Antenna Subtraction with Hadronic Initial States

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Hamburg, 17.10.2006

Subtraction

$$
\begin{aligned}
d\sigma^{\text{NLO}} &= d\sigma_m^{\text{V+CT}} + d\sigma_{m+1}^{\text{R}} \\
&= \left(d\sigma_m^{\text{V+CT}} + d\sigma_{m+1}^{\text{R,S}}\right) + \left(d\sigma_{m+1}^{\text{R}} - d\sigma_{m+1}^{\text{R,S}}\right)
\end{aligned}
$$

The subtraction term $d\sigma_{m+1}^{\text{R,S}}$

- reproduces the behaviour of the ME in all singular limits
- can be integrated analytically over the unresolved phase space
- does not introduce spurious singularities

Antenna subtraction

Many method available for NLO

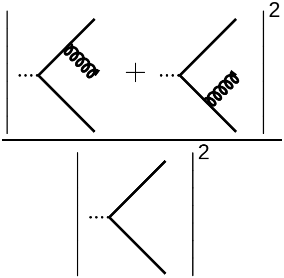
- $\mathcal{E}$ prescription [Frixione,Kunszt,Signer]
- Dipole formalism [Catani,Seymour]
- Antenna formalism [Kosower;Campbell,Cullen,Glover]
- ...

We use antenna functions to construct subtraction term

+ less complicated than dipoles (1 Antenna $\simeq$ 2 Dipoles)
+ extention to NNLO is feasible [Gehrmann-De Ridder,Gehrmann,Glover]
+ antenna functions are constructed from physical matrix elements
+ can be matched to parton shower [Giele,Kosower,Skands]
- no extension to initial state partons (so far)

Antenna functions

$X_{qg\bar{q}} =$

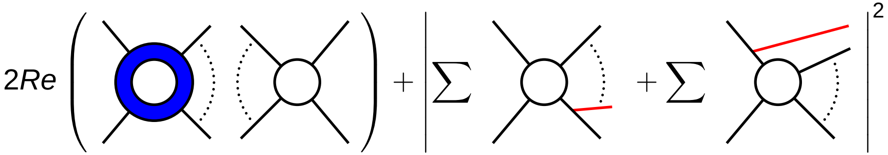


- contains by construction soft and collinear emission between two color connected hard radiators.
- are computed from ME of physical processes

Hadronic reaction

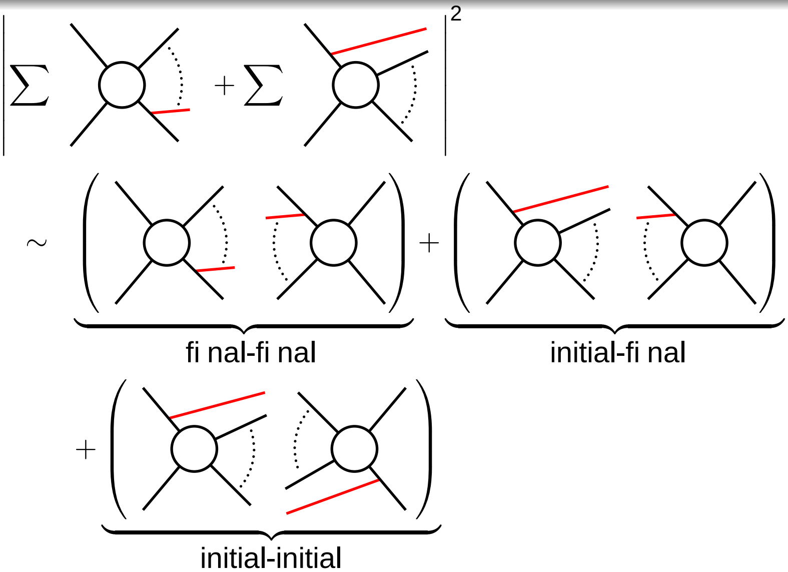
NLO corrections with hadronic initial state

$d\sigma^{\text{NLO}} =$



- final-state radiation → Virtual corrections
- initial-state radiation → Virtual corrections+pdf factorisation

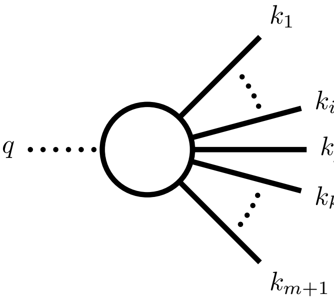
Hadronic reaction



Final-Final

We consider

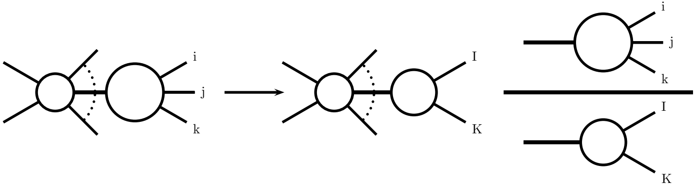
- No partons in the initial state
- Antenna function for hard radiator i, k in the final state
- j unresolved



Final state

ME squared: $\left|\mathcal{M}_{m+1}(k_1,\ldots,k_i,k_j,k_k,\ldots,k_{m+1};p_1,p_2)\right|^2$

Phasespace: $d\Phi_{m+1}(k_1,\ldots,k_i,k_j,k_k,\ldots,k_{m+1};p_1,p_2)$



ME squared, subtraction:
$\sum_j X_{ijk}(k_i,k_j,k_k)\left|\mathcal{M}_m(k_1,\ldots,K_I,K_K,\ldots,k_{m+1},p_1,p_2)\right|^2$

Phasespace:
$d\Phi_X(k_i,k_j,k_k)\times d\Phi_m(k_1,\ldots,K_I,K_K,\ldots,k_{m+1},p_1,p_2)$

Final-Final Subtraction term

Integrated antenna functions

$$\mathcal{X}_{ijk} = d\Phi_X(k_i, k_j, k_k) X_{ijk}(k_i, k_j, k_k) \qquad \mathcal{A}_{qg\bar{q}} = -2\mathbf{I}^{(1)}_{q\bar{q}} + \mathcal{O}(\epsilon^0)$$

For combining with virtual corrections

$$d\sigma^{R,S} = \mathcal{N} \sum_{i,j,k} \mathcal{X}_{ijk} \int d\Phi_m(\ldots, K_I, K_K, \ldots) \, |\mathcal{M}_m(\ldots, K_I, K_K, \ldots)|^2 \times J_m^{(m)}(k_1, \ldots, K_I, K_K, \ldots, k_{m+1})$$

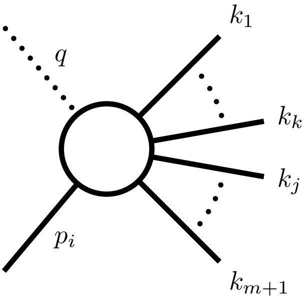
For combining with real corrections

$$d\sigma^{R,S} = \mathcal{N} \int d\Phi_{m+1}(\ldots k_i, k_j, k_l \ldots) \sum_{i,j,k} X_{ijk}(k_i, k_j, k_k) \, |\mathcal{M}_m(\ldots K_I, K_K \ldots)|^2 \times J_m^{(m)}(k_1, \ldots, K_I, K_K, \ldots, k_{m+1})$$

Initial-Final

We consider

- One parton in the initial state
- Antenna function for one initial *i* and one final *k* hard radiator
- j unresolved



Phasespace Mapping

$$\left.\begin{array}{c} p \\ k_j \\ k_k \end{array}\right\} \rightarrow \left\{\begin{array}{c} P = xp \\ K_K \end{array}\right.$$

$$x = \frac{s_{ij} + s_{ik} - s_{jk}}{s_{ij} + s_{ik}}$$

$$K_K = k_k + k_k - xp$$

Limits

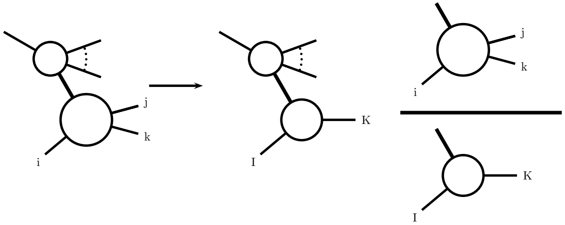
$$xp \rightarrow p, \quad K_K \rightarrow k_k, \quad \text{when j becomes soft}$$

$$xp \rightarrow p, \quad K_K \rightarrow k_j + k_k, \quad \text{when j collinear with } k_k$$

$$xp \rightarrow p - k_j, \quad K_K \rightarrow k_k, \quad \text{when j collinear with } p$$

ME squared: $\left|\mathcal{M}(k_1,\ldots,k_j,k_k,\ldots,k_{m+1};q,p)\right|^2$

Phasespace: $d\Phi_{m+1}(k_1,\ldots,k_j,k_k,\ldots,k_{m+1};q,p)$



ME squared, subtraction:

$X_{ijk}(k_j,k_k;p)\,|\mathcal{M}(k_1,\ldots,K_K,\ldots,k_{m+1},q,xp)|^2$

Phasespace:

$d\Phi_X(k_j,k_k;q,p)\mathcal{J}(k_j,k_k,p)d\Phi_m(k_1,\ldots,K_K,\ldots,k_{m+1},q,xp)$

Subtraction Term for Initial-Final

Integrated antenna functions

$$\mathcal{X}_{i,jk}(x) = d\Phi_X(k_j, k_k; p, q) X_{i,jk}(k_j, k_k; p) \mathcal{J}(k_j, k_k; p)$$

$$\mathcal{A}_{q,gq}(x) = -2\mathbf{I}^{(1)}_{q\bar{q}}\,\delta(1-x) + (Q^2)^{-\epsilon}\left[-\frac{1}{2\epsilon}P_{qq}(x) + \mathcal{O}(\epsilon^0)\right]$$

$$\begin{aligned} d\sigma^{NLO,S} \\ &= \mathcal{N}\sum_{ijk} \mathcal{X}_{i,jk}(x) \\ &\quad \times d\Phi_m(k_1,\ldots,K_K,\ldots,k_{m+1}; q, xp) \\ &\quad \times |\mathcal{M}_m(k_1,\ldots,K_K,\ldots,k_{m+1}; xp)|^2\, J_m^{(m)}(k_1,\ldots,K_K,\ldots,k_{m+1})\,. \end{aligned}$$

Cancellation of the divergencies

Hadronic Cross section

$$\sigma_h = \mathcal{N} \int_0^1 \frac{d\xi}{\xi} f_i(\xi) \hat{\sigma}(\xi p_i)$$

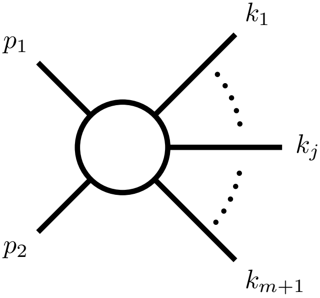
Mass factorization of the pdfs

$$f_i(\xi) = \sum_j \int_\xi^1 \frac{dx}{x} f_j\left(\frac{\xi}{x}\right) \left(\delta(1-x)\delta_{ij} + \frac{\alpha_s}{2\pi\epsilon} P_{ij}(x)\right)$$

Initial-Initial

We consider

- Two partons in the initial state
- Antenna function for initial p_1 and p_2 hard radiator
- j unresolved



Phasespace Mapping

$$\left.\begin{array}{c} p_1 \\ p_2 \\ k_j \end{array}\right\} \rightarrow \left\{\begin{array}{lcl} P_1 & = & x_1 p_1 + (1-x_2) p_2 - \frac{1-\beta}{2} k_j + \alpha t \\ \\ P_2 & = & (1-x_1) p_1 + x_2 p_2 - \frac{1+\beta}{2} k_j - \alpha t \end{array}\right.$$

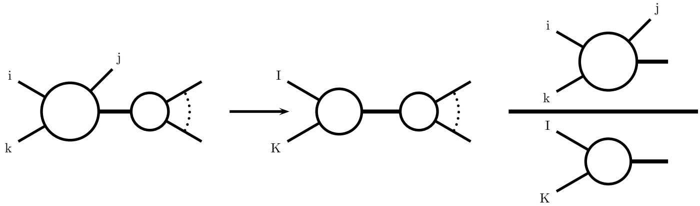
where $t \perp (p_1, p_2, k_j)$, $\quad t^\mu = \dfrac{\epsilon^{\mu\nu\rho\sigma} p_1^\nu p_2^\rho k_j^\sigma}{2 p_1 p_2}, \quad t^2 = -\frac{p_1 \cdot k_j \, p_2 \cdot k_j}{2 p_1 \cdot p_2}$

$$\beta = \frac{s_{2j} - s_{1j}}{s_{1j} + s_{2j}} \qquad \alpha \text{ free}$$

$$\begin{array}{lll} P_1 \rightarrow p_1, & P_2 \rightarrow p_2, & \text{when k becomes soft} \\ P_1 \rightarrow p_1 - k_j, & P_2 \rightarrow p_2, & \text{when j collinear with } p_1 \\ P_1 \rightarrow p_1, & P_2 \rightarrow p_2 - k_j, & \text{when j collinear with } p_2 \end{array}$$

ME squared: $\left|\mathcal{M}_{m+1}(k_1,\ldots,k_{j-1},k_j,k_{j+1},\ldots,k_{m+1};p_i,p_j)\right|^2$

Phasespace: $d\Phi_{m+1}(k_1,\ldots,k_{j-1},k_j,k_{j+1},\ldots,,k_{m+1};p_1,p_2)$



ME squared, subtraction:

$$\sum_k X_{12,k}(k_j;p_1,p_2)\left|\mathcal{M}_m(k_1,\ldots,k_{j-1},k_{j+1},\ldots,k_{m+1},P_1,P_2)\right|^2$$

Phasespace:

$$d\Phi_X(k_j;p_1,p_2)\mathcal{J}(k_j,p_1,p_2)d\Phi_m(k_1,\ldots,k_{j-1},k_{j+1},\ldots,k_{m+1};P_1,P_2)$$

Integrated Antenna function

Phase space integration freezes the value of x_1, x_2

$$\mathcal{A}_{q\bar{q},g} = -2\delta(1-x_1)\delta(1-x_2)\mathbf{I}^{(1)}_{q\bar{q},g} - s^{-\epsilon}\frac{P_{qq}(x_1)\delta(1-x_2)+P_{qq}(x_2)\delta(1-x_1)}{2\epsilon} + \mathcal{O}(\epsilon)$$

Final Form of The Subtraction term

$$
\begin{aligned}
d\sigma_{NLO}^{S,(ii)} \\
&= \mathcal{N} d\Phi_{m+1}(k_1, \ldots, k_{k-1}, k_k, k_{k+1}, \ldots, k_{m+1}; p_1, p_2) \\
&\quad \times \sum_k X^0_{12,k} \, |\mathcal{M}_m(k_1, \ldots, k_{k-1}, k_{k+1}, \ldots, k_{m+1}; P_1, P_2)|^2 \\
&\quad J_m^{(m)}(k_1, \ldots, k_{k-1}, k_{k+1}, \ldots, k_{m+1}) \\
&= \mathcal{N} \sum_k \mathcal{X}_{12,k}(x_1, x_2) d\Phi_m(k_1, \ldots, k_{k-1}, k_{k+1}, k_{m+1}; P_1, P_2) \\
&\quad \times |\mathcal{M}_m(k_1, \ldots, k_{k-1}, k_{k+1}, \ldots, k_{m+1}; P_1, P_2)|^2 \\
&\quad J_m^{(m)}(k_1, \ldots, k_{k-1}, k_{k+1}, \ldots, k_{m+1})
\end{aligned}
$$

Conclusion and Outlook

- formulated antenna subtraction for processes with initial state partons (at NLO)
- method can be extended to NNLO (work in progress)
- possible applications
  - matching to antenna-based parton shower
  - NNLO $t\bar{t}$ production
  - NNLO $pp \to 2j$ production
  - NNLO DIS $(2+1)j$ production