NNLO corrections to epsilon'/epsilon

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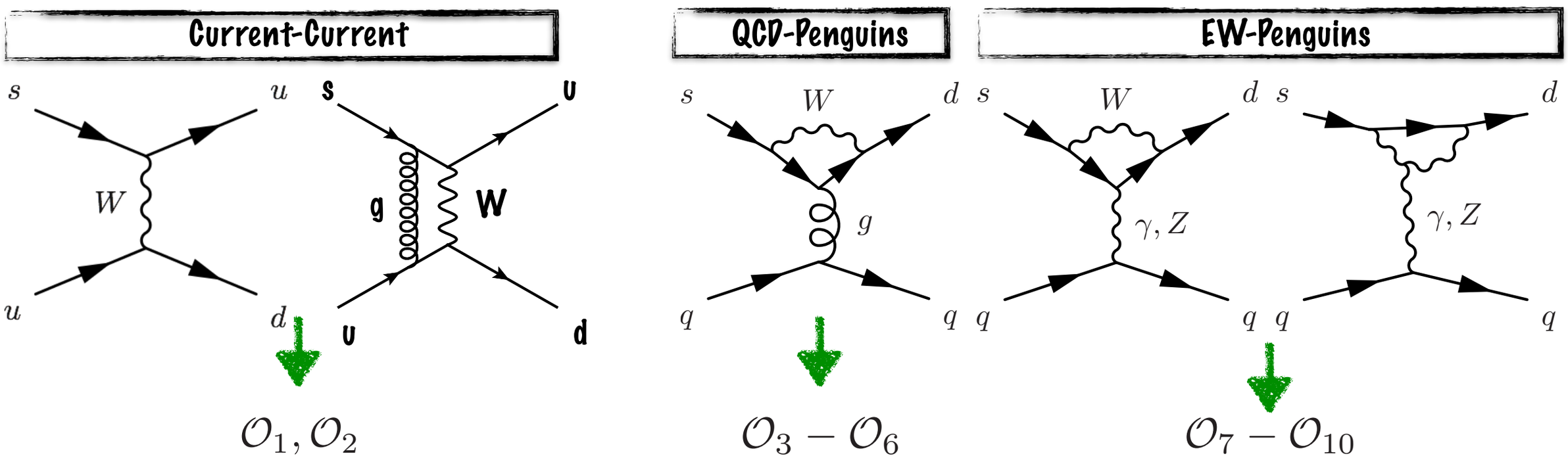
[Gorbahn, Jäger, Kokulu]

9th Annual Meeting of the Helmholtz Alliance
Physics at the Terascale
Nov 17-18, 2015

CPV in Kaon Decays

[Buras et.al., Ciuchini et. al. `92 `93]



$$V_{ij}^{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \sim \mathcal{O} \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \qquad \lambda = \mathcal{O}(0.2)$$

s → d Transition

$\lambda^5 \sim 10^{-4}$

The CP violation is small because of flavor suppression

Weak Effective Theory

Effective Hamiltonian at $\mu < m_c$

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_{i=1}^{10} (z_i(\mu) + \tau y_i(\mu)) \mathcal{O}_i(\mu), \quad \tau \equiv -\frac{V_{td} V_{ts}^*}{V_{ud} V_{us}}$$

perturbative Wilson coeffs.

Only the Imaginary part of solely τ responsible for CPV
(everything else is pure-real)

Short-Distance: [Buras et.al., Ciuchini et. al. `92 `93]

Wc's have been calculated at the NLO level.
Some pieces of NNLO corrections are also available.

Long-Distance:

Lattice QCD calculation of the Matrix Elements

[Blum et. al., Bai et. al. `15]

$$\langle (\pi\pi)_I | \mathcal{O}_i | K \rangle = \langle \mathcal{O}_i \rangle_I$$

by RBC-UKQCD.

epsilon'/epsilon in the SM

The CPV is parametrized as,

$$\mathcal{R}\mathrm{e}\left(\frac{\epsilon'}{\epsilon}\right) \simeq \frac{\epsilon'}{\epsilon} = -\frac{\omega}{\sqrt{2}|\epsilon_K|}\left[\frac{\mathcal{I}\mathrm{m}(A_0)}{\mathcal{R}\mathrm{e}(A_0)}(1-\Omega_{\mathrm{eff}}) - \frac{\mathcal{I}\mathrm{m}(A_2)}{\mathcal{R}\mathrm{e}(A_2)}\right]$$

[Buras, Gorbahn, Jäger, Jamin `15]

Adjusted to keep EW in Im(A0)

[Cirigliano, et.al. `11]

$$\left(\frac{\varepsilon'}{\varepsilon}\right)_{\mathrm{SM}} = (1.9 \pm 4.5) \times 10^{-4}$$

$$\left(\frac{\varepsilon'}{\varepsilon}\right)_{\mathrm{exp}} = (16.6 \pm 2.3) \times 10^{-4}$$

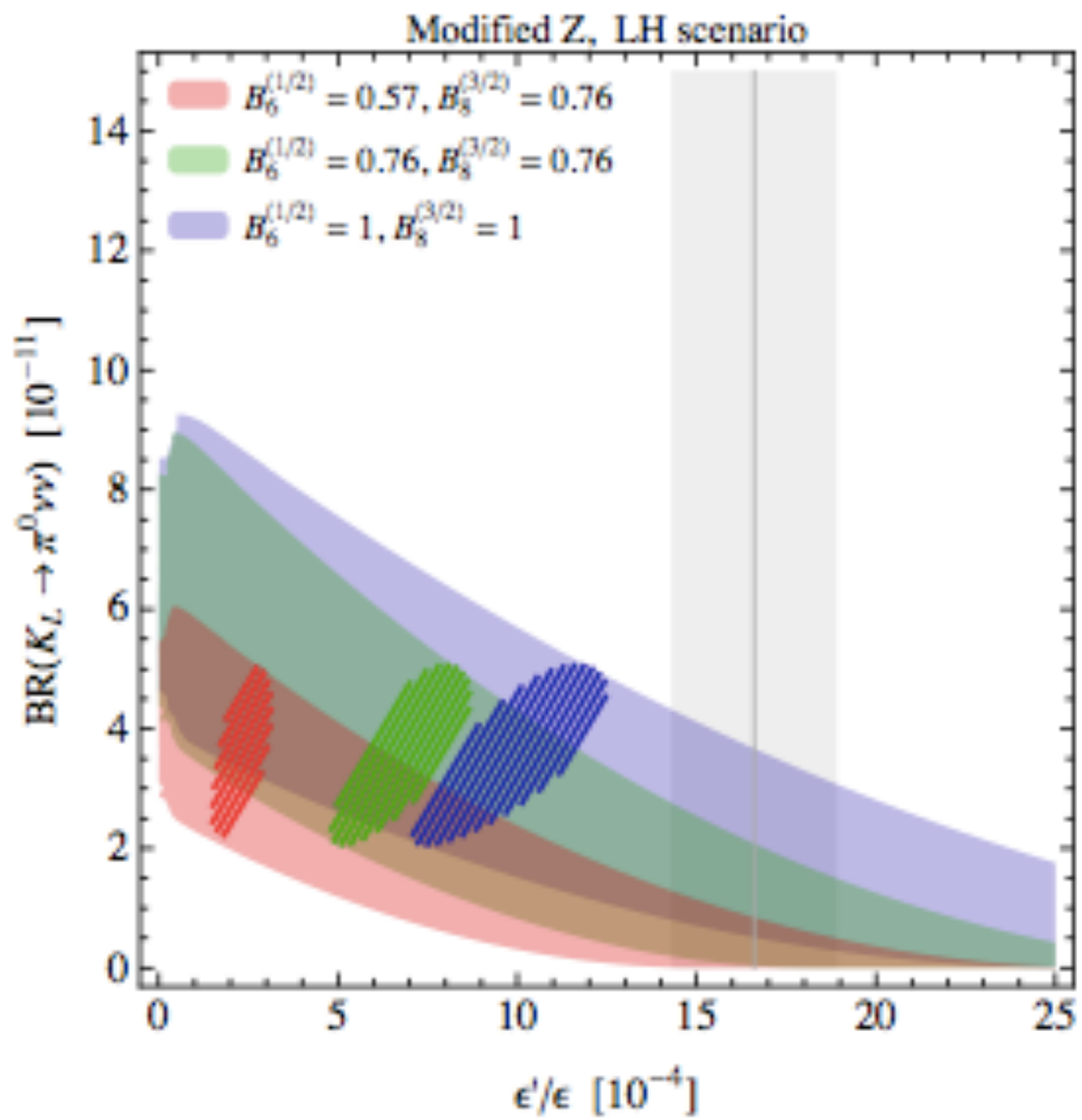
2.9σ → Tension between the theoretical prediction and the experimental data → NEW PHYSICS ???

Indirect searches for New Physics

[Buras et. al.`15]

If SM contribution is suppressed → NP sensitivity enhanced.

Additional K observables will help disentangle NP.



Computation of Im(A)/Re(A)

For I=0:

[Buras, Gorbahn, Jäger & Jamin `15]

$$\mathcal{R}e(\mathcal{A}_0) = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \left(z_+ \langle \mathcal{O}_+ \rangle_0 + z_- \langle \mathcal{O}_- \rangle_0 \right)$$

Fierz relations for (V-A)x(V-A) give, e.g.: $\langle \mathcal{O}_4 \rangle_0 = \langle \mathcal{O}_3 \rangle_0 + 2 \langle \mathcal{O}_- \rangle_0$

$$\left(\frac{\mathcal{I}m(\mathcal{A}_0)}{\mathcal{R}e(\mathcal{A}_0)} \right)_{\text{V-A}} = \mathcal{I}m(\tau) \frac{2y_4}{(1+q)z_-} + \mathcal{O}(p_3)$$

is only a function of the Wc's and the ratio

$$q \equiv \frac{z_+(\mu) \langle \mathcal{O}_+(\mu) \rangle_0}{z_-(\mu) \langle \mathcal{O}_-(\mu) \rangle_0}$$

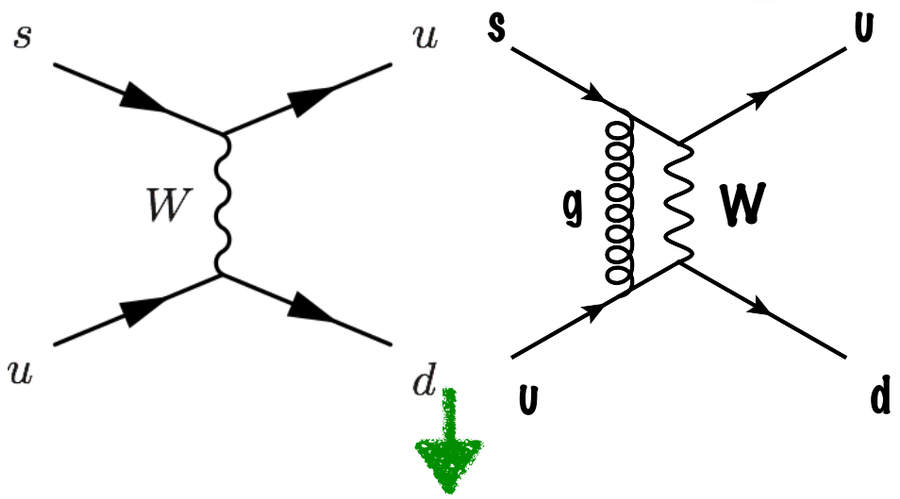
Sub-leading order contributions

For (V-A)x(V+A) operators:

$$\left(\frac{\mathcal{I}m(\mathcal{A}_0)}{\mathcal{R}e(\mathcal{A}_0)} \right)_{\text{V+A}} = -\frac{G_F}{\sqrt{2}} \mathcal{I}m\lambda_t \; y_6 \frac{\langle \mathcal{O}_6 \rangle_0}{\mathcal{R}e(\mathcal{A}_0)} + \mathcal{O}(p_5)$$

Operator Contributions

Current-Current



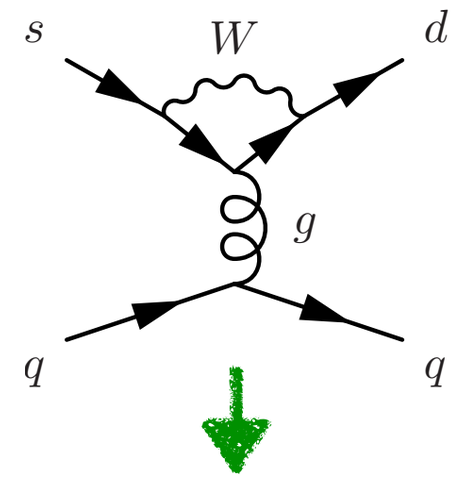
$\mathcal{O}_1, \mathcal{O}_2$

Dominate

$\mathcal{R}e(\mathcal{A}_0)\,\mathcal{R}e(\mathcal{A}_2)$

$$\mathcal{O}_{1,2/\pm} = (\bar{s}_i u_j)_{\text{V-A}} (\bar{u}_k d_l)_{\text{V-A}}$$

QCD-Penguins



$\mathcal{O}_4, \mathcal{O}_6$

Dominate

$\mathcal{I}m(\mathcal{A}_0)$

$$\mathcal{O}_{3,4} = (\bar{s}_i d_j)_{\text{V-A}} \sum_{q=u,d,s} (\bar{q}_k q_l)_{\text{V-A}}$$

$$\mathcal{O}_{5,6} = (\bar{s}_i d_j)_{\text{V-A}} \sum_{q=u,d,s} (\bar{q}_k q_l)_{\text{V+A}}$$

Prediction for epsilon'/epsilon

I=0 (V-A)x(V-A) I=2 (V-A)x(V-A)

$$\frac{\epsilon'}{\epsilon} = 10^{-4}\left[\frac{\mathcal{I}m\lambda_t}{1.4.10^{-4}}\right]\left[a(1-\hat{\Omega}_{\rm eff})(-4.1(8)+24.7B_6^{(1/2)})+1.2(1)-10.4B_8^{(3/2)}\right]$$

I=0 (V-A)x(V+A) I=2 (V-A)x(V+A)

(V-A)x(V+A) Matrix elements B_6=0.57(19) and B_8=0.76(5) from Lattice QCD.

normalization of $A_{0,2}$ Chiral suppression

$$\langle\mathcal{O}_6(\mu)\rangle_0 = -4h\left[\frac{m_K^2}{m_s(\mu)+m_d(\mu)}\right]^2(F_K-F_\pi)B_6^{(1/2)}$$

$$\langle\mathcal{O}_8(\mu)\rangle_2 = \sqrt{2}h\left[\frac{m_K^2}{m_s(\mu)+m_d(\mu)}\right]^2 F_\pi B_8^{(3/2)}$$

[Blum et. al., Bai et. al. `15]

Error budget

Quantity	error on ϵ'/ϵ
$B_6^{(1/2)}$	4.1
NNLO	1.6
$\hat{\Omega}_{\text{eff}}$	0.7
p_3	0.6
$B_8^{(3/2)}$	0.5
p_5	0.4
$m_s(m_c)$	0.3
$m_t(m_t)$	0.3

Accuracy of at least 10%

Perturbative uncertainty.

Improved Calculations of Isospin breaking effects

Higher precision would be beneficial

[Buras, MG, Jäger, Jamin `15]

RBC-UKQCD will reduce the statistical uncertainty.

1/N [Buras Gerard `15] consistent with RBC-UKQCD,
But, we still need an independent Lattice calculation.

NNLO vs NLO

i) [Misiak, Bobeth, Urban]
ii) [Gambino,Buras, U.H]
iii) [Gorbahn, Haisch]
iv) [Gorbahn, Brod]
v) [Buras, M.Jamin, M.E.L]
vi) [Blum et. al., Bai et. al. '15]

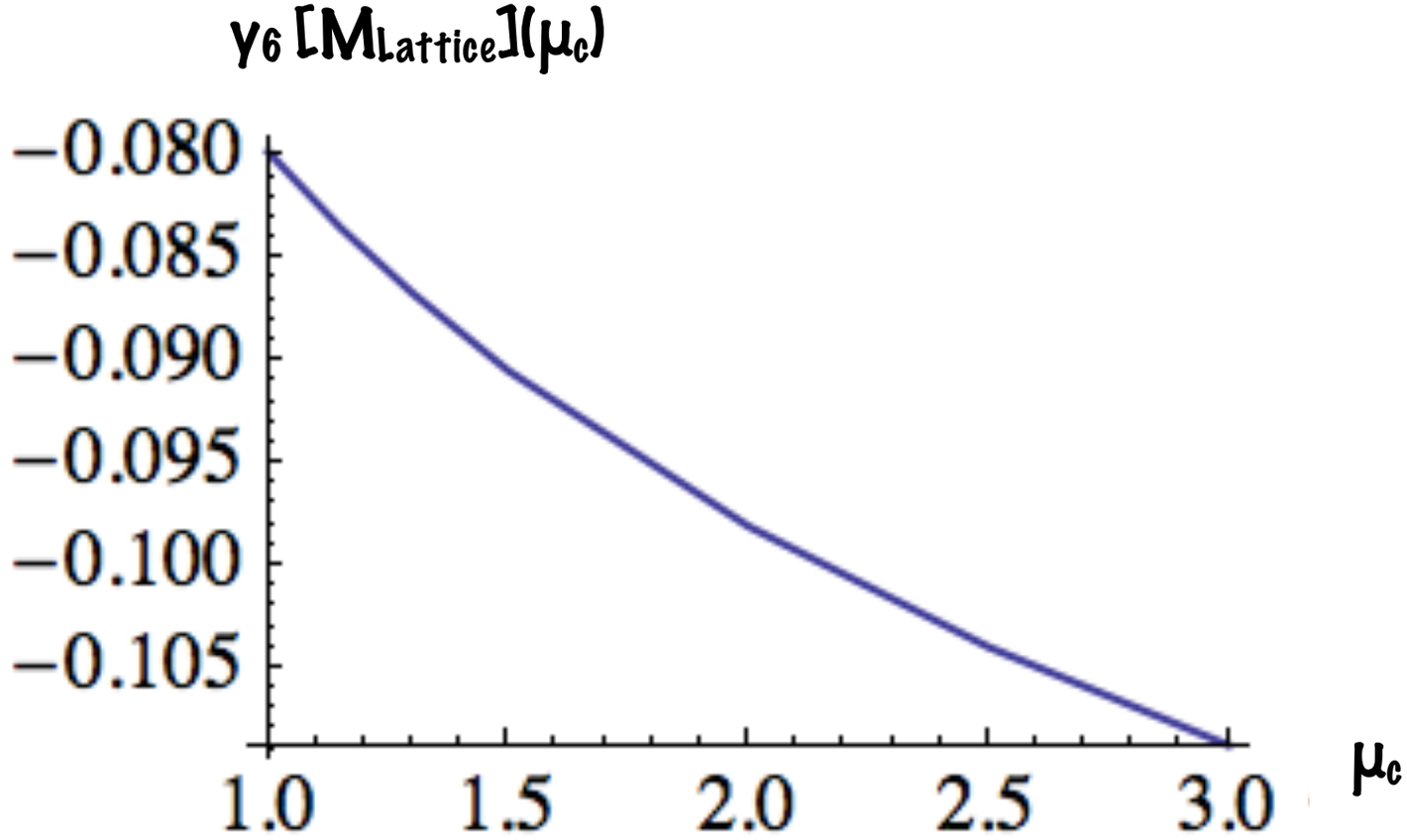
	Energy scale	Fields	Effective Theory	
Matching at μW	M_W	γ,g, H τ, μ, e, νi b, c, s, d, u	A5	i) NNLO QCD + Current-Current ii) NNLO EW penguins in the traditional basis iii) NNLO in QCD ADM/RGE
Matching at μb	m_b	γ,g τ, μ, e, νi c, s, d, u	A4	iv) NNLO QCD+ Current-Current
Matching at μc	m_c	γ,g μ, e, νi s, d, u	A3	V) NLO All operators in the traditional basis
Matching to Lattice QCD	$M_{Lattice}$	u, s, d	Lattice QCD	V) NLO for all operators except Q_{8g} in the traditional basis

y_6 ($M_{Lattice}$) at NLO

The (V-A) (V+A) $\times$ y_6 : largest contribution to $\mathrm{Im}(A_0)$ and ε'/ε

How well do we know y_6 at the scale of the Lattice matrix elements $M_{Lattice}$?

Integrating out m_c results in strong μ_c dependence.



For $\alpha_s(M_Z)$ =0.1185 GeV

Plot the residual μ_c dependence:

1, RGE: $y_6(\mu_b) \rightarrow y_6(\mu_c)$

2, Match $y_6^{(f=4)}(\mu_c) \rightarrow y_6^{(f=3)}(\mu_c)$

3, RGE: $y_6(\mu_c) \rightarrow y_6(M_{Lattice})$

The scale dependence in 1 & 3 is canceled by the $\log(\mu_c)$ in 2.

NNLO Operator Basis

The traditional basis requires the calculation of traces with $\gamma 5$.

$$\mathcal{O}_{5,6} = (\bar{s}_i d_j)_{\text{V-A}} \sum_{u,d,s} (\bar{q}_k q_l)_{\text{V+A}}$$

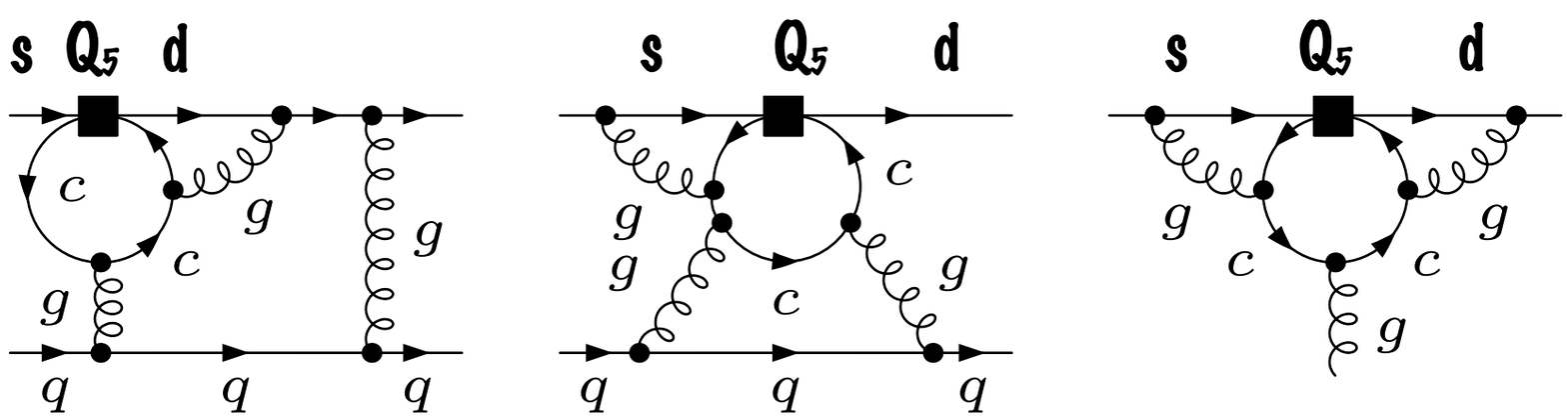
Issues with the treatment of the $\gamma 5$ in D dimensions

Higher order calculations can be significantly simplified if we use a different operator basis.

[Misiak et al.]

$$\mathcal{O}^m_{5,6} = (\bar{s}_i \gamma_\mu \gamma_\nu \gamma_\rho P_L d_j)_{\text{V-A}} \sum_{u,d,s} (\bar{q}_k \gamma^\mu \gamma^\nu \gamma^\rho q_l)$$

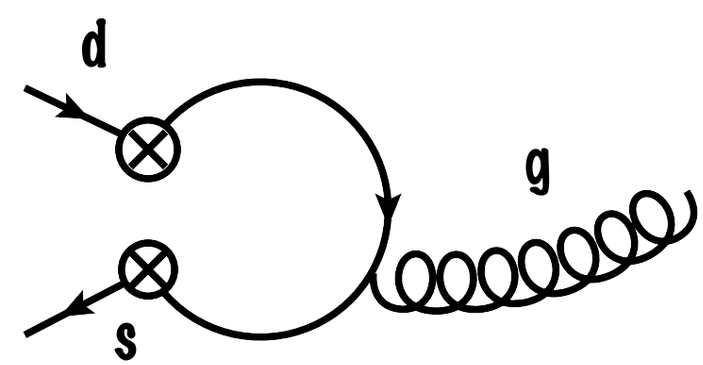
No trace of $\gamma 5$



Charm Matching

Q_1 & Q_2 have the largest Wilson Coefficients.

Only one type of $s \to d$ gluon diagram for Q_1 & Q_2



We perform an off-shell matching: expanding in external momentum $O(k^2)$

$$\mathcal{O}_{31} = \frac{1}{g}\bar{s}_L\gamma^\mu T^a b_L D^\nu G^a_{\mu\nu} + \mathcal{O}_4$$

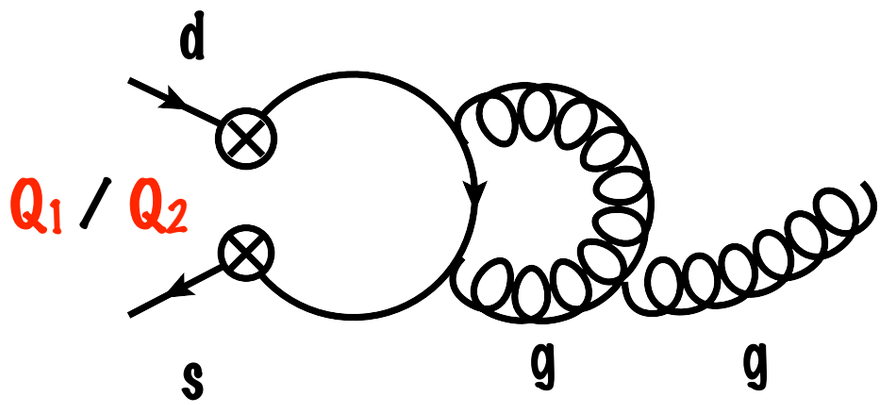
$$\mathcal{O}_4 = (\bar{s}_L\gamma^\mu T^a b_L)\sum_q(\bar{q}\gamma^\mu T^a q)$$

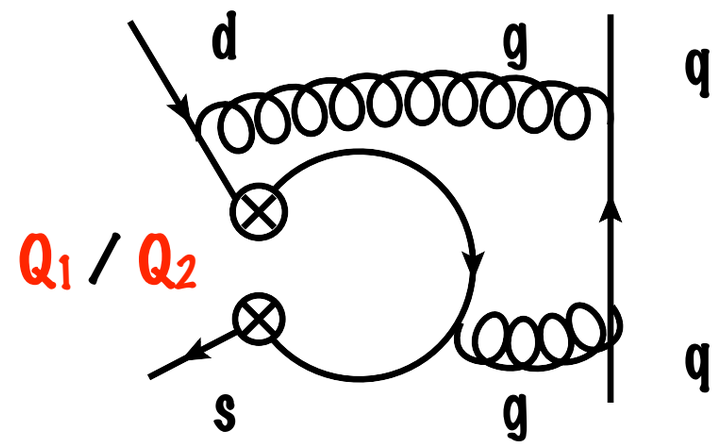
There are no one-light-particle-irreducible diagrams for $s \to d\,\bar{u}\,u$.

→ No evanescent operators are generated at one-loop.

NNLO Matching

There are Q_1 & Q_2 have the largest Wilson Coefficients.



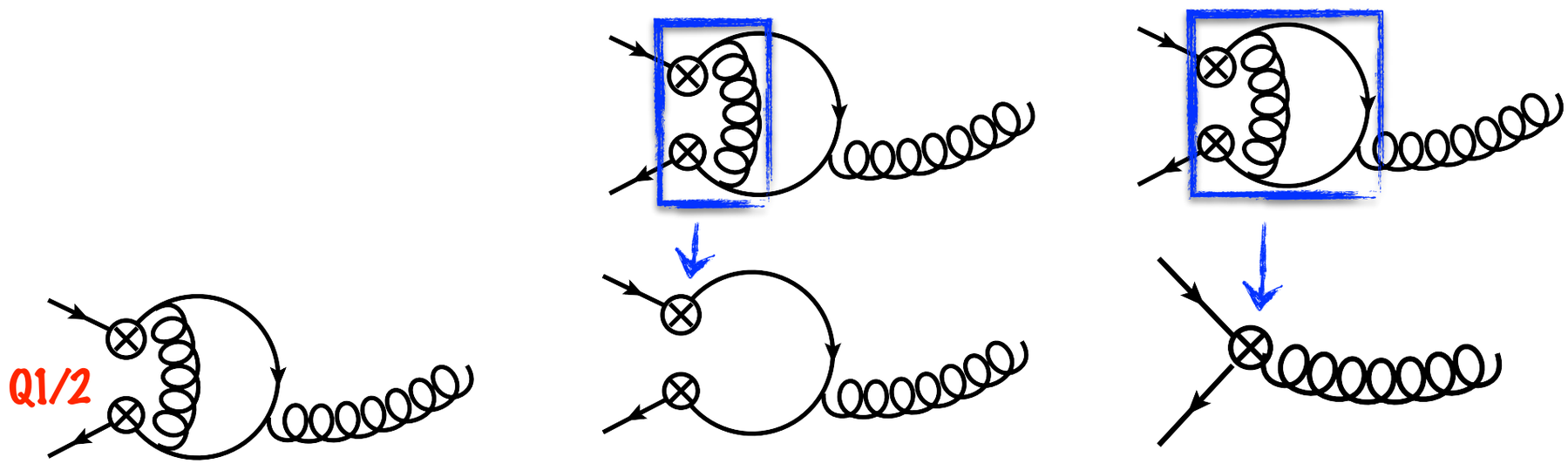


The calculation produces several types of structures,

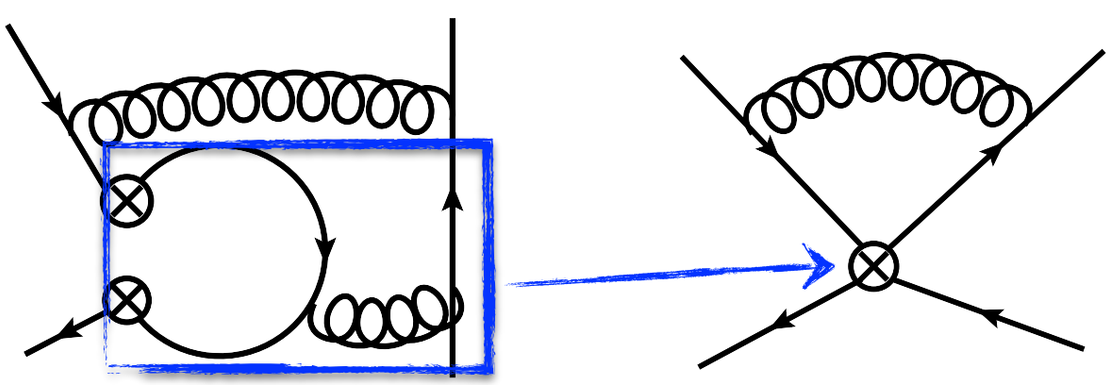
$$(\bar{s}_i \gamma^\mu P_L T^a_{ij} d_j) G^a_\mu k_1^2 \quad (\bar{s}_i \gamma_\nu T^a_{ij} P_L d_j) G^a_\mu k_1^\mu k_2^\nu \quad \ldots$$

- more than operators.

Renormalisation



Our procedure: Full (f=4) theory is still divergent after renormalisation.



Counterterm matrix element vanishing for $m_s = m_d = m_u = 0$

Renormalisation Q_1/Q_2 II

Divergencies in 4 flavor theory canceled by 3 flavor theory:

One-loop matching coefficient $\times$ one-loop operator mixing

$A_{full} = A_{eff}$ results then in finite threshold corrections for Q_3 - Q_6

Additional Check:
All results can be projected onto the Physical and EOM vanishing Operator Basis.

Note: Evanescent Operators only contribute in f=4 theory at NNLO

Rough numerics - considering only NNLO matching contributions of Q_1/Q_2 - suggest that perturbation theory seems to be OK

Conclusions

- There is a tension between the SM prediction of ε'/ε and experimental data.
- There are different potential sources of uncertainties in the prediction of ε'/ε
- Current perturbative calculation relies on removing the charm quark perturbatively.
- We perform this at NNLO and check the consistency of this approach.
- This could help to improve the SM prediction.
- And potentially motivate to look into new physics scenarios.

Thanks !!!!

[Buras, Gorbahn, Jäger & Jamin `15]

For I=2:

$$\mathcal{R}e(\mathcal{A}_2) = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* z_+ \langle \mathcal{O}_+ \rangle_2$$

For (V-A)x(V-A) structure

$$\left(\frac{\mathcal{I}m(\mathcal{A}_2)}{\mathcal{R}e(\mathcal{A}_2)}\right)_{\text{V-A}} = \mathcal{I}m(\tau) \frac{3(y_9+y_{10})}{2z_+}$$

Free from hadronic uncertainties.

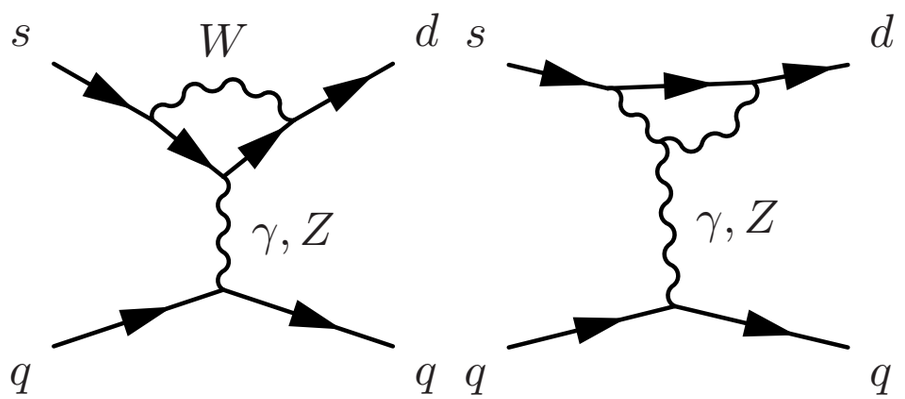
For (V-A)x(V+A) operators:

$$\left(\frac{\mathcal{I}m(\mathcal{A}_2)}{\mathcal{R}e(\mathcal{A}_2)}\right)_{\text{V+A}} = -\frac{G_F}{\sqrt{2}} \mathcal{I}m\lambda_t \, y_8^{\text{eff}} \frac{\langle \mathcal{O}_8 \rangle_2}{\mathcal{R}e(\mathcal{A}_2)}$$

Small effects of ME
Q7 I=2.

Operator Contributions

EW-Penguins



$\mathcal{O}_7, \mathcal{O}_8$

Dominate

$\mathcal{I}m(\mathcal{A}_2)$

$$\mathcal{O}_{7,8} = (\bar{s}_i d_j)_{\text{V-A}} \sum_{q=u,d,s} e_q (\bar{q}_k q_l)_{\text{V+A}}$$

$$\mathcal{O}_{9,10} = (\bar{s}_i d_j)_{\text{V-A}} \sum_{q=u,d,s} e_q (\bar{q}_k q_l)_{\text{V-A}}$$

Matching

At high energy scale

[Howard Georgi, HUTP 93 /A003 1/93]

$$\mathcal{A}_{\text{full}} = \mathcal{A}_{\text{eff}}$$

Up to Next to Next to Leading Order

$$\mathcal{A}_{\text{full}} = \langle \mathcal{O} \rangle^{(0)T} \left(\overline{\mathcal{A}}^{(0)} + \frac{\alpha_s(\mu_0)}{4\pi} \mathcal{A}^{(1)} + \left(\frac{\alpha_s(\mu_0)}{4\pi} \right)^2 \mathcal{A}^{(2)} \right)$$

$$\mathcal{A}_{\text{eff}} = \langle \mathcal{O} \rangle^{(0)T} \left(\hat{I} + \frac{\alpha_s(\mu_0)}{4\pi} \hat{r}^{(1)T} + \left(\frac{\alpha_s(\mu_0)}{4\pi} \right)^2 \hat{r}^{(2)T} \right) \overline{\mathcal{C}}(\mu_0)$$

Initial Conditions for Wc's up two-loops

At high energy scales:

$$\overline{\mathcal{C}}(\mu_0) = \overline{\mathcal{A}}^{(0)} + \frac{\alpha_s(\mu_0)}{4\pi}\left(\overline{\mathcal{A}}^{(1)} - \hat{r}^{(1)T}\overline{\mathcal{A}}^{(0)}\right) + \left(\frac{\alpha_s(\mu_0)}{4\pi}\right)^2\left(\overline{\mathcal{A}}^{(2)} - \hat{r}^{(1)T}\left[\overline{\mathcal{A}}^{(1)} - \hat{r}^{(1)T}\overline{\mathcal{A}}^{(0)}\right] - \hat{r}^{(2)T}\overline{\mathcal{A}}^{(0)}\right)$$

Evolving to a down scale:

$$\overline{\mathcal{C}}(\mu_0) = \hat{K}(\mu)\hat{U}^{(0)}(\mu,\mu_0)\left(\overline{\mathcal{A}}^{(0)} + \frac{\alpha_s}{4\pi}\left[\overline{\mathcal{A}}^{(1)} - \hat{R}^{(1)}\overline{\mathcal{A}}^{(0)}\right] + \left(\frac{\alpha_s(\mu_0)}{4\pi}\right)^2\left[\overline{\mathcal{A}}^{(2)} - \hat{R}^{(1)}\overline{\mathcal{A}}^{(1)} - \left(\hat{R}^{(2)} - (\hat{R}^{(1)})^2\right)\overline{\mathcal{A}}^{(0)}\right]\right)$$

with,

$$\hat{R}^{(1)} = \hat{r}^{(1)T} + \hat{J}^{(1)}, \quad \hat{R}^{(2)} = \hat{r}^{(2)T} + \hat{J}^{(2)} + \hat{r}^{(1)T}\hat{J}^{(1)}.$$

Threshold Corrections

The strong coupling constant is discontinuous beyond LO.

↓

Non-trivial matching correction at a flavor threshold.

$$
\begin{aligned}
\alpha_s^{(s)}(\mu_f) &= \alpha_s^{(f-1)}\left(1 + \frac{\alpha_s^{(f-1)}(\mu_f)}{4\pi}\frac{2}{3}\log\frac{\mu_f^2}{m_f^2}\right. \\
&- \left.\left(\frac{\alpha_s^{(f-1)}(\mu_f)}{4\pi}\right)^2\left(\frac{22}{9} - \frac{22}{3}\log\frac{\mu_f^2}{m_f^2} - \frac{4}{9}\log^2\frac{\mu_f^2}{m_f^2}\right)\right)
\end{aligned}
$$

Operator Basis

$$\mathcal{O}_1^{q'} = (\bar{s}_L \gamma_\mu T^a q'_L)(\bar{q'}_L \gamma^\mu T^a d_L), \quad \mathcal{O}_2^{q'} = (\bar{s}_L \gamma_\mu q'_L)(\bar{q'}_L \gamma^\mu d_L),$$

$$\mathcal{O}_3 = (\bar{s}_L \gamma_\mu d_L) \sum_q (\bar{q} \gamma^\mu q), \qquad \mathcal{O}_4 = (\bar{s}_L \gamma_\mu T^a d_L) \sum_q (\bar{q} \gamma^\mu T^a q),$$

$$\mathcal{O}_5 = (\bar{s}_L \gamma_{\mu\nu\rho} d_L) \sum_q (\bar{q} \gamma^{\mu\nu\rho} q), \qquad \mathcal{O}_6 = (\bar{s}_L \gamma_{\mu\nu\rho} T^a d_L) \sum_q (\bar{q} \gamma^{\mu\nu\rho} T^a q),$$

$$\mathcal{O}_3^Q = (\bar{s}_L \gamma_\mu d_L) \sum_q (\bar{q} \gamma^\mu q), \qquad \mathcal{O}_4^Q = (\bar{s}_L \gamma_\mu T^a d_L) \sum_q (\bar{q} \gamma^\mu T^a q),$$

$$\mathcal{O}_5^Q = (\bar{s}_L \gamma_{\mu\nu\rho} d_L) \sum_q (\bar{q} \gamma^{\mu\nu\rho} q), \qquad \mathcal{O}_6^Q = (\bar{s}_L \gamma_{\mu\nu\rho} T^a d_L) \sum_q (\bar{q} \gamma^{\mu\nu\rho} T^a q).$$