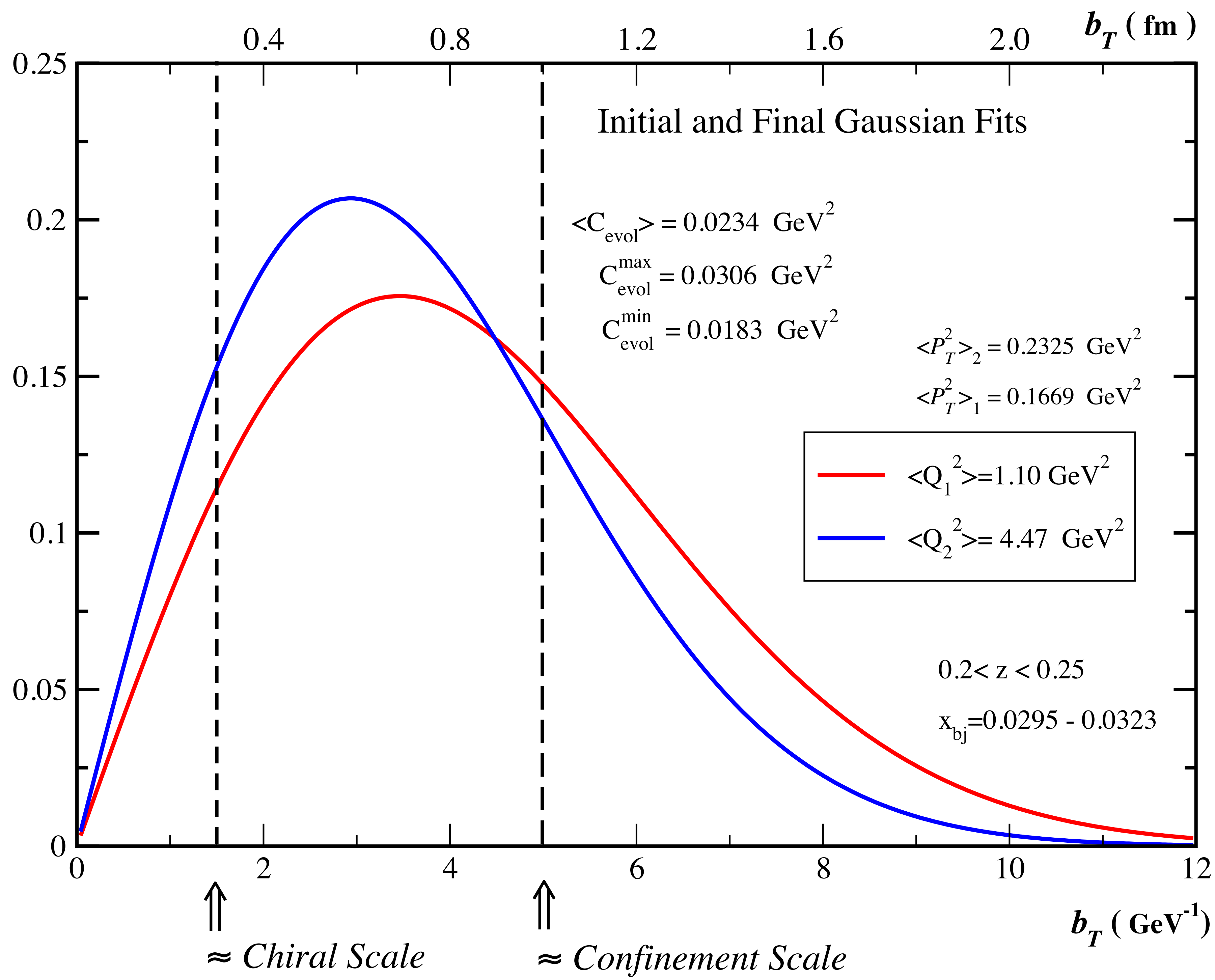
Looking for maximum range on Q to perform study

LIMITS ON TRANSVERSE MOMENTUM DEPENDENT …



$$\Delta\langle P_T^2\rangle(Q_1, Q_2) \approx 4C_{\text{evol}} \ln\left(\frac{Q_2}{Q_1}\right)$$

Quantify Broadening but in b-space



From the general features of Fig.we conclude that, for the differential cross section in the limit of small P_T, the relevant range of b_T nearly dominated by the non-perturbative region of b_T for $Q \sim 1.0$ GeV to $b_T \sim 2.0$ GeV. This is the scale of nucleon structure

Comments

The only aspect of TMD factorization that we have used to parametrize broadening is CS equation & observation that one can fit COMPASS multiplicities w/ Gaussians parameterization

Specifically, we have applied it to the case of the COMPASS data for the small range of Q where the P_{T} distribution appears to remain approximately Gaussian even after evolution to obtain

$$\frac{d\sigma}{dP_T^2} \propto \text{F.T.} \exp\left\{-\frac{b_T^2}{4}\left(\langle P_T^2\rangle_0 + 4C_{\text{evol}} \ln\left(\frac{Q_2}{Q_1}\right)\right)\right\}$$

Now we will address the question of whether evolution is governed primarily by perturbative or nonperturbative b_{T} dependence.

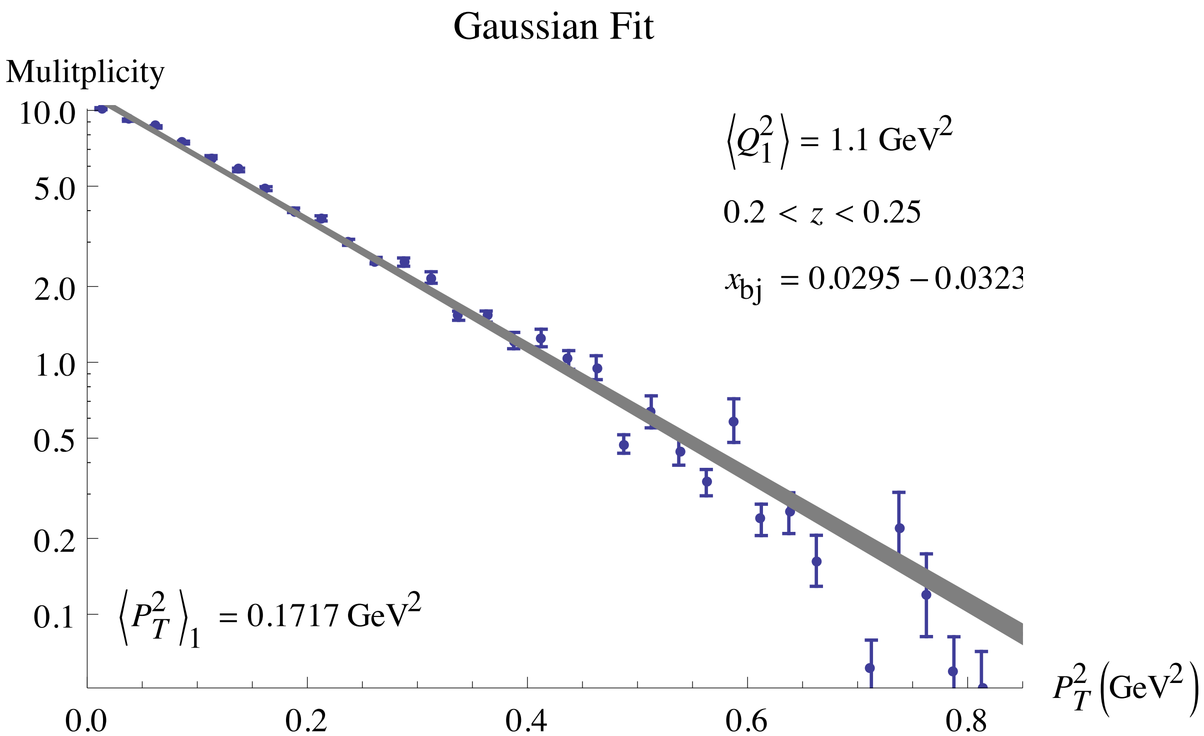
source of error

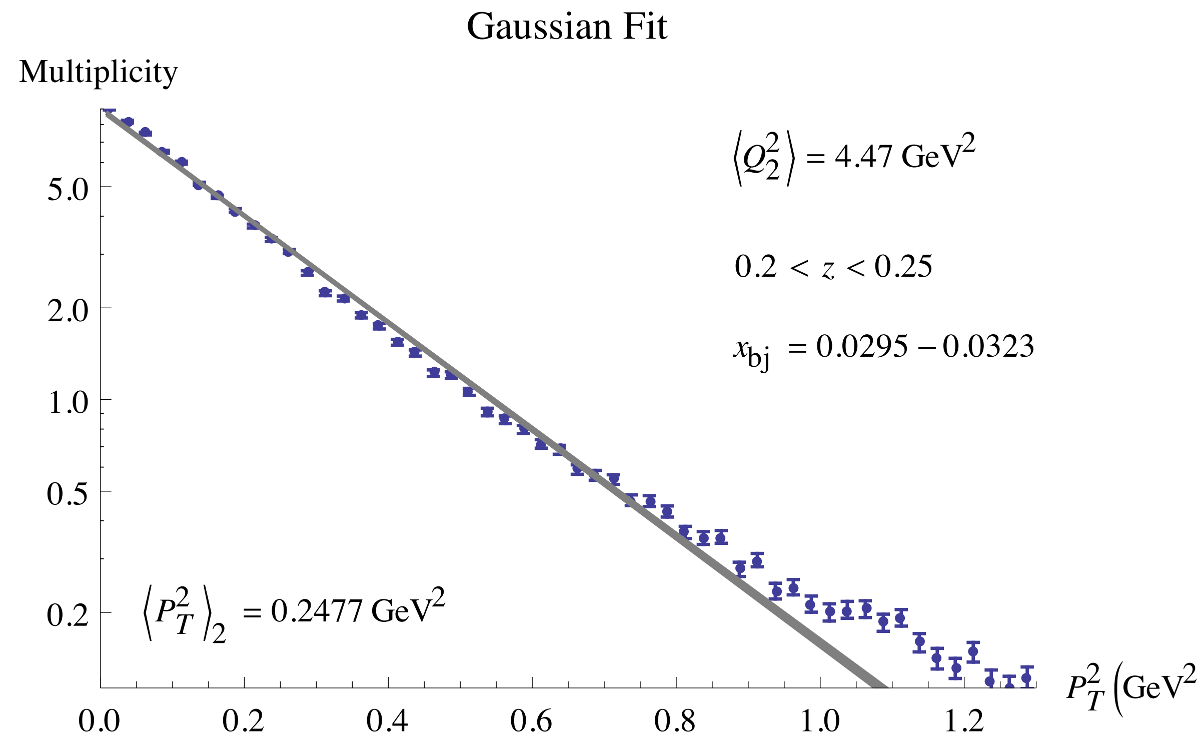
The cutoff at $P_T = 0.85$ GeV in the fits of COMPASS Data where the Gaussian description starts to break down.

One could speculate that including more of the large P_T tail might result in an enhanced relative contribution from small b_T .

To address this, we have performed our own fit of the Gaussian form using the same data from COMPASS DATA that gave the two curves for $Q = 1.049$ GeV and $Q = 2.114$ GeV Fig. but now for the entire range of P_T (up to $P_T \sim 1.0$ GeV).

Refit-momentum space





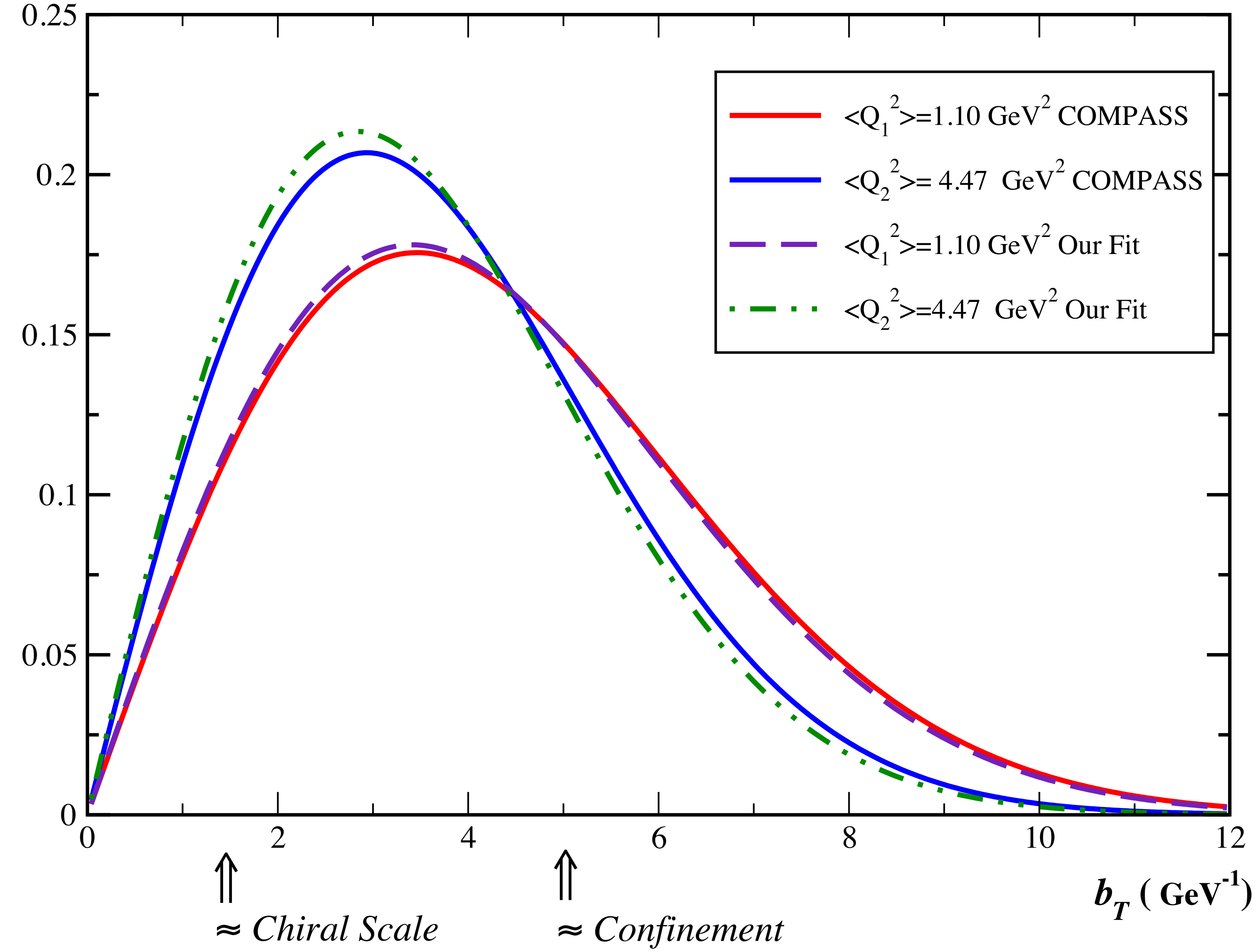
$$\langle P_T^2 \rangle^{\mathrm{New\ Fits}}_{Q_1=1.049\,\mathrm{GeV}} = 0.1717 \pm 0.0011\,\mathrm{GeV}^2\,; \qquad \langle P_T^2 \rangle^{\mathrm{New\ Fits}}_{Q_2=2.114\,\mathrm{GeV}} = 0.2477 \pm 0.0008\,\mathrm{GeV}^2$$

Was

$$\langle P_T^2 \rangle^{\mathrm{Old\ Fits}}_{Q_1=1.049\,\mathrm{GeV}} = 0.1669 \pm \ldots \qquad \langle P_T^2 \rangle^{\mathrm{Old\ Fits}}_{Q_1=2.114\,\mathrm{GeV}} = 0.2325 \pm \ldots\ \mathrm{GeV}^2$$

Refit b space

Little change when we include “large” P_T data



The solid red and blue curves are the same as those in previous Fig. in where fit is restricted to region of $P_T \leq 0.85$ GeV.

Purple dashed and green dot-dashed curves are from the refit Gaussian curves above that use all P_T and correspond to Eq. (32) with the initial and final P_T from Eq. (34)

$$\langle P_T^2 \rangle^{\text{New Fits}}_{Q_1=1.049\,\text{GeV}} = 0.1717 \pm 0.0011\,\text{GeV}^2\,; \qquad \langle P_T^2 \rangle^{\text{New Fits}}_{Q_2=2.114\,\text{GeV}} = 0.2477 \pm 0.0008\,\text{GeV}^2$$

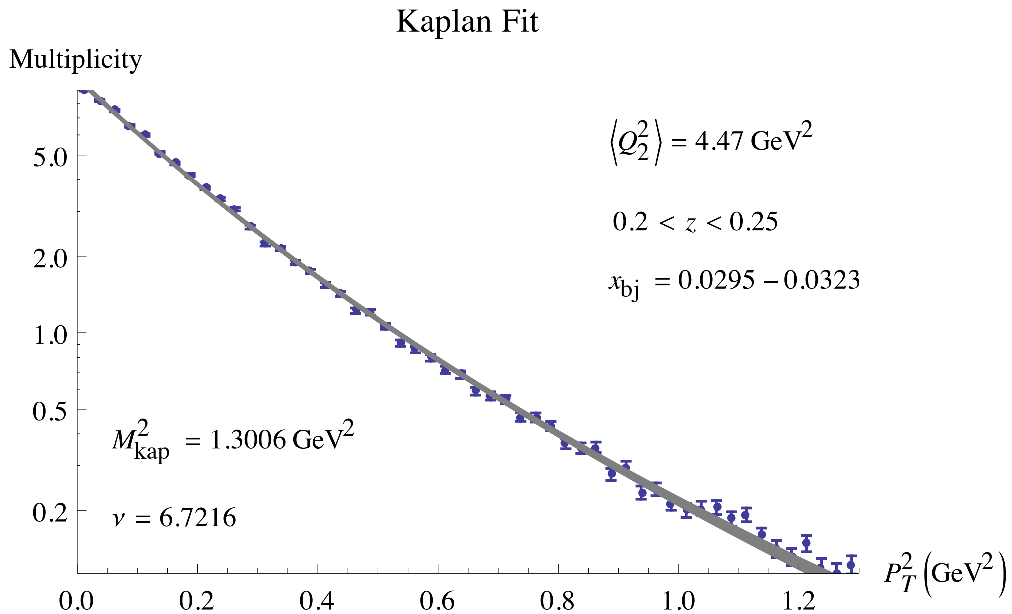
$$\langle P_T^2 \rangle^{\text{Old Fits}}_{Q_1=1.049\,\text{GeV}} = 0.1669 \pm \ldots \qquad \langle P_T^2 \rangle^{\text{Old Fits}}_{Q_1=2.114\,\text{GeV}} = 0.2325 \pm \ldots\ \text{GeV}^2$$

Also ....

A critique could be made regarding the use of a Gaussian form on the grounds that analyticity considerations imply a power law fall-off for the large PT behavior of TMD correlation functions.

Moreover, a power law behavior $1/P_T^2$ (up to logarithmic corrections and the effects of evolution of collinear PDFs) is a prediction of pQCD .

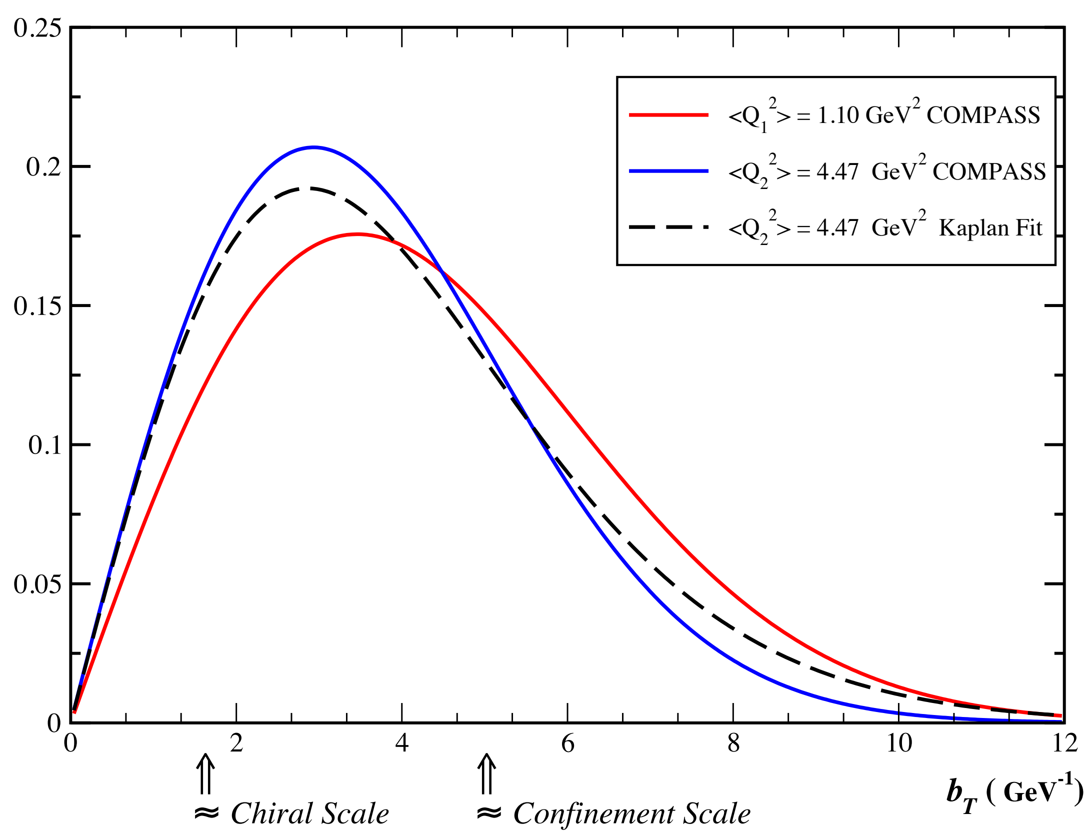
The true large P_T behavior of the TMD functions is not directly meaningful at very large P_T , since TMD factorization (without the Y term) is inapplicable once the P_T is comparable with Q. Clearly, the Y -term will be need be incorporated in the future to deal with these issues.



$$\frac{d\sigma}{dP_T^2} \propto \frac{1}{\left(1+\frac{P_T^2}{M_{\mathrm{kap}}^2}\right)^{\nu}}$$

FT

$$\frac{2b_T^{\nu} M_{\mathrm{kap}}}{\Gamma(\nu)} \left(\frac{M_{\mathrm{kap}}}{2}\right)^{\nu} K_{1-\nu}\left(b_T M_{\mathrm{kap}}\right)$$



The black dashed curve shows the b_T space function for Q_2 = 2.114GeV. This corresponds to the fit obtained in transverse momentum space using the Kaplan function in momentum space

The fits themselves yield parameters M^2 = 1.3006 GeV^2 and ν = 6.7216.

For comparison, we have again included the solid red and blue curves corresponding to the original fits obtained by the COMPASS collaboration at $\langle Q_1 \rangle$ = 1.049 GeV and $\langle Q \rangle$ = 2.114 GeV, respectively

Comparison w TMD Evolution

Next, we examine the evolved formula to estimate how well it matches the change in widths of the Gaussian fits observed in under different assumptions for g_{K}

$$b_T \tilde{\sigma}(b_T, \ldots) =$$

$$\frac{b_T}{N(Q)} \exp\Bigg\{ -g_{\mathrm{PDF}}(x, b_T; b_{\mathrm{max}}) - g_{\mathrm{FF}}(z, b_T; b_{\mathrm{max}}) - 2 g_K(b_T; b_{\mathrm{max}}) \ln\left(\frac{Q}{Q_0}\right)$$
$$+ 2 \ln\left(\frac{Q}{\mu_b}\right) \tilde{K}(b_*; \mu_b) + \int_{\mu_b}^{Q} \frac{d\mu'}{\mu'} \left[\gamma_{\mathrm{PDF}}(\alpha_s(\mu'); 1) + \gamma_{\mathrm{FF}}(\alpha_s(\mu'); 1) - 2 \ln\left(\frac{Q}{\mu'}\right) \gamma_K(\alpha_s(\mu')) \right] \Bigg\}$$

We will require that for Q = Q_0=1.049 GeV, AND

$b_T \tilde{\sigma}(b_T, \ldots)$ reduces to the Q = 1.049 GeV COMPASS Gaussian fit

Input to evolution kernel

The anomalous dimensions to order $\alpha_s(\mu)$ are the same for the TMD PDF and the TMD fragmentation function:

$$\gamma_{\rm PDF}(\alpha_s(\mu), \zeta_{\rm PDF}/\mu^2) = 4C_{\rm F}\left(\frac{3}{2} - \ln\left(\frac{\zeta_{\rm PDF}}{\mu^2}\right)\right)\left(\frac{\alpha_s(\mu)}{4\pi}\right) + \mathcal{O}(\alpha_s(\mu)^2), \tag{A1}$$

$$\gamma_{\rm FF}(\alpha_s(\mu), \zeta_{\rm FF}/\mu^2) = 4C_{\rm F}\left(\frac{3}{2} - \ln\left(\frac{\zeta_{\rm FF}}{\mu^2}\right)\right)\left(\frac{\alpha_s(\mu)}{4\pi}\right) + \mathcal{O}(\alpha_s(\mu)^2). \tag{A2}$$

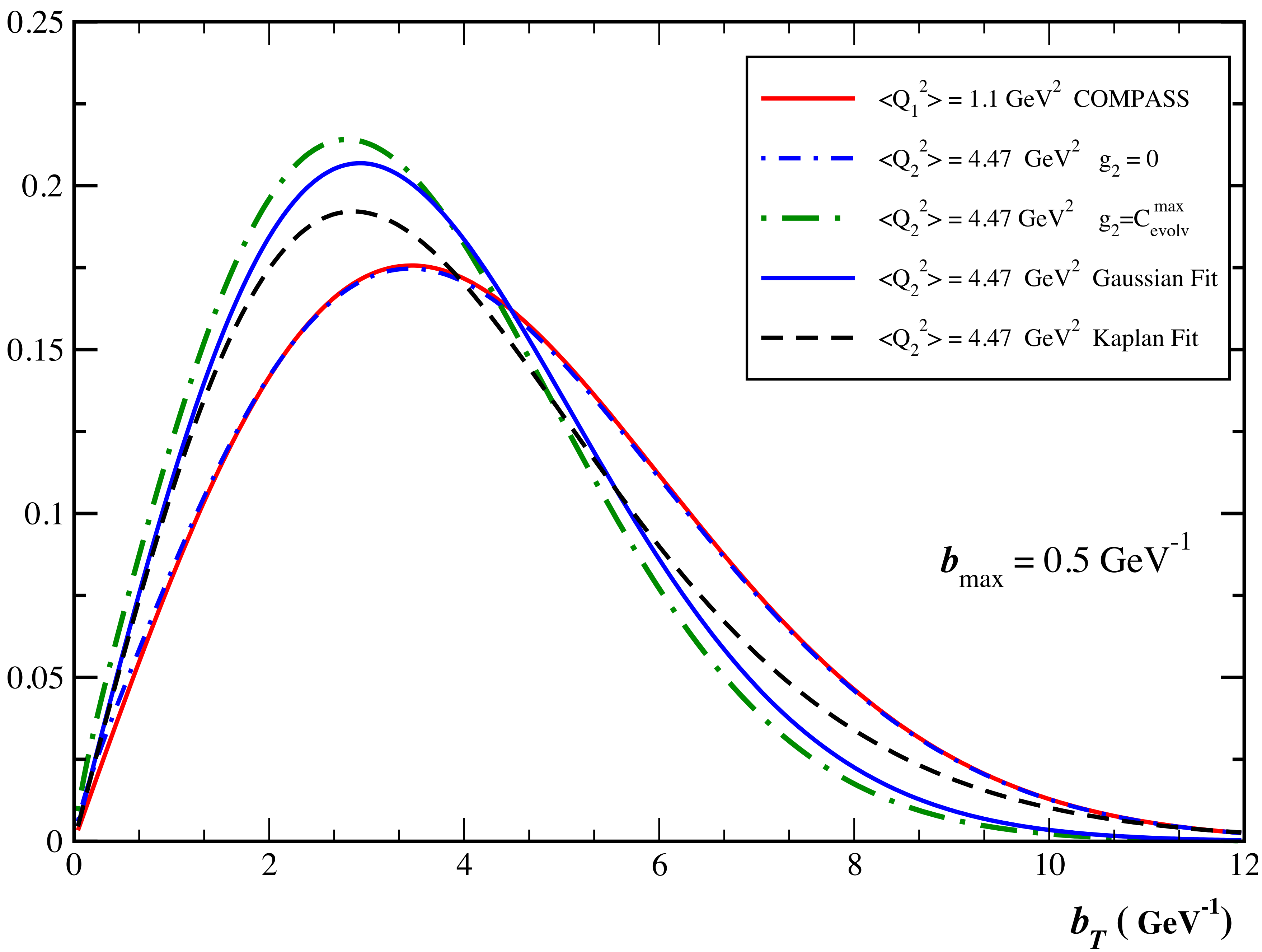
The $\overline{\rm MS}$ anomalous dimension of the CS kernel to one loop is

$$\gamma_K(\alpha_s(\mu)) = 8C_F\left(\frac{\alpha_s(\mu)}{4\pi}\right) + \mathcal{O}(\alpha_s(\mu)^2)\,. \tag{A3}$$

$$\tilde{K}(b_{\rm T};\mu) = -\frac{\alpha_s(\mu)}{\pi}C_F\left[\ln\frac{b_{\rm T}^2\mu^2}{4} + 2\gamma_E\right] + O(\alpha_s(\mu)^2),$$

$\tilde{K}(b_*;\mu_b)$ vanishes exactly when a choice of $C_1 = 2e^{-\gamma_{\rm E}}$ is made.

First Turn off g_k $g_2(b_{\max}) = 0$

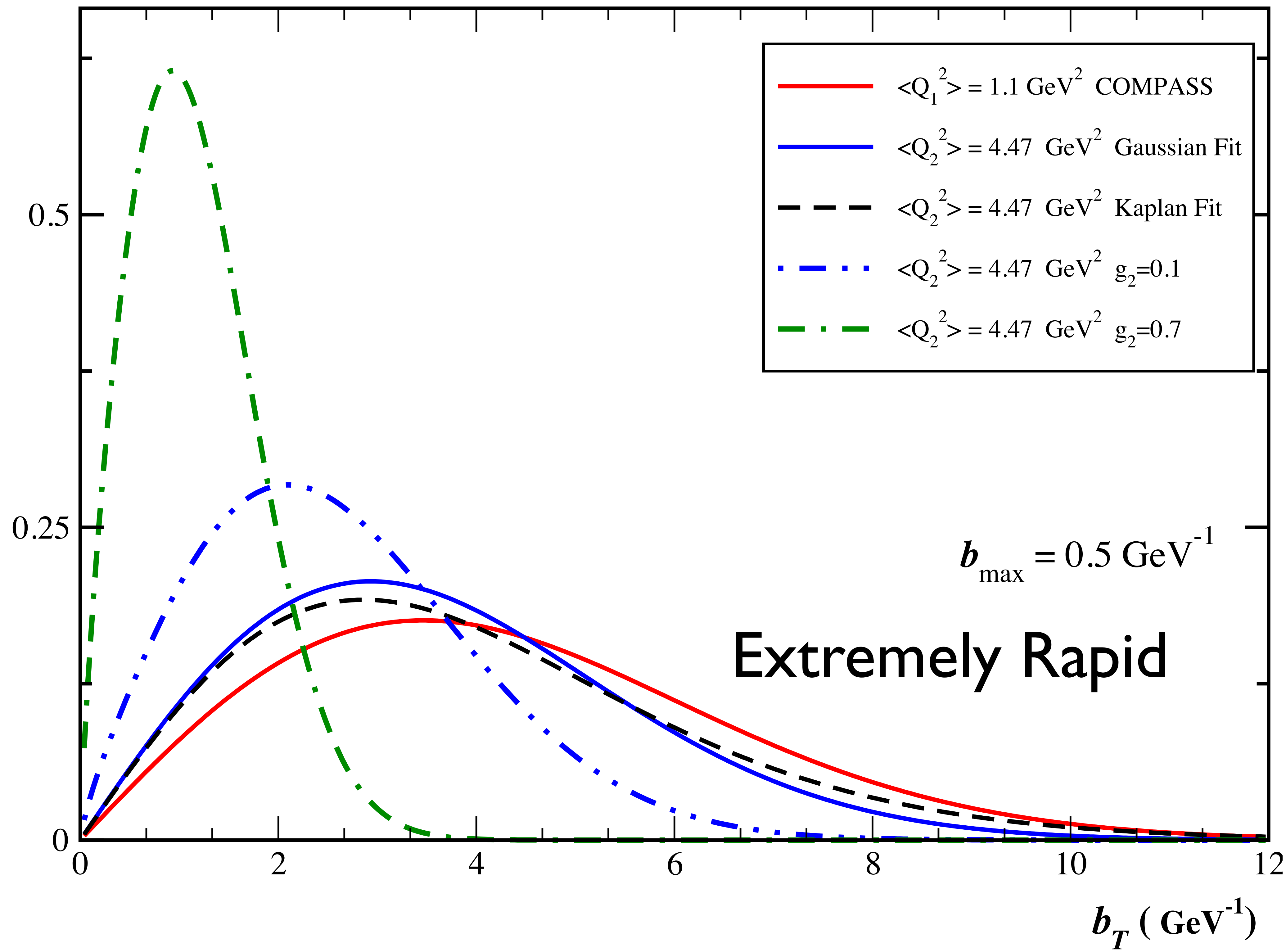


$$g_2(b_{\max}) \lesssim C^{\max}_{\text{evolv}}$$

very weak evolution

vs.

$$g_2(b_{\max}) \lesssim C_{\rm evolv}^{\max} \quad \textbf{vs.} \quad g_2(b_{\max}) \geq 0.1\,\mathrm{GeV}^2$$



COMMENTS

- Thus, if we demand the Gaussian ansatz in for the form of $g_K(b_T; b_{max})$ for all b_T, then we estimate that the true value of g_2, at least for the kinematics of our fit must lie roughly in the range of $0 < g_2 < 0.03$ GeV^2.
- Because of the strong universality of $g_K(b_T; b_{max})$, these results seem on the surface to indicate a discrepancy between the low Q data and detailed and successful fits of the past that focus on larger Q, which tend to find $g_2 > 0.1$ GeV^2

Setup g_K to respect *DY* fits

$$g_K(b_T; b_{\max}) = \frac{g_2(b_{\max}) b_{\rm NP}^2}{2} \ln\left(1 + \frac{b_T^2}{b_{\rm NP}^2}\right)$$

$$b_T \ll b_{\rm NP}$$

$$g_K(b_T; b_{\max}) \approx g_2(b_{\max}) \frac{1}{2} b_T^2 - g_2(b_{\max}) \frac{1}{4 b_{\rm NP}^2} b_T^4 + \ldots$$

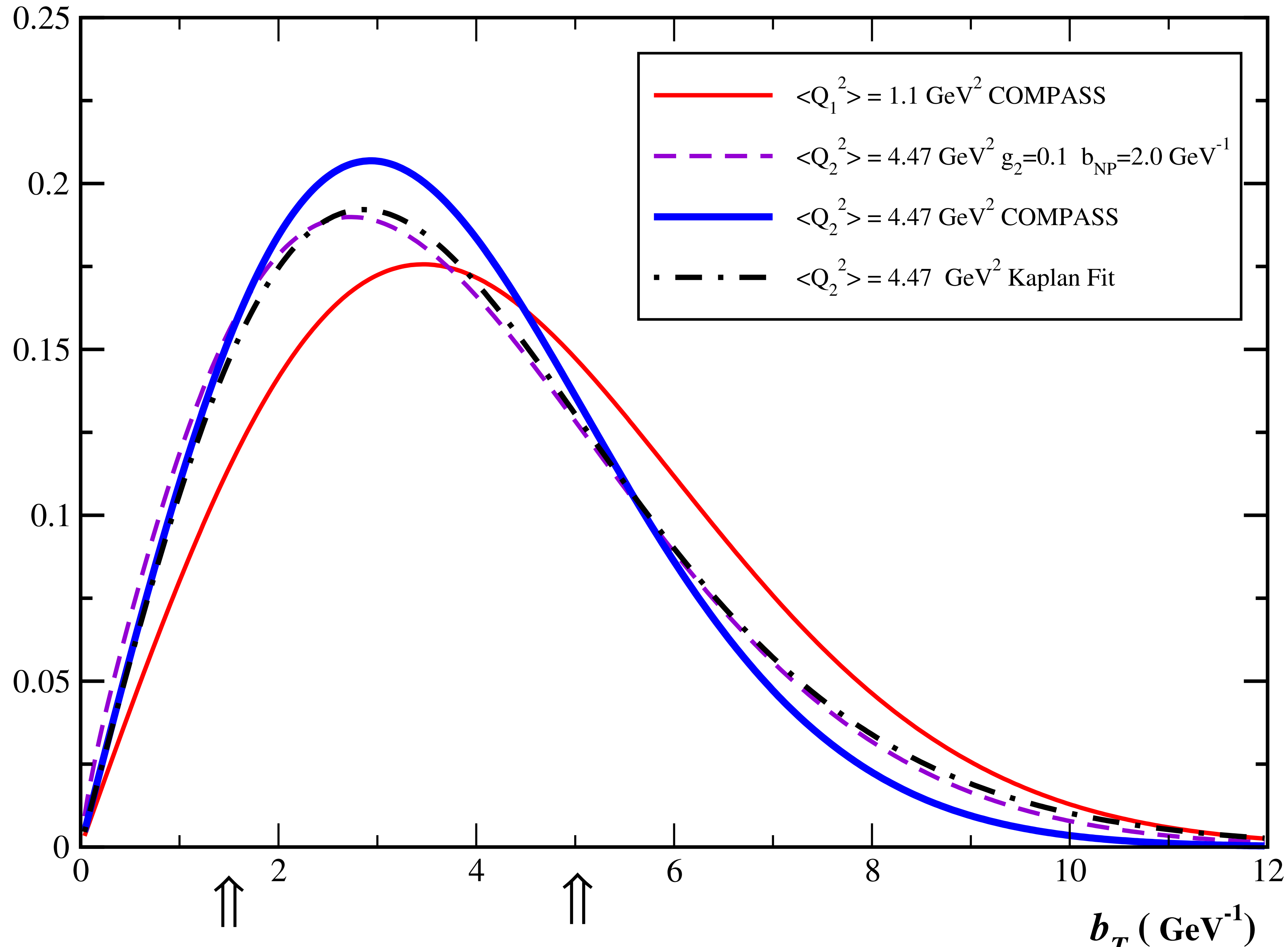
$b_{\max} = 0.5\,\text{GeV}$, $g_2 = 0.1\,\text{GeV}^2$ and $b_{\rm NP} = 2.0\,\text{GeV}^{-1}$

P. M. Nadolsky, D. Stump, and C. Yuan, Phys. Rev. D **61**, 014003 (1999).

P. M. Nadolsky, D. Stump, and C. Yuan, Phys. Rev. D **64**, 114011 (2001).

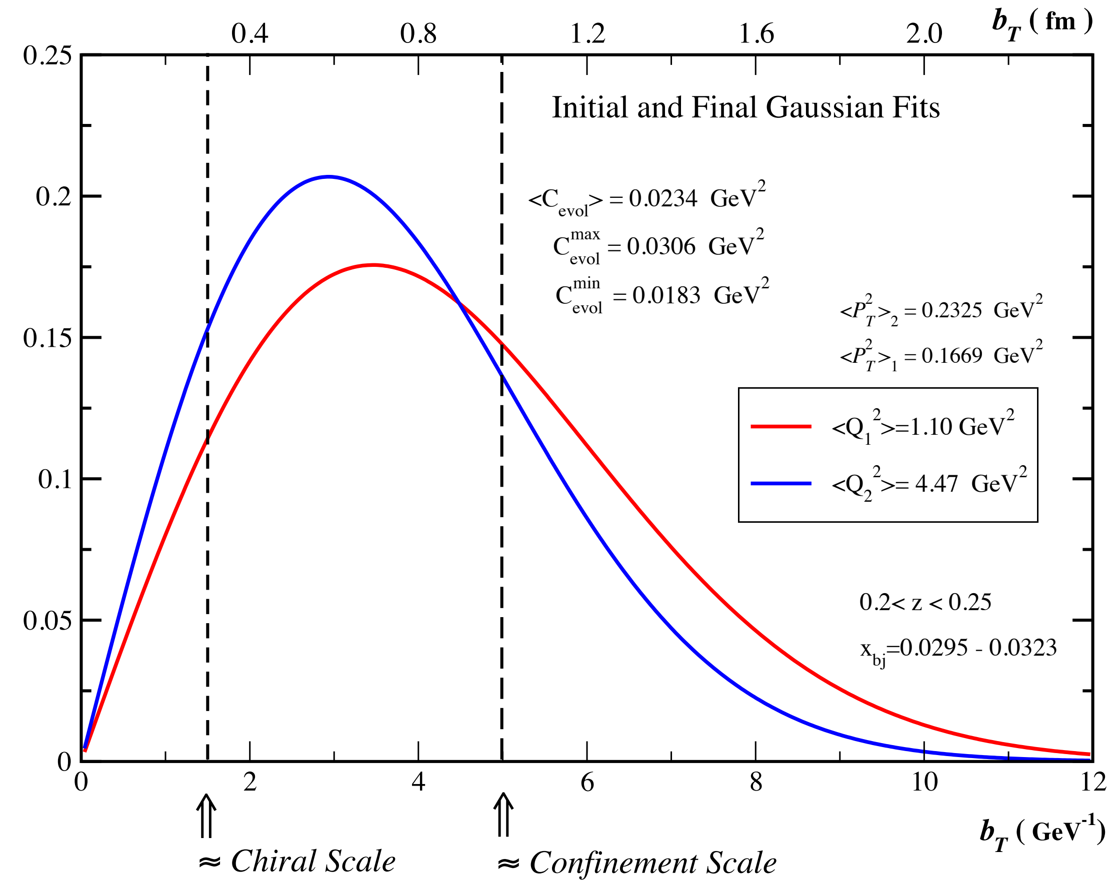
$b_{\max} = 0.5\,\mathrm{GeV}$, $g_2 = 0.1\,\mathrm{GeV}^2$ and $b_{\mathrm{NP}} = 2.0\,\mathrm{GeV}^{-1}$

$$\frac{b_T}{N(Q)} \exp\left\{ -g_{\mathrm{PDF}}(x, b_T; b_{\max}) - g_{\mathrm{FF}}(z, b_T; b_{\max}) - 2 g_K(b_T; b_{\max}) \ln\left(\frac{Q}{Q_0}\right) + 2 \ln\left(\frac{Q}{\mu_b}\right) \tilde{K}(b_*; \mu_b) + \int_{\mu_b}^{Q} \frac{d\mu'}{\mu'} \left[\gamma_{\mathrm{PDF}}(\alpha_s(\mu'); 1) + \gamma_{\mathrm{FF}}(\alpha_s(\mu'); 1) - 2 \ln\left(\frac{Q}{\mu'}\right) \gamma_K(\alpha_s(\mu')) \right] \right\}$$

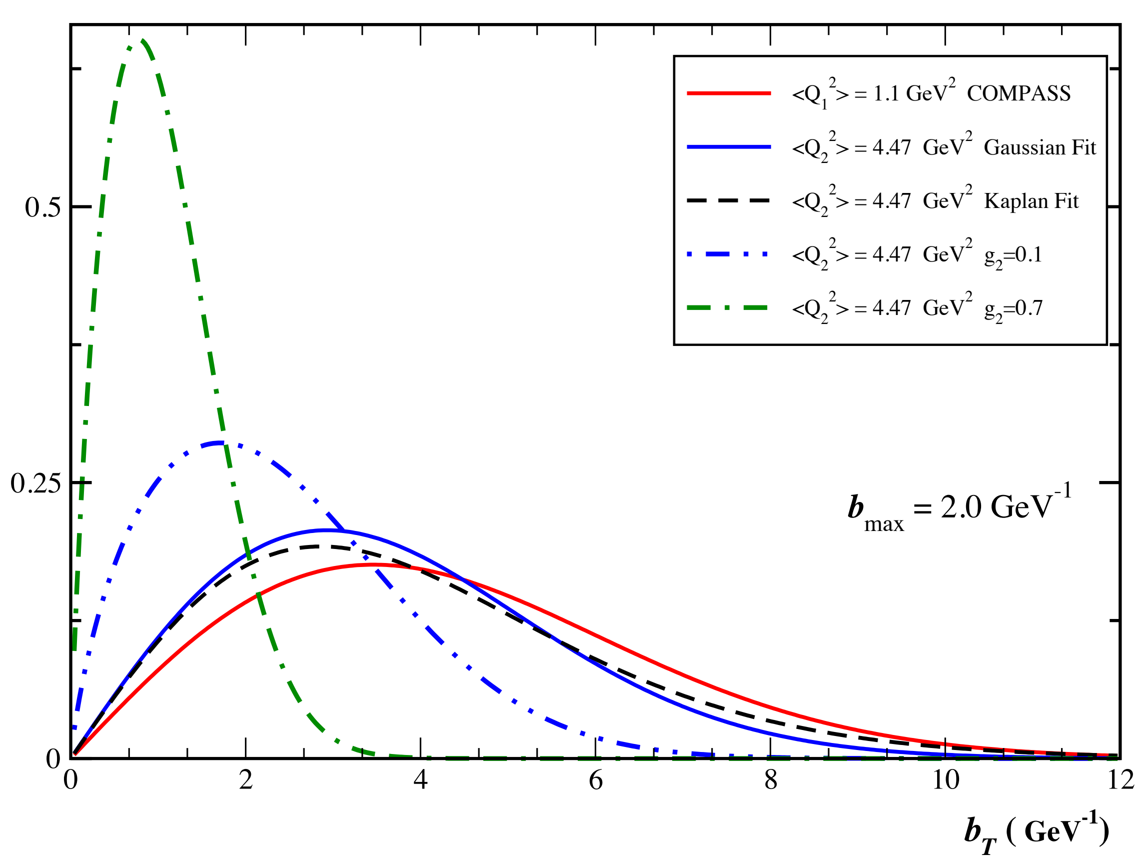


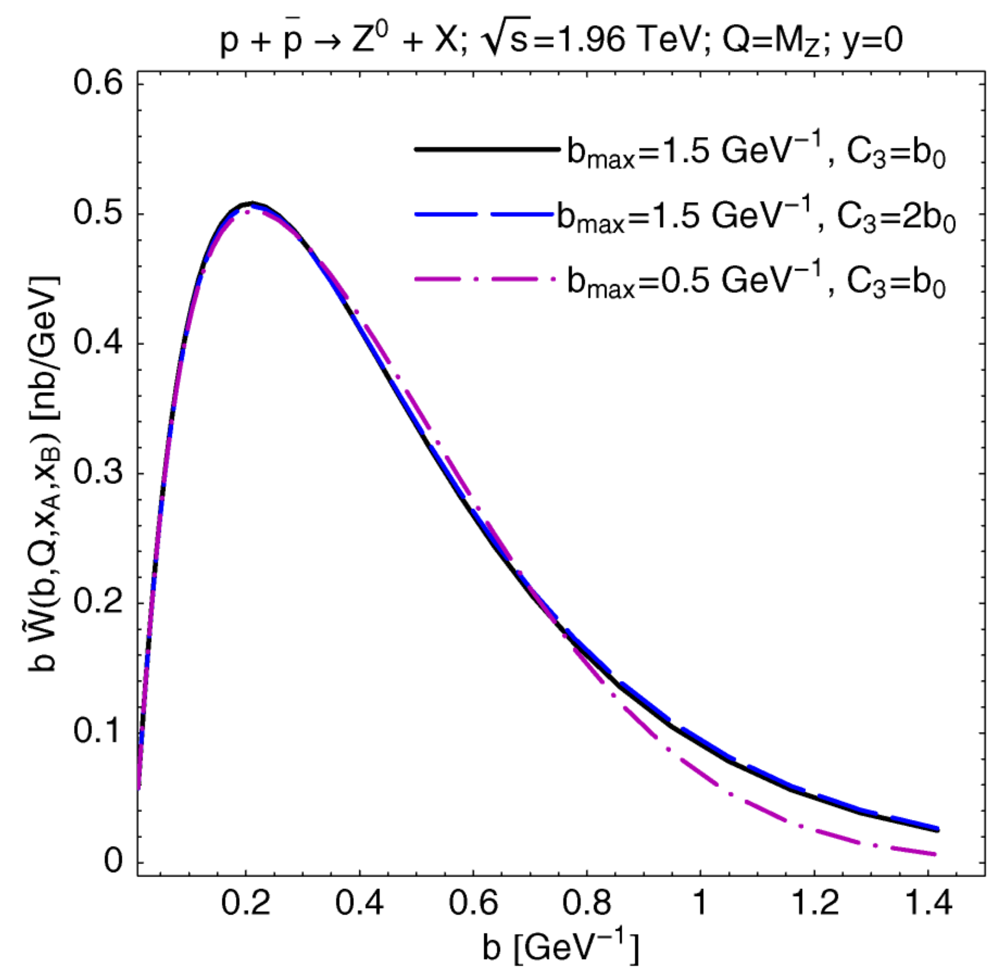
⇑ ≈ *Chiral Scale*

⇑ ≈ *Confinement*



See, for example, Fig. of Konychev and Nadolsky and compare this with Fig. 3, where contributions from bT < 2.0 GeV^{-1} dominate.





Comments Factorization

- This strong form of universality is, an important basic test of the TMD factorization theorem. It is related to the soft factors—the vacuum expectation values of Wilson loops—that are needed in the TMD definitions for consistent factorization with a minimal number of arbitrary cutoffs.

- Constraining the nonperturbative component of the evolution probes fundamental aspects of soft QCD.

- CSS/JCC TMD-factorization formalism is tailored to the treatment of the individual, well-defined operator definitions for the TMDs, and it maps directly onto the partonic picture displayed in the TMD factorization

Conclusions

- Even with the small variations in Q discussed in this paper, however, one is able to constrain general properties of $g_K(b_T; b_{\max})$
- That the data are atrelatively low Q helps especially to constrain the form ofthe nonperturbative evolution function $g_K(b_T; b_{\max})$
- We find much greater sensitivity to the details of NP large b_{T} structure rather than evidence that nonperturbative contributions to evolution are unnecessary
- By accounting for nonperturbative behavior from at large b_{T} we find it is not difficult to reconcile past large Q fits e.g. from DY and SIDIS data

What about *Y*-term matching
$M << p_T << Q$???

- Problems seen Sun et. al, arXiv 1406.3073, Boglione et al JHEP 2015

$$d\sigma = W_{\rm TMD}(P_T, Q) + Y(P_T, Q) + O\left(\frac{\Lambda}{Q}\right)^c d\sigma$$

work in progress ...

Extras

★ CS has simple *S/T* interpretation--multipole expansion in terms of $b_T\,[\mathrm{GeV}^{-1}]$ conjugate to $\boldsymbol{P}_{h\perp}$

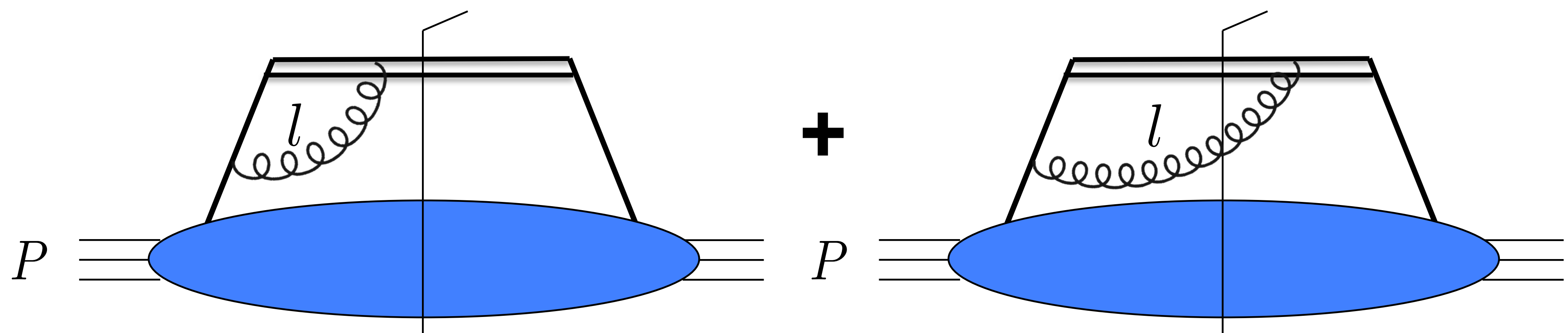
Boer, Gamberg,Musch,Prokudin JHEP 2011

$$
\begin{aligned}
\frac{d\sigma}{dx_B\, dy\, d\phi_S\, dz_h\, d\phi_h\, |\boldsymbol{P}_{h\perp}| d|\boldsymbol{P}_{h\perp}|} = {} &
\frac{\alpha^2}{x_B y Q^2} \frac{y^2}{(1-\varepsilon)} \left(1+\frac{\gamma^2}{2x_B}\right) \int \frac{d|\boldsymbol{b}_T|}{(2\pi)} |\boldsymbol{b}_T| \Big\{ J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{F}_{UU,T} + \varepsilon\, J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{F}_{UU,L} \\
& + \sqrt{2\,\varepsilon(1+\varepsilon)}\, \cos\phi_h\, J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{F}_{UU}^{\cos\phi_h} + \varepsilon \cos(2\phi_h)\, J_2(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{F}_{UU}^{\cos(2\phi_h)} \\
& + \lambda_e \sqrt{2\,\varepsilon(1-\varepsilon)}\, \sin\phi_h\, J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{F}_{LU}^{\sin\phi_h} \\
& + S_\parallel \Big[\sqrt{2\,\varepsilon(1+\varepsilon)}\, \sin\phi_h\, J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{F}_{UL}^{\sin\phi_h} + \varepsilon \sin(2\phi_h)\, J_2(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{F}_{UL}^{\sin 2\phi_h} \Big] \\
& + S_\parallel \lambda_e \Big[\sqrt{1-\varepsilon^2}\, J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{F}_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\, \cos\phi_h\, J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{F}_{LL}^{\cos\phi_h} \Big] \\
& + |\boldsymbol{S}_\perp| \Big[\sin(\phi_h-\phi_S)\, J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|) \left(\mathcal{F}_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\, \mathcal{F}_{UT,L}^{\sin(\phi_h-\phi_S)} \right) \\
& \quad + \varepsilon\, \sin(\phi_h+\phi_S)\, J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{F}_{UT}^{\sin(\phi_h+\phi_S)} \\
& \quad + \varepsilon\, \sin(3\phi_h-\phi_S)\, J_3(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{F}_{UT}^{\sin(3\phi_h-\phi_S)} \\
& \quad + \sqrt{2\,\varepsilon(1+\varepsilon)}\, \sin\phi_S\, J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{F}_{UT}^{\sin\phi_S} \\
& \quad + \sqrt{2\,\varepsilon(1+\varepsilon)}\, \sin(2\phi_h-\phi_S)\, J_2(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{F}_{UT}^{\sin(2\phi_h-\phi_S)} \Big] \\
& + |\boldsymbol{S}_\perp| \lambda_e \Big[\sqrt{1-\varepsilon^2}\, \cos(\phi_h-\phi_S)\, J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{F}_{LT}^{\cos(\phi_h-\phi_S)} \\
& \quad + \sqrt{2\,\varepsilon(1-\varepsilon)}\, \cos\phi_S\, J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{F}_{LT}^{\cos\phi_S} \\
& \quad + \sqrt{2\,\varepsilon(1-\varepsilon)}\, \cos(2\phi_h-\phi_S)\, J_2(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{F}_{LT}^{\cos(2\phi_h-\phi_S)} \Big] \Big\} \qquad .)
\end{aligned}
$$

unpolarized

Sivers

Factorization and Lightcone Divergences



$$\int_0^\infty dl^+$$

- Divergent contribution at $l^+ = 0$.
- Cancelation in the integral over all l_t.
- What if we don't integrate?

Evolved TMD formalism for entire range of P_T

$$\frac{d\sigma}{dP_T^2} \propto \mathcal{H}(\alpha_s(Q)) \int d^2 b_T e^{i b_T \cdot P_T} \; \tilde{F}_{H_1}(x, b_T; Q, Q^2) \, \tilde{D}_{H_2}(z, b_T; Q, Q^2) \;\; + \;\; Y_{\rm SIDIS}$$

$$\frac{d\sigma}{dP_T^2} \propto \; {\rm F.T.} \exp\left\{ -g_{\rm PDF}(x, b_T; b_{\rm max}) - g_{\rm FF}(z, b_T; b_{\rm max}) - 2 g_K(b_T; b_{\rm max}) \ln\left(\frac{Q}{Q_0}\right) + \right.$$

$$\left. + \; 2\ln\left(\frac{Q}{\mu_b}\right) \tilde{K}(b_*; \mu_b) + \int_{\mu_b}^{Q} \frac{d\mu'}{\mu'} \left[\gamma_{\rm PDF}(\alpha_s(\mu'); 1) + \gamma_{\rm FF}(\alpha_s(\mu'); 1) - 2\ln\left(\frac{Q}{\mu'}\right) \gamma_K(\alpha_s(\mu')) \right] \right\}$$

$$+ \; Y_{\rm SIDIS}$$

$$g_{\rm PDF}(x, b_T; b_{\rm max}) \equiv g_1(x, b_T; b_{\rm max}) \; - \; \ln(\tilde{F}_{H_1}(x, b_*; \mu_b, \mu_b^2))$$