Double parton scattering without double counting

M. Diehl

Deutsches Elektronen-Synchroton DESY

9th Annual Meeting of the Helmholtz Alliance
"Physics at the Terascale"
18 November 2015

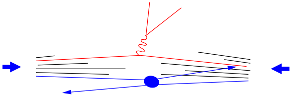
Single and multiple hard scattering

- standard description of hard proton-proton collisions:

  cross sect = parton distributions × parton-level cross sect

example: Z production

$pp \to Z + X \to \ell^+\ell^- + X$



- always have interactions between “spectator” partons
  - effects cancel in inclusive cross sections thanks to unitarity
  - predominantly low-p_T scattering $\rightsquigarrow$ underlying event
  - at high c.m. energy can be hard $\rightsquigarrow$ multiple hard scattering

- factorisation formula for double parton scattering:

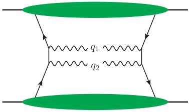
  cross sect = double parton distributions × parton-level cross sect’s

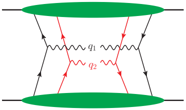
  have several elements of a factorisation proof

  see MD, J Gaunt, D Ostermeier. P Plößl, A Schäfer 2015

Single vs. double hard scattering (SPS vs. DPS)

- example: prod'n of two gauge bosons, transverse momenta $\boldsymbol{q}_1$ and $\boldsymbol{q}_2$





single scattering:

$|\boldsymbol{q}_1|$ and $|\boldsymbol{q}_2| \sim$ hard scale Q^2

$|\boldsymbol{q}_1 + \boldsymbol{q}_2| \ll Q^2$

double scattering:

both $|\boldsymbol{q}_1|$ and $|\boldsymbol{q}_2| \ll Q^2$

- for transv. momenta $\sim \Lambda \ll Q$:

$$\frac{d\sigma_{\text{SPS}}}{d^2\boldsymbol{q}_1\, d^2\boldsymbol{q}_2} \sim \frac{d\sigma_{\text{DPS}}}{d^2\boldsymbol{q}_1\, d^2\boldsymbol{q}_2} \sim \frac{1}{Q^4\,\Lambda^2}$$

but single scattering populates larger phase space :

$$\sigma_{\text{SPS}} \sim \frac{1}{Q^2} \gg \sigma_{\text{DPS}} \sim \frac{\Lambda^2}{Q^4}$$

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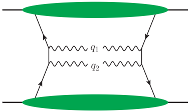

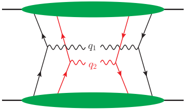

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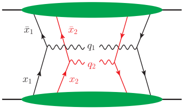
- for small parton mom. fractions x
  double scattering enhanced by parton luminosity
- depending on process: enhancement or suppression
  from parton type (quarks vs. gluons), coupling constants, etc.
  example: like-sign W pairs, $pp \to W^+W^+ + X$
  A Kulesza, J Stirling 2009; J Gaunt et al 2010; E Berger et al 2011

Double parton scattering

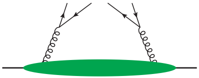
$$\frac{d\sigma_{\text{DPS}}}{dx_1\, d\bar{x}_1\, dx_2\, d\bar{x}_2} = \frac{1}{C}\, \hat{\sigma}_1\, \hat{\sigma}_2 \int d^2\boldsymbol{y}\; F(x_1, x_2, \boldsymbol{y})\, F(\bar{x}_1, \bar{x}_2, \boldsymbol{y})$$



$$\begin{aligned} C &= \text{combinatorial factor} \\ \hat{\sigma}_i &= \text{parton-level cross sections} \\ F(x_1, x_2, \boldsymbol{y}) &= \text{double parton distribution (DPD)} \\ \boldsymbol{y} &= \text{transv. distance between partons} \end{aligned}$$

- at higher orders in α_s get usual convolution integrals over x_i in $\hat{\sigma}_i$ and F
- analogous formulation for measured $\boldsymbol{q}_1$ and $\boldsymbol{q}_2$
  $\rightsquigarrow$ transverse-momentum dependent DPDs
- for $\boldsymbol{y} \ll 1/\Lambda$ can compute

$$F(x_1, x_2, \boldsymbol{y}) \sim \frac{1}{\boldsymbol{y}^2} \text{ splitting fct} \otimes \text{usual PDF}$$



Double parton scattering: ultraviolet problem

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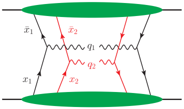

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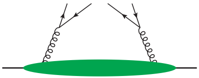
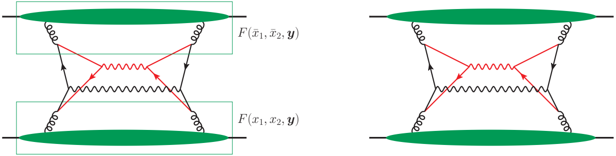


gives UV divergent cross section $\propto \int d^2\boldsymbol{y}/\boldsymbol{y}^4$

in fact, formula not valid for $|\boldsymbol{y}| \sim 1/Q$

. . . and more problems

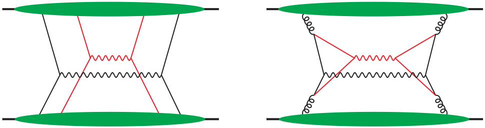


- double counting problem between double scattering with splitting and single scattering at loop level

MD, Ostermeier, Schäfer 2011; Gaunt, Stirling 2011; Gaunt 2012
Blok, Dokshitzer, Frankfurt, Strikman 2011; Ryskin, Snigirev 2011, 2012
already noted by Cacciari, Salam, Sapeta 2009

A consistent solution

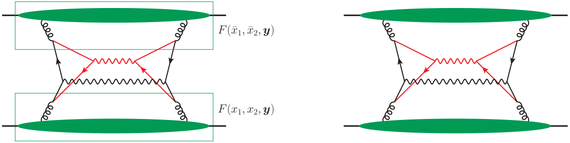
MD, J. Gaunt work in progress



- regulate DPS: $\sigma_{\text{DPS}} \propto \int d^2\boldsymbol{y}\, \Phi(\nu y)\, F(x_1, x_2, \boldsymbol{y})\, F(\bar{x}_1, \bar{x}_2, \boldsymbol{y})$
  - $\Phi \to 0$ for $u \to 0$ and $\Phi \to 1$ for $u \to \infty$, e.g. $\Phi(u) = \theta(u-1)$
  - cutoff scale $\nu \sim Q$
  - $F(x_1, x_2, \boldsymbol{y})$ has both splitting and non-splitting contributions

  analogous regulator for transverse-momentum dependent DPDs
- full cross section: $\sigma = \sigma_{\text{DPS}} - \sigma_{\text{sub}} + \sigma_{\text{SPS}}$
  - subtraction σ_{sub} to avoid double counting:
    $= \sigma_{\text{DPS}}$ with F computed for small $\boldsymbol{y}$ in fixed order perturb. theory
    (much simpler computation than σ_{SPS} at given order)

Subtraction formalism at work



$$\sigma = \sigma_{\text{DPS}} - \sigma_{\text{sub}} + \sigma_{\text{SPS}}$$

- for $y \lesssim 1/Q$ have $\sigma_{\text{DPS}} \approx \sigma_{\text{sub}}$
  because pert. computation of F gives good approx. at considered order
  $\Rightarrow \quad \sigma \approx \sigma_{\text{SPS}}$
- for $y \gg 1/Q$ have $\sigma_{\text{sub}} \approx \sigma_{\text{SPS}}$
  because DPS approximations work well in box graph
  $\Rightarrow \quad \sigma \approx \sigma_{\text{DPS}}$ with regulator fct. $\Phi(\nu y) \approx 1$
- subtraction formalism works order by order in perturb. theory
  J. Collins, Foundations of Perturbative QCD, Chapt. 10

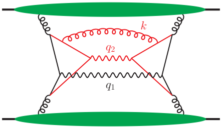
Added benefit: DGLAP logarithms

- define DPDs as matrix elements of renormalised twist-two operators:

$$F(x_1, x_2, \boldsymbol{y}; \mu_1, \mu_2) \sim \langle p| \mathcal{O}_1(\mathbf{0}; \mu_1)\, \mathcal{O}_2(\boldsymbol{y}; \mu_2) |p\rangle \qquad f(x;\mu) \sim \langle p| \mathcal{O}(\mathbf{0}; \mu) |p\rangle$$

$\Rightarrow$ separate DGLAP evolution for partons 1 and 2:

$$\frac{d}{d \log \mu_i} F(x_i, \boldsymbol{y}; \mu_i) = P \otimes_{x_i} F \qquad \text{for } i = 1, 2$$



- for $Q_1 \ll Q_2$ higher orders in box graph give logarithms $\alpha_s^n \log^n (Q_2/Q_1)$ of DGLAP type from region $Q_1 \ll |\boldsymbol{k}_1| \ll \cdots \ll |\boldsymbol{k}_n| \ll Q_2$
  - resummed by DPD evolution if take $\nu \sim \mu_1 \sim Q_1$, $\mu_2 \sim Q_2$ and appropriate initial conditions, e.g. $F = F_{\text{split}} + F_{\text{non-split}}$

$$F_{\text{split}}(x_1, x_2, \boldsymbol{y}; 1/y^*, 1/y^*) = F_{\text{perturb.}}(y^*)\, e^{-y^2 \Lambda^2} \quad \text{with} \quad 1/y^{*2} = 1/y^2 + 1/y_{\text{max}}^2$$

$$F_{\text{non-split}}(x_1, x_2, \boldsymbol{y}; \mu_0, \mu_0) = f(x_1; \mu_0) f(x_2; \mu_0)\, e^{-y^2 \Lambda^2}$$

Summary

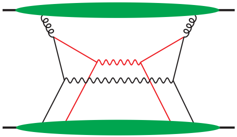
- double parton scattering important
  in specific kinematics/for specific processes
- recent progress:
  towards a systematic formulation of factorisation in QCD
- solution for UV problem of DPS $\leftrightarrow$ double counting with SPS
  - simple UV regulator for DPS using distance y between partons
  - simple subtraction term to avoid double counting order by order in perturbation theory

  correctly resums DGLAP logarithms
- distinction between "splitting" and "non-splitting" in DPD
  necessary in ansatz for DPD (inevitable model dependence)
  but not in formulation of factorisation

Backup: 1 vs 2 contribution

- in addition to DPS and SPS
  also have graphs with splitting in one proton only: "1 vs 2"

  $$\sim \int d^2\boldsymbol{y}/\boldsymbol{y}^2 \times F_{\text{non-split}}(x_1, x_2, \boldsymbol{y})$$

  B Blok et al 2011-13

  J Gaunt 2012

  B Blok, P Gunnellini 2015

- naturally included in DPS term $\propto (F_{\text{split}} + F_{\text{non-split}})\,(F_{\text{split}} + F_{\text{non-split}})$
  and regulated by $\Phi(\nu y)$ in our scheme
- in full cross section: $\sigma = \sigma_{\text{DPS}} - \sigma_{\text{sub (SPS + tw2}\times\text{tw4)}} + \sigma_{\text{SPS}} + \sigma_{\text{tw2}\times\text{tw4}}$
  may add twist 2 $\times$ twist 4 contribution with subtraction

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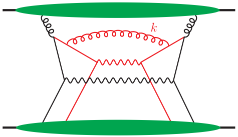

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  may add twist 2 $\times$ twist 4 contribution with subtraction
- lowest order 1 vs 2 term $\propto \log(Q/\Lambda)$
  additional logs $\alpha_s^n \log^{n+1}(Q/\Lambda)$ from $\Lambda \ll |\boldsymbol{k}_1| \ll \cdots \ll |\boldsymbol{k}_n| \ll Q$
  - correctly resummed by DPD evolution if take $\nu \sim \mu_1 \sim \mu_2 \sim Q$
    with same initial conditions for F as before
  - with $\nu \sim Q$ have no $\log(Q/\Lambda)$ in $\sigma_{\text{tw2}\times\text{tw4}} - \sigma_{\text{sub (tw2}\times\text{tw4)}}$