Matching the NLL threshold resummation with fixed-order QCD corrections in top-pair production

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in collaboration with

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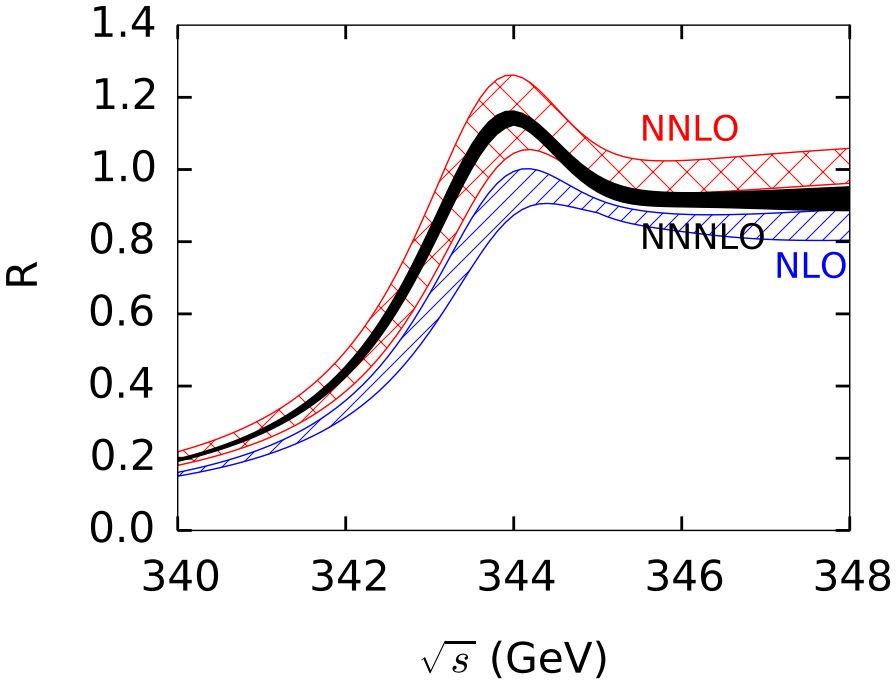
QCD corrections to $t\bar{t}$ S-wave production at lepton colliders known to $\mathrm{N^3LO}$ [Beneke et al. 2015] and resummed to NNLL [Hoang and Stahlhofen 2014]



Very sensitive to top mass and width.

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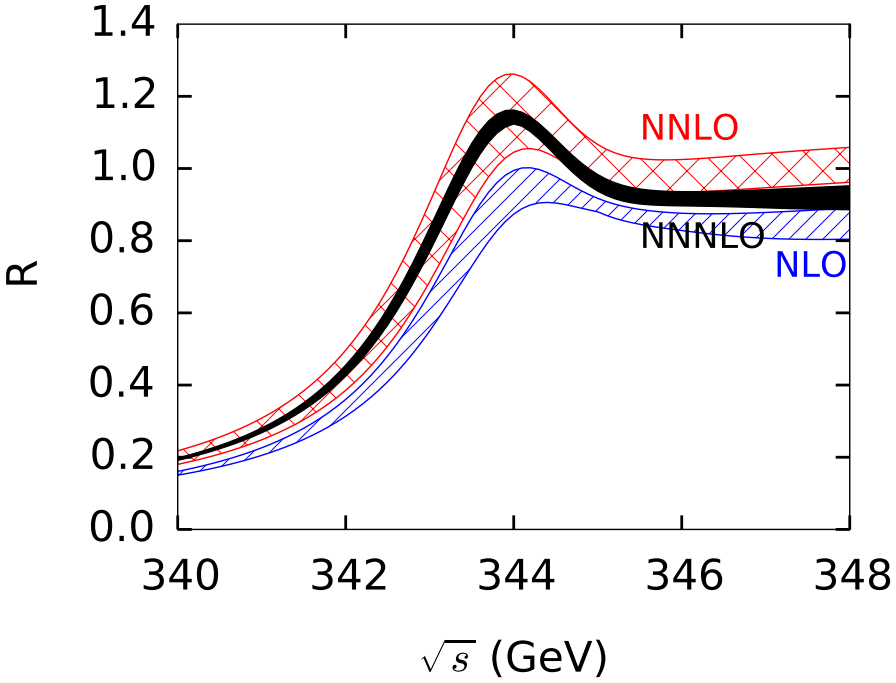
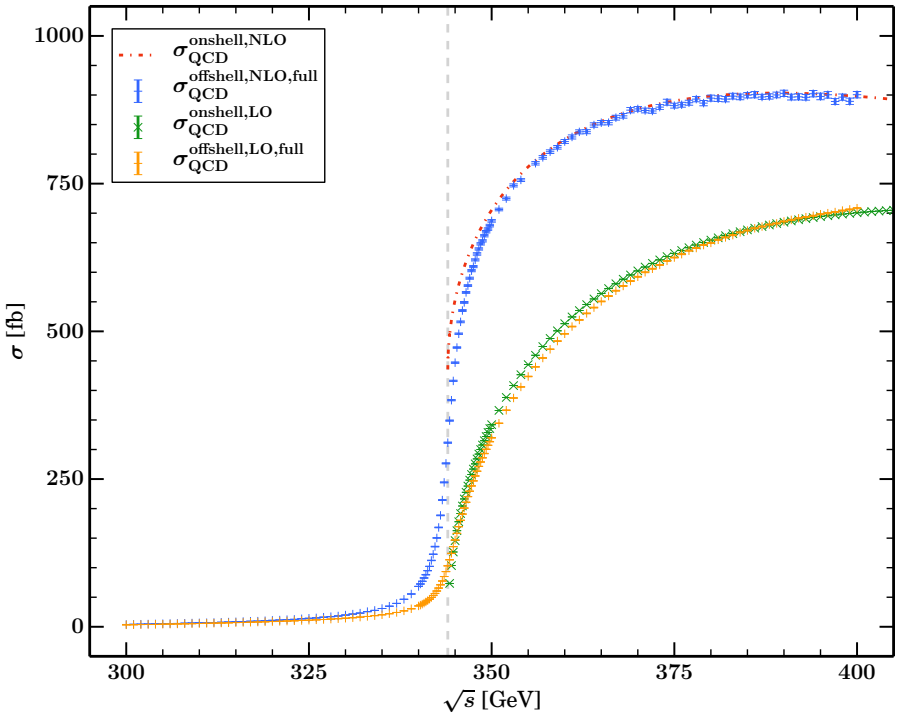


Very sensitive to top mass and width.

However, this is not the full story.

How to combine inclusive NRQCD with differential, fixed-order predictions for $e^+e^- \to W^+bW^-\bar{b}$?



What about beamstrahlung or inital state radiation?

Outline

Threshold resummation within WHIZARD
- What is WHIZARD?
- NRQCD in a small nutshell
- Validation with analytic results
- Benefits of WHIZARD+NRQCD

Matching threshold with continuum
- Gauge invariance
- Switching off resummation & matching formula
- Widths and results

Threshold resummation within WHIZARD

The WHIZARD Event Generator

- Universal event generator for multi-particle states in SM and BSM physics
- Object-Oriented implementation in `Fortran 2003` and `OCaml`
- Convenient analysis setup using own script-language (`Sindarin`)
- Supported event formats: `LHEF`, `HepMC`, `LCIO`, `StdHEP`, various `ASCII` formats

Current Release: WHIZARD 2.2.7, August 11th, 2015

Web address: http://projects.hepforge.org/whizard
Standard reference: Kilian/Ohl/Reuter, EPJC (2011) 1742, arXiv:0708.4233

WHIZARD - Physics Features

- Optimized, automatic tree-level matrix elements for arbitrary processes (O'MEGA) [Moretti, Ohl, and Reuter 2001]
- Efficient phasespace parameterization with multi-channel Monte-Carlo integration (VAMP) [Ohl 1999]
- Excellent support for lepton collider studies:
  - Effects of correlated beamstrahlung using CIRCE2 [Ohl 1996], which makes GUINEAPIG [Schulte et al. 2007] suitable for event generation
  - Resummation of QED ISR radiation off leptons
  - Beam polarizations, photon-induced processes
- Interfaces to LHAPDF5, LHAPDF6, HOPPET as well as built-in PDFs
- Parton shower and hadronization:
  analytic shower [Kilian et al. 2012], PYTHIA6, PYTHIA8 (soon)

Status of automated NLO QCD corrections

- Automated construction of radiation phase space of all $n+1$ particle processes
- IR divergencies are handled with FKS subtraction [Frederix et al. 2009]
- Virtual amplitudes come from one-loop-providers:
  GoSam [Cullen et al. 2014], OpenLoops [Cascioli, Maierhöfer, and Pozzorini 2012]
- Real and Born amplitudes can be computed with O'Mega, GoSam and OpenLoop $\Rightarrow$ good cross-checking
- Most lepton-collider processes can be computed out-of-the-box by specifying `nlo_calculation = "Full"`
- Infrastructure for hadron collider processes mostly implemented

Non-Relativistic QCD framework (NRQCD)

see also talk by Jan Piclum

- close to $t\bar{t}$-threshold: top velocity $v \sim \alpha_s$

$$v \equiv \sqrt{\frac{\sqrt{s} - 2m + i\Gamma}{m}}$$

- NRQCD is an EFT that simultaneously expands in v and α_s
- Resummation of singular terms close to $v = 0$, see e.g. [Hoang et al. 2001]

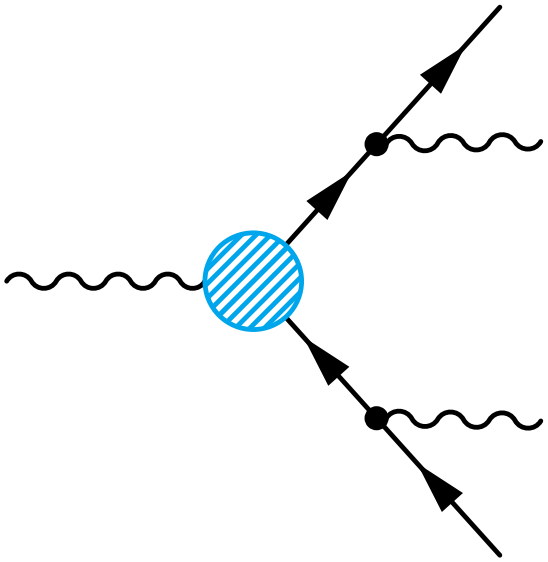
$$R = \frac{\sigma_{t\bar{t}}}{\sigma_{\mu^+\mu^-}} = v \sum_k \left(\frac{\alpha_s}{v}\right)^k \sum_i (\alpha_s \ln v)^i \times$$

$$\left\{1\,(\mathrm{LL});\ \alpha_s, v\,(\mathrm{NLL});\ \alpha_s^2,\ \alpha_s v,\ v^2\,(\mathrm{NNLL}); \ldots\right\}$$

v : $t\bar{t}$ phasespace,
Coulomb summation of powers of α_s/v (gluon ladders),

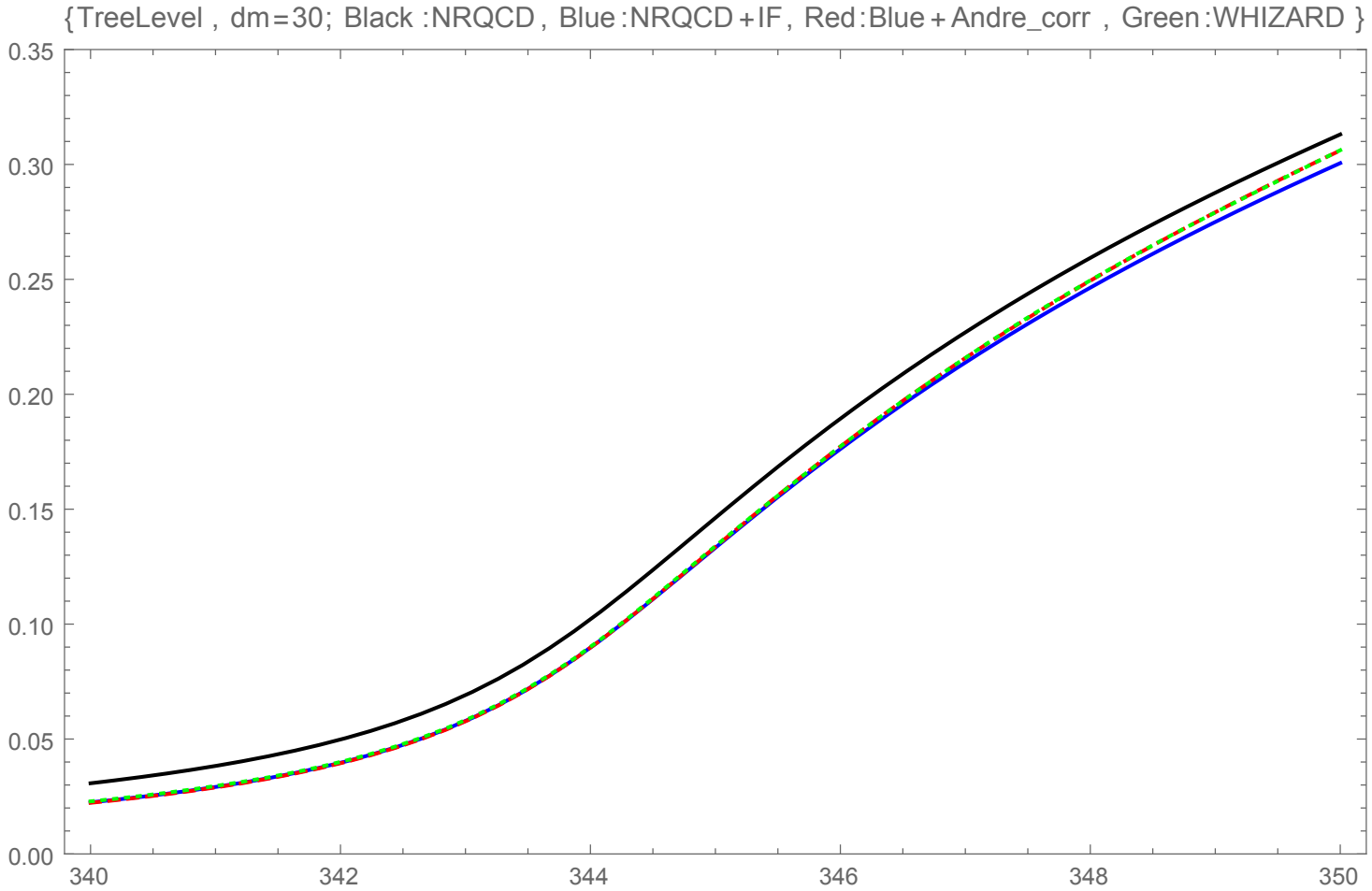
velocity renormalization group improved (vNRQCD)

Implementation in WHIZARD



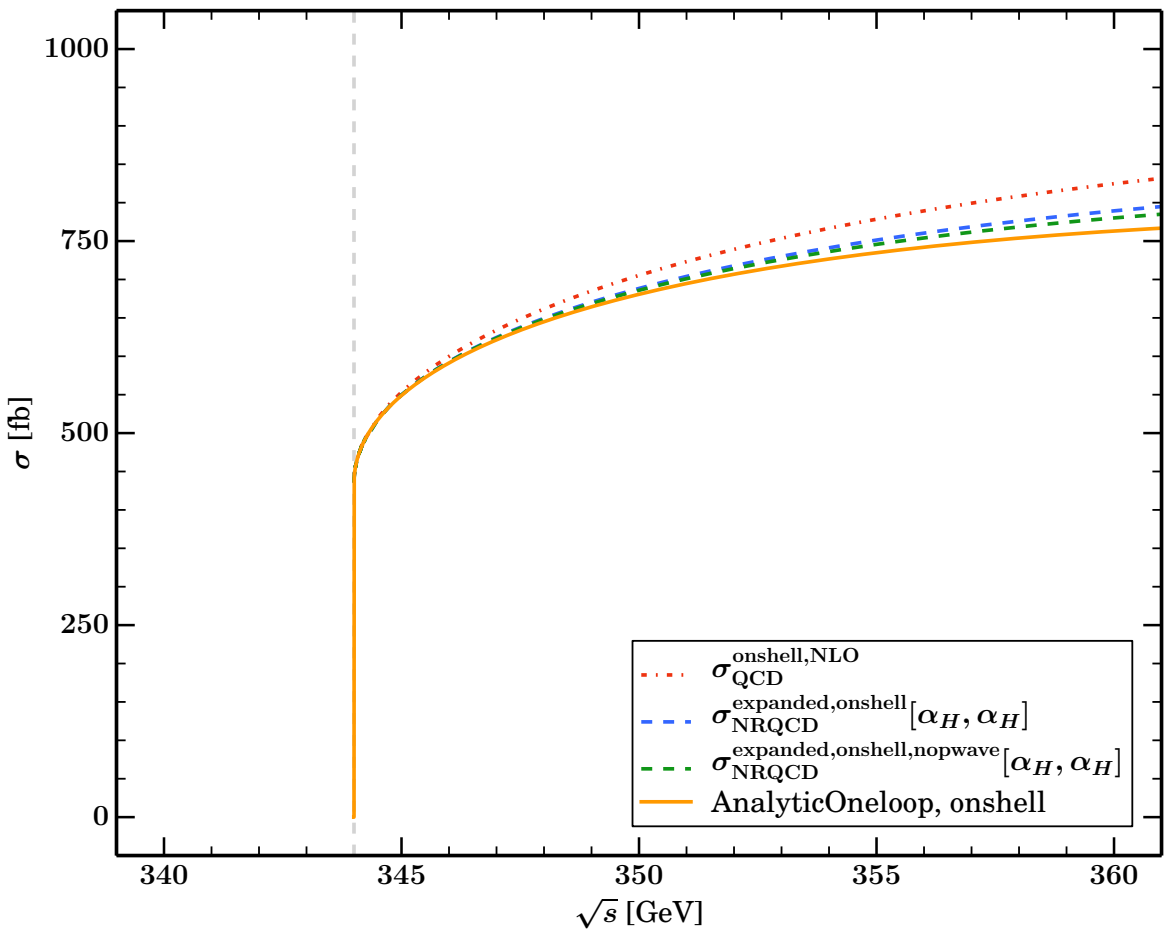
- vNRQCD modifies the vector $t\gamma_\mu\bar{t}$ and axial-vector $t\gamma_\mu\gamma_5\bar{t}$ currents
- We can use TOPPIK [Hoang and Teubner 1999] to obtain an NLL improved ttV form factor
- Also contains p-wave contributions
  $\rightarrow$ talk by Thomas Rauh
  very small change in σ but important for A_{fb}
- Rest of the diagram as well as all other diagrams in $W^+bW^-\bar{b}$ obtained from O'MEGA as in an ordinary relativistic LO prediction
- The α_s expansion of this term behaves like a virtual correction without divergence

Comparison with analytical NRQCD computation



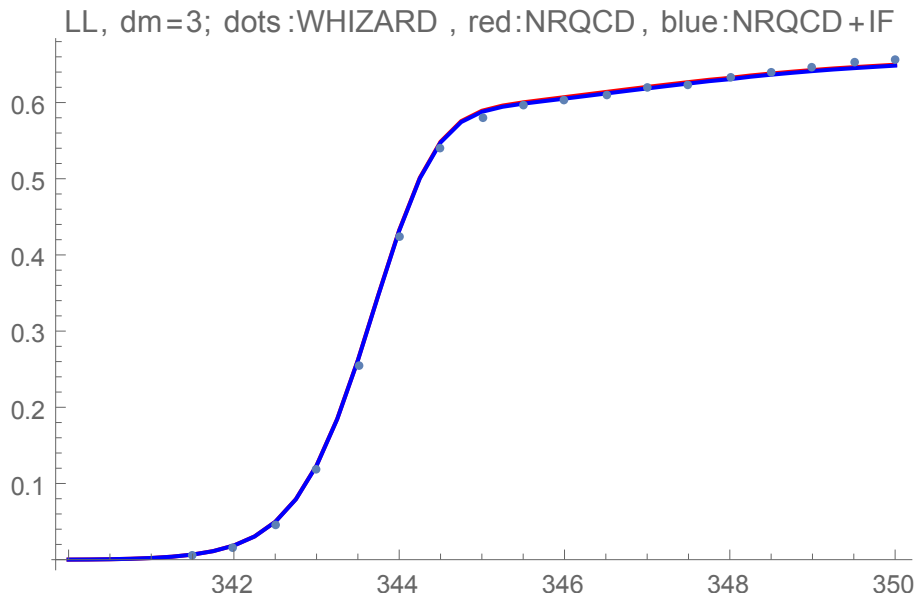
- On tree-level, the relativistic corrections can be implemented on the analytic side. On higher orders, not straightforward.
- Taking it into account gives perfect agreement

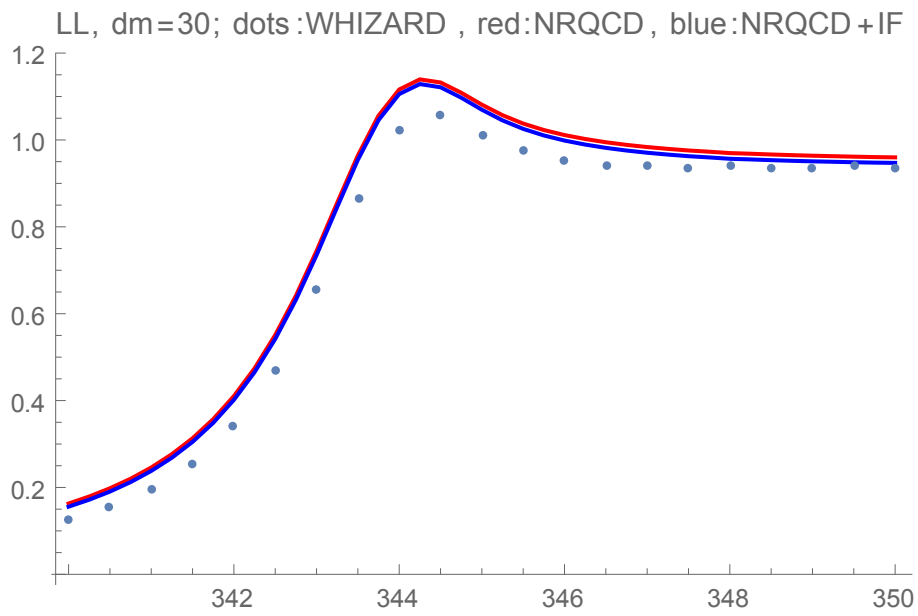
Comparison with analytical NRQCD computation



- Checking the onshell result of the α_s expansion of the LL
- Even for this fairly trivial case, we see that the nonrelativsitic phase space systematically underestimates the full phase space

Comparison with analytical NRQCD computation



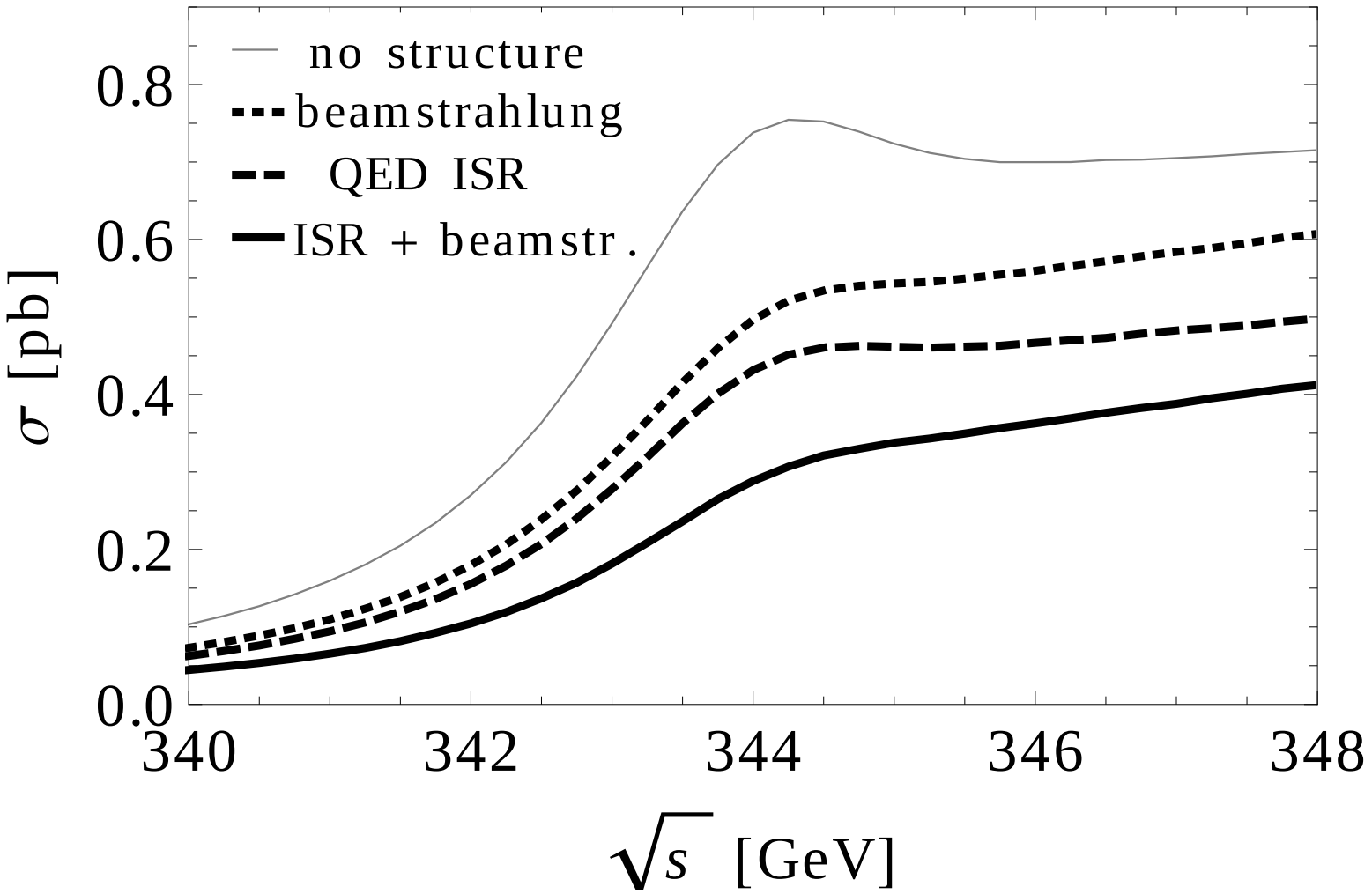


- For (N)LL, differences vanish for

$$\Delta M = |m_{Wb} - m_t| \to 0$$

- It is thus an offshell effect
- But offshell effects are expected to be strongly surpressed at threshold
- We would thus like to understand this better

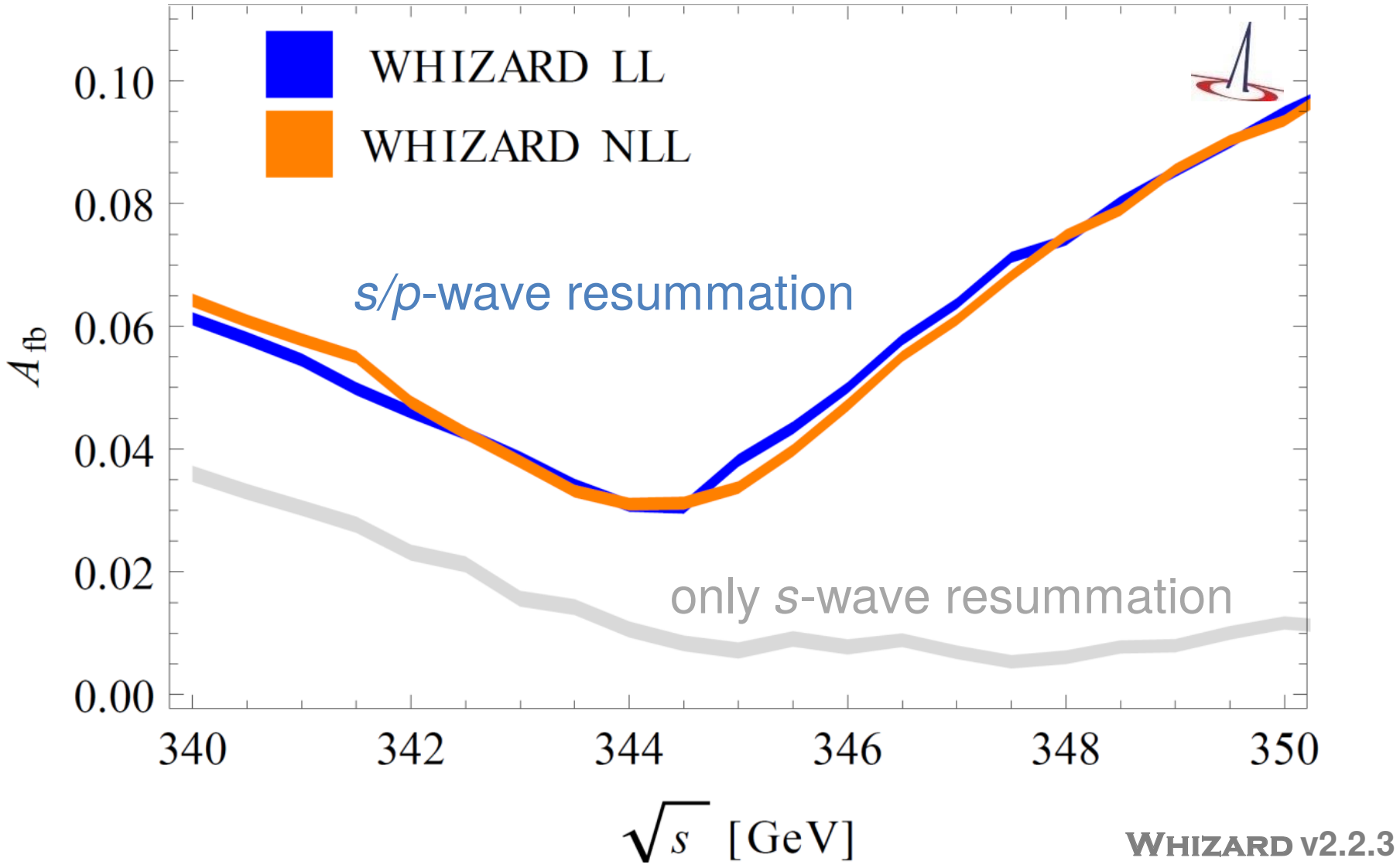
A more realistic picture of the threshold



NLL with $\Delta M = 30\,\text{GeV}$.

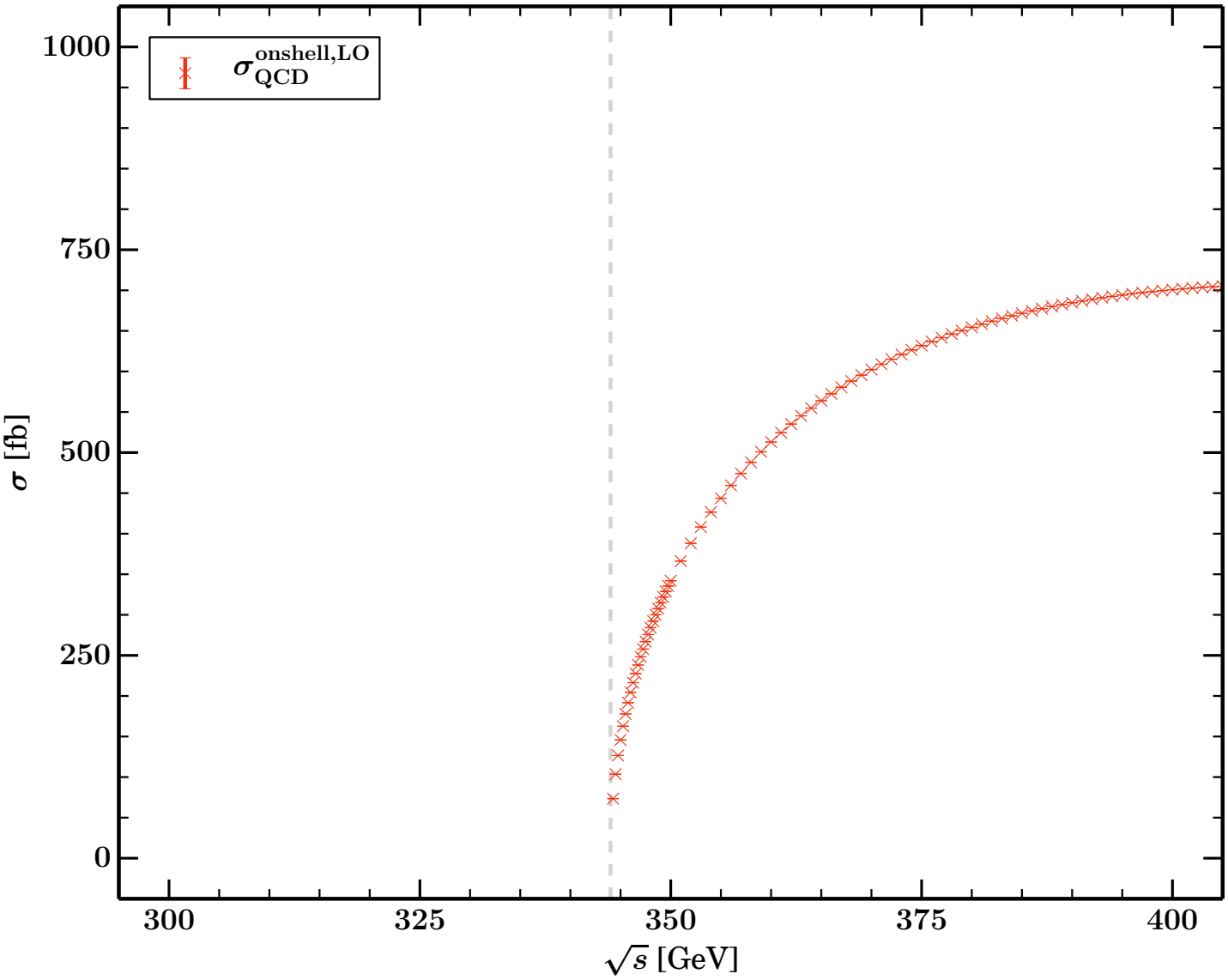
Also doable by folding TOPPIK with beam spectra (e.g. [Seidel et al. 2013])

Forward-backward asymmetry $A_{fb} = \frac{\sigma(p_z^t>0)-\sigma(p_z^t<0)}{\sigma(p_z^t>0)+\sigma(p_z^t<0)}$



Matching threshold with continuum

Fixed-order landscape at threshold



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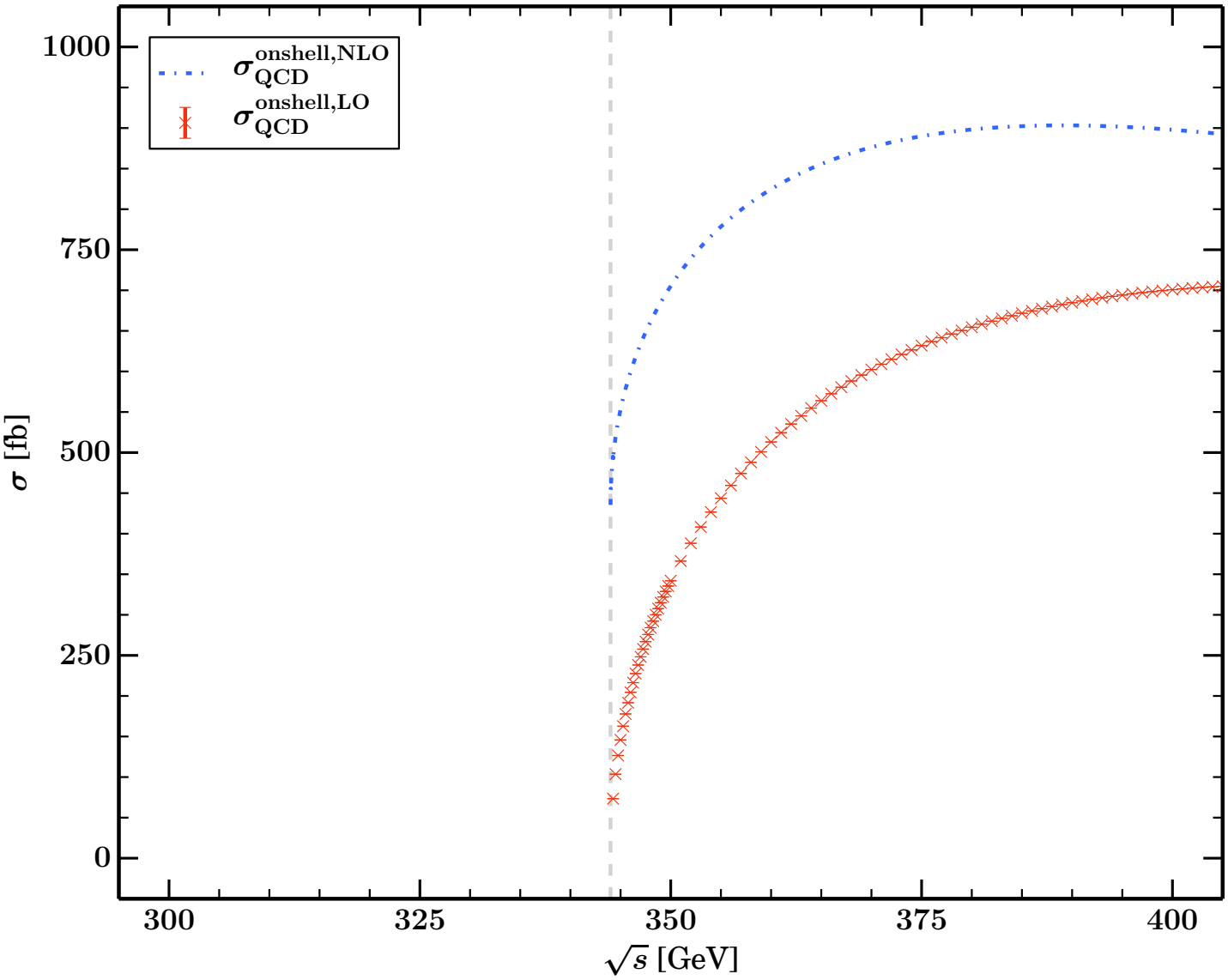

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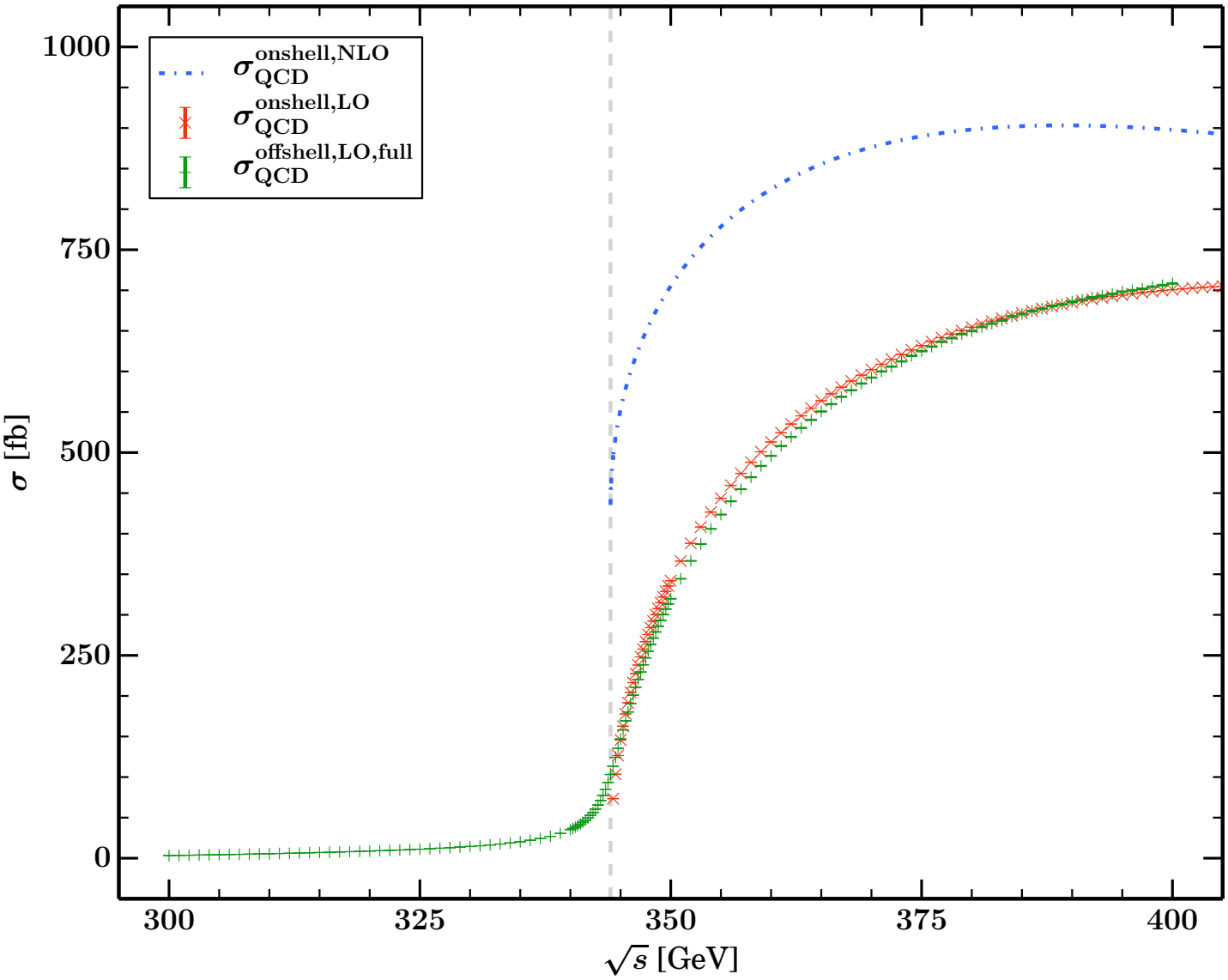

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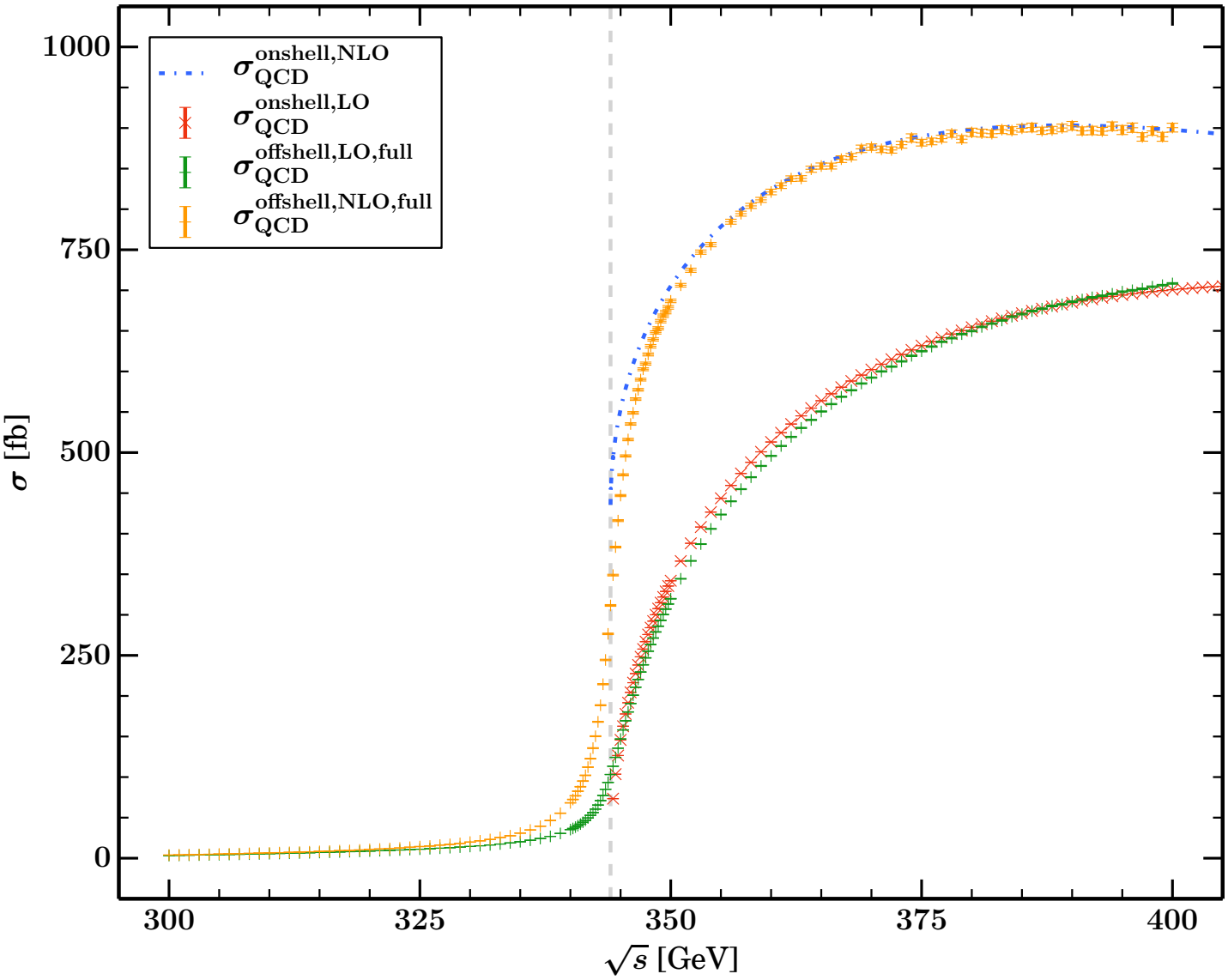

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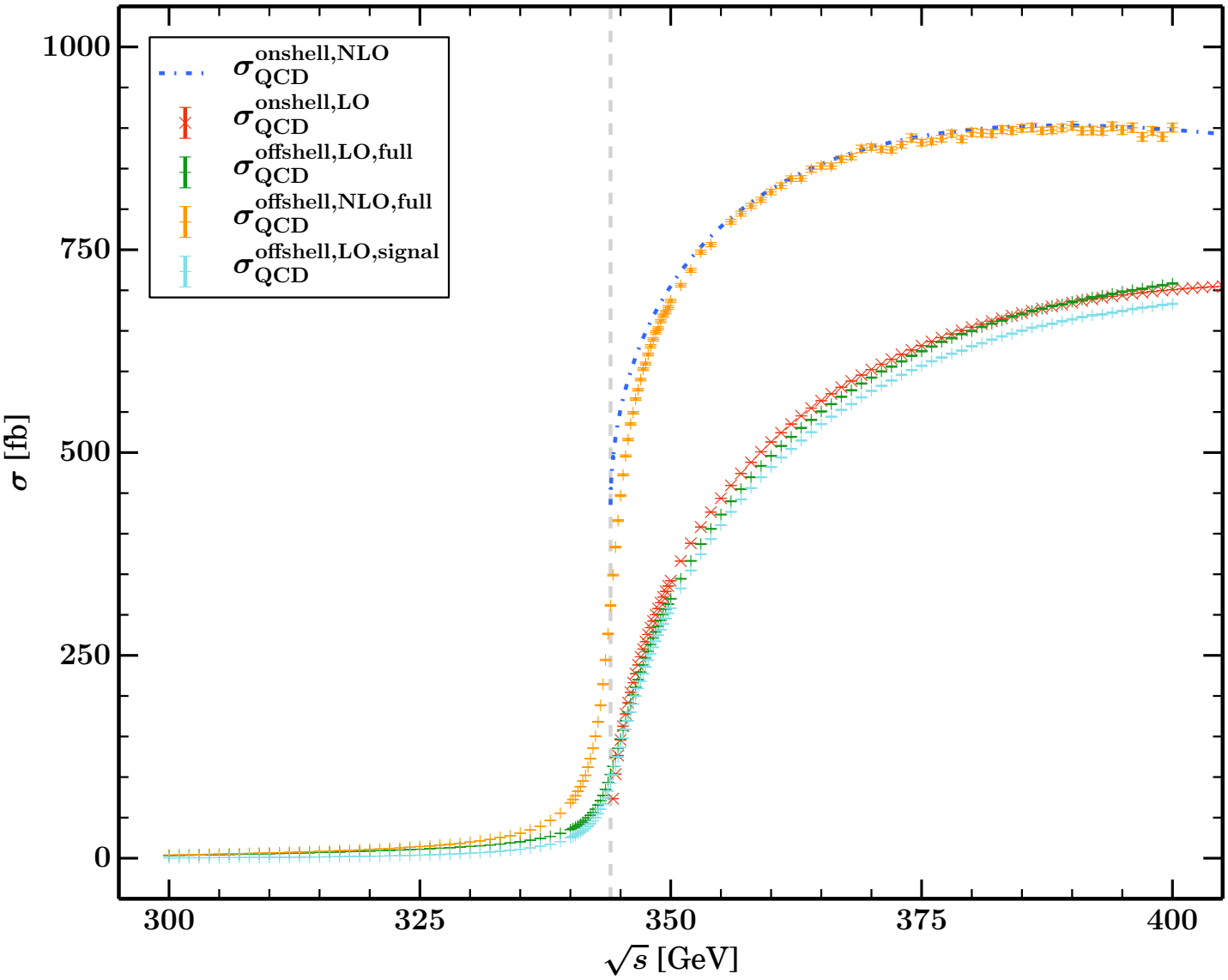

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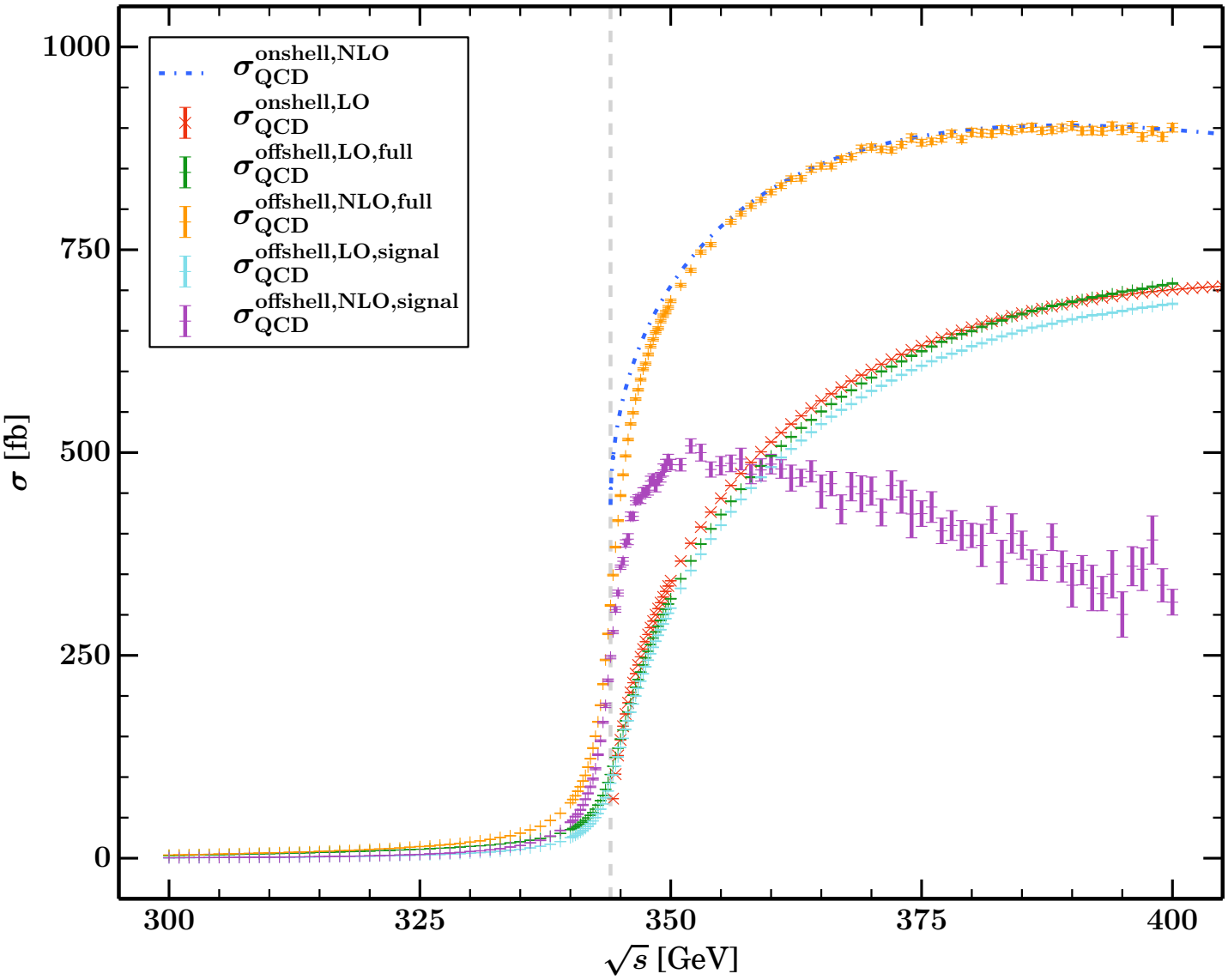
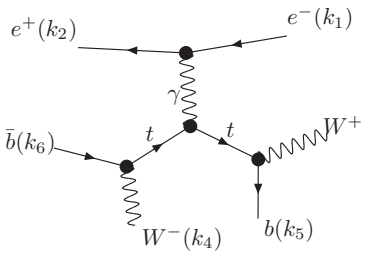


Selecting diagrams vs gauge-invariance



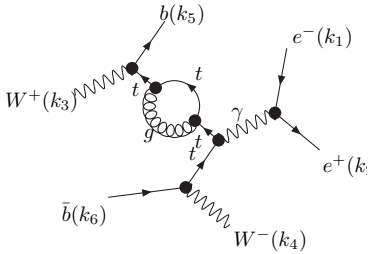
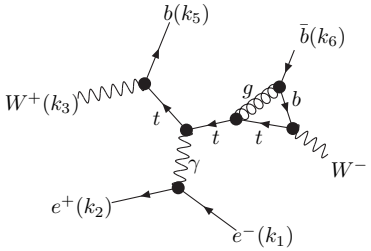


Diagram 249
$S' = S^{\{1,2,4\}}_{Q\to -q}$, rk = 2



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$S' = S^{\{4,5\}}_{Q\to g-(-k6)}$, rk = 2

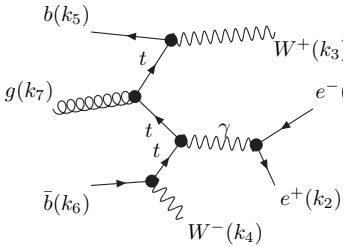

Diagram 79

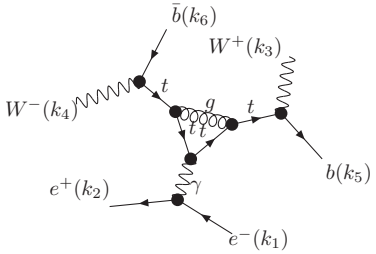

Diagram 187
$S' = S^{\{2,4\}}_{Q\to -q-(-k3-k5)}$, rk = 2

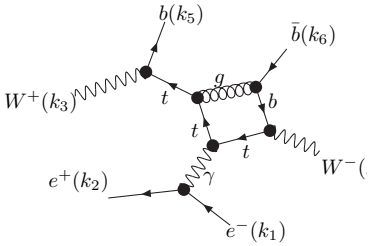

Diagram 351
$S' = S^{\{4\}}_{Q\to q-(-k6)}$, rk = 3

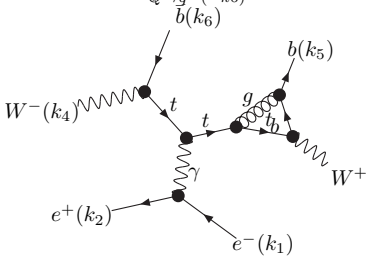

Diagram 420
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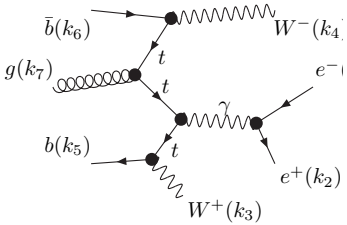


Diagram 87

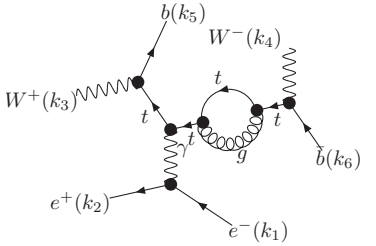


Diagram 245
$S' = S^{\{2,4,5\}}$, rk = 2

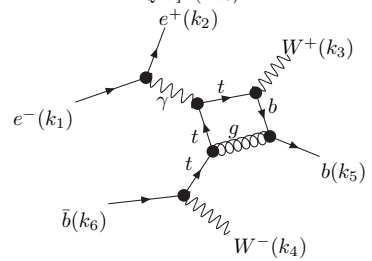


Diagram 355
$S' = S^{\{2\}}_{Q\to -q-(-k5)}$, rk = 3

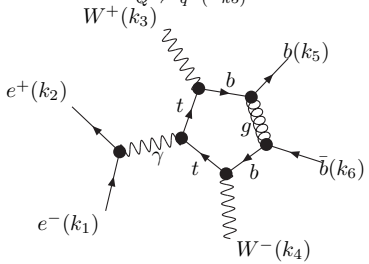


Diagram 1595
$S' = S_{Q\to -q}$, rk = 4

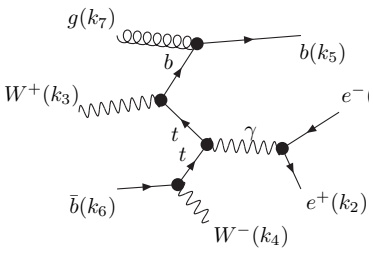


Diagram 104

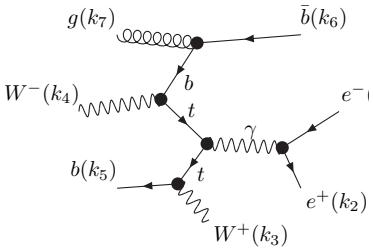
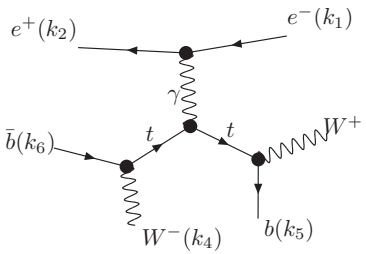
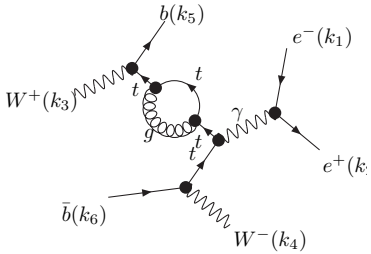


Diagram 120

- No theorem shields us from arbitrary large gauge cancellations with other diagrams
- Not feasible to modify pentagon and box diagrams with $F(p)$

Selecting diagrams vs gauge-invariance





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$S' = S^{\{1,2,4\}}_{Q\to -q}$, rk = 2

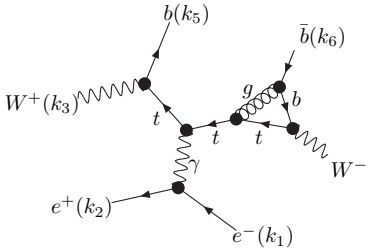

Diagram 416
$S' = S^{\{4,5\}}_{Q\to g-(-k6)}$, rk = 2

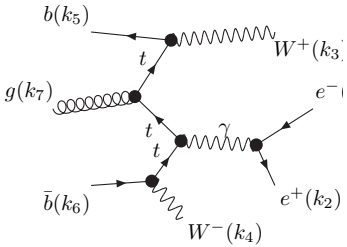

Diagram 79

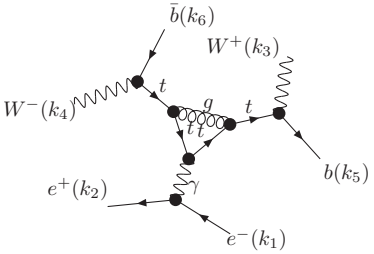

Diagram 187
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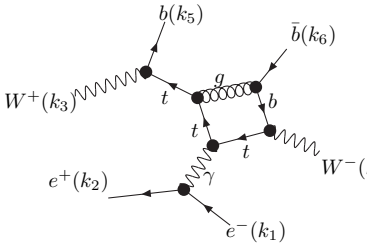
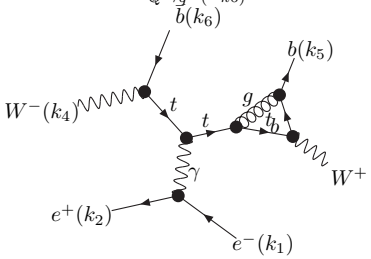


Diagram 351
$S' = S^{\{4\}}_{Q\to q-(-k6)}$, rk = 3



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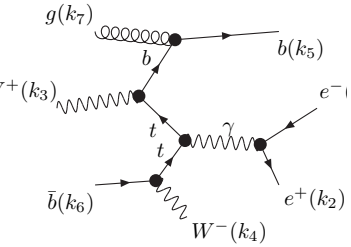
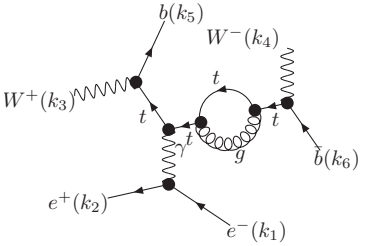


Diagram 104



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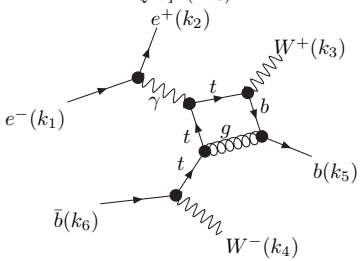

Diagram 355
$S' = S^{\{2\}}_{Q\to -q-(-k5)}$, rk = 3

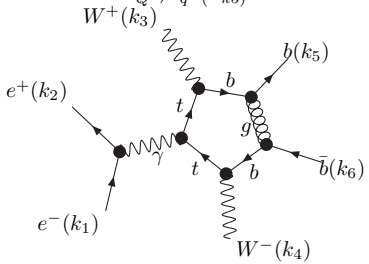

Diagram 1595
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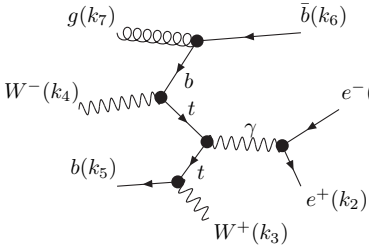
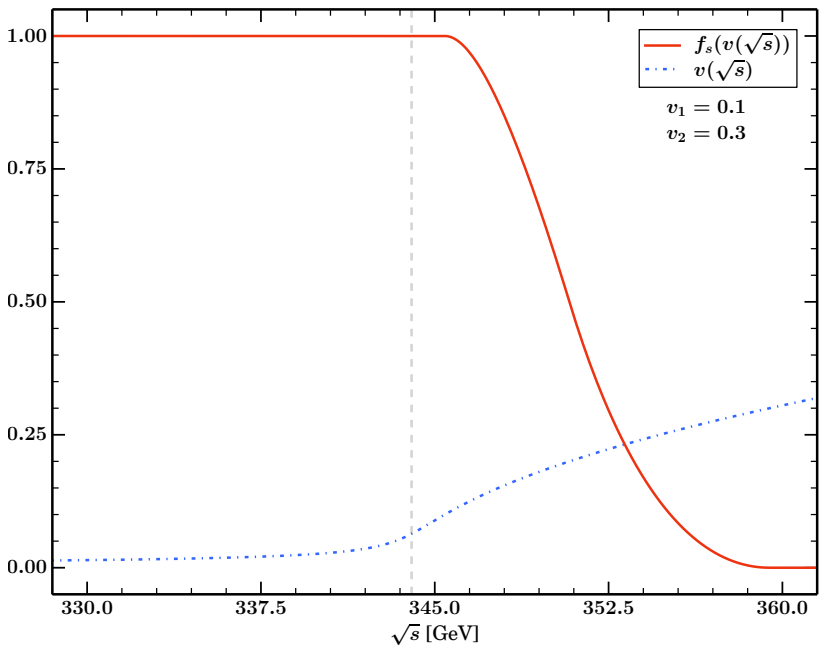


Diagram 120

- No theorem shields us from arbitrary large gauge cancellations with other diagrams
- Not feasible to modify pentagon and box diagrams with $F(p)$

Resummed prediction keeps rising (unphysically) with $\sqrt{s}$.
Contains the nonrelativistic assumption that $v \sim \alpha_S$.
We need an $f_s(v)$ with

$$f_s(0) = 1 \quad \text{and} \quad f_s(1) = 0$$



Use e.g.

$$f_s(v) = \begin{cases} 1 & v < v_1 \\ 1 - 2\frac{(v-v_1)^2}{(v_2-v_1)^2} & v_1 < v < \frac{v_1+v_2}{2} \\ 2\frac{(v-v_2)^2}{(v_2-v_1)^2} & \frac{v_1+v_2}{2} < v < v_2 \\ 0 & v > v_2 \end{cases}$$

$$v = \mathrm{Re}\sqrt{\frac{\sqrt{s} - 2m + \mathrm{i}\Gamma}{m}}$$

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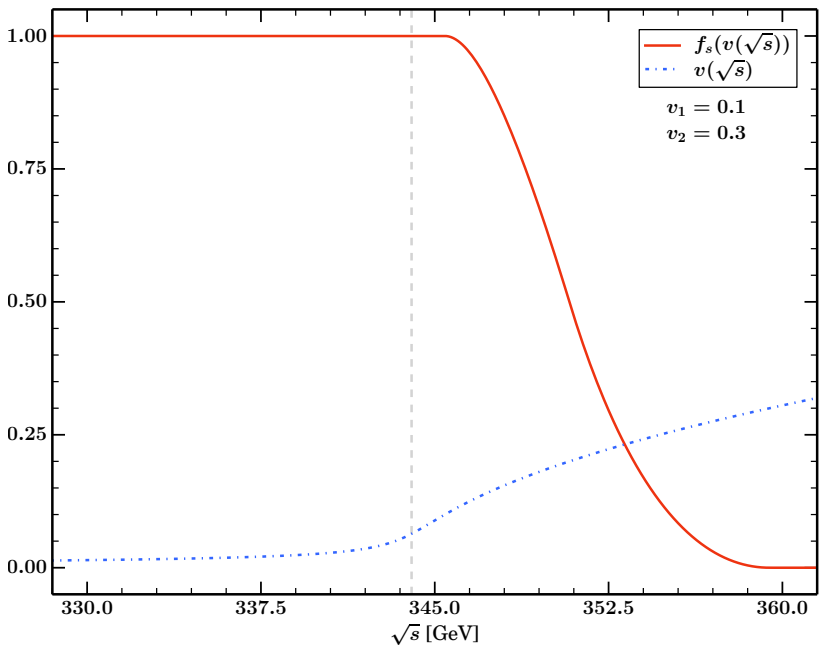

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Matching

- $\sigma_{\mathrm{NRQCD}}^{\mathrm{expanded}} = \sigma_{\mathrm{NRQCD}}^{\mathrm{resummed}}\Big|_{\mathcal{O}(\alpha_s)}$ and contains a form factor

$$F^{\mathrm{expanded}}\left[\alpha_{\mathrm{H}},\ \alpha_{\mathrm{S}}\right] = \alpha_{\mathrm{H}}\left(-\frac{2\,C_F}{\pi}\right) + \alpha_{\mathrm{S}}\left(\frac{\mathrm{i}\,C_F\,m\log\frac{mv+p}{mv-p}}{2p}\right)$$

- Aim: Remove double counting, keep α_s-correctness in fixed-order regime and maintain NLL-correctness in threshold region

$$\begin{aligned}\sigma_{\mathrm{matched}} &= \sigma_{\mathrm{QCD}}\left[\alpha_{\mathrm{H}}\right] - \sigma_{\mathrm{NRQCD}}^{\mathrm{expanded}}\left[\alpha_{\mathrm{H}},\ \alpha_{\mathrm{H}}\right] \\ &\quad + \sigma_{\mathrm{NRQCD}}^{\mathrm{expanded}}\left[\alpha_{\mathrm{H}},\ f_s\,\alpha_{\mathrm{S}} + (1-f_s)\,\alpha_{\mathrm{H}}\right] \\ &\quad + \sigma_{\mathrm{NRQCD}}^{\mathrm{resummed}}\left[f_s\,\alpha_{\mathrm{H}},\ f_s\,\alpha_{\mathrm{S}},\ f_s\,\alpha_{\mathrm{US}}\right] - \sigma_{\mathrm{NRQCD}}^{\mathrm{expanded}}\left[f_s\,\alpha_{\mathrm{H}},\ f_s\,\alpha_{\mathrm{S}}\right]\end{aligned}$$

$$\begin{aligned}&= \sigma_{\mathrm{QCD}}\left[\alpha_{\mathrm{H}}\right] - \sigma_{\mathrm{NRQCD}}^{\mathrm{expanded}}\left[f_s\,\alpha_{\mathrm{H}},\ f_s\,\alpha_{\mathrm{H}}\right] \\ &\quad + \sigma_{\mathrm{NRQCD}}^{\mathrm{resummed}}\left[f_s\,\alpha_{\mathrm{H}},\ f_s\,\alpha_{\mathrm{S}},\ f_s\,\alpha_{\mathrm{US}}\right]\end{aligned}$$

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\end{aligned}$$

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\end{aligned}$$

Which width where . . . ?

- Recall that the offshell cross section depends on the width in the propagator

$$\frac{1}{(p-m)^2 + m^2\Gamma^2} \underset{\text{NWA}}{\longrightarrow} \frac{\pi}{m\Gamma}\,\delta(p^2 - m^2)$$

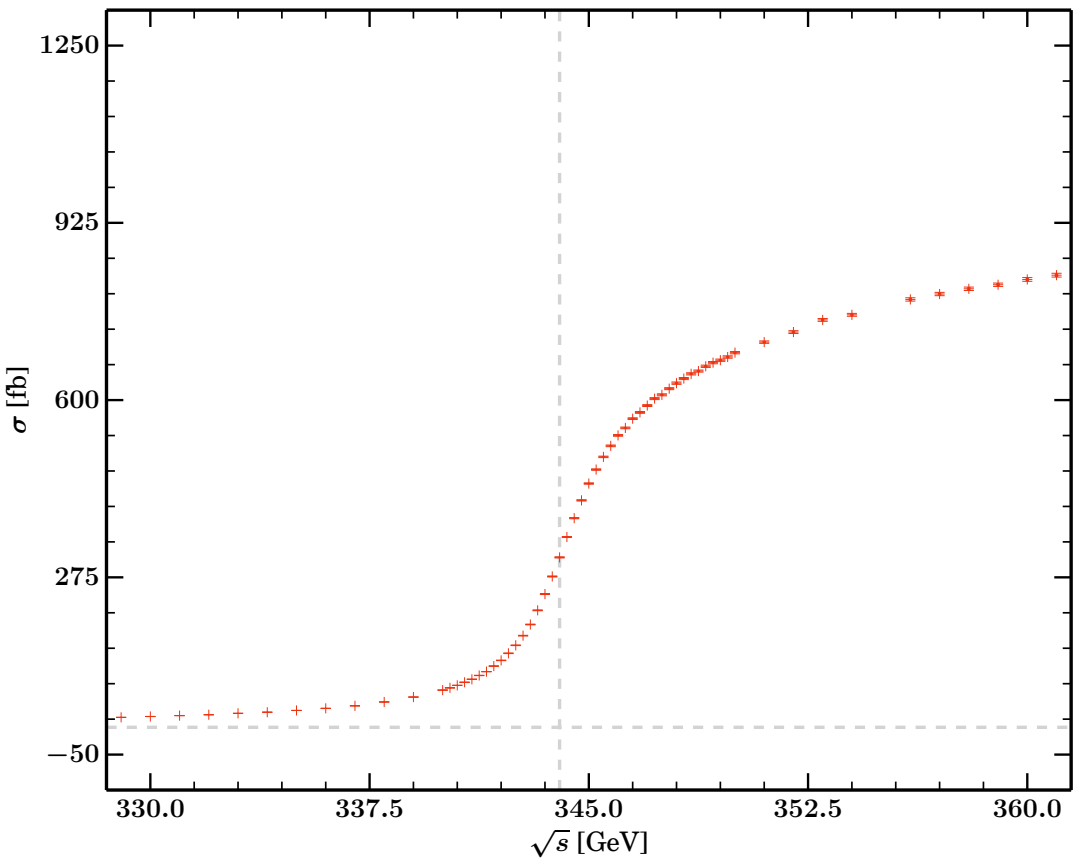
- At the same time, Γ is computed indirectly via the offshell top decay $t^* \to Wb$.
- Therefore, one should use the NLO width in an NLO computation and the LO width in an LO calculation for a consistent computation
- Especially, the **width should not be treated as free parameter** in a MC simulation ($\Rightarrow$ Unphysical results!). Instead vary V_{tb}, Γ_{invis}
- But what about the matched computation?

Which width where in the matched computation?

$$\sigma_{\text{matched}} = \sigma_{\text{QCD}}\left[\alpha_{\text{H}}\right] - \sigma_{\text{NRQCD}}^{\text{expanded}}\left[f_s\,\alpha_{\text{H}},\ f_s\,\alpha_{\text{H}}\right] + \sigma_{\text{NRQCD}}^{\text{resummed}}\left[f_s\,\alpha_{\text{H}},\ f_s\,\alpha_{\text{S}},\ f_s\,\alpha_{\text{US}}\right]$$

- σ_{QCD} needs Γ_{NLO}
- $\sigma_{\text{NRQCD}}^{\text{expanded}}$ agrees well with σ_{QCD} at threshold when Γ_{NLO} is used because it contains the dominant Coulomb term
- $\sigma_{\text{NRQCD}}^{\text{resummed}}$ contains higher order terms and enhances the cross section strongly
- NLO corrections to the t decay in σ_{QCD} cannot keep up with $\sigma_{\text{NRQCD}}^{\text{resummed}}$ containing only the Born part of the decay
- Need Γ_{LO} for this additional contribution

Matching



$$\sigma_{\text{matched}} = \boxed{\sigma_{\text{QCD}}\left[\alpha_{\text{H}}\right] - \sigma_{\text{NRQCD}}^{\text{expanded}}\left[\alpha_{\text{H}},\ \alpha_{\text{H}}\right]}$$
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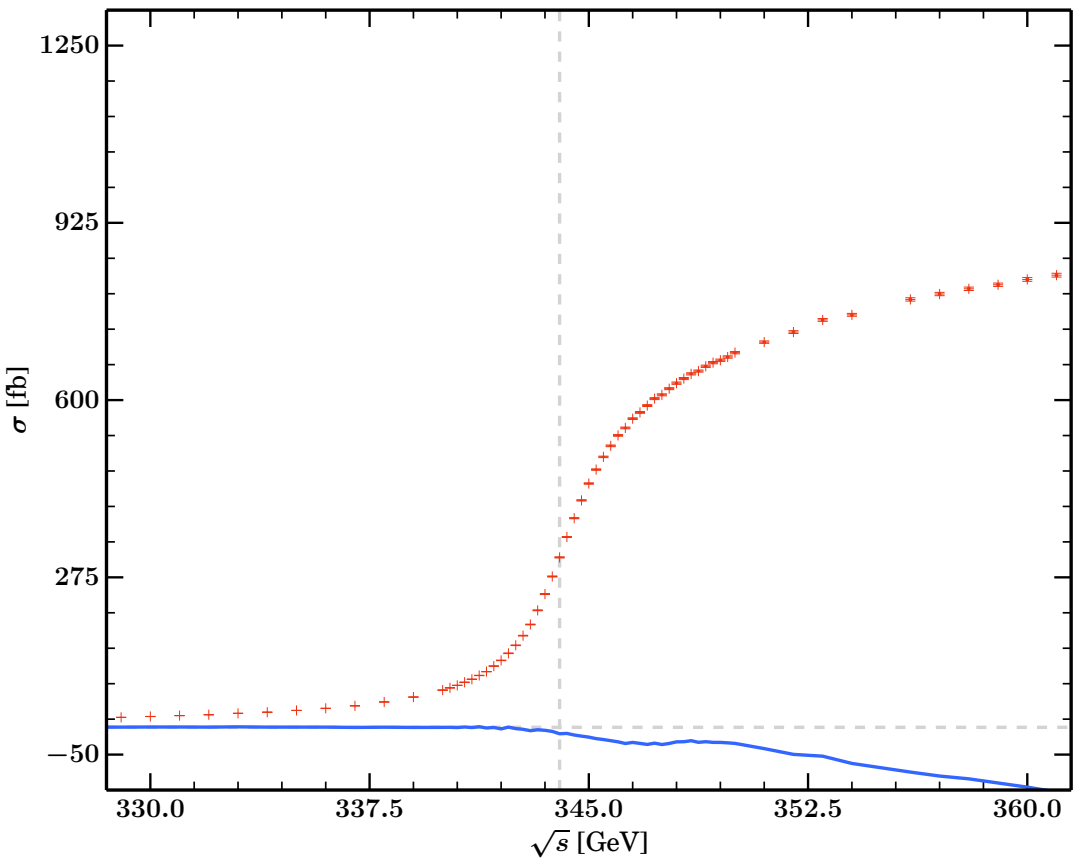

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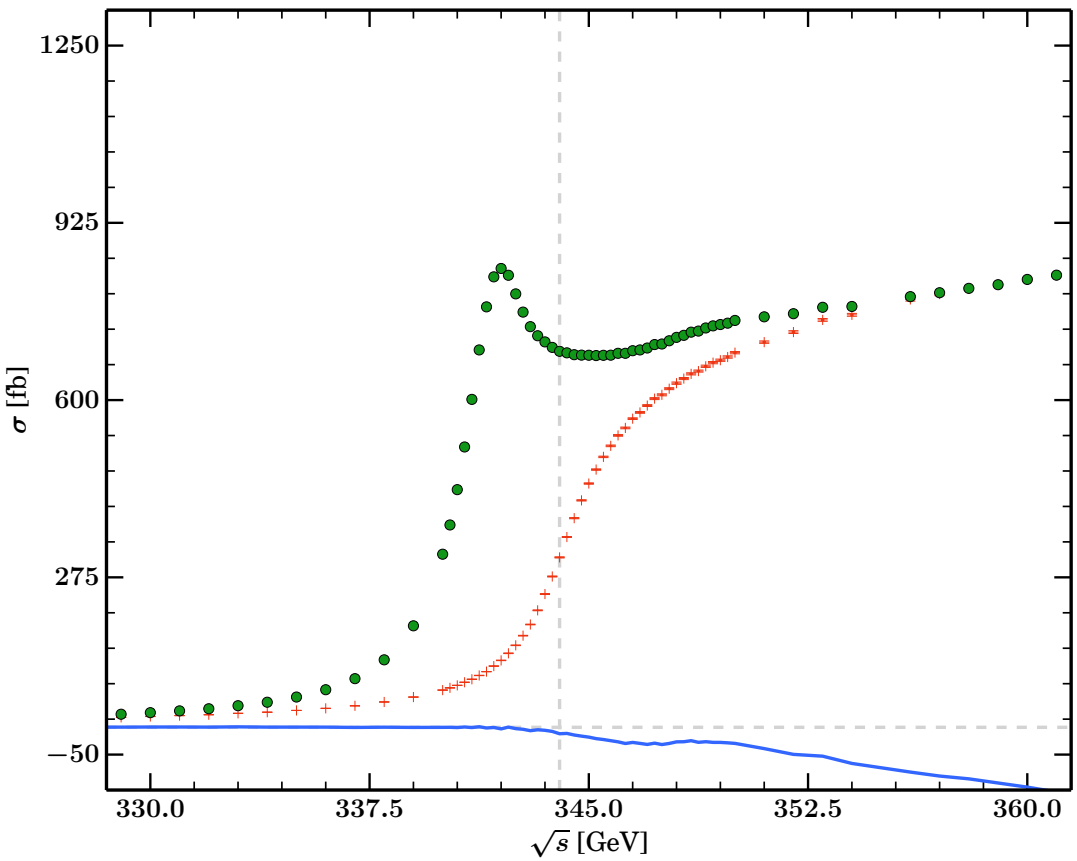

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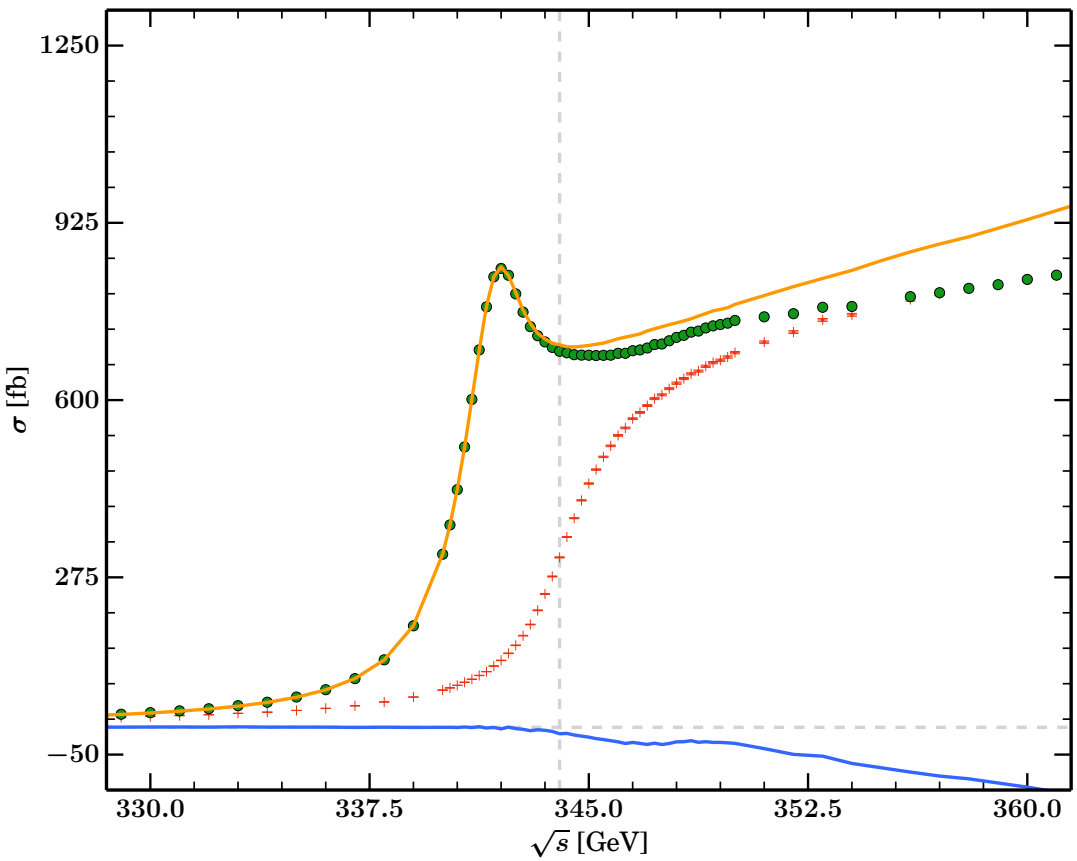

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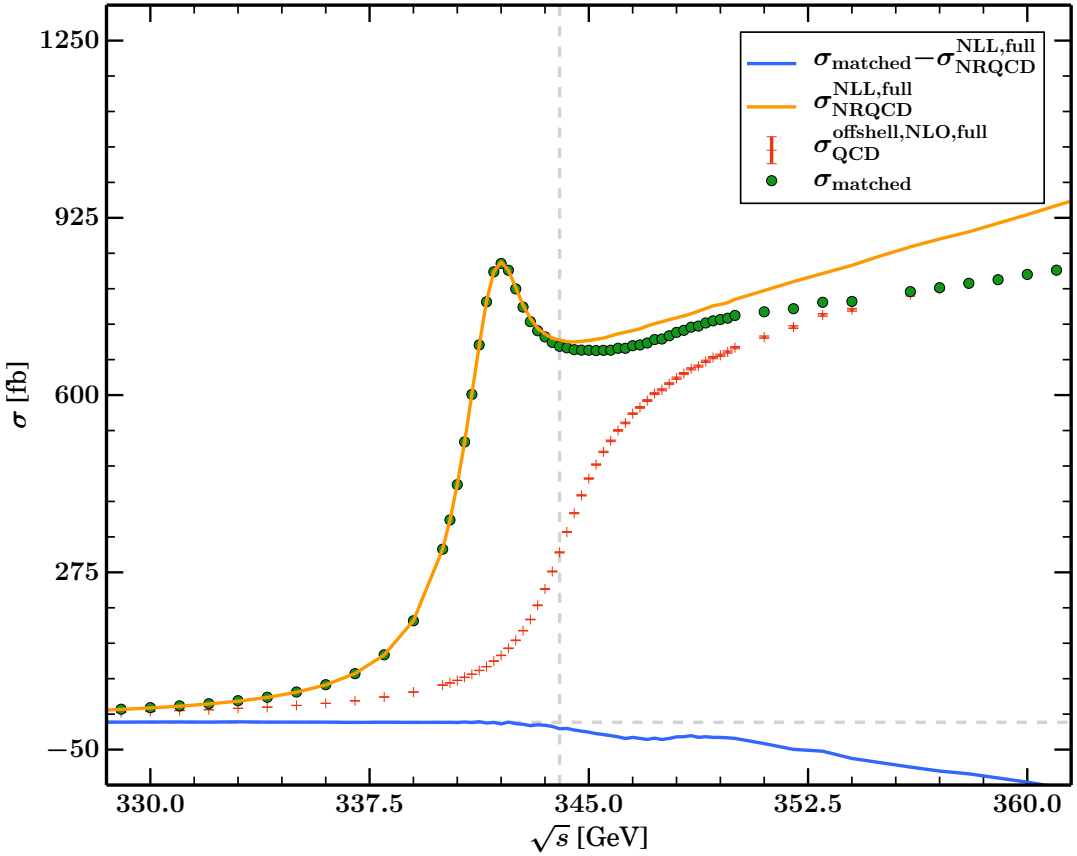

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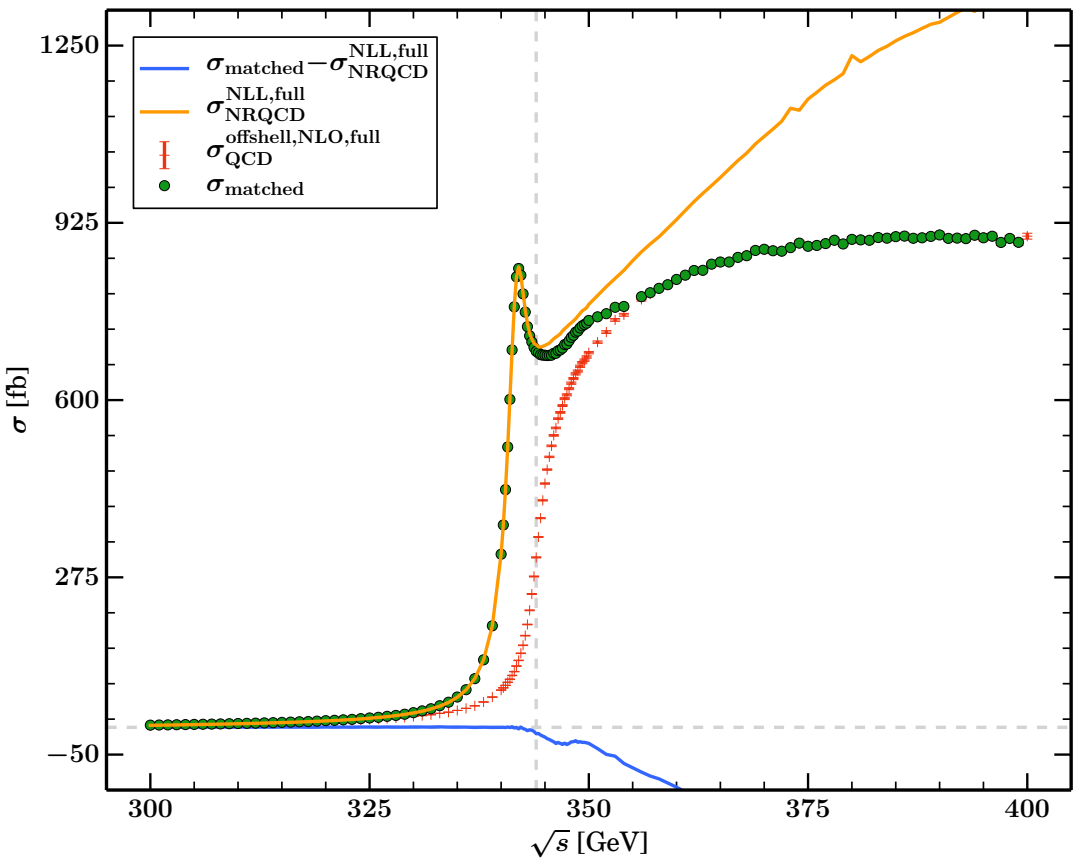

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Conclusions

- NLL threshold resummation can be used directly within WHIZARD. Includes interference effects with all tree-level diagrams of $WbWb$
- Consistent embedding of the NRQCD formfactor in the relativistic fixed-order NLO computation

Outlook

- Study the effects of polarized beams, Beamstrahlung and ISR on and distributions of the matched prediction
- Include NLO t decay also in threshold region by using an offshell factorized approach

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To ease cross-checking, we used m_{pole} for now.
But there is no problem to compute
$m_{\text{pole}}(m_{1S}, \alpha_s)$ and $\Gamma(m_{\text{pole}}(m_{1S}, \alpha_s))$ event-by-event

Backup

Parameters used

```
mZ = 91.188
mW = 80.419
mb = 4.2
alphas = 0.118 (Z pole, NLL running)
alpha_em_i = 125.924 (not running)
m_top = 172.
wtop = 1.54
ee = 0.315901
cw = 0.881903
sw = 0.47143
v = 240.024
```

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