Resummation in b- or p_T-space.

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Outline

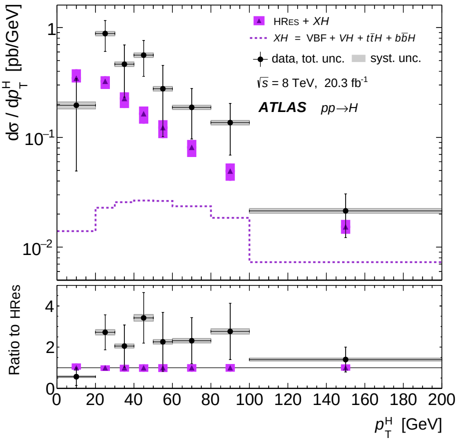
Introduction.

Motivation: Higgs p_T spectrum.

Status of Higgs measurements:

- Important observable of the LHC Higgs program
- Theory and experiment are ~compatible
- Large uncertainties currently limited by statistics
- $\sim$20 times more data in the LHC run 2

  $\rightarrow$ Theory uncertainties important soon



[ATLAS]

Need precision predictions for the Higgs p_T-spectrum.

Motivation: Higgs p_T spectrum.

Theory status:

- Spectrum contains large logarithms $\ln \frac{p_T}{m_H}$
- Need to be resummed to all orders for $p_T \ll m_H$
- Current results:
  - [de Florian, Ferrera, Grazzini, Tommasini '12]: HRes (NNLO + NNLL)
  - [Becher, Lübbert, Neubert, Wilhelm]: CuTe (NNLO + N^3LL_{partial})
  - [Neill, Rothstein, Vaidya '15] (NNLO + NNLL)
  - [Echevarria, Kasemets, Mulders, Pisano '15] (NNLL)
  - Resummation is performed (partially) in impact parameter space: Resums logarithms $\ln(b\, m_H)$ instead of $\ln \frac{p_T}{m_H}$

Central question:

Is it possible to perform resummation directly in p_T-space?

Schematic overview of p_T-resummation.

- DDT-formula [Dokshitzer, Dyakonov, Troyan '80]:

$$\mathcal{S}(m_H, p_T) = -\int_{p_T}^{m_H} \frac{\mathrm{d}\mu'}{\mu'} \left[A(\alpha_s(\mu')) \ln \frac{m_H^2}{\mu'^2} + B(\alpha_s(\mu')) \right]$$

  - Exponentiates $\ln \frac{p_T}{m_H}$ directly
  - Only accurate to (N?)LL

- CSS-formula [Collins, Soper, Sterman '85]

$$\tilde{\mathcal{S}}(m_H, b) = -\int_{1/b}^{m_H} \frac{\mathrm{d}\mu'}{\mu'} \left[\tilde{A}(\alpha_s(\mu')) \ln \frac{m_H^2}{\mu'^2} + \tilde{B}(\alpha_s(\mu')) \right]$$

  - Based on impact parameter space $b \sim 1/p_T$ [Parisi, Petronzio '79]
  - Exponentiates $\ln \frac{p_T}{m_H}$ indirectly through $\ln(b\, m_H)$

- SCET [Becher, Neubert '10; Chiu, Jain, Neill, Rothstein '12; Echevarria, Idilbi, Scimemi '12]
  - Very general setup
  - Allows to rederive CSS-formalism
  - Typically carried out in impact parameter space

The b-space formalism.

Advantages of b-space: [Parisi, Petronzio '79]

- Easy calculation
- Claim: Allows calculation for $p_T \to 0$ (?)

Technical difficulties: [Ellis, Veseli '98; Frixione, Nason, Ridolfi '99; Kulesza, Stirling '99]

- Matching onto fixed order:

$$\frac{d\sigma}{dp_T} = \underbrace{\frac{d\sigma^{\text{resummed}}}{dp_T}}_{b\text{-space}} + \underbrace{\frac{d\sigma^{\text{non-sing}}}{dp_T}}_{p_T\text{-space}}$$

- b-integral hits Landau pole $\alpha_s(1/b)$
  $\to b_*$-prescription [Collins, Soper, Sterman '85]

Our motivation: conceptual issues

- Resummation of $\ln(b\,m_H)$ rather than $\ln \frac{p_T}{m_H}$
  $\to$ Implication on theory uncertainty?
- Theory should allow direct resummation in p_T-space

Goal: Attempt resummation directly in momentum space.

Transverse momentum distribution in SCET.

Overview: Resummation.

(Non)singular cross section:

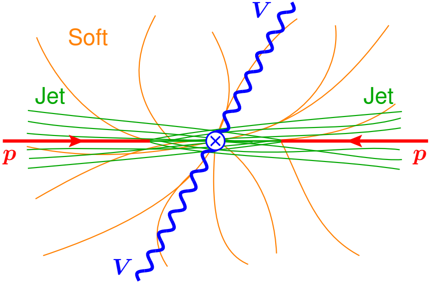
- Split cross section: $\sigma = \sigma^{\text{sing}} + \sigma^{\text{non-sing}}$
- Nonsingular piece: $\sigma^{\text{non-sing}} \sim \ln \frac{p_T^2}{m_H^2} + \ldots$
  - Integrable for $p_T \to 0$
  - Obtained from fixed-order calculations
  - Irrelevant for present discussion
- Singular piece: $\sigma^{\text{sing}} \sim \frac{1}{p_T^2} + \frac{1}{p_T^2} \ln \frac{p_T^2}{m_H^2} + \ldots$
  - Divergent for $p_T \to 0$
  - Logarithms $\ln \frac{p_T}{m_H}$ must be resummed to all orders
  - Focus of present discussion

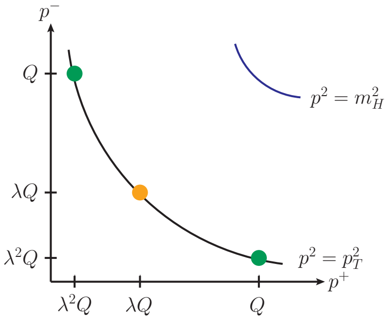
Resummation of singular piece:

- Collins-Soper-Sterman equation (CSS)
- Soft-Collinear Effective Theory (SCET)
  $\to$ We focus on SCET

Higgs production in SCET.

Illustration of factorization in SCET:





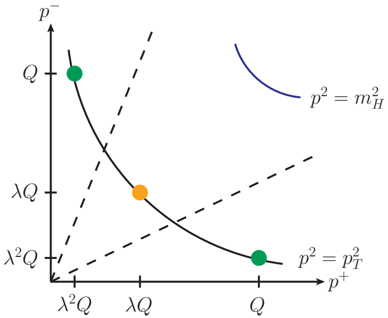
$$\sigma = H(B_1 \otimes B_2 \otimes S)(\vec{p}_T)$$

- Hard function: Higgs produced $p^2 \sim m_H^2, \; p \sim Q(1,1,1)$
- Beam function: collinear radiation $p^2 \sim p_T^2, \quad p \sim Q(1,\lambda^2,\lambda)$
- Soft function: isotropic radiation $p^2 \sim p_T^2, \quad p \sim Q(\lambda,\lambda,\lambda)$

The rapidity regulator.

Rapidity divergences:

- Beam and soft momenta overlap: Mix under Lorentz boosts
- New regulator needed



Rapidity regulators:

- Induces new scale ν (in CSS: ζ)
- Associated anomalous dimension: $\gamma^{(\nu)}$ (in CSS: evolution factor $\tilde{K}$)
- Gives rise to the *rapidity renormalization group*
- ν is associated with rapidity logarithms $\ln \frac{p_T}{m_H}$:
  $\rightarrow$ Allows to resum these logs to all orders

Interlude: Rapidity regulators.

Many equivalent regulators available:

- Non-light like axial gauge [Collins, Soper '81]
- Wilson lines off light-cone [Collins '11]
- Modify propagators through α-regulator [Becher, Neubert '10]
- Modify propagators through δ-regulator [Echevarria, Idilbi, Scimemi '12]
- Modify Wilson lines through η-regulator [Chiu, Jain, Neill, Rothstein '11]

Our choice: The η-regulator [Chiu, Jain, Neill, Rothstein '11]

- Rapidity regulator modifies Wilson lines:

$$W_n = \sum_{\text{perms}} \exp\left[-\frac{g}{\bar{n}\cdot\mathcal{P}} w^2 \frac{|\bar{n}\cdot\mathcal{P}_g|^{-\eta}}{\nu^{-\eta}} \bar{n}\cdot A_n \right] ,$$

$$S_n = \sum_{\text{perms}} \exp\left[-\frac{g}{n\cdot\mathcal{P}} w \frac{|2\mathcal{P}_{g3}|^{-\eta/2}}{\nu^{-\eta/2}} \bar{n}\cdot A_s \right]$$

- Preserves gauge invariance
- Consistent with non-Abelian exponentiation
- ν, η correspond to μ, ϵ in dimensional regularization

RG structure of the cross section.

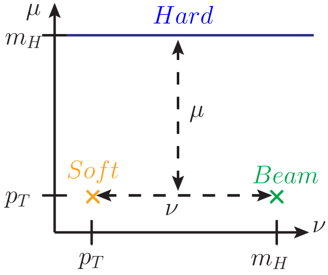
- After regularizing rapidity divergences:

$$\sigma = H(\mu)\big[B_1(\mu,\nu) \otimes B_2(\mu,\nu) \otimes S(\mu,\nu)\big](\vec{p}_T)$$

  - μ: Usual renormalization scale
  - ν: Rapidity renormalization scale
- Logarithms are split into

$$\ln^2 \frac{p_T}{m_H} = \ln^2 \frac{m_H}{\mu} + \ln \frac{p_T}{\mu} \ln \frac{\nu^2}{m_H^2} + \ln \frac{p_T}{\mu} \ln \frac{\mu\, p_T}{\nu^2}$$

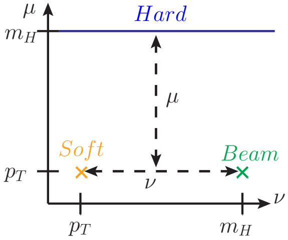
- μ and ν give rise to renormalization group equations (RGE)
- RGEs allow to resum all large logarithms
  - General setup
  - Previous results can be recovered through specific scale choices



RG-evolved cross section.

$$\sigma = \underbrace{U(\mu_H, \mu_B, \mu_S)}_{\mu\text{-evolution}} \underbrace{V(\vec{p}_T, \nu_B, \nu_S)}_{\nu\text{-evolution}}$$

$$\otimes \underbrace{H(\mu_H)\left[B^2(\mu_B, \nu_B) \otimes S(\mu_S, \nu_S)\right]}_{\text{Evaluated at canonical scales}}(\vec{p}_T)$$



- Canonical scale choice:

$$\mu_H = m_H \qquad \mu_B = p_T \qquad \mu_S = p_T$$

$$\nu_B = m_H \qquad \nu_S = p_T$$

- Deviating scale choices in literature:
  - [de Florian, Ferrera, Grazzini, Tommasini '12]: CSS-formula, corresponding to $\mu_B = \mu_S = \nu_S = 1/b$
  - [Neill, Rothstein, Vaidya '15]: Choose $\mu_B = \mu_S = \nu_S = 1/b$
  - [Becher, Neubert, Wilhelm '13]: Corresponds to $\mu_B = \mu_S = q_* + p_T, \qquad \nu_S = 1/b$

Impact parameter space

Goal: Use canonical scale choices in momentum space.

The rapidity evolution factor.

$$\sigma = \underbrace{U(\mu_H, \mu_B, \mu_S)}_{\mu\text{-evolution}} \underbrace{V(\vec{p}_T, \nu_B, \nu_S)}_{\nu\text{-evolution}} \otimes \underbrace{H\left[B_1 \otimes B_2 \otimes S\right](\vec{p}_T)}_{\text{Evaluated at canonical scales}}$$

- $H\left[B_1 \otimes B_2 \otimes S\right]$: Calculable in fixed-order SCET ✓
- μ-evolution ✓

$$U(\mu_H, \mu_B, \mu_S) = \exp\left[\int_{\mu_H}^{\mu} \frac{\mathrm{d}\mu'}{\mu'} \gamma_H^{(\mu)}(\mu') + 2\int_{\mu_B}^{\mu} \frac{\mathrm{d}\mu'}{\mu'} \gamma_B^{(\mu)}(\mu') + \int_{\mu_S}^{\mu} \frac{\mathrm{d}\mu'}{\mu'} \gamma_S^{(\mu)}(\mu')\right]$$

- ν-evolution ?

$$\nu \frac{\mathrm{d}V(\vec{p}_T, \nu, \nu_0)}{\mathrm{d}\nu} = \gamma^{(\nu)}(\vec{p}_T, \mu) \otimes V(\vec{p}_T, \nu, \nu_0) \quad \Rightarrow \quad V(\vec{p}_T, \nu_B, \nu_S) = ?$$

Only complication:

Momentum space solution of rapidity RGE.

Rapidity evolution.

Rapidity evolution.

Impact parameter space:

- Problem of resumming the p_T-spectrum reduced to solving

$$\nu \frac{\mathrm{d}V(\vec{p}_T, \nu, \nu_0)}{\mathrm{d}\nu} = \int \frac{\mathrm{d}^2\vec{q}_T}{(2\pi)^2} \gamma^{(\nu)}(\vec{p}_T - \vec{q}_T, \mu) V(\vec{q}_T, \nu, \nu_0)$$

- Easily solved after Fourier transformation (*Impact Parameter Space*):

$$\nu \frac{\mathrm{d}V(b, \nu, \nu_0)}{\mathrm{d}\nu} = \gamma^{(\nu)}(b, \mu) V(b, \nu, \nu_0)$$

$$\Rightarrow \qquad V(b, \nu_B, \nu_S) = \exp\left[\gamma^{(\nu)}(b, \mu) \ln \frac{\nu_B}{\nu_S}\right]$$

- p_T-space solution:

$$V(\vec{p}_T, \nu_B, \nu_S) = \int \mathrm{d}^2\vec{b}\, e^{i\vec{p}_T \cdot \vec{b}} \exp\left[\gamma^{(\nu)}(b, \mu) \ln \frac{\nu_B}{\nu_S}\right]$$

Rapidity evolution.

Example: 1-loop

- ν-anomalous dimension:

$$\gamma^{(\nu)}(b,\mu) = -\frac{2C_A\alpha_s}{\pi}\ln\left(\frac{b^2\mu^2 e^{2\gamma_E}}{4}\right)$$

- p_T-space solution:

$$V(\vec{p}_T,\nu_B,\nu_S) = \int \mathrm{d}^2\vec{b}\, e^{i\vec{p}_T\cdot\vec{b}}\left(\frac{b^2\mu^2 e^{2\gamma_E}}{4}\right)^{-\omega_s}$$

$$\propto e^{-2\gamma_E\omega_s}\frac{\Gamma(1-\omega_s)}{\Gamma(1+\omega_s)}$$

where

$$\omega_s = 2\frac{C_A\alpha_s(\mu)}{\pi}\ln\frac{\nu_B}{\nu_S} \sim 2\frac{C_A\alpha_s(p_T)}{\pi}\ln\frac{m_H}{p_T} = \mathcal{O}(1)$$

- p_T-space solution diverges for $\omega_s \to 1$

Rapidity evolution.

CSS solution of the divergence:

- Divergence from $b \to 0$:

$$V(\vec{p}_T, \nu_B, \nu_S) = \int \mathrm{d}^2\vec{b}\, e^{i\vec{p}_T\cdot\vec{b}} \left(\frac{b^2\mu^2 e^{2\gamma_E}}{4}\right)^{-\omega_s} \sim \int \frac{\mathrm{d}b^2}{(\mu^2 b^2)^{\omega_s}}$$

- Simple solution (CSS): $\mu \sim 1/b$
- Resums logarithms $\ln(b\, m_H)$ instead of $\ln \frac{p_T}{m_H}$
  (In principle fine since $b \sim p_T^{-1}$; in practice affects estimating uncertainties)

Goal:

Solve problem of $b \to 0$ divergence for $\mu \sim p_T$

- Previous attempts (CSS): [Ellis, Veseli '98], [Kulesza, Stirling '99]
- Previous attempts (SCET): [Becher, Neubert, Wilhelm '12]

Rapidity evolution.

Solution without b-space:

- Iterative solution to $\nu \frac{\mathrm{d}V(\vec{p}_T,\nu,\nu_0)}{\mathrm{d}\nu} = \gamma^{(\nu)}(\vec{p}_T,\mu) \otimes V(\vec{p}_T,\nu,\nu_0)$:

$$V(\vec{p}_T,\nu_B,\nu_S) = (2\pi)^2\delta^2(\vec{p}_T) + \sum_{n=1}^{\infty} \frac{1}{n!} \ln^n\left(\frac{\nu_B}{\nu_S}\right) (\gamma^{(\nu)}\otimes^n)(\vec{p}_T)$$

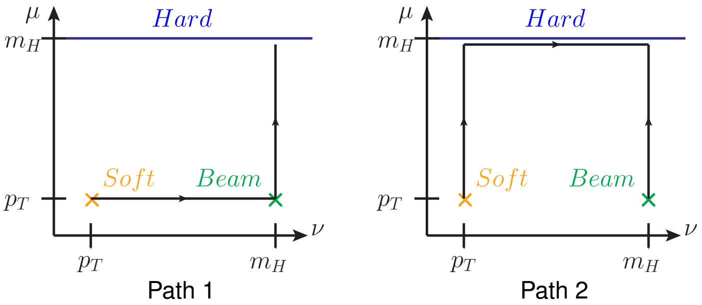
- Multiple convolutions:

$$(\gamma^{(\nu)}\otimes^n)(\vec{p}_T) \propto \underbrace{n\,\mathcal{L}_{n-1}\left(\frac{p_T^2}{\mu^2}\right)}_{\text{leading log (?)}} \underbrace{- \psi^{(2)}\mathcal{L}_{n-4}\left(\frac{p_T^2}{\mu^2}\right) + \ldots}_{\text{subleading log(?)}}$$

- Removing "subleading logs" by hand removes divergence $\Gamma(1-\omega_s)$
(Similar to approach in [Ellis, Veseli '98])
- Conclusion: Divergent part due to incorrectly captured subleading logs
(Already noted in [Frixione, Nason, Ridolfi '99])
- Problems:
  - Consistent log counting?
  - Generalization to higher orders?

Consistency relation.

- Freedom to evolve in (μ, ν)-space:



- Expressed through commutativity $[\mathrm{d}/\mathrm{d}\mu, \mathrm{d}/\mathrm{d}\nu] = 0$
- Induced consistency relation:

$$\mu \frac{\mathrm{d}\gamma^{(\nu)}(\vec{q}_T, \mu)}{\mathrm{d}\mu} = -(4\pi)^2 \Gamma_{\mathrm{C}}[\alpha_s(\mu)] \delta^2(\vec{q}_T)$$

- Naively irrelevant when evolving along path 1

Consistency relation.

- Consistency relation in impact parameter space:

$$\mu \frac{d\gamma^{(\nu)}(b,\mu)}{d\mu} = -4\Gamma_{\mathrm{C}}[\alpha_s(\mu)]$$

- Solving consistency resums logarithms $\ln(b\,\mu)$ *inside* $\gamma^{(\nu)}$
  $\to$ should correctly capture subleading logs
- Simple solution in b-space:

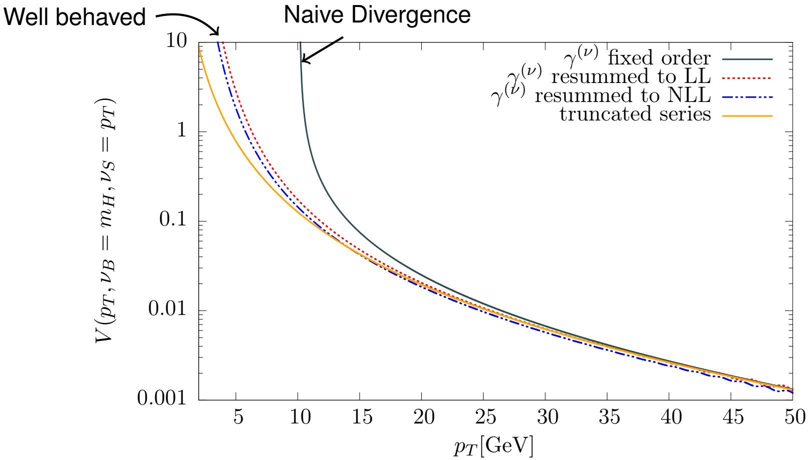
$$\underbrace{\gamma^{(\nu)}(b,\mu)}_{\text{resummed anom dim}} = -4\int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'}\Gamma_{\mathrm{C}}[\alpha_s(\mu')] + \underbrace{\gamma^{(\nu)}(b,\mu_b)}_{\text{calculable in fixed order}}$$

  - Regulates the $b \to 0$ divergence ✓
  - Introduces *new scale* $\mu_b \sim 1/b$!
  - At large b: Evolution kernel is non-perturbative [Collins, Soper, Sterman '85]

- Direct solution in p_T-space nontrivial due to distributional character
  $\to$ work in progress

Rapidity evolution.

Comparison of current results:



- Rapidity evolution factor well behaved for $\boldsymbol{p_T} \gtrsim \boldsymbol{\Lambda}_{\text{QCD}}$
- $\Rightarrow$ Resummation of $\gamma^{(\nu)}$ allows scale choice in momentum space ✓

Conclusion.

Conclusion.

Transverse momentum spectra:

- Large logarithms $\ln \frac{p_T}{m_H}$ require resummation for $p_T \ll m_H$
- Standard approach: scale setting in impact parameter space
  - Resums logarithms $\ln(b\, m_H)$ instead of $\ln \frac{p_T}{m_H}$
- SCET allows scale setting in momentum space:
  - Directly resums logarithms $\ln \frac{p_T}{m_H}$
  - Requires resummation of anomalous dimension $\gamma^{(\nu)}$ to correctly capture subleading logarithms

Open tasks:

- Resum $\gamma^{(\nu)}$ directly in p_T-space
- Numerical comparison to scale choices in literature
- Numerical study of uncertainties

Thank you for your attention!

Backup slides.

Higgs production in SCET

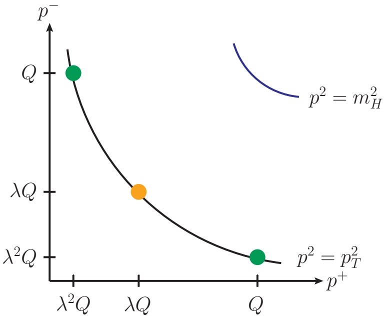
Light-cone coordinates

- Reference vector: $n^\mu, \bar{n}^\mu = (1, 0, 0, \pm 1)$
- Split any momentum into

$$p^\mu = \underbrace{p^+ n^\mu}_{\text{along } \hat{z}} + \underbrace{p^- \bar{n}^\mu}_{\text{along } -\hat{z}} + \vec{p}_\perp \quad \to \quad p = (p^+, p^-, \vec{p}_T)$$

Relevant modes:

- Classify modes using $\lambda \sim \frac{p_T}{m_H}$
- Hard mode:
  $p^2 \sim m_H^2, \;\; p \sim Q(1, 1, 1)$
- n-collinear mode:
  $p^2 \sim p_T^2, \quad p \sim Q(1, \lambda^2, \lambda)$
- $\bar{n}$-collinear mode:
  $p^2 \sim p_T^2, \quad p \sim Q(\lambda^2, 1, \lambda)$
- Soft mode:
  $p^2 \sim p_T^2, \quad p \sim Q(\lambda, \lambda, \lambda)$



TMDPDF in SCET.

- Bare TMDPDF:

$$B_g^{\mathrm{B}\mu\nu}(\omega = zp_n, \vec{p}_T)$$
$$= \omega \langle P(p_n)| \operatorname{Tr}\{\mathcal{B}^\mu_{n,\perp}(0)\delta(\omega - \overline{\mathcal{P}}_n)\delta^2(\vec{p}_T - \mathcal{P}_{\perp n})\mathcal{B}^\nu_{n,\perp}(0)\} |P(p_n)\rangle$$

where $\mathcal{B}^\mu_{n,\perp}$ are collinear gluon fields in SCET

- Renormalization:

$$B_g^{\mathrm{B}\mu\nu}(z, \vec{p}_T) = Z^{\mathrm{B}}(\mu, \omega/\nu, \vec{p}_T) \otimes B_g^{\mu\nu}(z, \vec{p}_T, \mu, \omega/\nu)$$

- RGEs:

$$\mu \frac{B(z, \vec{p}_T, \mu, \omega/\nu)}{\mathrm{d}\mu} = \gamma_B^\mu(\mu, \omega/\nu) B(z, \vec{p}_T, \mu, \omega/\nu)\,,$$
$$\nu \frac{B(z, \vec{p}_T, \mu, \omega/\nu)}{\mathrm{d}\nu} = -\frac{1}{2}\gamma^{(\nu)}(\vec{p}_T, \mu) \otimes B(z, \vec{p}_T, \mu, \omega/\nu)\,.$$

- Matching onto PDFs:

$$B_g^{\rho\sigma}(z, \vec{p}_T, \mu, \omega/\nu) = \sum_k I_{gk}^{\rho\sigma}(z, \vec{p}_T, \mu, \omega/\nu) \otimes f_k(z, \mu)$$

Soft function in SCET.

- Bare soft function:

$$S(\vec{p}_T) = \frac{1}{N_c^2 - 1} \langle 0| \,\mathrm{Tr}\{\bar{T}[S^\dagger_{n\perp}(0) S_{\bar{n}\perp}(0)] \delta^2(\vec{p}_T - \mathcal{P}_{\perp s}) T[S^\dagger_{\bar{n}\perp}(0) S_{n\perp}(0)]\}$$

  where $S_{\bar{n}\perp}$ is a soft Wilson line

- Renormalization:

$$S^{\mathrm{B}}(\vec{p}_T) = Z^{\mathrm{S}}(\mu, \mu/\nu, \vec{p}_T) \otimes S(\vec{p}_T, \mu, \mu/\nu)$$

- RGEs:

$$\mu \frac{S(\vec{p}_T, \mu, \mu/\nu)}{\mathrm{d}\mu} = \gamma_S^{(\mu)}(\mu, \mu/\nu) S(\vec{p}_T, \mu, \mu/\nu)$$

$$\nu \frac{S(\vec{p}_T, \mu, \mu/\nu)}{\mathrm{d}\nu} = \gamma^{(\nu)}(\vec{p}_T, \mu) \otimes S(\vec{p}_T, \mu, \mu/\nu)$$