Non-factorizable photonic corrections to resonant production and decay of many unstable particles

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Processes with Unstable Particles

(Heavy) unstable particles: Z, $\mathrm{W}^{\pm}$, H, t, ...

Physics: $pp \to \ldots$

- Z, W: Drell–Yan, precise measurement of the Z-mass, W-mass
- $\mathrm{Z}\gamma, \mathrm{W}\gamma$: Triple gauge boson couplings (TGC)
- $\mathrm{W}^{+}\mathrm{W}^{-}$: W pair production: TGC
- $\mathrm{W}^{\pm}\mathrm{W}^{\pm}jj$: Vector boson scattering (VBS), test of symmetry breaking mechanism, quartic gauge couplings (QGC)
- $\mathrm{WWZ}, \mathrm{WWW}$: Triple gauge boson production: QGC
- $\mathrm{Ht\bar{t}}$: Higgs Yukawa couplings to top quarks

Challenge

With subsequent decays complicated: $2 \to 2, 3, 4, 6, 8$

Aim: Calculate approximate electroweak corrections

Electroweak Corrections

- $\alpha_s^2(M_Z) \approx 1.8 \cdot \alpha(M_Z) \rightarrow$ EW effects small, but still relevant ($\sim$ 10 %):
- Electroweak Sudakov logs large in tails of distributions, e.g.

$$\log^2 \frac{s}{M_W^2} \approx 25 \quad \text{at } \sqrt{s} = 1\,\text{TeV}$$

- And: Near thresholds also important
- change distibutions in shape $\rightarrow$ important for measurement of anomalous couplings

Strategy of the Calculation

$$\sigma = \int_n \mathrm{d}\sigma_{\mathrm{LO}} + \int_{n+1} \mathrm{d}\sigma_{\mathrm{real}} + \int_n \mathrm{d}\sigma_{\mathrm{virt}}$$

- Calculate σ_{LO}, σ_{real} with full off-shell effects
- Approximate σ_{virt} using pole approximation
- Cancellation of IR singularities $\rightarrow$ backup slide
- Use complex-mass scheme

$\rightarrow$ In the following: Pole Approximation (PA) for virtual corrections

Pole Approximation (I)

Idea at LO: Pole expansion around the singularities at $\overline{k}_i^2 = M_i^2$, i.e.

$$\mathcal{M}_{\text{LO,PA}} = \sum_{\lambda_1,\ldots,\lambda_r} \left(\prod_{i=1}^{r} \frac{1}{K_i} \right) \left[\mathcal{M}_{\text{LO}}^{I\to N,R} \left(\prod_{j=1}^{r} \mathcal{M}_{\text{LO}}^{j\to R_j} \right) \right]_{\left\{\overline{k}_l^2 = M_l^2\right\}} .$$

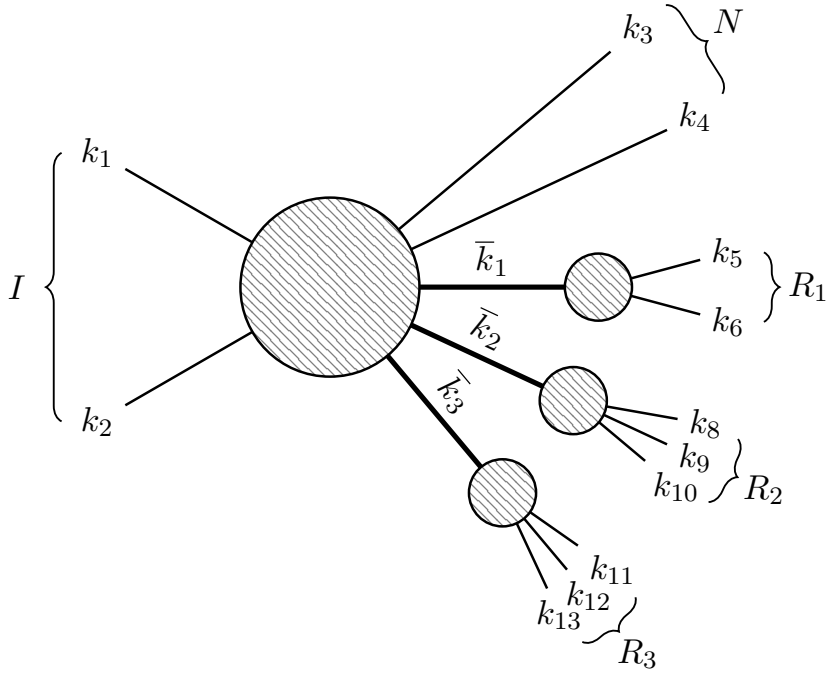
Similar to Narrow-width approximation, i.e.

- On-shell momenta in the $\mathcal{M}$: $\overline{k}_i^2 = M_i^2$
- Gauge invariant

but also

- Full spin correlation
- Full momentum dependence in $K_i = \overline{k}_i^2 - M_i^2 + \mathrm{i} M_i \Gamma_i$

$\rightarrow$ Faster to evaluate



On-Shell Projection

- Matrix elements must be evaluated with resonances on-shells, i.e. $\overline{k}_i^2 = M_i^2$,
- but phase space generator produces external momenta so that the resonances are usually off-shell

$$\overline{k}_i^2 = \left(\sum_{a \in R_i} k_a\right)^2 \neq M_i^2$$

This leads to ...

On-shell projection

$$\begin{aligned} \left\{k_a\right\} &\rightarrow \left\{\hat{k}_a\right\} \\ \left\{\overline{k}_i^2 \neq M_i^2, k_a^2 = m_a^2\right\} &\rightarrow \left\{\hat{\overline{k}}_i^2 = M_i^2, k_a^2 = m_a^2\right\} \end{aligned}$$

For example, $2 \rightarrow 2$:

$$\hat{s} = \frac{M^2}{s} s = M^2, \hat{t} = \frac{M^2}{s} t, \hat{u} = \frac{M^2}{s} u$$

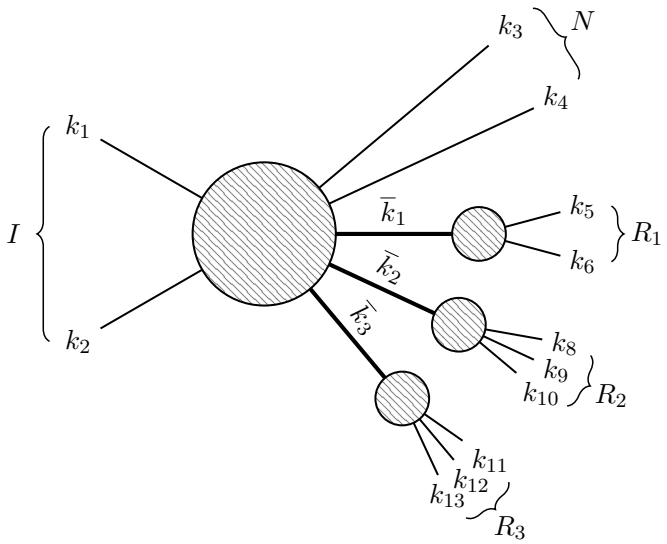
Different on-shell projections possible

NLO: Factorizable Corrections

So far pole approximation (PA) at LO, for

$$2\,\mathrm{Re}\left\{\mathcal{M}^*_{\mathrm{LO,PA}}\mathcal{M}_{\mathrm{virt,PA}}\right\},$$

generalization to virtual corrections straightforward:

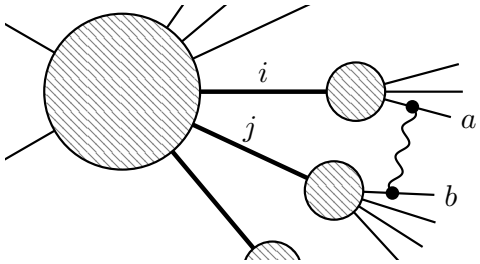


$$\mathcal{M}_{\mathrm{virt,fact,PA}} = \sum_{\lambda_1,\ldots,\lambda_r}\left(\prod_{i=1}^{r}\frac{1}{K_i}\right)\left[\mathcal{M}^{I\to N,R}_{\mathrm{virt}}\prod_{j=1}^{r}\mathcal{M}^{j\to R_j}_{\mathrm{LO}} + \mathcal{M}^{I\to N,R}_{\mathrm{LO}}\sum_{k=1}^{r}\mathcal{M}^{k\to R_k}_{\mathrm{virt}}\prod_{j\neq k}^{r}\mathcal{M}^{j\to R_j}_{\mathrm{LO}}\right]_{\left\{\bar{k}_l^2=M_l^2\right\}}$$

$\rightarrow$ Factorizable Corrections

NLO: Non-Factorizable Corrections (I)

At 1-loop there are also resonant contributions different from the factorizable ones: Non-factorizable corrections, e.g. from

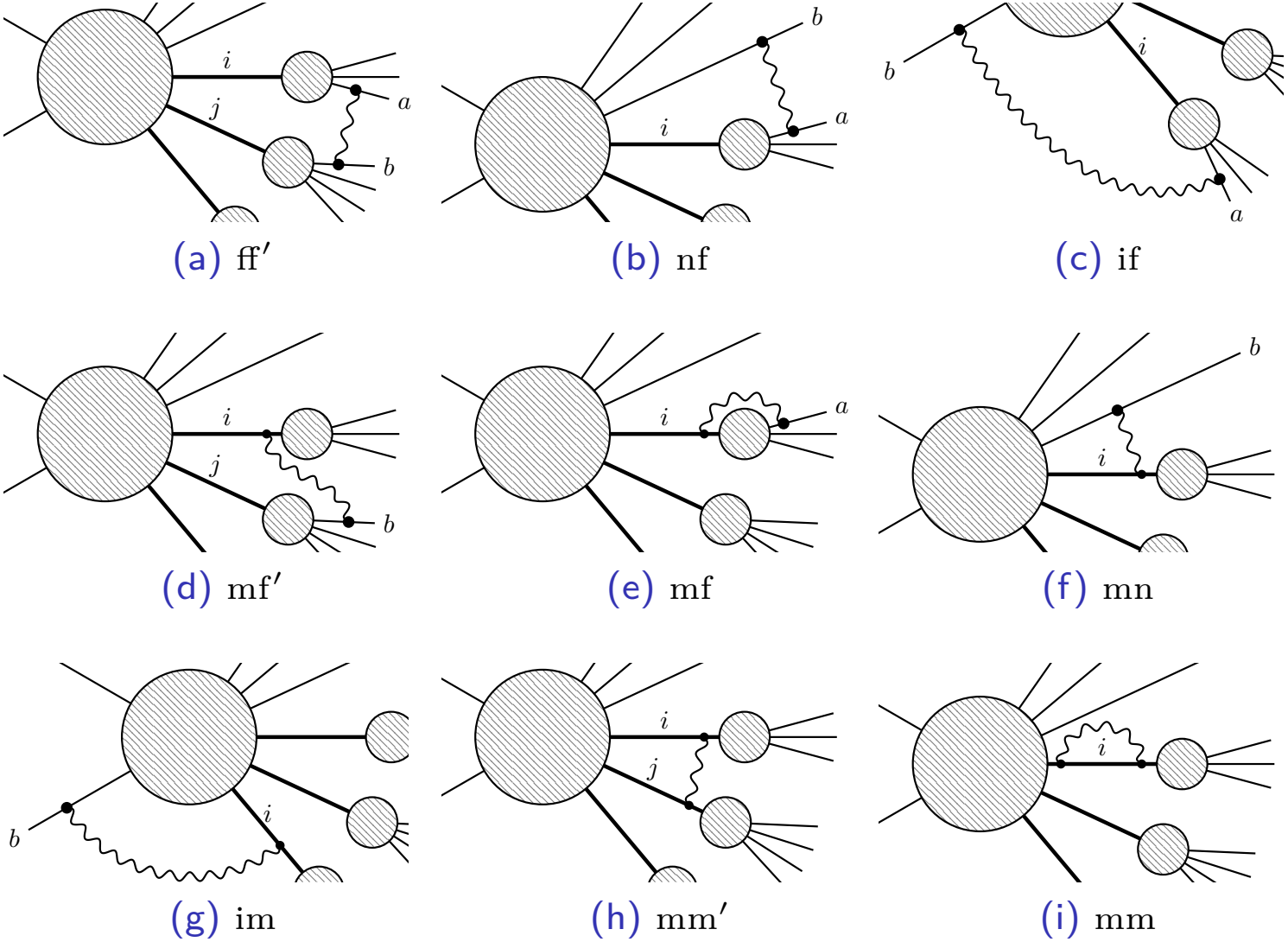


$$\sim \left[\mathcal{M}_{\mathrm{LO,PA}} \int \frac{\mathrm{d}^D q}{(2\pi)^D} \frac{K_i}{q^2 + 2q\overline{k}_i + K_i} \times \frac{K_j}{q^2 - 2q\overline{k}_j + K_j} \frac{1}{q^2 - m_\gamma^2} \frac{1}{q^2 + 2qk_a} \frac{1}{q^2 - 2qk_b} \right]_{\left\{\overline{k}_l^2 = M_l^2\right\}}$$

with $K_i = \overline{k}_i^2 - M_i^2 + \mathrm{i} M_i \Gamma_i$

- this is a purely soft effect $\rightarrow$ consider loops with photons
- "non-factorizable" concerns the structure of the resonant propagators, not the diagrams!

NLO: Non-Factorizable Corrections (II)



(a) ff′ (b) nf (c) if

(d) mf′ (e) mf (f) mn

(g) im (h) mm′ (i) mm

NLO: Non-Factorizable Corrections (III)

Non-factorizable corrections factor from LO:

$$\mathcal{M}_{\text{virt,nfact,PA}} = \mathcal{M}_{\text{LO,PA}}\delta_{\text{nfact}}$$

δ_{nfact} is universal, for arbitrary number r of resonances and non-resonant particles:

$$\delta_{\text{nfact}} = -\sum_{i=1}^{r}\sum_{j=i+1}^{r}\sum_{a\in R_i}\sum_{b\in R_j}\sigma_a\sigma_b Q_a Q_b \frac{\alpha}{\pi}\,\text{Re}\left\{\Delta\right\}$$
$$-\sum_{i=1}^{r}\sum_{a\in R_i}\sum_{b\in N\cup I}\sigma_a\sigma_b Q_a Q_b \frac{\alpha}{\pi}\,\text{Re}\left\{\Delta'\right\}$$

with

$$\Delta(i,a;j,b) = \Delta_{\text{mm}} + \Delta_{\text{mf}} + \Delta_{\text{mm}'} + \Delta_{\text{mf}'} + \Delta_{\text{ff}'},$$
$$\Delta'(i,a;b) = \Delta'_{\text{mm}} + \Delta'_{\text{mf}} + \Delta_{\text{xf}} + \Delta_{\text{xm}},$$

$$\begin{aligned}\mathrm{d}\sigma_{\text{virt}} &\approx \mathrm{d}\sigma_{\text{virt,nfact,PA}} + \mathrm{d}\sigma_{\text{virt,fact,PA}}\\ &= \mathrm{d}\phi_n\left(2\,\text{Re}\left\{\mathcal{M}^*_{\text{LO,PA}}\mathcal{M}_{\text{virt,fact,PA}}\right\} + \left|\mathcal{M}_{\text{LO,PA}}\right|^2\delta_{\text{nfact}}\right)\end{aligned}$$

Comparison between PA and full off-shell

EW corrections for $\sigma(\mathrm{e}^+\mathrm{e}^- \to \mathrm{W}^+\mathrm{W}^- \to \nu_\tau \tau^+ \mu^- \overline{\nu}_\mu)$

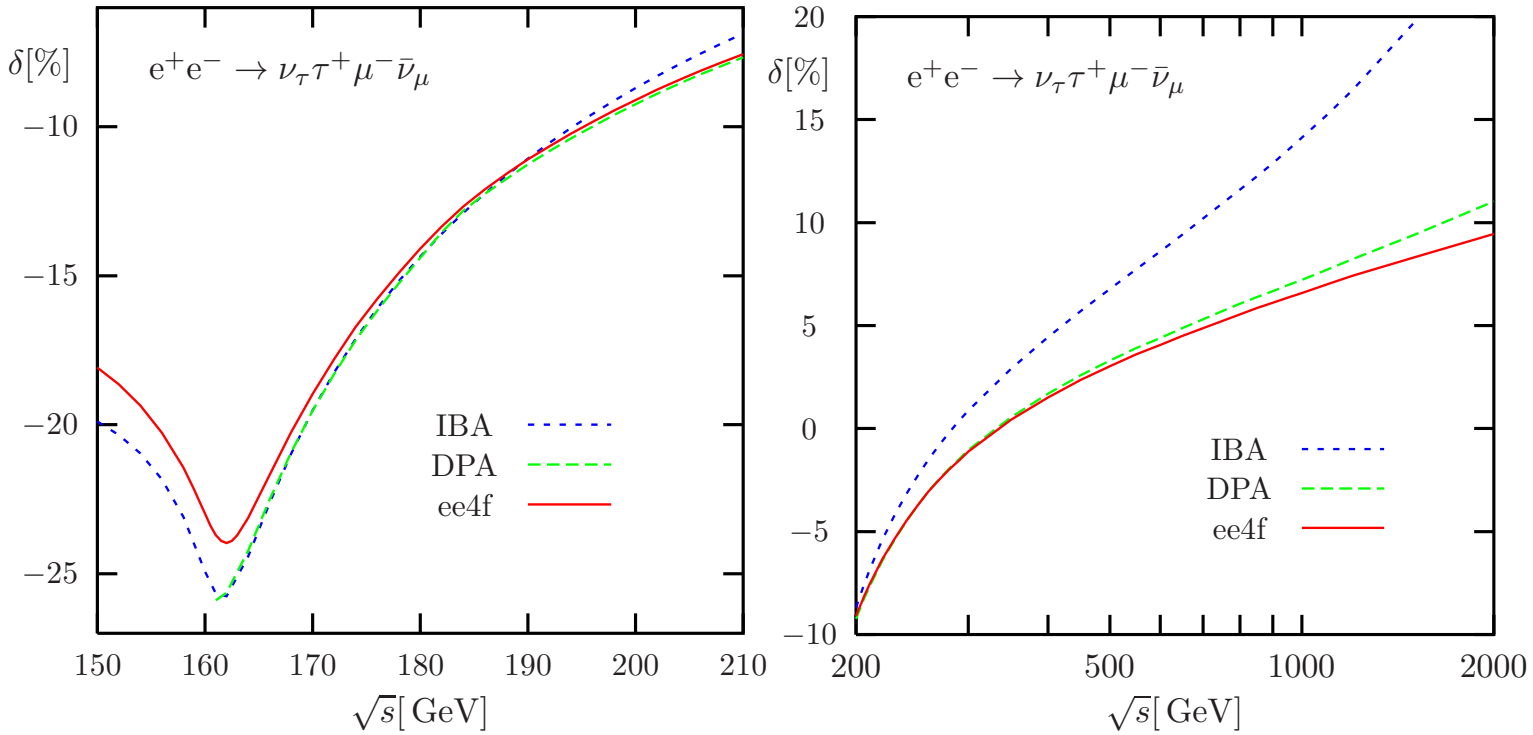


Figure: A.Denner, S.Dittmaier, M.Roth, L.H.Wieders (2011)

- PA valid only several Γ above threshold
- At threshold difference is 2.1 %
- At $\sqrt{s} = 1\,\mathrm{TeV}$ difference is 0.6 %

Summary

- Electroweak corrections are important for precision measurements, can be several 10 %, change the shape of distributions
- Approximation that captures the largest parts is the pole approximation (PA)
- Apply PA only for the virtual correction, full off-shell real correction
- This divides corrections into factorizable, and non-factorizable corrections
- The latter can be computed in a universal way for an arbitrary number of particles: arxiv:1511.01698

Pole Approximation: Resonance expansion

PA basically a "multi-dimensional Laurent series expansion" cut off after the leading term:

$$f(z) = \frac{f(\hat{z})}{z - \hat{z}} + \dots$$

- in more than one dimension, with
- $z \to \overline{k}^2$ and $\hat{z} \to M^2$
- generalization leads to product of the resonances,
- but generalization of the residue is not unambiguous:

$$\frac{\mathcal{M}(\hat{s} = M^2, \hat{t} = ?, \hat{u} = ?)}{s - M^2}$$

Narrow-Width Approximation

$$\left| \frac{1}{p^2 - M^2 + \mathrm{i}M\Gamma} \right|^2 \xrightarrow{\Gamma \to 0} \frac{\pi}{M\Gamma} \delta(p^2 - M^2) \tag{1}$$

- naively no spin correlation
- no interference between production and decay $\rightarrow$ factorization into production matrix elements and branching ratios
- error $\mathcal{O}(\Gamma/M)$

Complex-Mass Scheme

Input-parameter scheme, e.g. with parameters $M_{\mathrm{W}}, M_{\mathrm{Z}}, \Gamma_{\mathrm{W}}, \Gamma_{\mathrm{Z}}, G_{\mu}, \ldots$ from which one obtains the complex, squared masses

$$\overline{M}_{\mathrm{W}}^2 = M_{\mathrm{W}}^2 - \mathrm{i} M_{\mathrm{W}} \Gamma_{\mathrm{W}},$$
$$\overline{M}_{\mathrm{Z}}^2 = M_{\mathrm{Z}}^2 - \mathrm{i} M_{\mathrm{Z}} \Gamma_{\mathrm{Z}},$$

with depending parameters also complex, e.g.

$$\cos^2 \overline{\theta}_{\mathrm{w}} \equiv \frac{\overline{M}_{\mathrm{W}}^2}{\overline{M}_{\mathrm{Z}}^2} = \frac{M_{\mathrm{W}}^2 - \mathrm{i} M_{\mathrm{W}} \Gamma_{\mathrm{W}}}{M_{\mathrm{Z}}^2 - \mathrm{i} M_{\mathrm{Z}} \Gamma_{\mathrm{Z}}}.$$

and

$$\overline{\alpha} = \frac{\sqrt{2}}{\pi} G_{\mu} \overline{M}_{\mathrm{W}}^2 \left(1 - \cos^2 \overline{\theta}_{\mathrm{w}}\right)$$

Cancellation of IR Singularities

In dipole subtraction decompose

$$\int_1 \mathrm{d}\sigma^{\text{real}} = \ldots + \mathrm{d}\sigma^{\text{real,endp}}$$

where the singular part (in mass regularization) is

$$\mathrm{d}\sigma^{\text{real,endp}} = -\mathrm{d}\sigma^{\text{Born}} \frac{\alpha}{2\pi} \sum_i \sum_{j>i} (-1)^{i+j} Q_i Q_j \times \\ \times \left[\mathcal{L}(s_{ij}, m_i^2) + \mathcal{L}(s_{ij}, m_j^2) + C_{ij} + C_{ji} \right],$$

with

$$\mathcal{L}(s, m^2) = \ln\left(\frac{m^2}{s}\right) \ln\left(\frac{m_\gamma^2}{s}\right) + \ln\left(\frac{m_\gamma^2}{s}\right) - \frac{1}{2} \ln^2\left(\frac{m^2}{s}\right) + \frac{1}{2} \ln\left(\frac{m^2}{s}\right)$$

and C_{ij} constants depending on the type of external particles i, j.

$\rightarrow$ For IR singularities of the virtual corrections use approximated $\mathrm{d}\sigma_{\text{virt,sing}} \approx \mathrm{d}\sigma_{LO,PA} \ldots$ and on-shell projected invariants $s_{ij} \rightarrow \hat{s}_{ij}$

Analytic Form of the Non-Factorizable Corrections

$$\delta_{\text{nfact}} = -\sum_{i=1}^{r}\sum_{j=i+1}^{r}\sum_{a\in R_i}\sum_{b\in R_j}\sigma_a\sigma_b Q_a Q_b \frac{\alpha}{\pi}\,\text{Re}\left\{\Delta\right\}$$
$$-\sum_{i=1}^{r}\sum_{a\in R_i}\sum_{b\in N\cup I}\sigma_a\sigma_b Q_a Q_b \frac{\alpha}{\pi}\,\text{Re}\left\{\Delta'\right\}$$

$$\Delta(i,a;j,b) = \Delta_{\mathrm{mm}} + \Delta_{\mathrm{mf}} + \Delta_{\mathrm{mm'}} + \Delta_{\mathrm{mf'}} + \Delta_{\mathrm{ff'}},$$
$$\Delta'(i,a;b) = \Delta'_{\mathrm{mm}} + \Delta'_{\mathrm{mf}} + \Delta_{\mathrm{xf}} + \Delta_{\mathrm{xm}},$$

$$\Delta_{\mathrm{mm}}(i;j) = \Delta'_{\mathrm{mm}}(i) + \Delta'_{\mathrm{mm}}(j),$$
$$\Delta_{\mathrm{mf}}(i,a;j,b) = \Delta'_{\mathrm{mf}}(i,a) + \Delta'_{\mathrm{mf}}(j,b),$$
$$\Delta_{\mathrm{mf'}}(i,a;j,b) = \Delta'_{\mathrm{mf'}}(i;j,b) + \Delta'_{\mathrm{mf'}}(j;i,a),$$

Non-Factorizable Contributions

$$\Delta_{\mathrm{ff}'} \sim -(s_{ab} - m_a^2 - m_b^2) K_i K_j E_0 \left(k_a, \overline{k}_i, -\overline{k}_j, -k_b, m_\gamma^2, m_a^2, \overline{M}_i^2, \overline{M}_j^2, m_b^2 \right),$$

$$\Delta'_{\mathrm{mf}'} \sim -(\overline{s}_{ib} - M_i^2 - m_b^2) K_j D_0 \left(\overline{k}_i, -\overline{k}_j, -k_b, m_\gamma^2, \overline{M}_i^2, \overline{M}_j^2, m_b^2 \right),$$

$$\Delta_{\mathrm{xf}} \sim -(s_{ab} - m_a^2 - m_b^2) K_i D_0 \left(k_a, \overline{k}_i, -k_b, m_\gamma^2, m_a^2, \overline{M}_i^2, m_b^2 \right).$$

$$\Delta_{\mathrm{mm}'} \sim -\left(\overline{\overline{s}}_{ij} - M_i^2 - M_j^2\right) \Big\{ C_0 \left(\overline{k}_i^2, \overline{\overline{s}}_{ij}, \overline{k}_j, 0, \overline{M}_i^2, \overline{M}_j^2 \right) - C_0 \left(M_i^2, \overline{\overline{s}}_{ij}, M_j^2, m_\gamma^2, M_i^2, M_j^2 \right) \Big\},$$

$$\Delta'_{\mathrm{mm}} \sim 2M_i^2 \left\{ \frac{1}{K_i} \left[B_0 \left(\overline{k}_i^2, 0, \overline{M}_i^2 \right) - B_0 \left(\overline{M}_i^2, m_\gamma^2, \overline{M}_i^2 \right) \right] - B_0' \left(M_i^2, m_\gamma^2, M_i^2 \right) \right\},$$

$$\Delta'_{\mathrm{mf}} \sim -(\tilde{s}_{ia} - M_i^2 - m_a^2) \Big\{ C_0 \left(\overline{k}_i^2, \tilde{s}_{ia}, m_a^2, 0, \overline{M}_i^2, m_a^2 \right) - C_0 \left(M_i^2, \tilde{s}_{ia}, m_a^2, m_\gamma^2, M_i^2, m_a^2 \right) \Big\},$$

$$\Delta_{\mathrm{xm}} \sim -\left(\overline{s}_{ib} - M_i^2 - m_b^2\right) \Big\{ C_0 \left(\overline{k}_i^2, \overline{s}_{ib}, m_b^2, 0, \overline{M}_i^2, m_b^2 \right) - C_0 \left(M_i^2, \overline{s}_{ib}, m_b^2, m_\gamma^2, M_i^2, m_b^2 \right) \Big\}.$$

Expression for Δ'

$$\Delta' \sim 2\left[\ln\left(\frac{M_i^2 - \widetilde{s}_{ia}}{M_i^2}\right) + \ln\left(\frac{M_i^2 - \overline{s}_{ib}}{-s_{ab}}\right) - 1\right]\ln\left(\frac{-K_i}{m_\gamma M_i}\right) + \mathcal{L}\mathrm{i}_2\left(\frac{M_i^2 - \widetilde{s}_{ia}}{M_i^2}, \frac{M_i^2 - \overline{s}_{ib}}{-s_{ab}}\right) + 2.$$