

# QCD corrections to top-pair production near threshold

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AEC  
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FOR FUNDAMENTAL PHYSICS

based on

M. Beneke, Y. Kiyo, P. Marquard, A. Penin, JP, M. Steinhauser,  
PRL 115 (2015) 192001, arXiv:1506.06864

direct reconstruction:

[ATLAS, CDF, CMS, D0 2014]

LHC + Tevatron:  $m_t = 173.34 \pm 0.76$  GeV

# Top Mass Determinations

direct reconstruction:

[ATLAS, CDF, CMS, D0 2014]

$$\text{LHC} + \text{Tevatron: } m_t = 173.34 \pm 0.76 \text{ GeV}$$

extract pole mass from cross section:

[CMS 2013; ATLAS 2014]

$$\text{ATLAS: } m_t = 172.9^{+2.5}_{-2.6} \text{ GeV}$$

$$\text{CMS: } m_t = 176.7^{+3.0}_{-2.8} \text{ GeV}$$

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[Alekhin, Blümlein, Moch 2013]

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$$\text{ABM12: } m_t = 171.2 \pm 2.4 \text{ GeV}$$

extract pole mass from flavour physics and electroweak data:

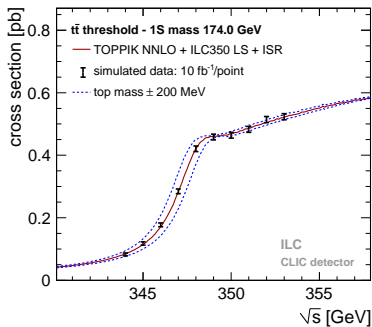
[Giudice, Paradisi, Strumia 2015]

$$m_{t,\text{flavour}} = 175.1 \pm 8.0 \text{ GeV}$$

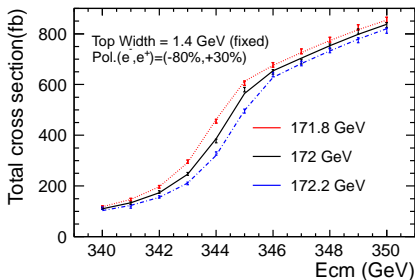
$$m_{t,\text{EW}} = 177.0 \pm 2.6 \text{ GeV}$$

perform threshold scan

↪ comparison with theoretical prediction in well-defined mass scheme (e.g. PS or 1S)

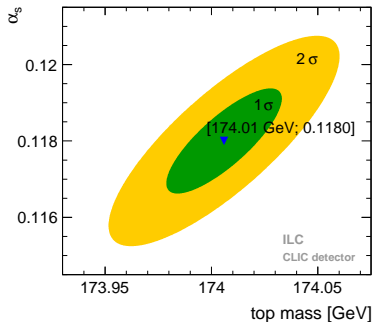
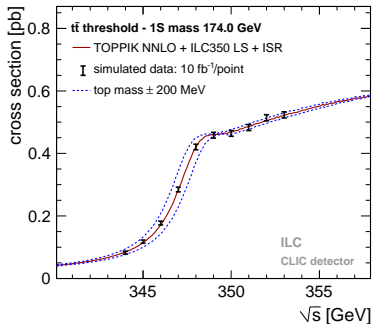


[Seidel, Simon, Tesař, Poss 2013]



[Horiguchi et al. 2013]

[Seidel, Simon, Tesař, Poss 2013]



for  $10 \text{ fb}^{-1}$  per point:

	$\delta m_t$	$\delta \alpha_s$
stat. error	27 MeV	0.0008
theory (3%)	9 MeV	0.0022

see also Frank Simon's LCWS talk for update with NNNLO result

# Threshold Production

relative velocity of quark-antiquark pair is small:

$$\sqrt{s} = E + 2m_Q \approx 2m_Q \quad \Rightarrow \quad v = \sqrt{\frac{E}{m_Q}} \ll 1; \quad v \sim \alpha_s(m_Q v)$$

- multi-scale problem: mass  $m_Q$ , momentum  $m_Q v$ , energy  $m_Q v^2$
- perturbation theory breaks down due to terms proportional to  $\frac{\alpha_s}{v}$   
 $\rightsquigarrow$  Coulomb resummation
- formation of bound states below threshold

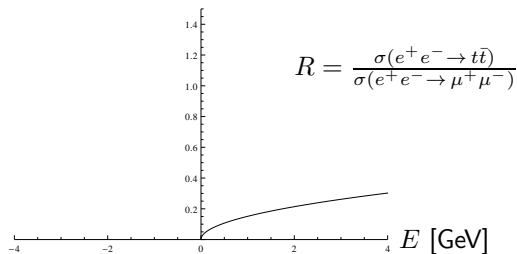
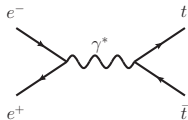


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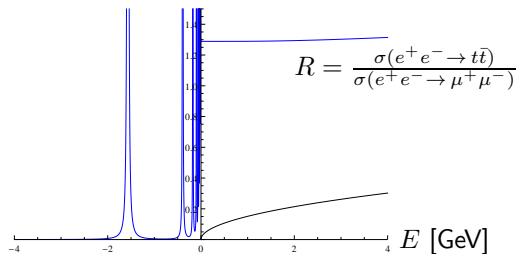
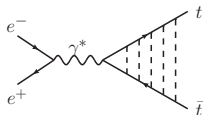


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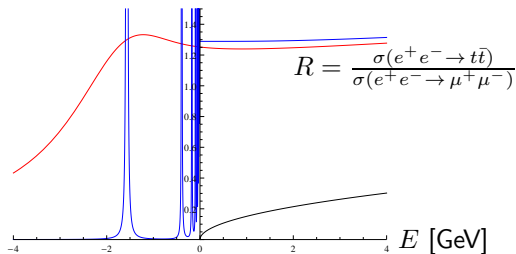
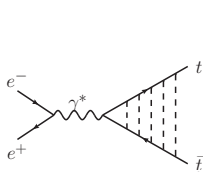


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- formation of bound states below threshold
- $b\bar{b}$ : bound-state resonances
- $t\bar{t}$ : large width prevents existence of bound states



[Thacker, Lepage; Lepage, Magnea, Nakhleh, Magnea, Hornbostel; Bodwin, Braaten, Lepage]

[Pineda, Soto; Beneke, Signer, Smirnov; Brambilla, Pineda, Soto, Vairo]

scale hierarchy:  $m_t \gg m_tv \gg m_tv^2 \gg \Lambda_{\text{QCD}}$

QCD

full theory

# Effective Theory Setup

[Thacker, Lepage; Lepage, Magnea, Nakhleh, Magnea, Hornbostel; Bodwin, Braaten, Lepage]  
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integrate out hard modes:  $k^0 \sim k^i \sim m_t$   
hard subgraphs become point-like vertices  
 $\rightsquigarrow$  hard matching coefficients

NRQCD

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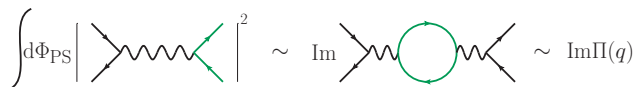
integrate out soft modes:  $k^0 \sim k^i \sim m_tv$   
soft subgraphs become instantaneous, non-local vertices  
 $\rightsquigarrow$  potentials

pNRQCD

contains potential quarks and ultrasoft gluons

# Calculating the Cross Section

use optical theorem:

$$\int d\Phi_{\text{PS}} \left| \text{Im} \left[ \text{Diagram} \right] \right|^2 \sim \text{Im} \left[ \text{Diagram} \right] \sim \text{Im} \Pi(q)$$


compute polarisation function in pNRQCD:  $\Pi(q) \sim c_v^2 G(\vec{0}, \vec{0}; E)$

- compute **hard matching coefficients**
- compute **potential**
- solve Schrödinger equation
- compute **ultrasoft** corrections

power counting:

$$\sum_n \left( \frac{\alpha_s}{v} \right)^n \times \left\{ 1; \underbrace{\alpha_s, v}_{\text{NLO}}; \underbrace{\alpha_s^2, \alpha_s v, v^2}_{\text{NNLO}}; \underbrace{\alpha_s^3, \alpha_s^2 v, \alpha_s v^2, v^3}_{\text{NNNLO}}; \dots \right\}$$

- NNLO

[Hoang, Teubner; Melnikov, Yelkhovsky; Yakovlev; Beneke, Signer, Smirnov;  
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- NNNLO
  - potential contributions [Beneke, Kiyo, Schuller; Kniehl, Penin, Smirnov, Steinhauser]
  - 3-loop static potential [Anzai, Kiyo, Sumino; Smirnov, Smirnov, Steinhauser]
  - ultrasoft corrections [Beneke, Kiyo, Penin]
  - 3-loop contribution to  $C_V$  [Marquard, JP, Seidel, Steinhauser]
  - P-wave contribution [Penin, Pivovarov; Beneke, JP, Rauh]

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- electroweak corrections [Guth, Kühn; Hoang, Reißer; Eiras, Steinhauser; Kiyo, Seidel, Steinhauser]
- finite-width effects [Fadin, Khoze; Hoang, Reißer, Ruiz-Femenía; Beneke, Jantzen, Ruiz-Femenía; Penin, JP]
- Higgs contribution [Harlander, Ježabek, Kühn; Eiras, Steinhauser; Beneke, Maier, JP, Rauh]

- NNNLO → this talk

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- 3-loop static potential
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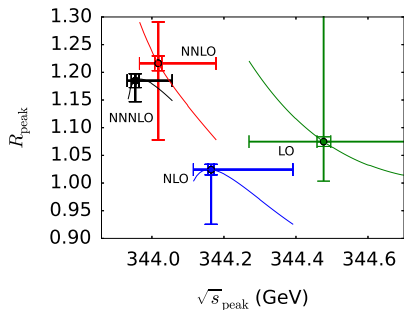
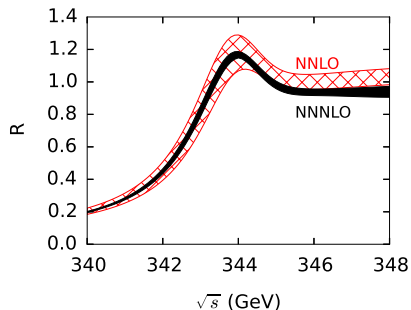
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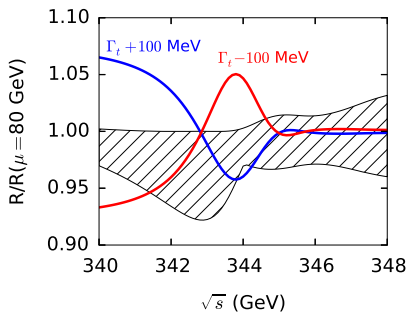
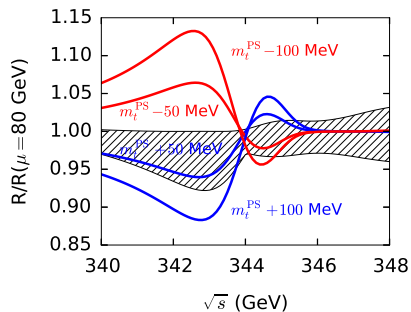
[Harlander, Ježabek, Kühn; Eiras, Steinhauser; Beneke, Maier, JP, Rauh]

→ following talk by Thomas Rauh



PS scheme:  $m_t^{\text{PS}}(\mu_f = 20 \text{ GeV}) = 171.5 \text{ GeV}$   
 $\Gamma_t = 1.33 \text{ GeV}$   
 $\alpha_s(M_Z) = 0.1185 \pm 0.0006$   
 $\mu = 50 - 350 \text{ GeV}$

# Parameter Dependence



(see Frank Simon's LCWS talk for more detailed analysis of mass determination)

- threshold production allows for very precise and theoretically clean determination of top-quark mass
- very high theoretical precision is required
- NNNLO QCD corrections are done
- scale uncertainty  $\pm 3\%$
- $\pm 50$  MeV uncertainty in top mass seems feasible

lots of work still to be done, e.g.:

- electroweak/finite-width corrections  $\rightarrow$  see next talk
- combination of fixed-order and resummed calculations