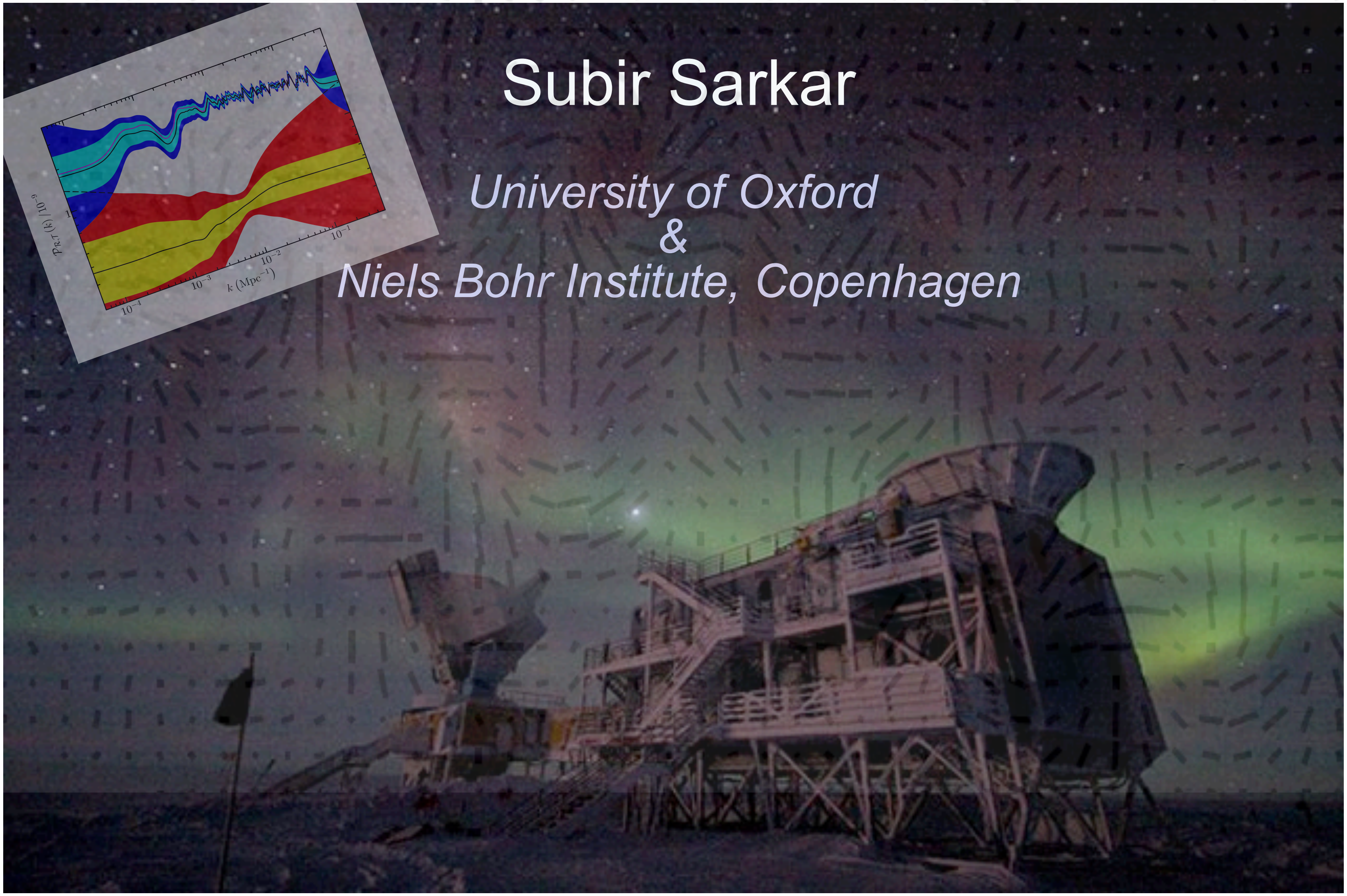
Testing cosmic inflation



Subir Sarkar

University of Oxford
&
Niels Bohr Institute, Copenhagen

Physics Seminar DESY 2015

Hamburg 6th October / Zeuthen 7th October

STANFORD UNIVERSITY

DEPARTMENT OF PHYSICS

Direct evidence of cosmic inflation

Mon, 2014-03-17

The detection of gravitational waves by the BICEP2 experiment at the South Pole supports the cosmic inflation theory of how the universe came to be. The discovery, made in part by

CERN COURIER

Apr 30, 2014

BICEP2 finds evidence of cosmic inflation

CfA HARVARD-SMITHSONIAN CENTER FOR ASTROPHYSICS

First Direct Evidence of Cosmic Inflation

Monday, March 17, 2014 - 10:45am

Telescope captures view of gravitational waves

nature

SKY & TELESCOPE

BBC

17 March 2014 Last updated at 14:46

Cosmic inflation: 'Spectacular' discovery hailed

Researchers with an experiment based at the South Pole have discovered the long-sought "smoking gun" for inflation.

BICEP2 COLLABORATORS CONFIRM COSMIC INFLATION

NSF-funded BICEP2 collaborators confirm cosmic inflation, a cataclysmic event that followed a fraction of a second after the Big Bang.

Gravitational Waves from Big Bang Detected

SCIENTIFIC AMERICAN

Big Bang breakthrough announced; gravitational waves detected

By Elizabeth Landau, CNN

03/17/2014

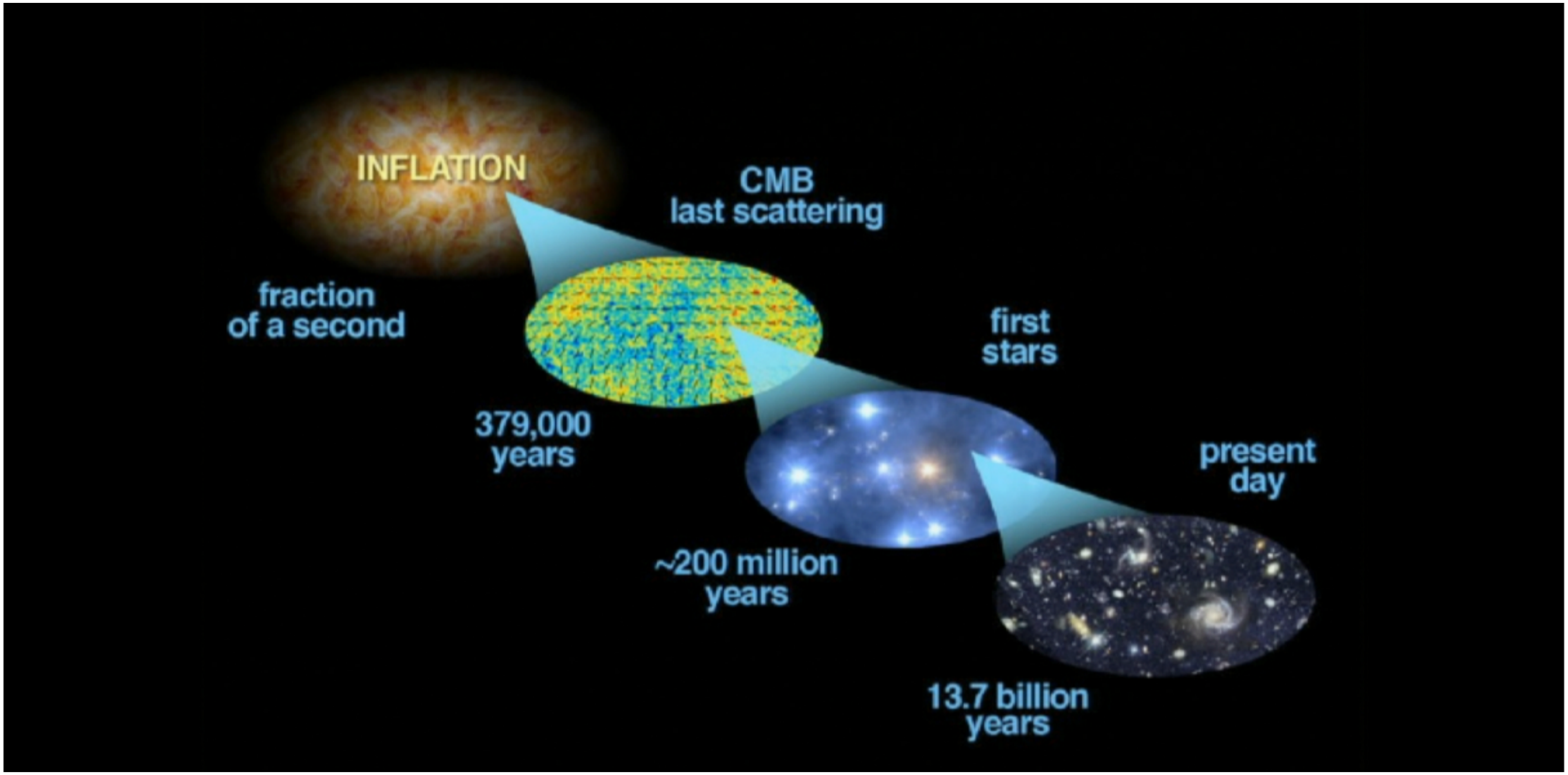
BICEP2 Discovers First Direct Evidence of Inflation and Primordial Gravitational Waves

Caltech

physicsworld.com

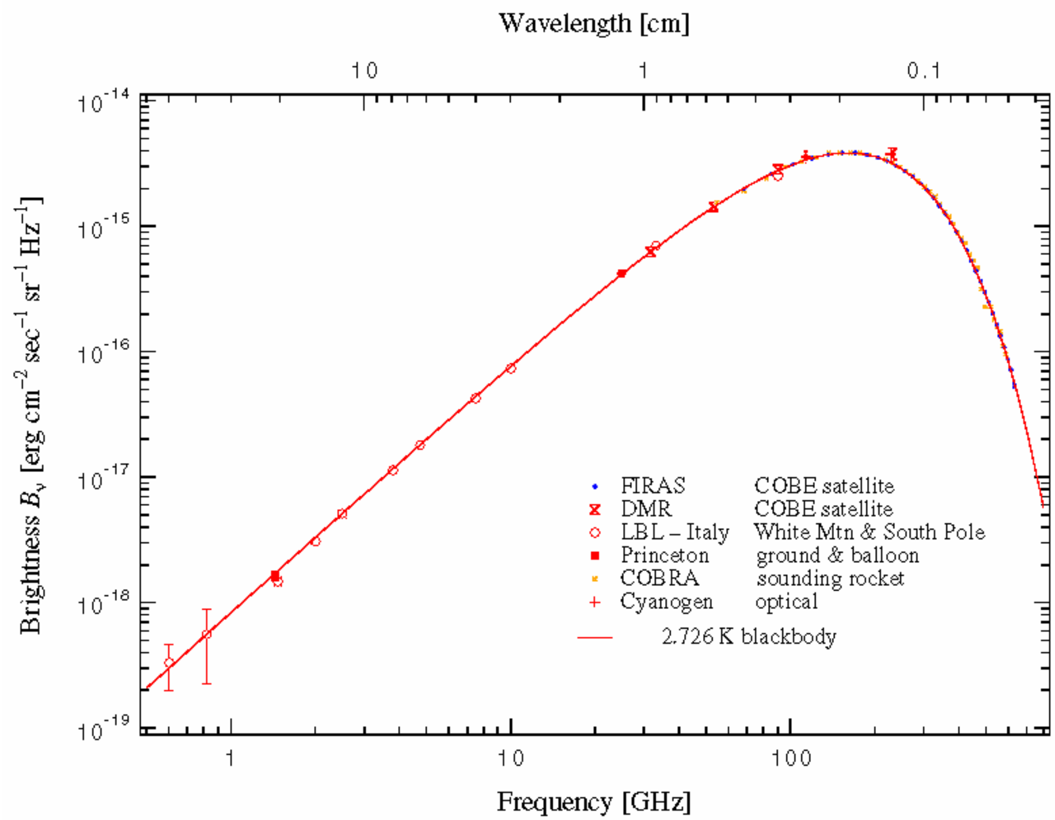
BICEP2 finds first direct evidence of cosmic inflation

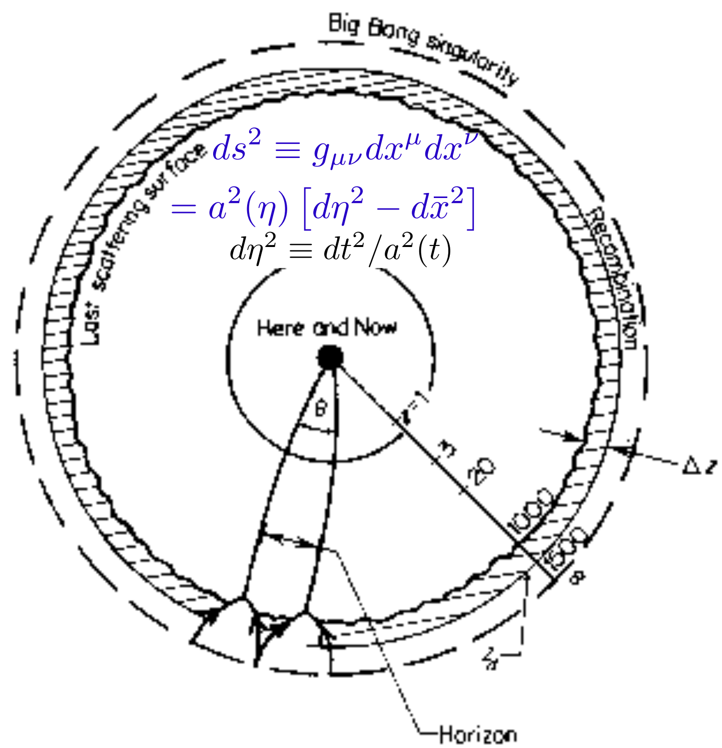
Why did cosmologists get so very excited?



We have a nearly complete picture of the growth of **large-scale structure** through **gravitational instability** in a sea of **dark matter**, starting with **scalar density perturbations** which we have detected imprinted on the **cosmic microwave background** … if these were created by '**inflation**' then seeing the associated **tensor perturbations** would *prove* that inflation actually occurred!

The discovery of the cosmic microwave background radiation established the standard 'Big Bang' model in which the universe has a finite age



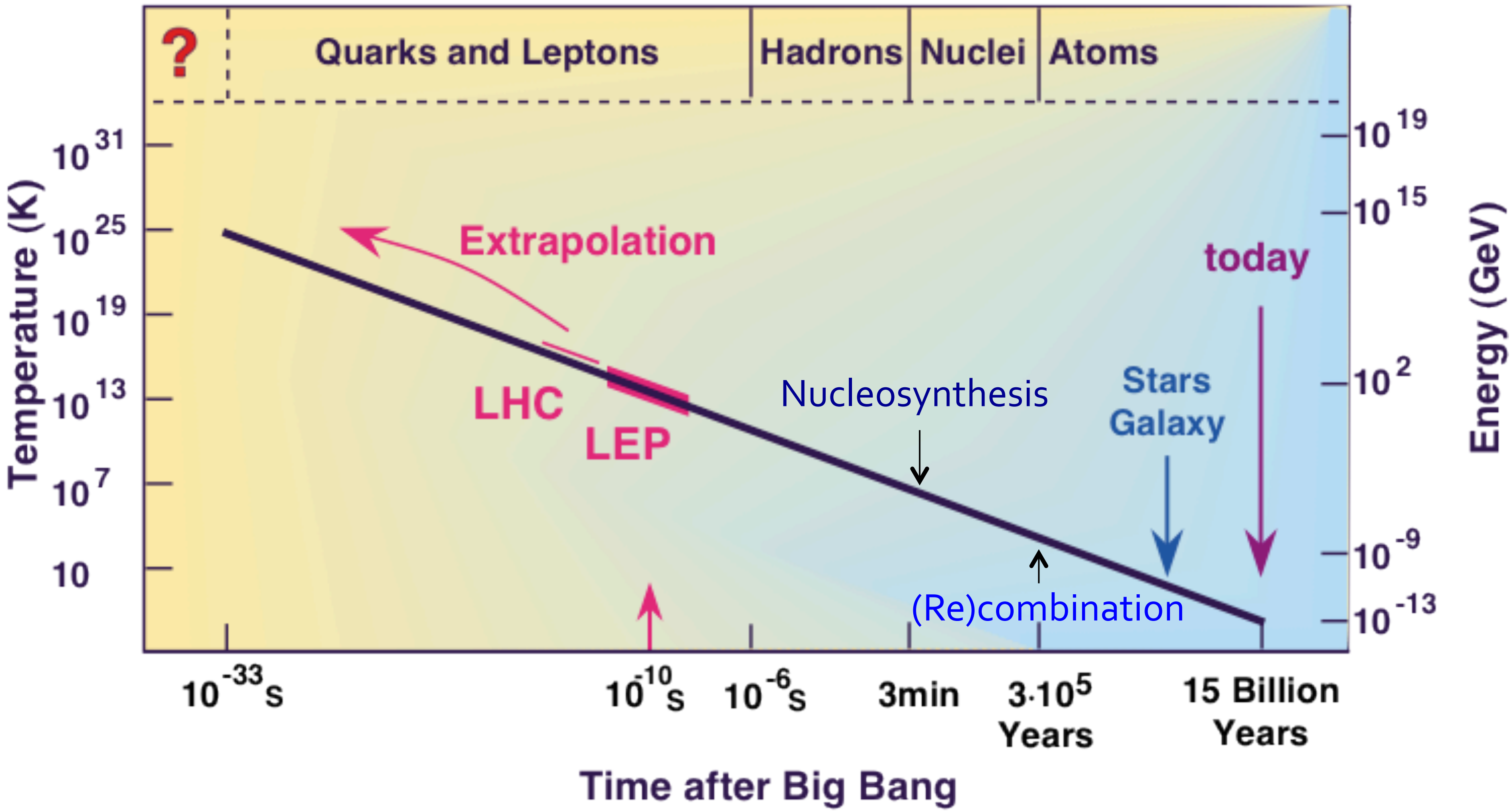


This ~perfect blackbody is testimony to our hot, dense past and shows that the expansion was *adiabatic* (with negligible energy release) back at least to $t \sim 1$ day

However the temperature is not quite the same over the sky – it varies by 1 part in ~10^4

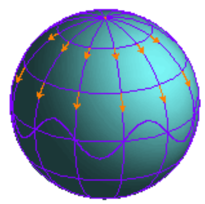
… these fluctuations *cannot* have been generated by any causal process in the standard cosmology so they are evidence for *new* physics in the very early universe

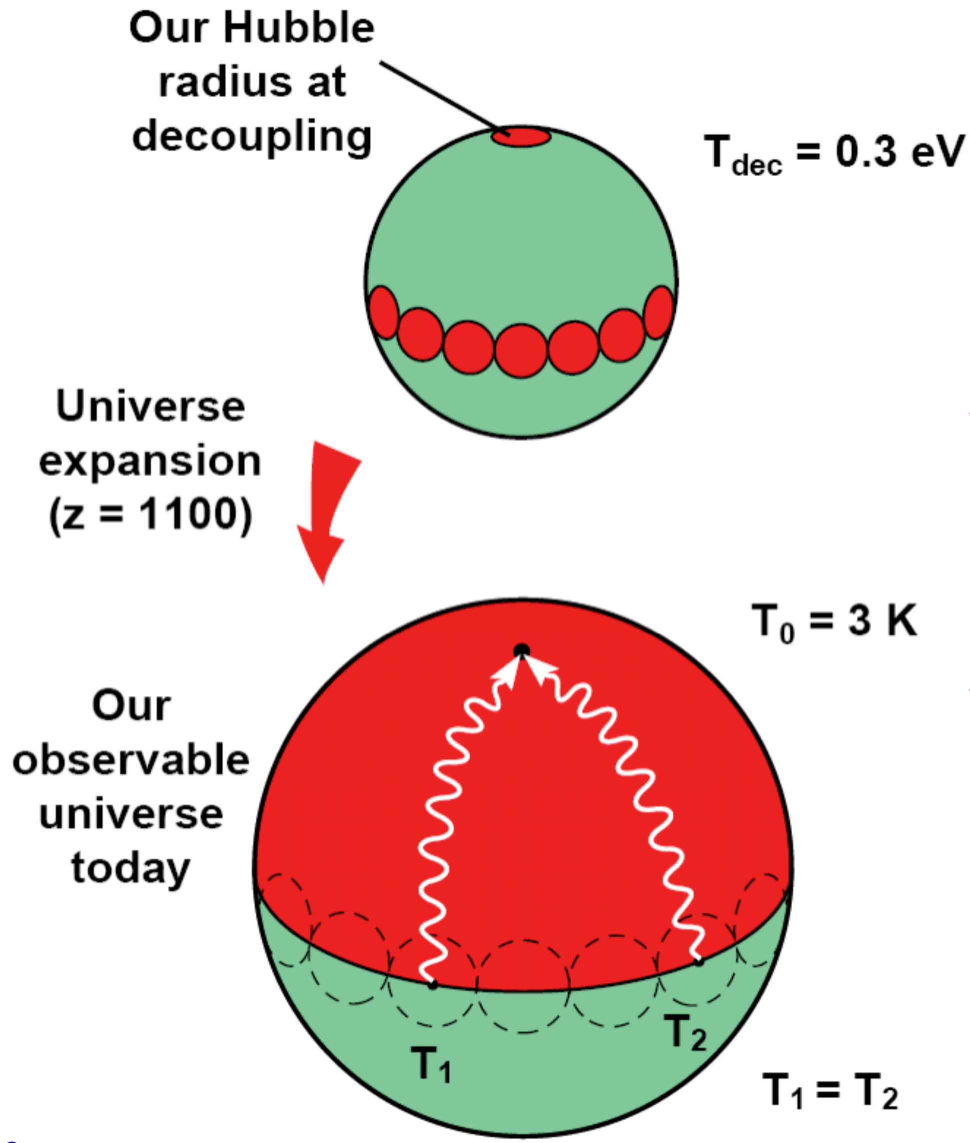
The blackbody temperature can be used as a clock (assuming adiabatic expansion: aT = constant), so our thermal history can be reconstructed



The furthest we 'see' directly is back to $t \sim 1$ s (when light elements were synthesised) but the small variations in CMB temperature must have been generated *much* earlier

Why is the temperature nearly the same over the sky?





(Courtesey Wayne Hu)

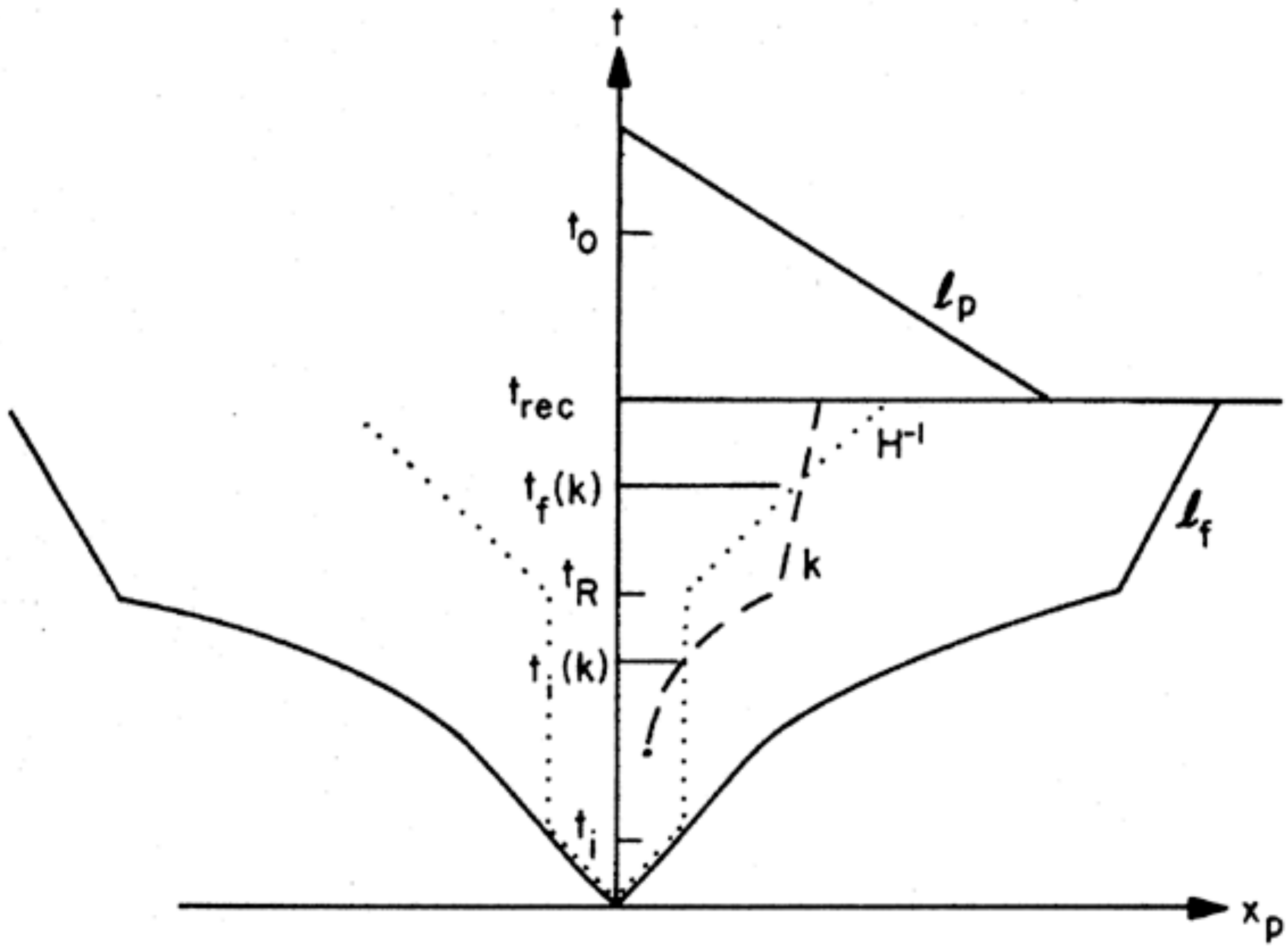
Distance travelled by light since the BB:

$$\eta \equiv \int \frac{\mathrm{d}t}{a(t)} = \int (aH)^{-1} \mathrm{d}\ln a$$

$$\text{so: } \eta_2 - \eta_1 = \int_{z_1}^{z_2} \frac{\mathrm{d}z}{H(z)}$$

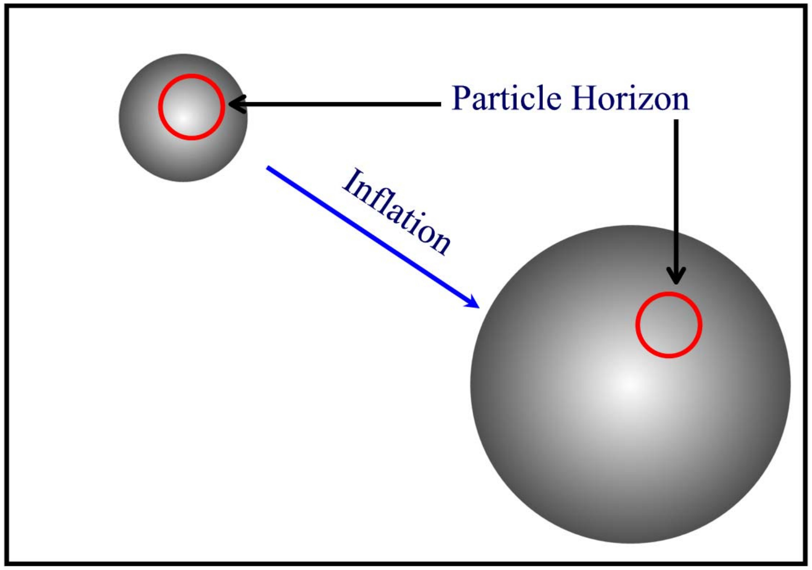
If $H(z)$ is monotonically *decreasing*, the `particle horizon' at recombination ($d_{\mathrm{hor}} = \eta_{rec}$) should be much *smaller* than the sky observable today … i.e. **there should be *no* correlations on scales larger than:** $\frac{\eta_{\mathrm{rec}} - \eta_{\mathrm{BB}}}{\eta_0 - \eta_{\mathrm{rec}}} \simeq 1.2^\circ$

One solution to the 'horizon problem' is a period of inflation



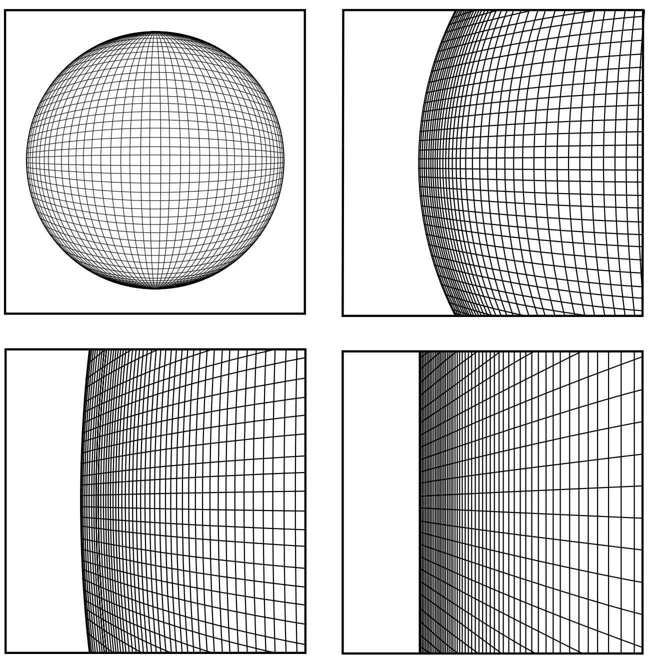
An *exponential* increase of the scale factor/particle horizon at some early time enables causal linking of apparently disconnected regions on the sky

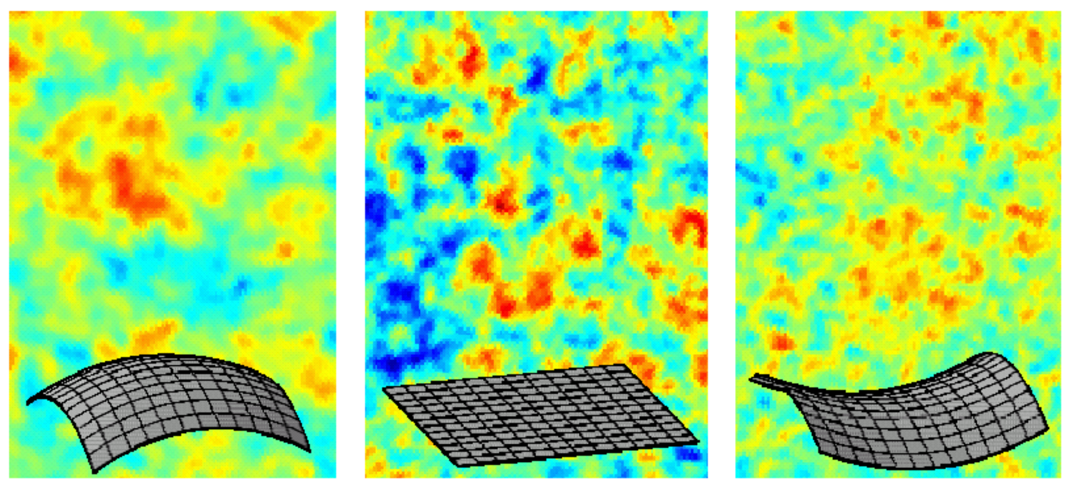
… this can happen if the energy density is briefly ~constant, e.g. via domination by the vacuum energy of a scalar field which is displaced from the minimum of its potential



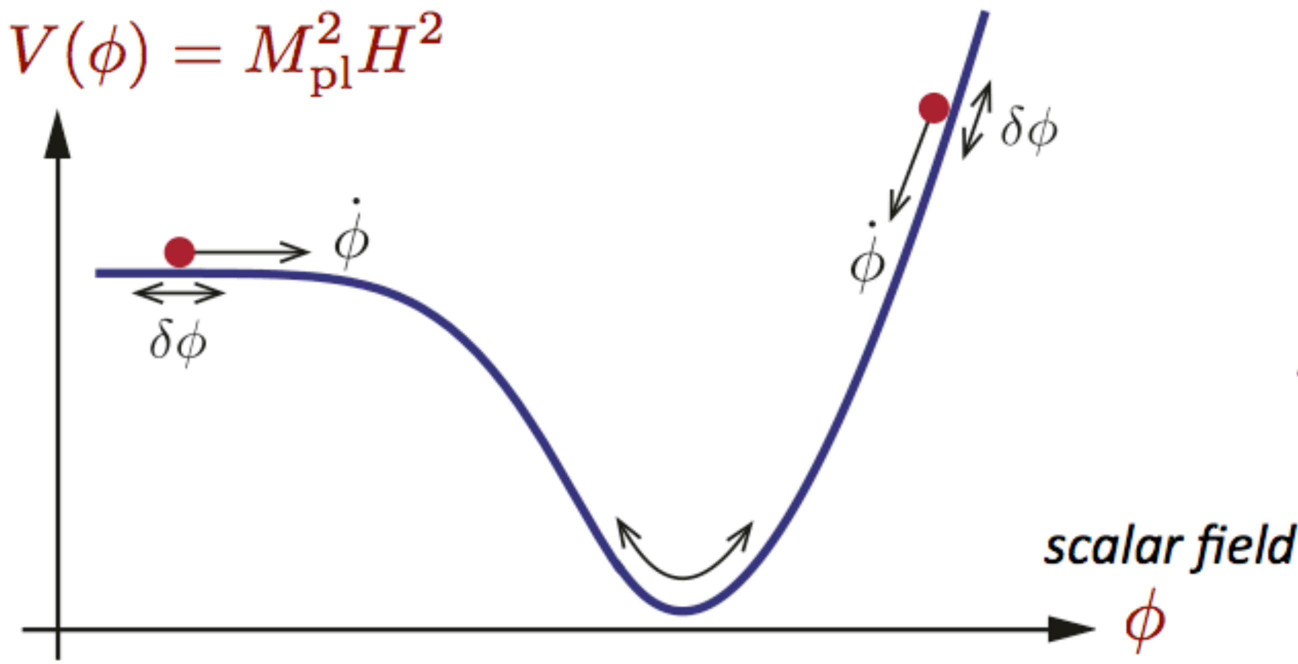
A period of *accelerated* expansion which blew up the scale-factor by $> e^{50\text{-}60}$ $(\sim 10^{30})$ can explain why causally (*apparently*) disconnected regions are, in fact, correlated …

This will also drive the curvature of space to zero … in accordance with observations of the angular scale of characteristic fluctuations in the CMB





(Boomerng Collaboration)



Toy model of slow-roll inflation:

$$\varepsilon = \frac{M_{\rm pl}^2}{2}\left(\frac{V'}{V}\right)^2 \ll 1$$

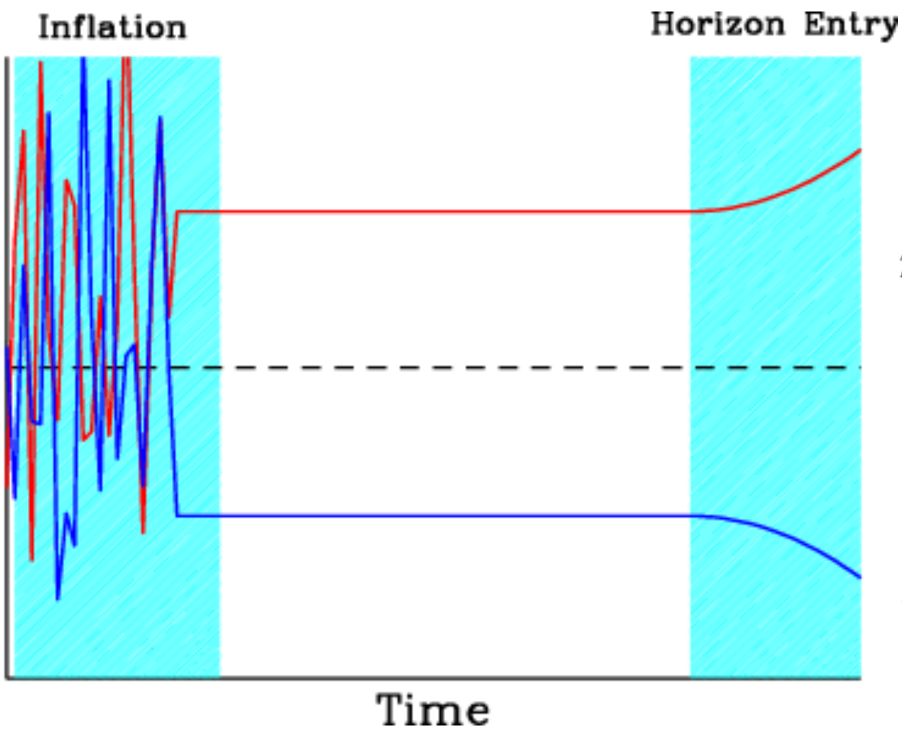
$$\eta = M_{\rm pl}^2 \frac{V''}{V} \ll 1$$

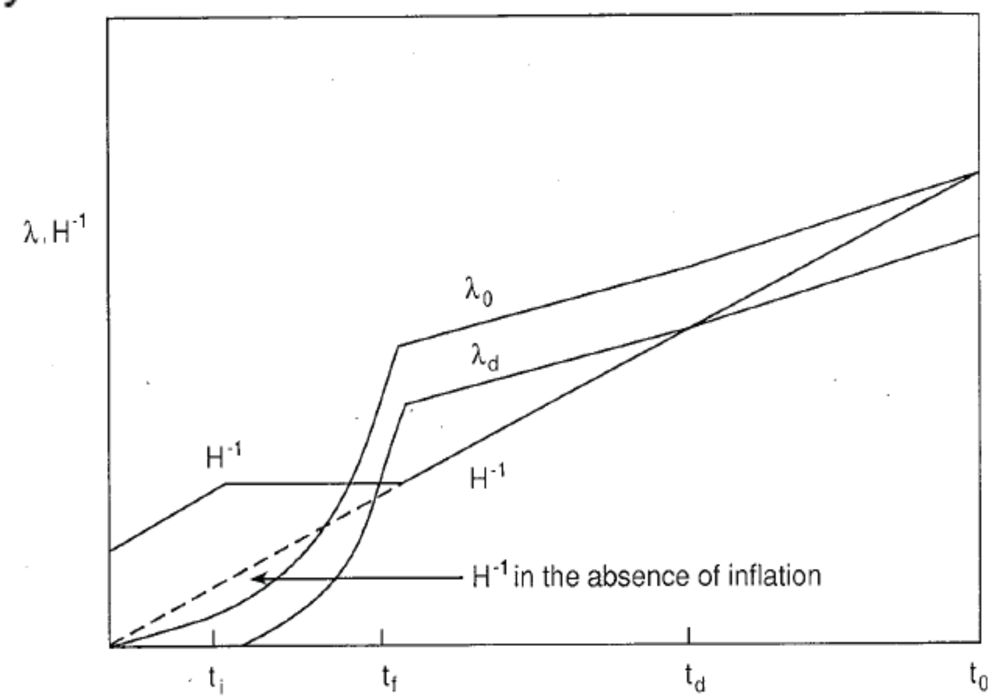
The slow evolution of a scalar field down a nearly flat part of its potential during which its vacuum energy is nearly constant so:

$$a \propto \mathrm{e}^{H_{\rm infl} t}, \quad \text{with} \quad H_{\rm infl} = \sqrt{\frac{8\pi G_{\rm N}}{3} V_0}$$

If the number of e-folds $N(\phi) = \int_{\phi_{\rm ini}}^{\phi_{\rm end}} \frac{H_{\rm infl}}{\dot\phi} \mathrm{d}\phi$ exceeds ~50-60, the region within the present Hubble radius would have been causally connected at the inflationary epoch, thus solving the 'horizon problem' (also the 'monopole problem')

- Quantum mechanical fluctuations: $\langle \Psi(\mathbf{k})\, \Psi(\mathbf{k}') \rangle = (2\pi)^3\, \delta^3(\mathbf{k}-\mathbf{k}')\, P_\Psi(\mathbf{k})$
- Inflation *stretches* wavelength beyond horizon: $\Psi(\mathbf{k}, \mathbf{t})$ becomes constant (until horizon reentry after inflation ends – *first out, last in*)
- Infinite number of independent perturbations with independent amplitudes, but … inflation synchronizes all modes!





During inflation, Ψ fluctuates quantum mechanically around a smooth background ... its *mean* value is zero, but its variance is:

$$\begin{aligned}\langle\Psi^2(\vec{x})\rangle &= \int\frac{d^3k}{(2\pi)^3}\int\frac{d^3k'}{(2\pi)^3}e^{i\vec{k}\cdot\vec{x}}e^{i\vec{k}'\cdot\vec{x}}\langle\tilde{\Psi}(\vec{k})\tilde{\Psi}(\vec{k}')\rangle\\ &= \int\frac{dk}{k}\frac{k^3P_\Psi(k)}{2\pi^2}\end{aligned}$$

... so get equal contributions on all scales if $P_\Psi \propto k^{-4+n}$ with $n = 1$ ("scale-invariant" spectrum)

In the toy model of inflation the slope of the potential (which provides the 'arrow of time' induces a 'tilt' to the spectrum of scalar density perturbations which have amplitude:

$$\Delta_s^2 \equiv \left(\frac{H^2}{2\pi\dot{\psi}}\right)^2$$

Inflation also generates a spectrum of tensor perturbations (gravitational waves) with amplitude:

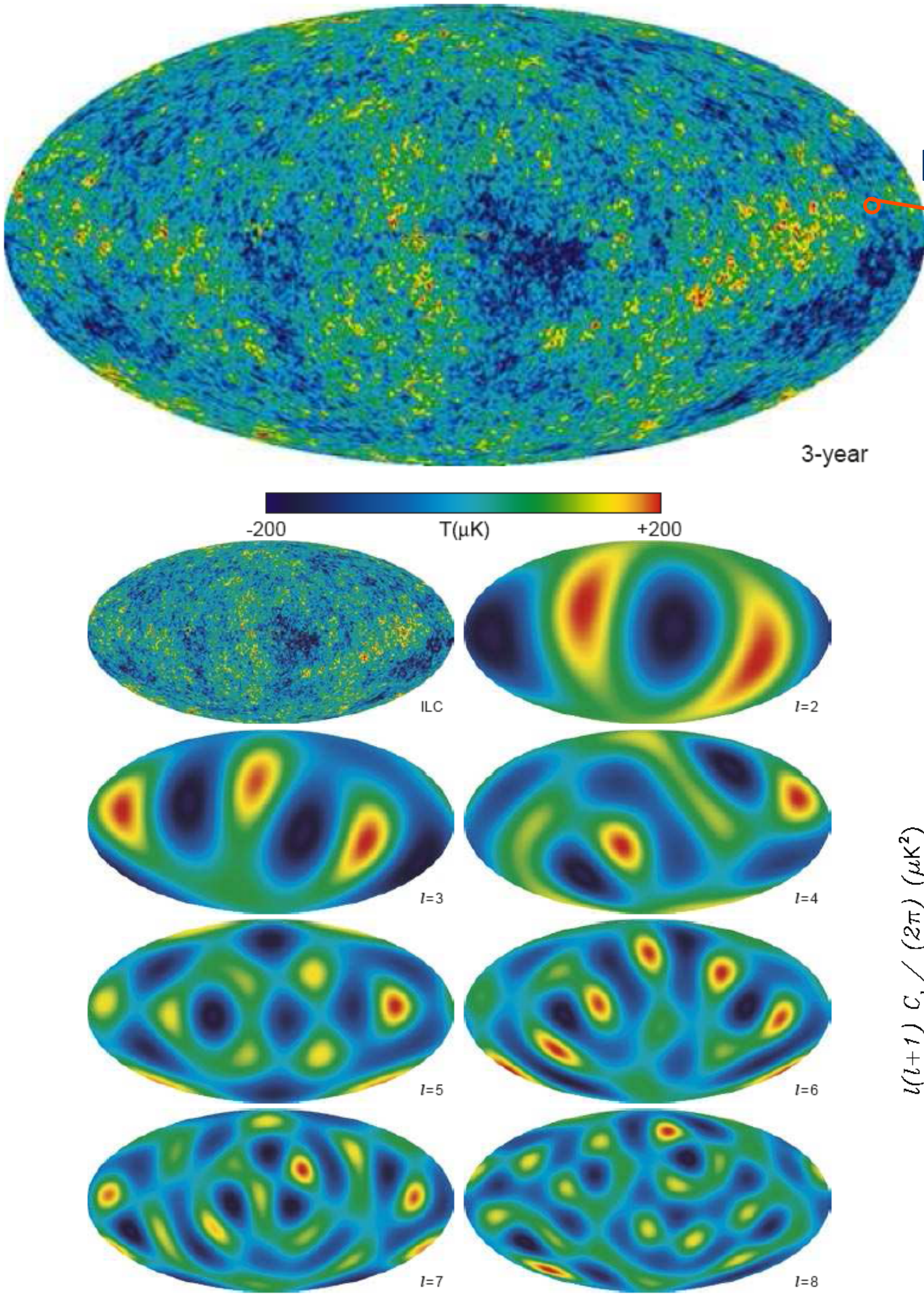
$$\Delta_t^2 \equiv \frac{2}{\pi^2}\frac{H^2}{M_{\mathrm{Pl}}^2}$$

The ratio of tensor to scalar perturbations is therefore: (characteristic of the inflationary potential)

$$r \equiv \frac{\Delta_t^2}{\Delta_s^2} = \frac{8}{M_{\mathrm{pl}}^2}\left(\frac{\dot{\phi}}{H}\right)^2$$

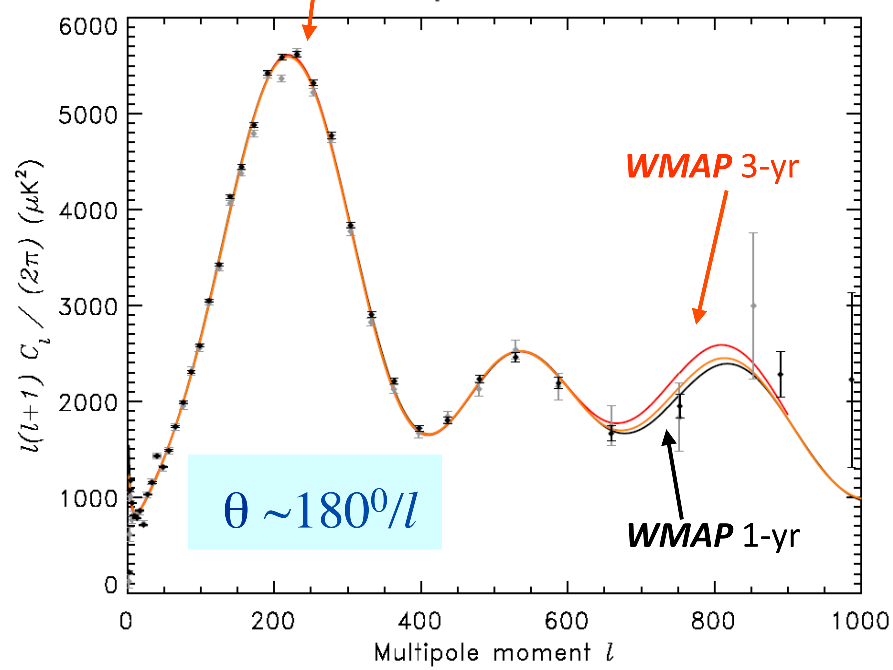
Coherent oscillations in photon-baryon plasma, excited by density perturbations on *super*-horizon scales …

(Hubble radius at t_{rec})

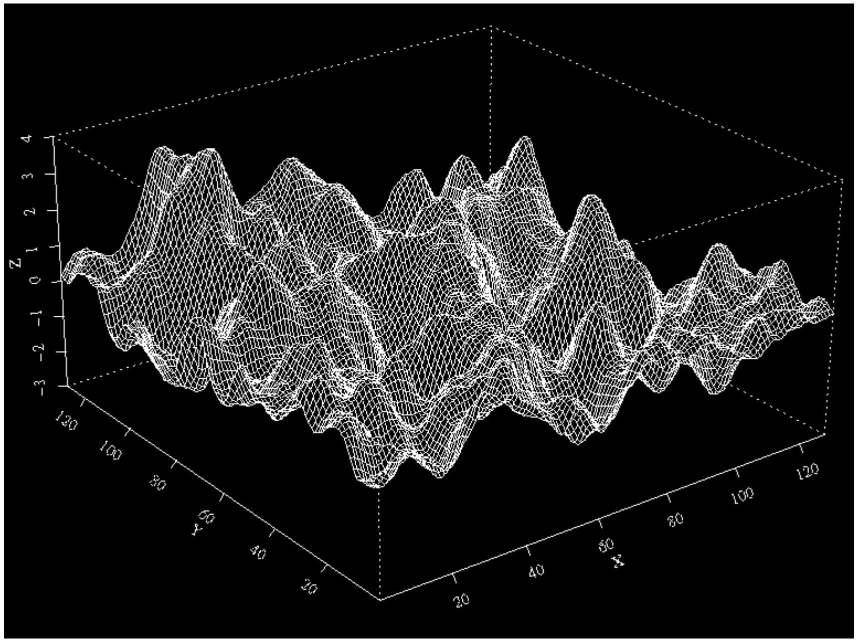


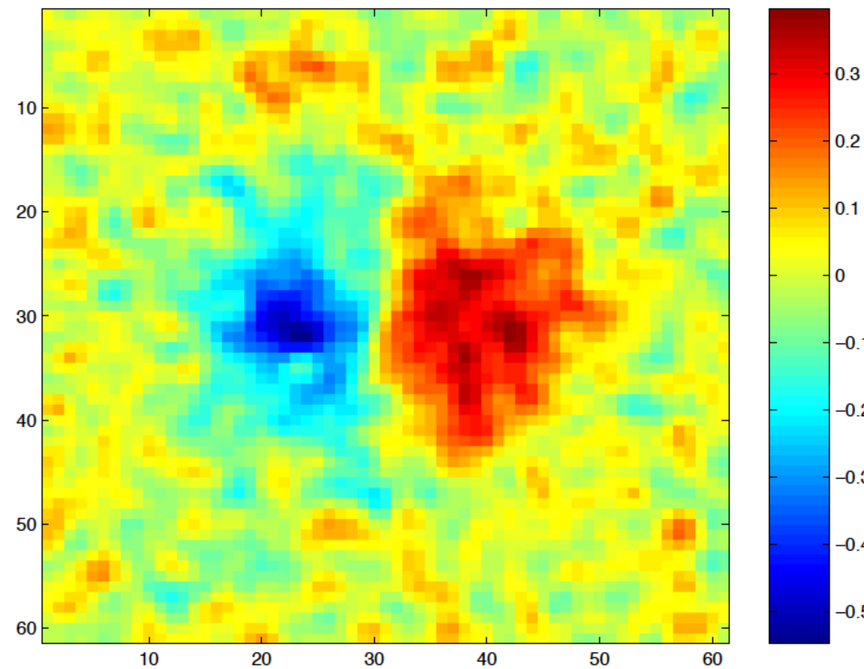
$$\Delta T(\mathbf{n}) = \sum a_{lm} Y_{lm}(\mathbf{n})$$

$$C_l \equiv \frac{1}{2l+1} \sum |a_{lm}|^2$$



O(10$^{7)}$ pixels can be reduced to *O*(10^3) multipoles *only* by assuming that the fluctuations are a **random Gaussian density field**

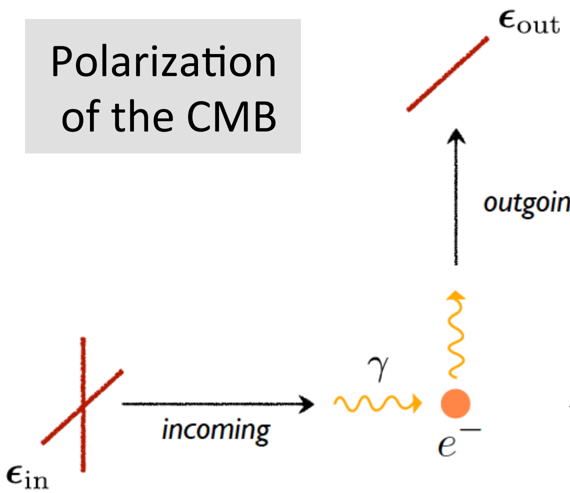




... and *O*(10^3) multipoles can be characterized by just two parameters (amplitude and slope of a power-law spectrum) *only* by assuming that the primordial perturbations are close to **scale-invariant**

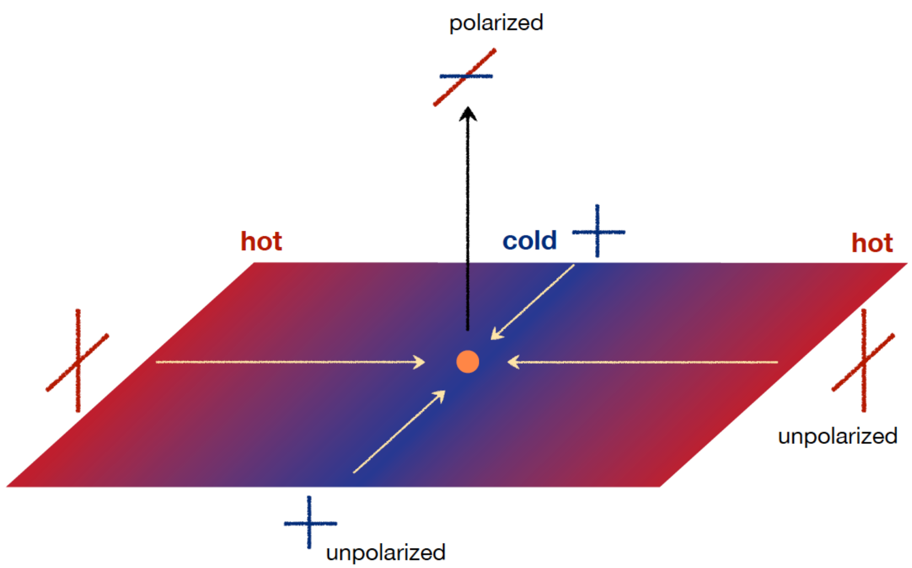
Gaussianity & scale –invariance are characteristic of the **quantum fluctuations** of a free massless scalar field in a ~De Sitter background … so we implicitly *assume* that slow-roll inflation is the origin of CMB temperature fluctuations

Polarization of the CMB

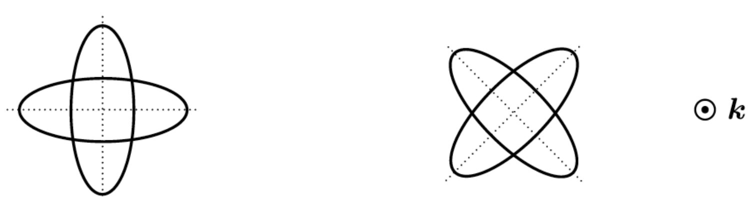


$$\frac{d\sigma}{d\Omega} \propto \left|\epsilon_{in} \cdot \epsilon_{out}\right|^2$$

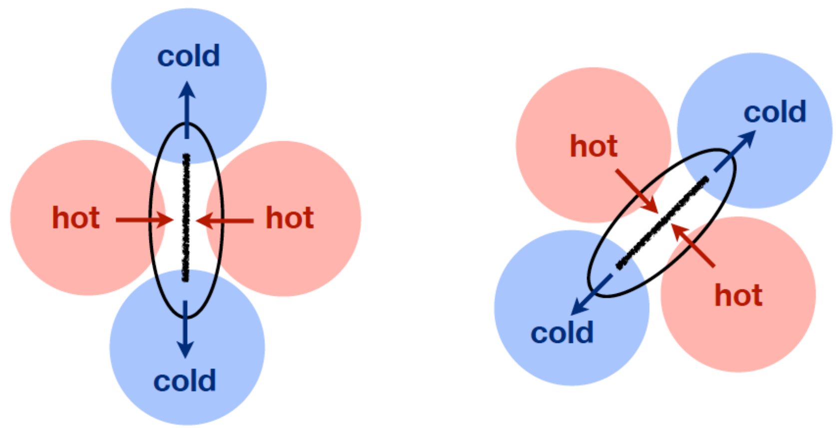
Quadrupolar temperature anisotropy leads to linear polarization:



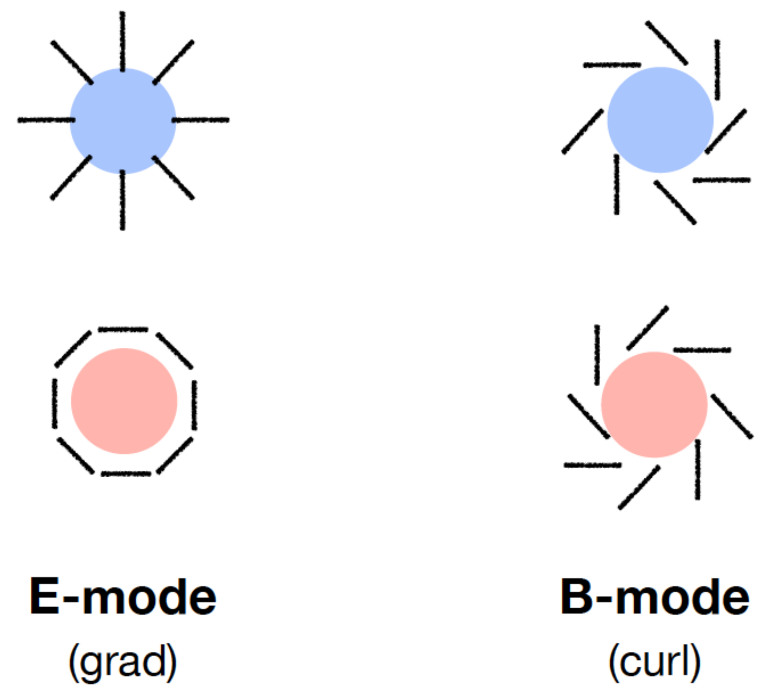
Recall the two polarization modes of a gravitational wave:



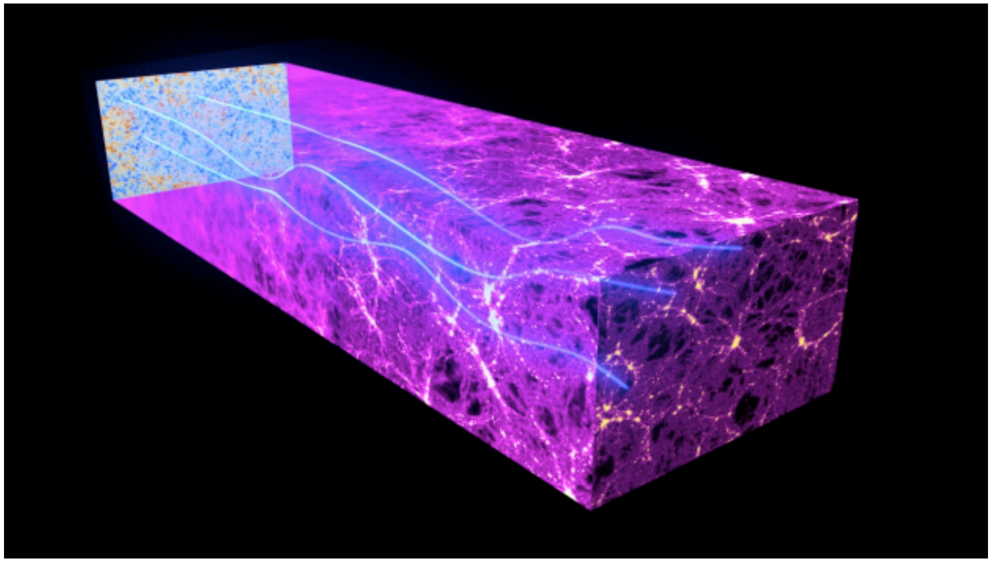
The anisotropic stretching of space induces a temperature quadrupole and scattering produces two types of polarization

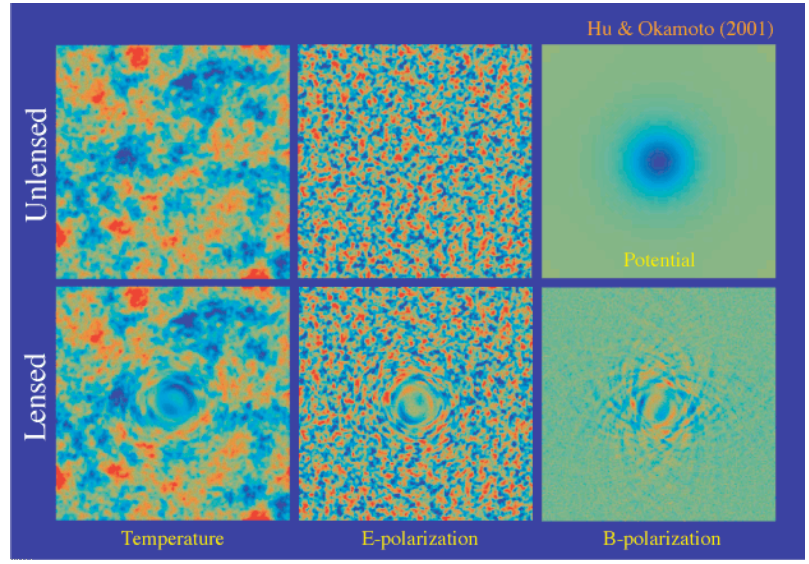


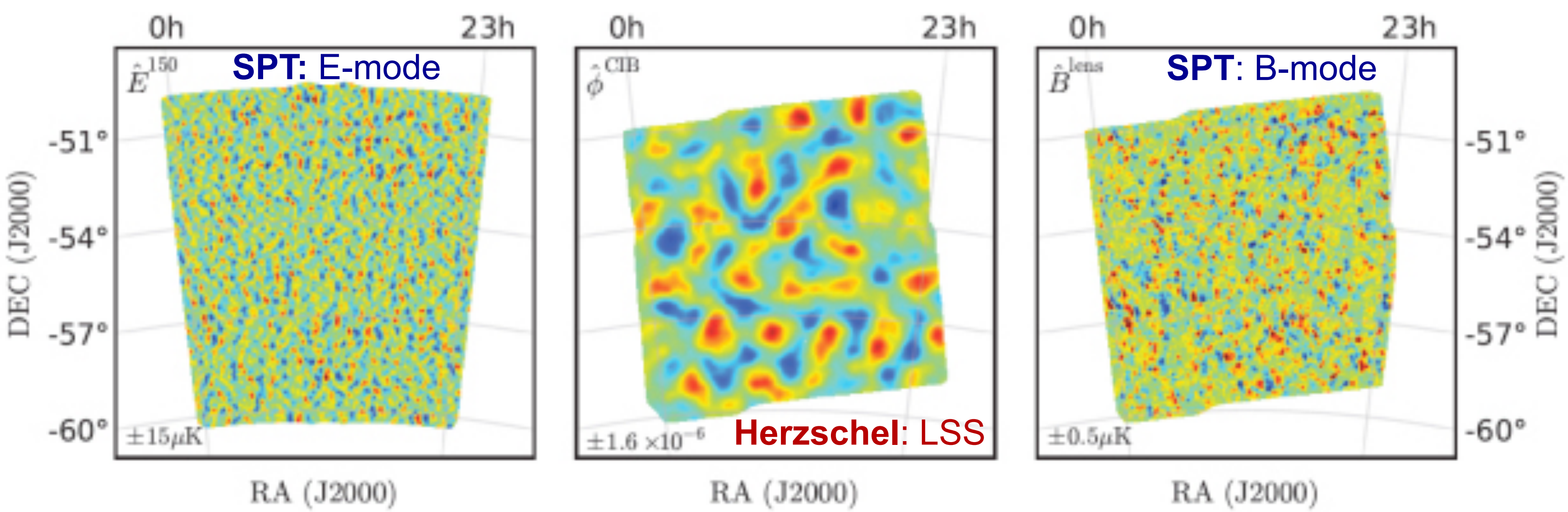
Summing over many waves, we get the following polarization patterns around **hot** and **cold** spots:



E mode -> B mode through gravitational lensing of the CMB



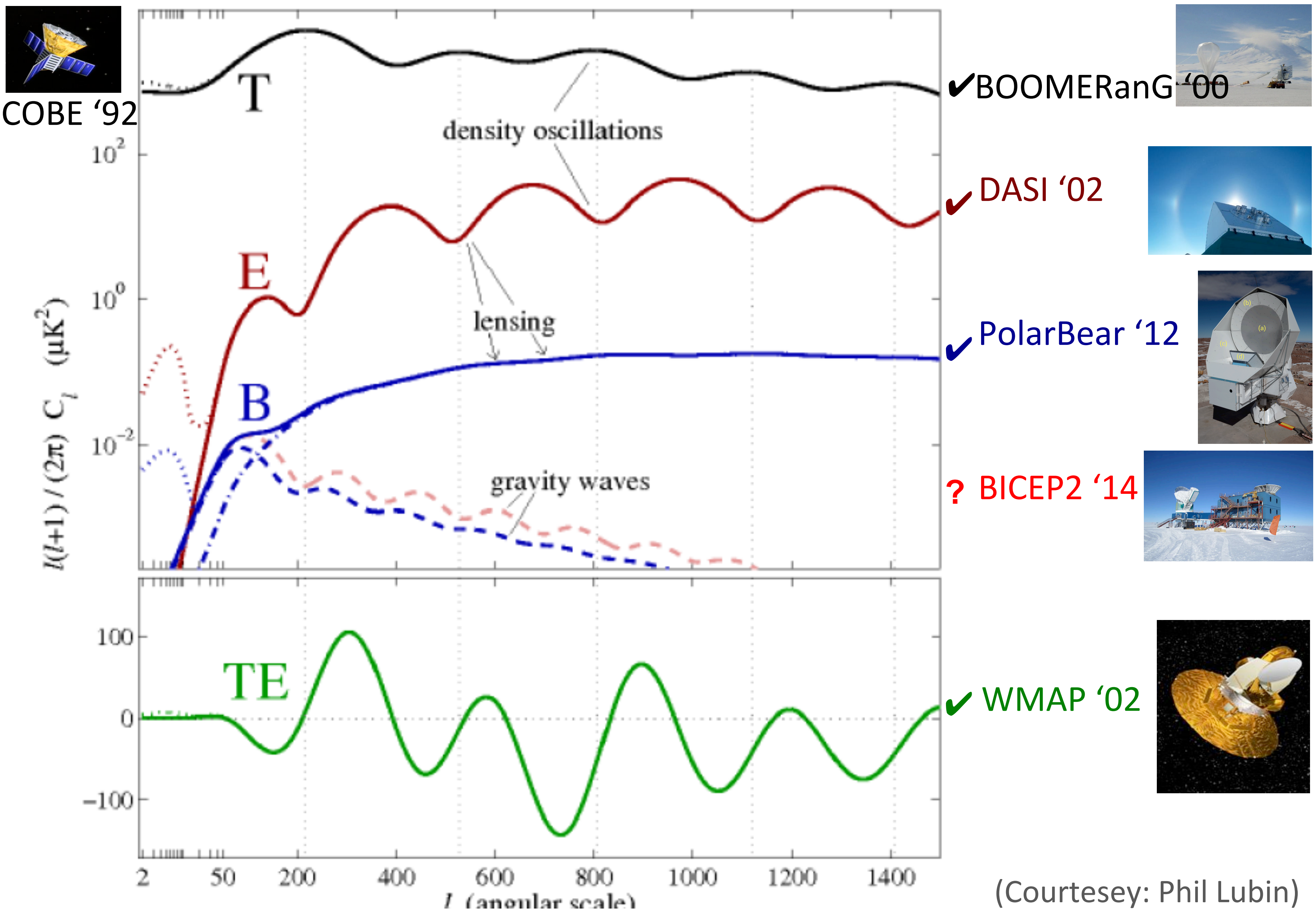




http://sci.esa.int/jump.cfm?oid=52905

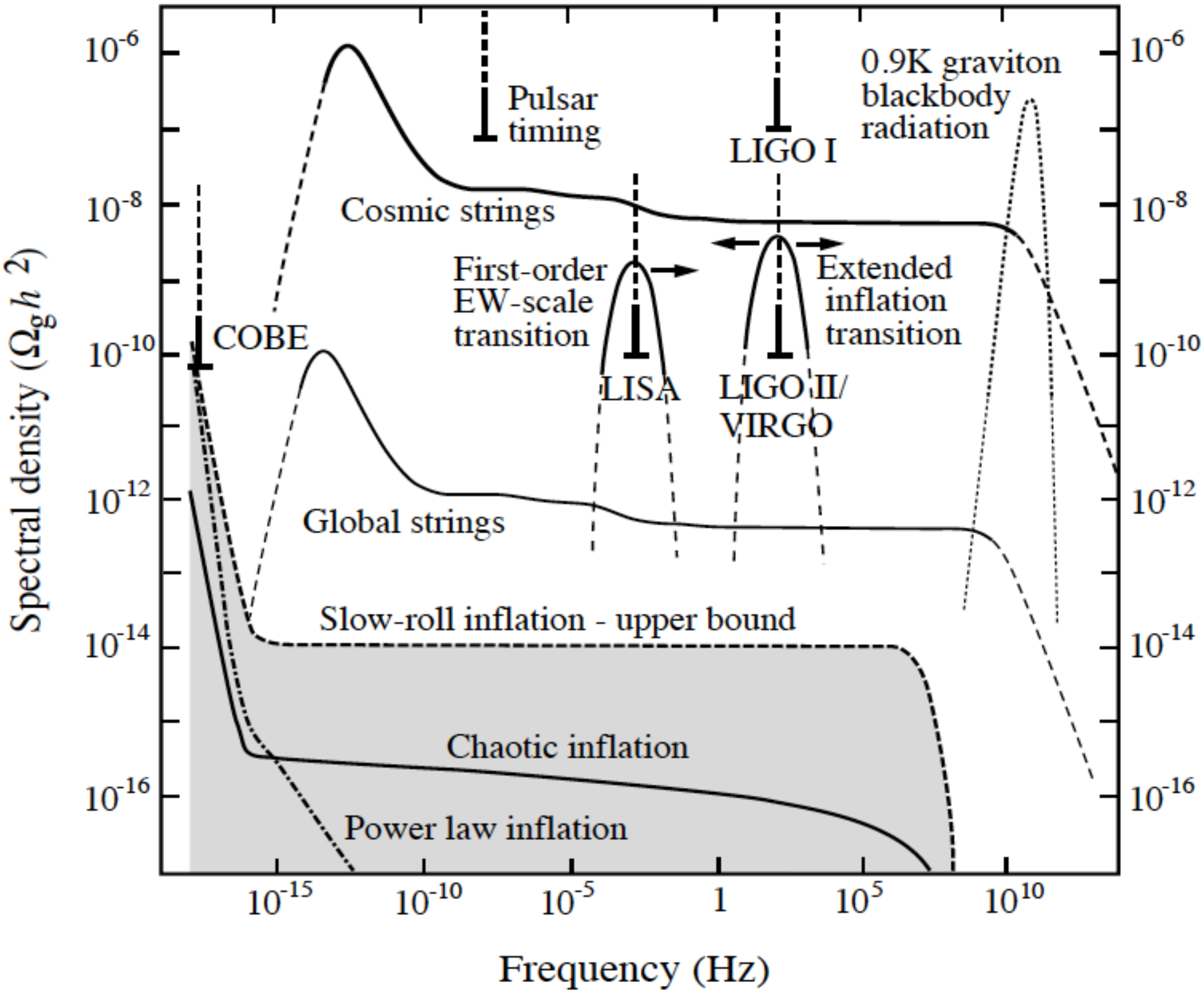
Depicts: E-modes and B-modes in the CMB polarisation (left and right panels, respectively) and the gravitational potential of the large-scale distribution of matter that is lensing the CMB (central panel)

Inflationary predictions for (adiabatic) CMB fluctuations



(Courtesey: Phil Lubin)

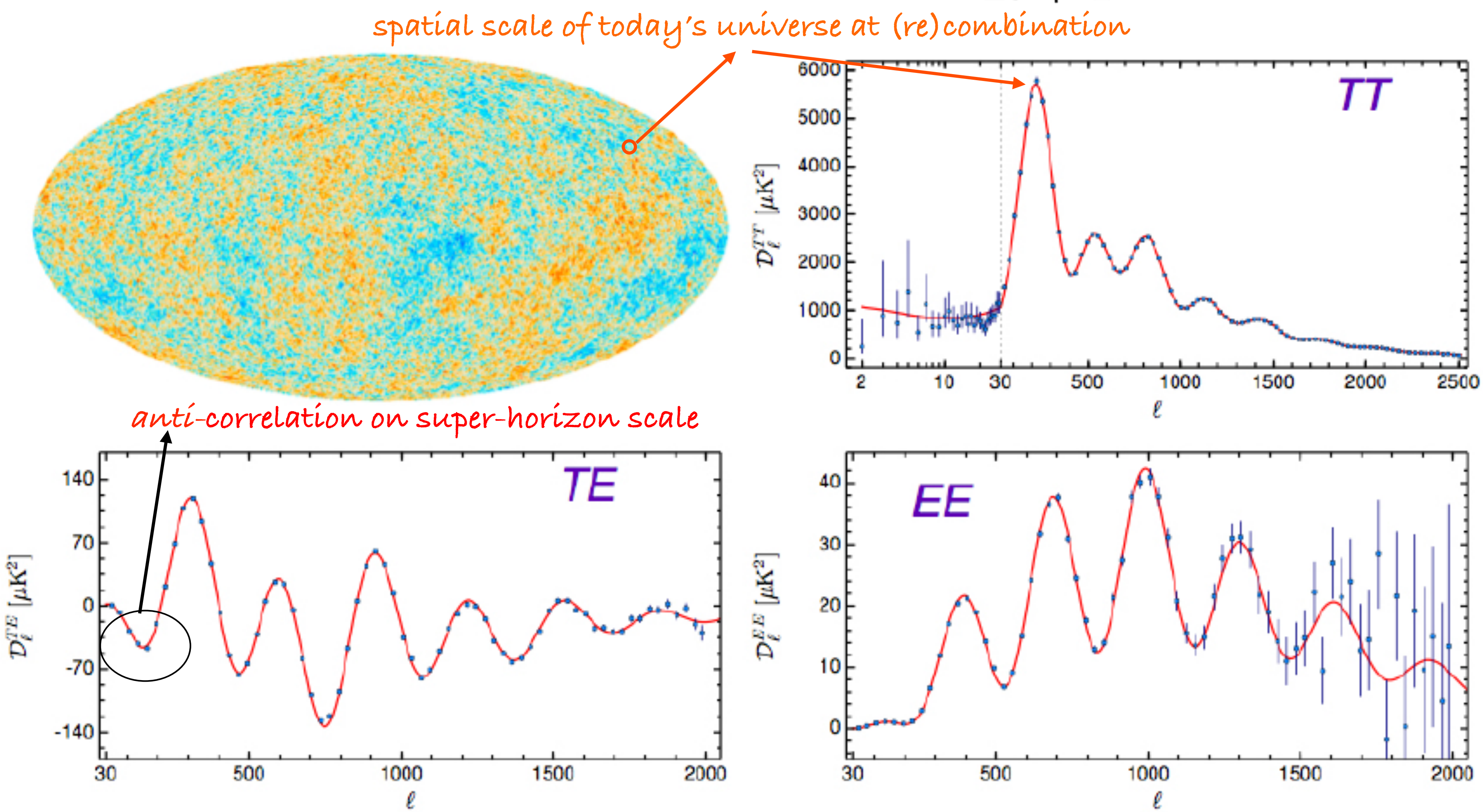
… well below the sensitivity of gravitational wave detectors



Battye & Shellard 1996

Planck data release II – 1st December 2014

$$\Delta T(\mathbf{n}) = \sum a_{lm} Y_{lm}(\mathbf{n}) \qquad C_l \equiv \frac{1}{2l+1} \sum |a_{lm}|^2$$



Coherent oscillations in a photon+baryon plasma excited by primordial scalar density perturbations on *super*-horizon length scales

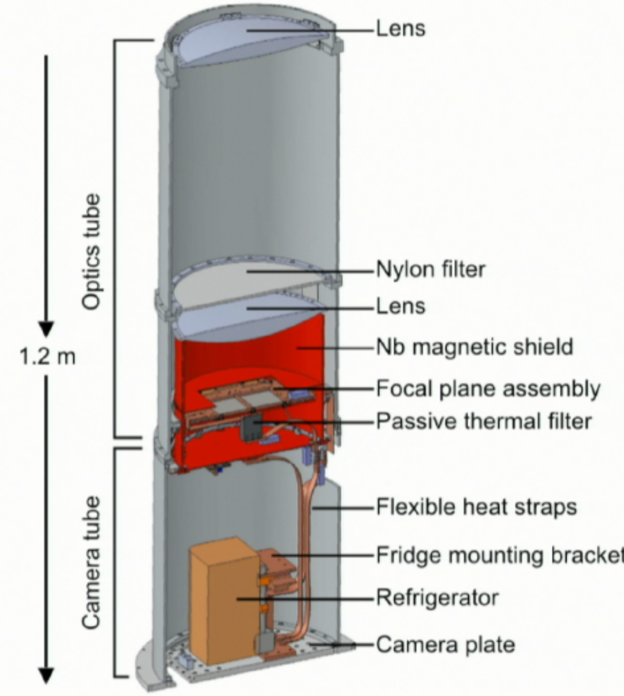
The BICEP2 Telescope

Telescope as compact as possible while still having the angular resolution to observe degree-scale features.

On-axis, refractive optics allow the entire telescope to rotate around boresight for polarization modulation.

Liquid helium cools the optical elements to 4.2 K.

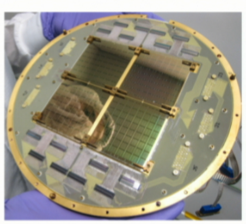
A 3-stage helium sorption refrigerator further cools the detectors to 0.27 K.

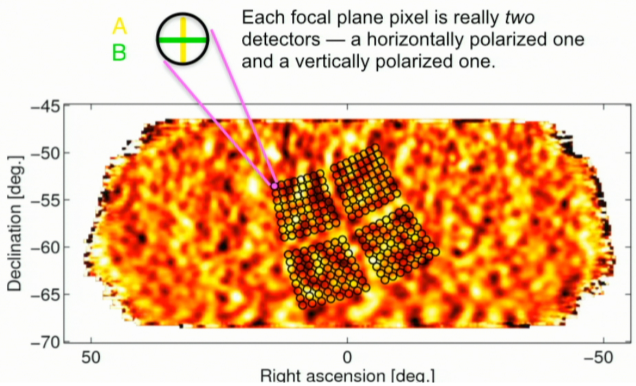


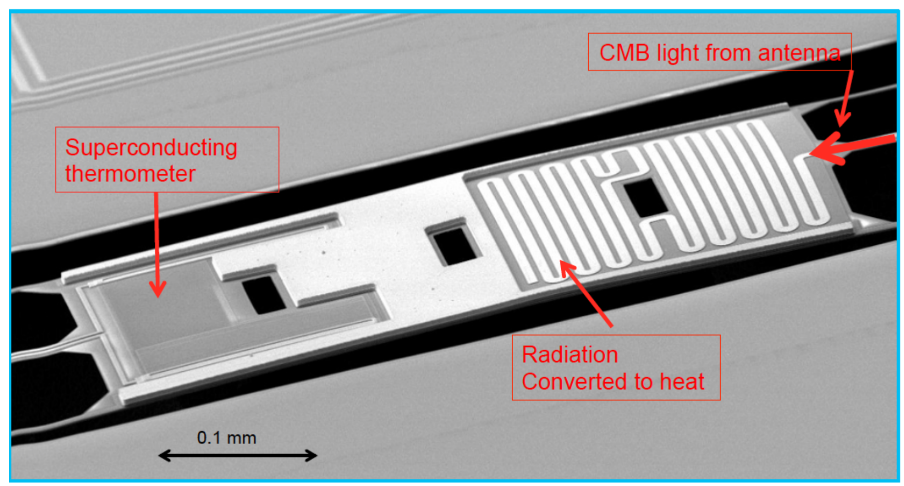
Scan the telescope back and forth on the sky.

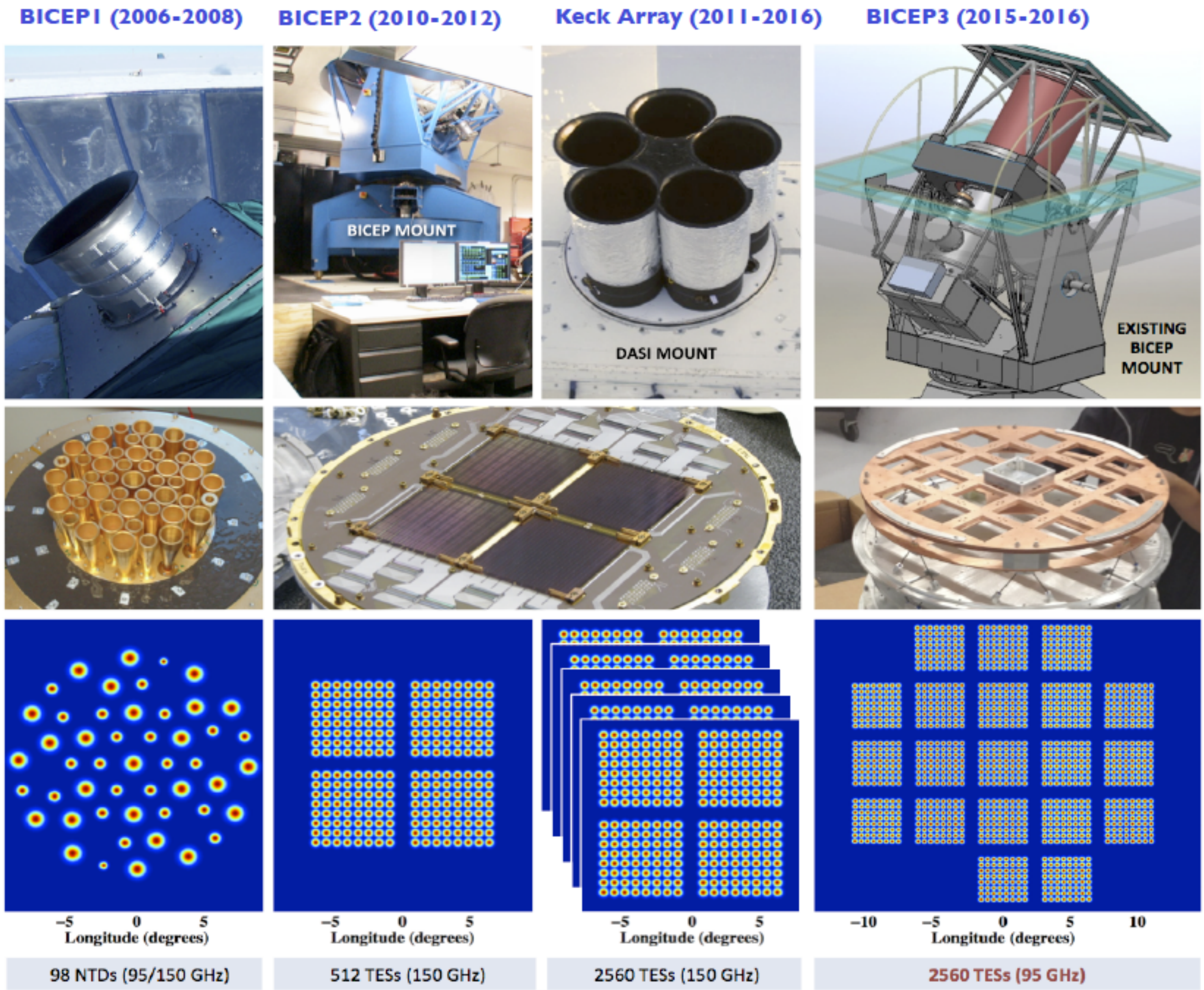
Measure CMB T by summing the signal from orthogonally polarized detector pairs.

Measure CMB polarization by differencing the signal.








BICEP1 (2006-2008)
BICEP2 (2010-2012)
Keck Array (2011-2016)
BICEP3 (2015-2016)
BICEP MOUNT
DASI MOUNT
EXISTING BICEP MOUNT
−5 0 5
Longitude (degrees)
−5 0 5
Longitude (degrees)
−5 0 5
Longitude (degrees)
−10 −5 0 5 10
Longitude (degrees)
98 NTDs (95/150 GHz)
512 TESs (150 GHz)
2560 TESs (150 GHz)
2560 TESs (95 GHz)

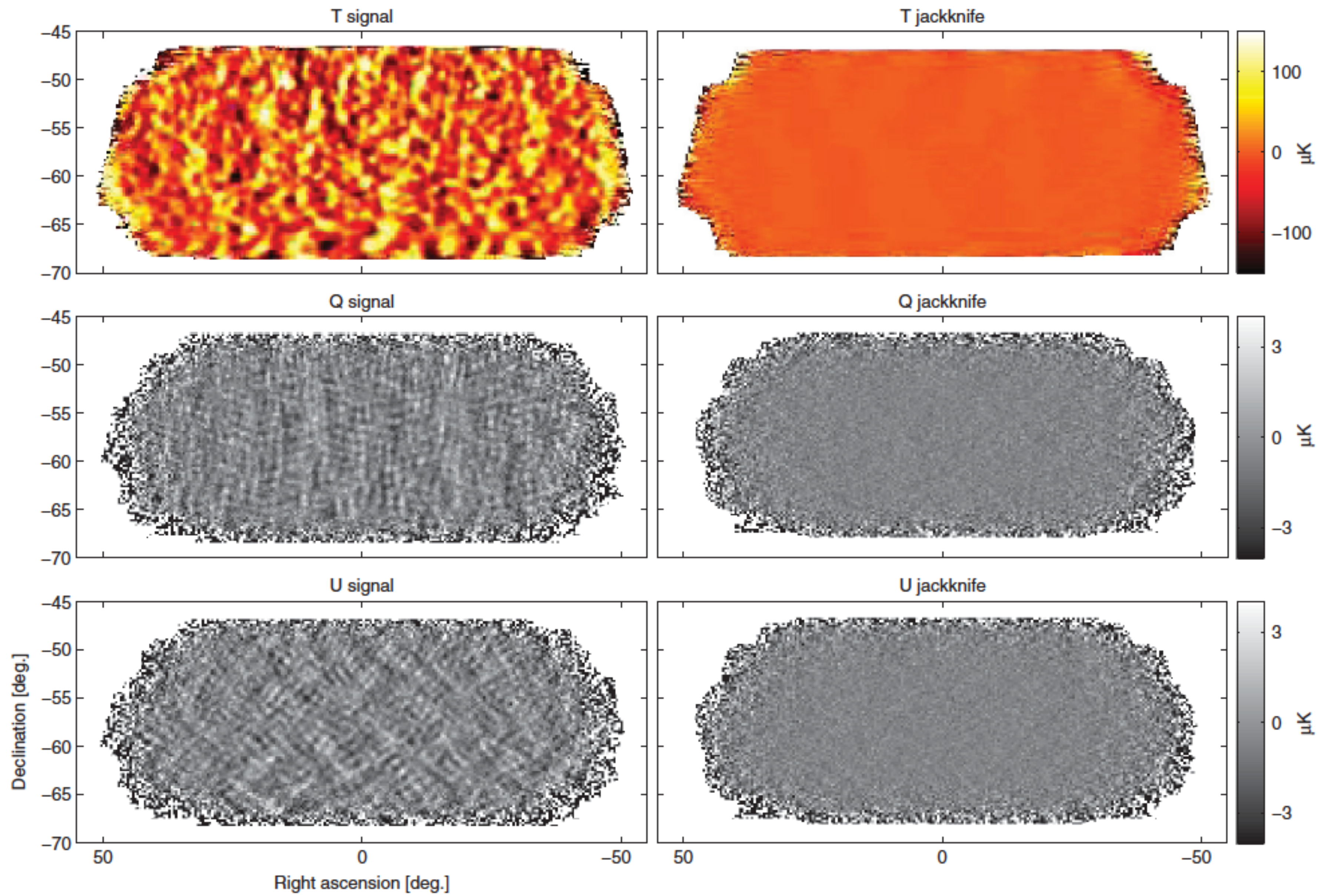
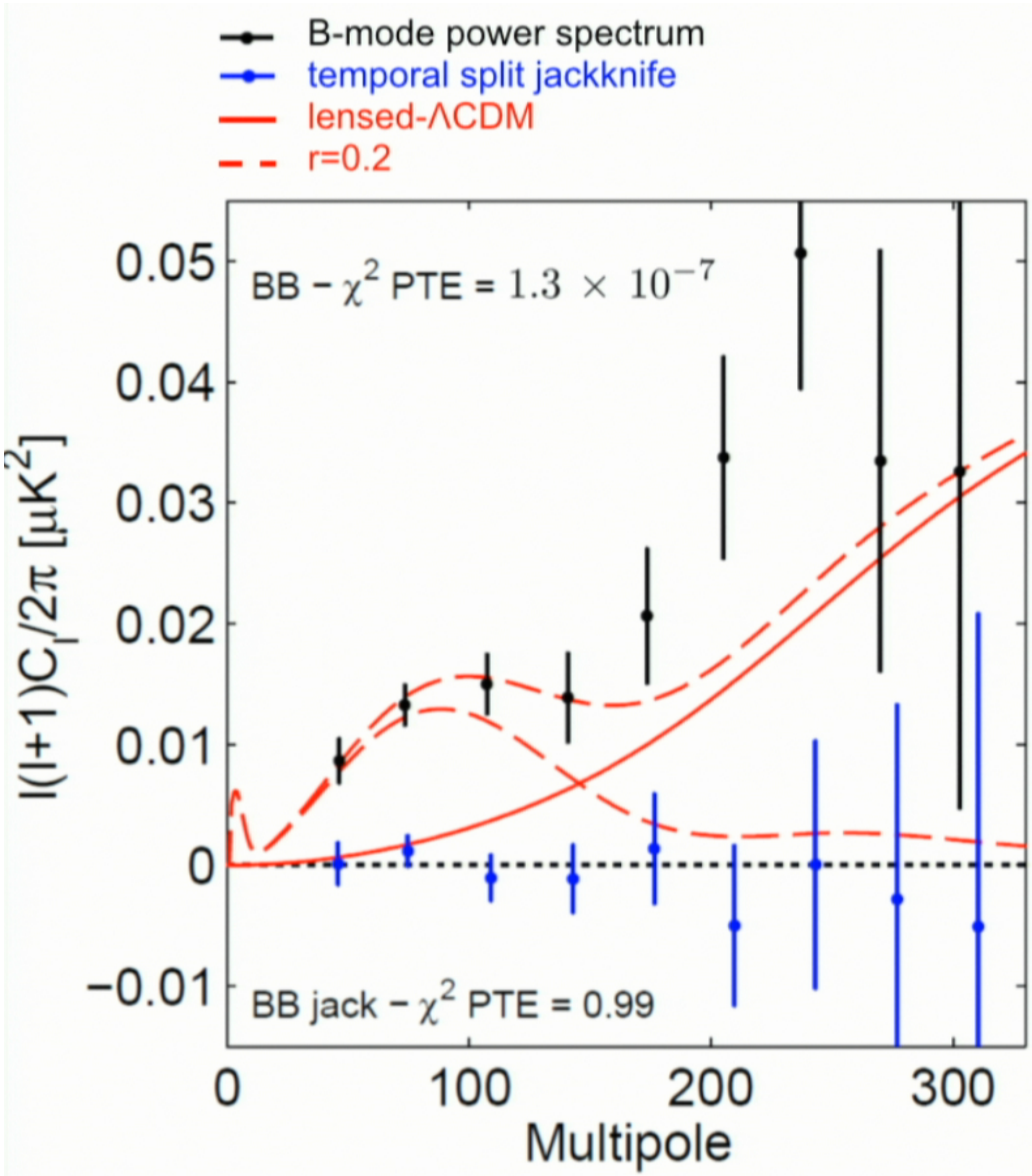


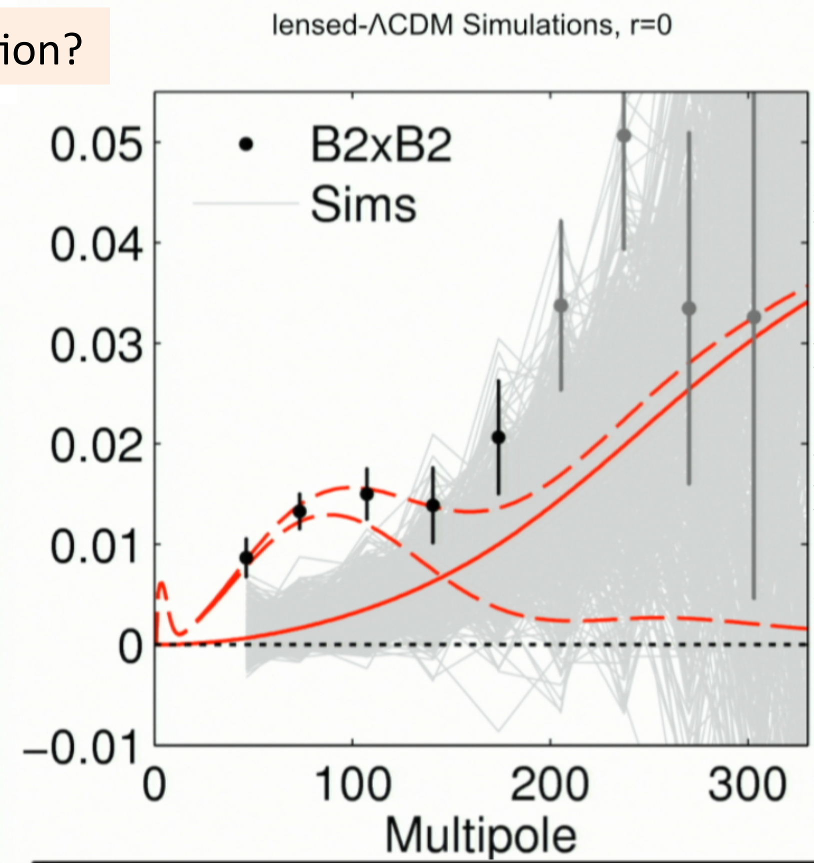
FIG. 1 (color). BICEP2 T, Q, U maps. The left column shows the basic signal maps with 0.25° pixelization as output by the reduction pipeline. The right column shows difference (jackknife) maps made with the first and second halves of the data set. No additional filtering other than that imposed by the instrument beam (FWHM 0.5°) has been done. Note that the structure seen in the Q and U signal maps is as expected for an E-mode dominated sky.

Ade *et al*, PRL **112**:241101,2014

What is the actual significance of the *B*-mode detection?



Ade *et al*, PRL **112**:241101,2014

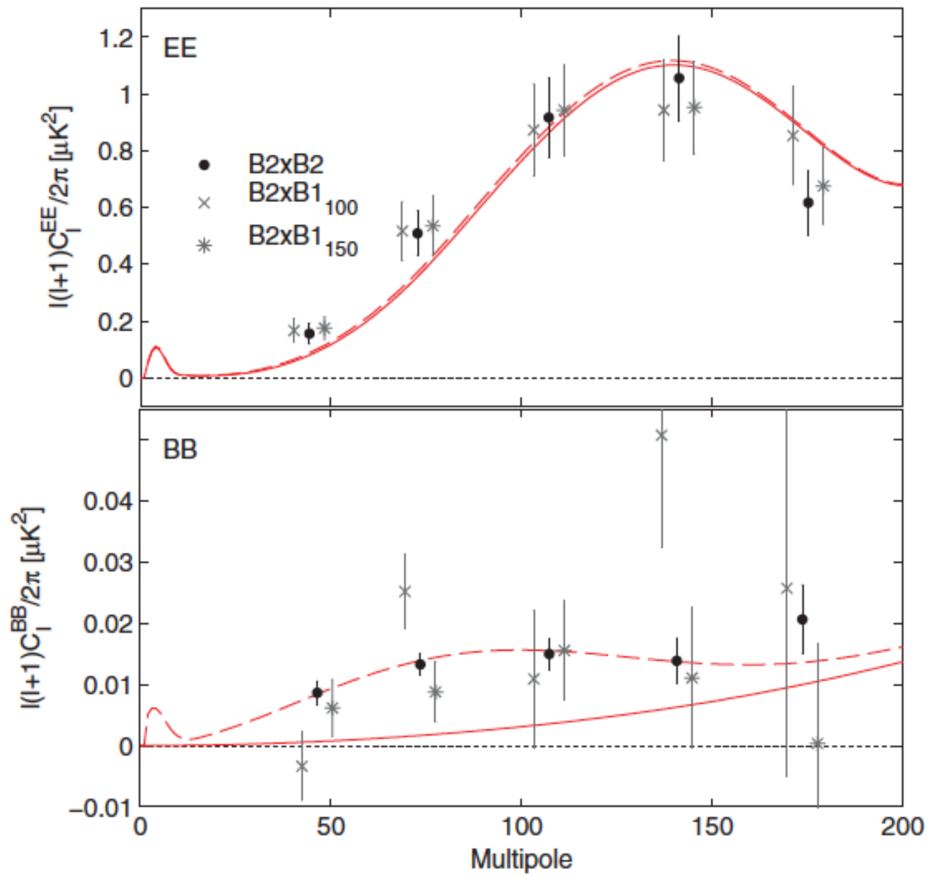


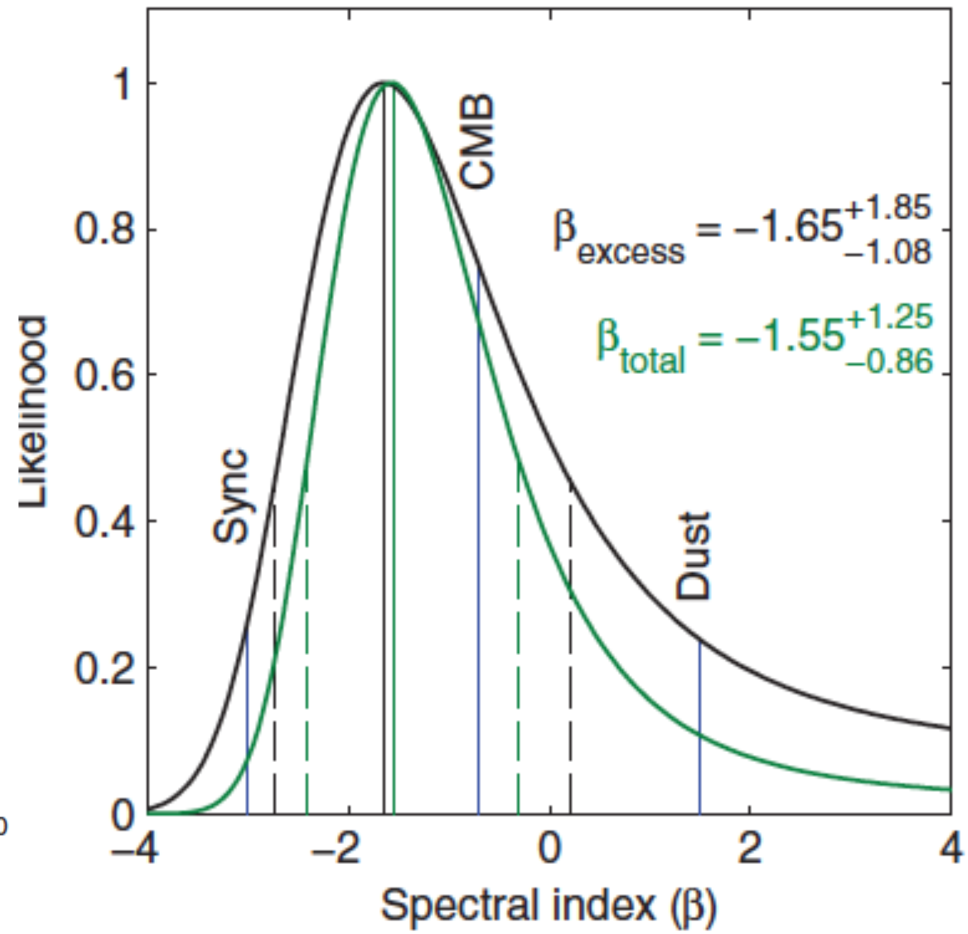
(Kendrick Smith, PI talk)

χ^2 PTE	1.3×10^{-7}
significance	$5.2\,\sigma$

This is just the chance probability of the observed B-mode signal to arise as a fluctuation of the lensed E-mode signal …
it is *not* a '>5σ detection' of a CMB signal

"*We can use the BICEP2 auto and BICEP2xBICEP1$_{100}$ spectra to constrain the frequency dependence of the nominal signal, If the signal at 150 GHz were due to synchrotron we would expect the frequency cross spectrum to be much larger in amplitude than the BICEP2 auto spectrum. Conversely if the 150 GHz power were due to polarized dust emission we would not expect to see a significant correlation with the 100 GHz sky pattern.*" Ade *et al*, PRL **112**:241101,2014

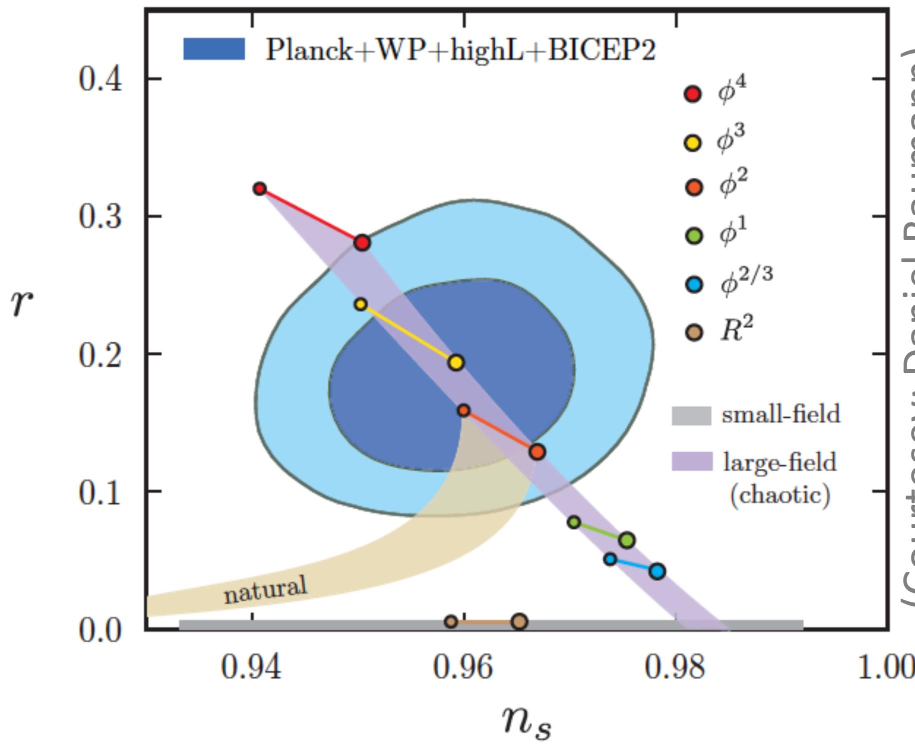




… so the significance with which the observed signal was likely to be **CMB** ($\beta \sim -0.7$) rather than either **synchrotron** ($\beta \sim -3$) or **dust** ($\beta \sim 1.5$) emission was *only* $(1.6\text{–}1.7)\sigma$

If this is all true *then* ...

- The energy scale of inflation is: $V^{1/4} \approx 2.1\text{x}10^{16}\,\text{GeV}\,(r/0.2)^{1/4} \sim M_{\text{GUT}}$
- The field excursion was super-Planckian: $\Delta\phi \approx 4\, M_{\text{Pl}}\,(r/0.2)^{1/2}$

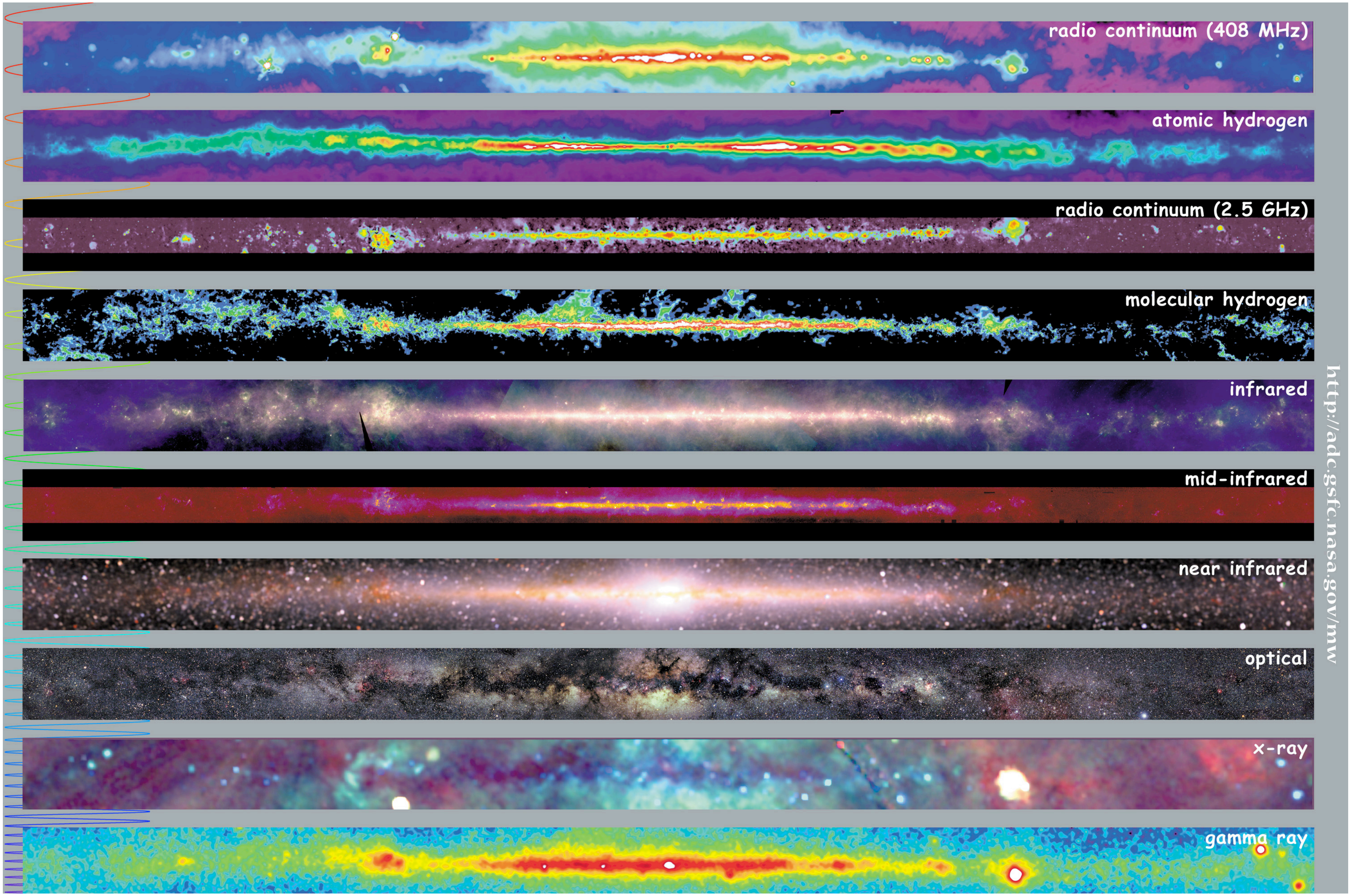


(Courtesey: Daniel Baumann)

- **The vacuum energy was cancelled to 1 part in 10^{112} after inflation!**

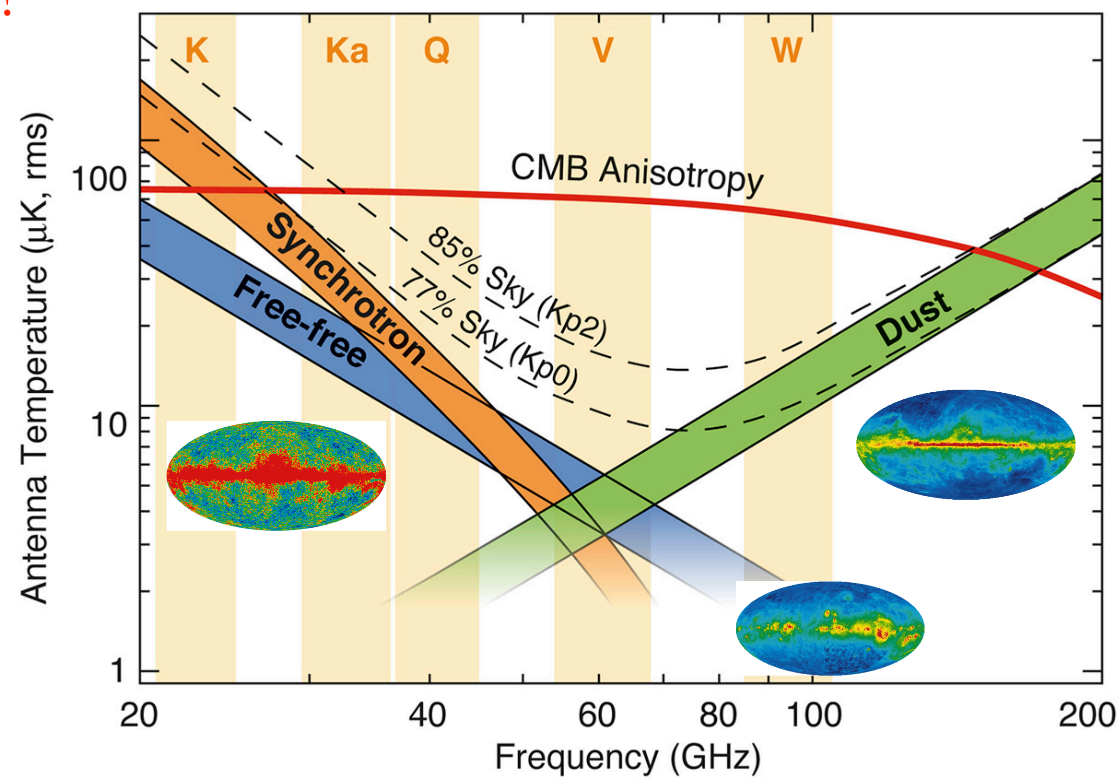
So we ought to be *very* cautious about interpreting the observational result given its momentous implications … e.g. could it just be some astrophysical foreground?

Diffuse Milky Way foregrounds



At CMB frequencies the most important sources of foregrounds are:

- Synchrotron radiation by cosmic ray electrons in the (ordered + turbulent) Galactic magnetic field (strongly polarised)
- Free-free emission from ionised hydrogen (unpolarised)
- Thermal dust emission (weakly polarised) + 'spinning dust' (unpolarised)
- what else?!



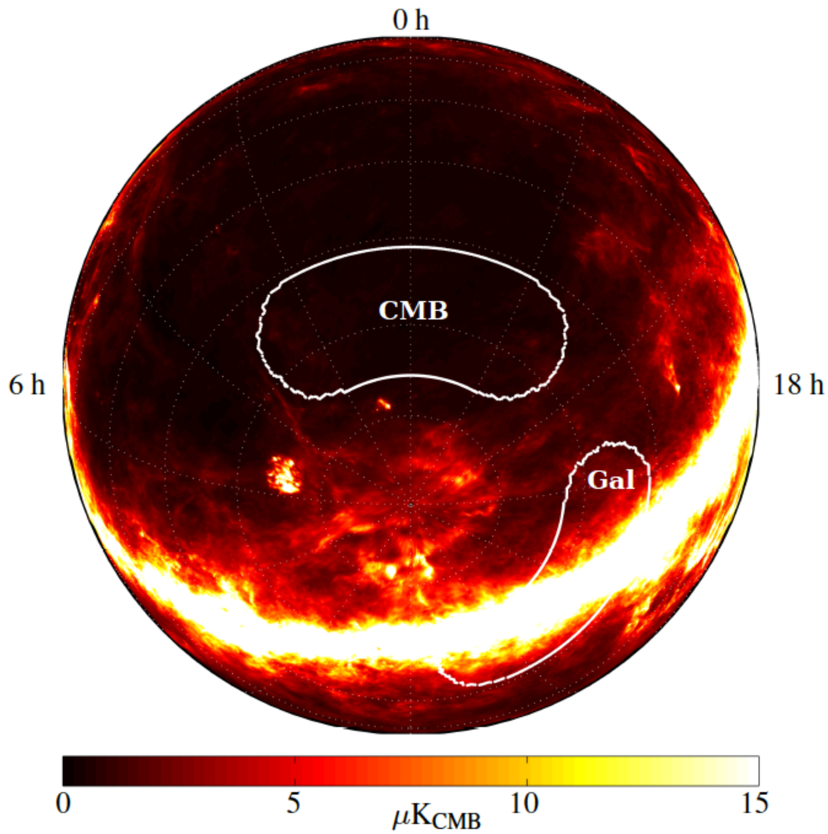
Bennett *et al.*, ApJS 148 (2003) 97

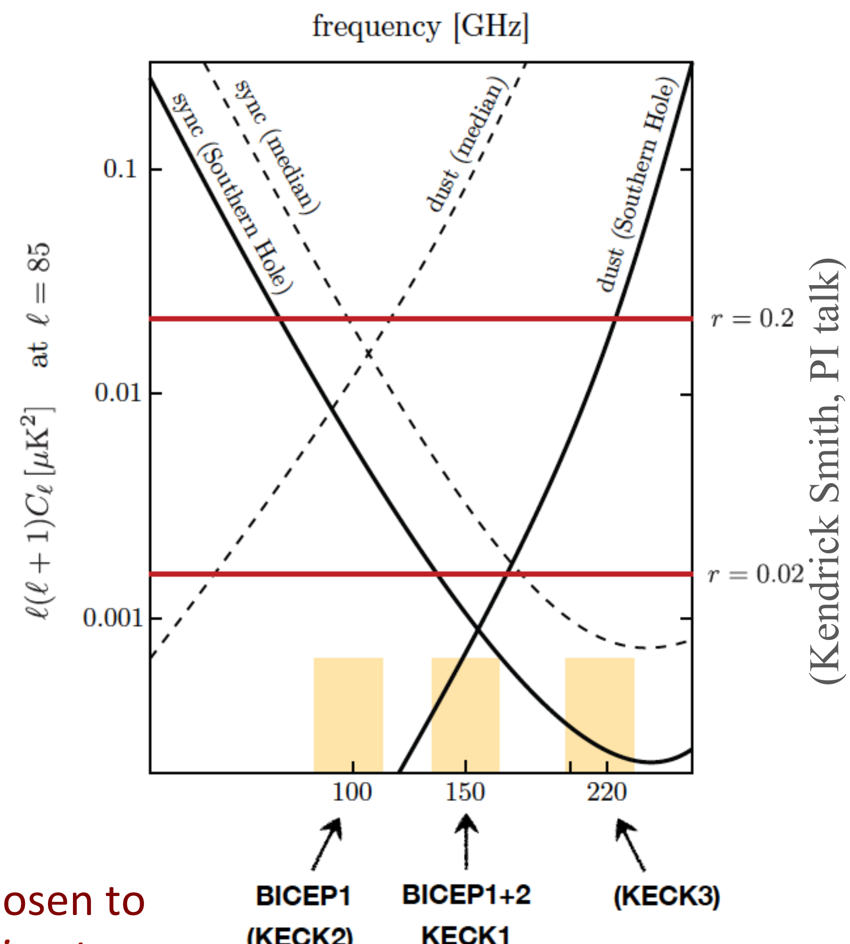
To subtract out the foregrounds, observe at multiple frequencies and isolate the CMB by its blackbody spectrum … and/or look at high galactic latitude away from Milky Way

The important astrophysical *polarised* foregrounds at CMB frequencies are:

- Synchrotron radiation from relativistic cosmic ray electrons gyrating in the Galactic magnetic field (polarised perpendicular to local field direction)

- Thermal emission from interstellar dust (also polarised perpendicular to magnetic field due to tendency of grains to align along the field)

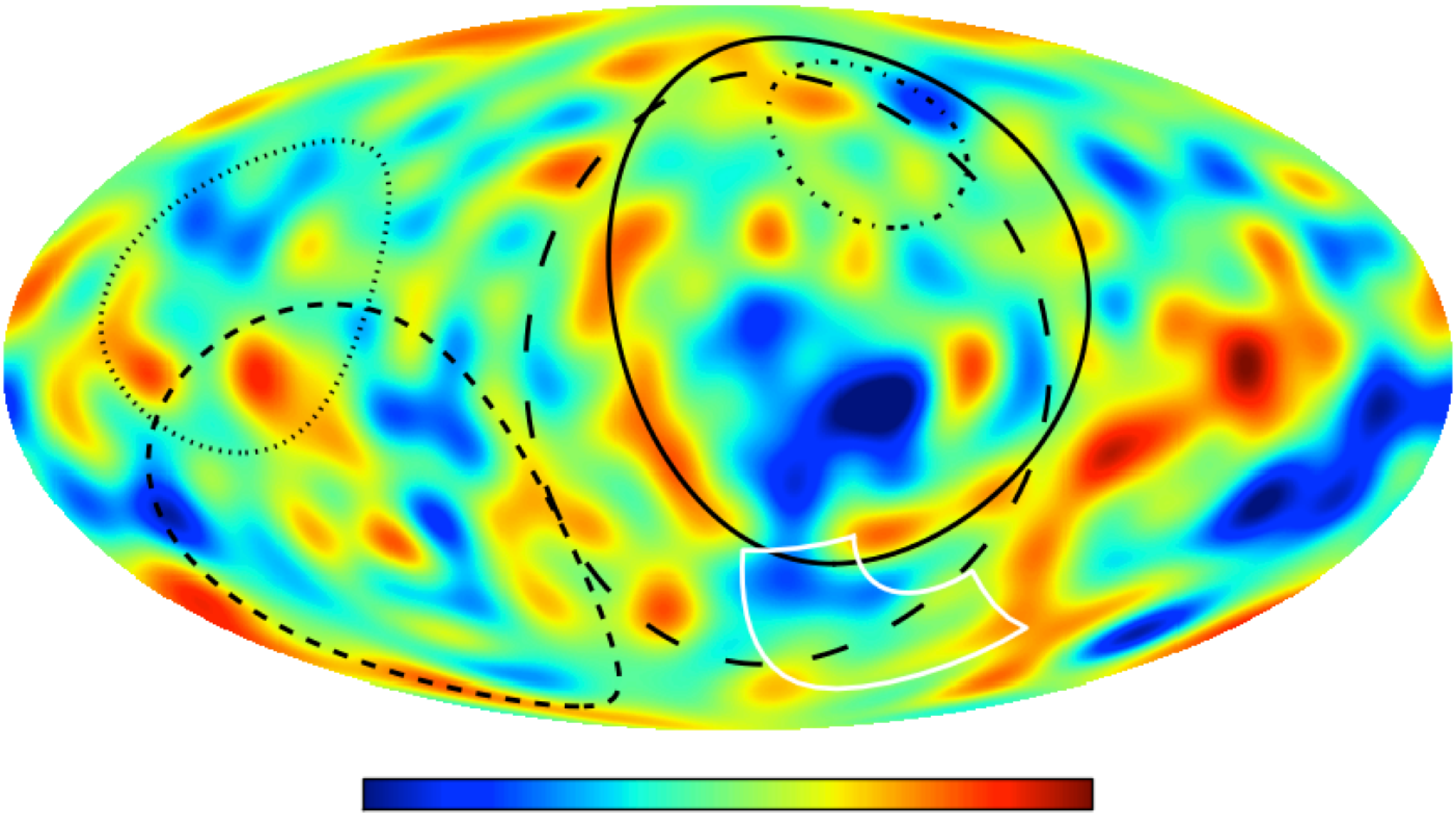




BICEP2 observes a small patch of high-latitude sky chosen to minimise these foregrounds … but the levels are *estimates*

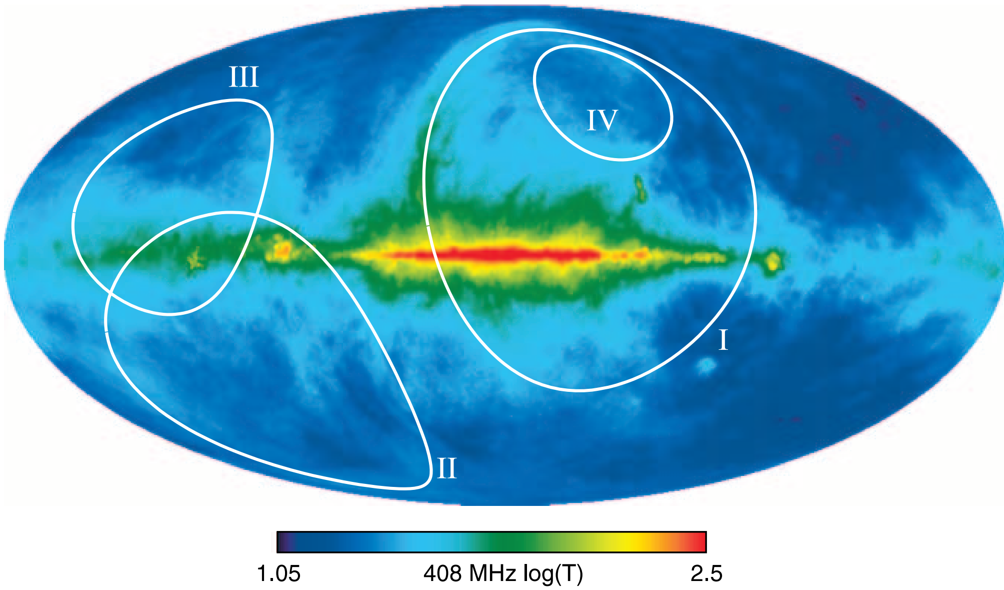
This particular patch of sky was chosen to be observed because:

"... such ultra clean regions are very special – at least an order of magnitude cleaner than the average b >50^0 level" Ade *et al*, PRL **112**:241101,2014

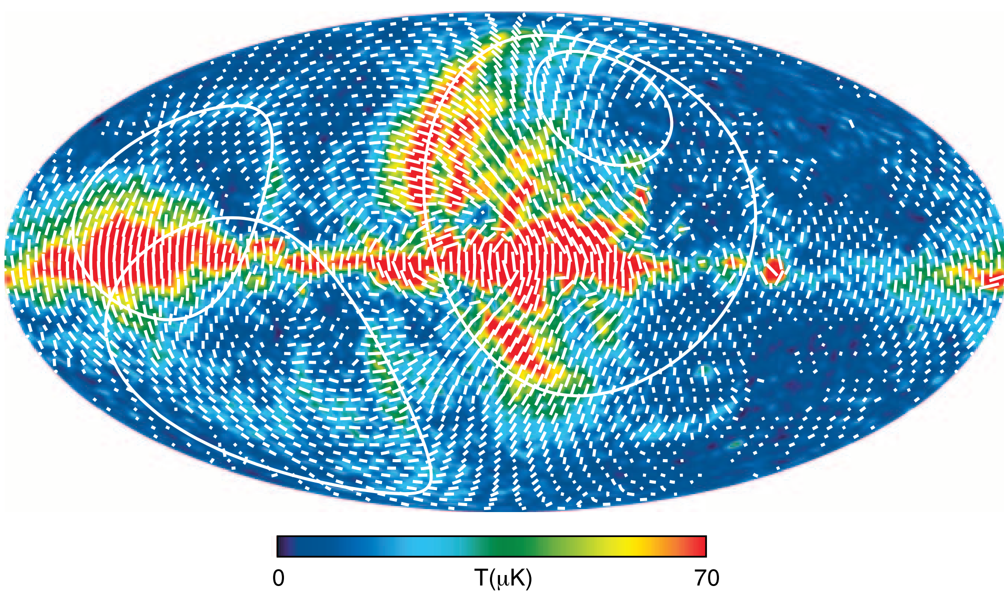


However it is in fact crossed by a galactic 'radio loop'!

What are the ‘radio loops’?



Haslam *et al*, A&AS **47**:1,1982



Page *et al*, ApJS **170**:335,2007

- ✧ Probably the radiative shells of very old supernova remnants
- ✧ Can see only 4 of these in the 408 MHz radio sky

  Berkhuijsen *et al*, A&A **14**:252,1971
- ✧ However there must be *several thousand* loops in the Galaxy which cannot be resolved against the galactic radio background … indeed they probably constitute much of the ‘diffuse’ background

  Sarkar, MNRAS **199**:97,1982

Boosted emissivity in old SNRs

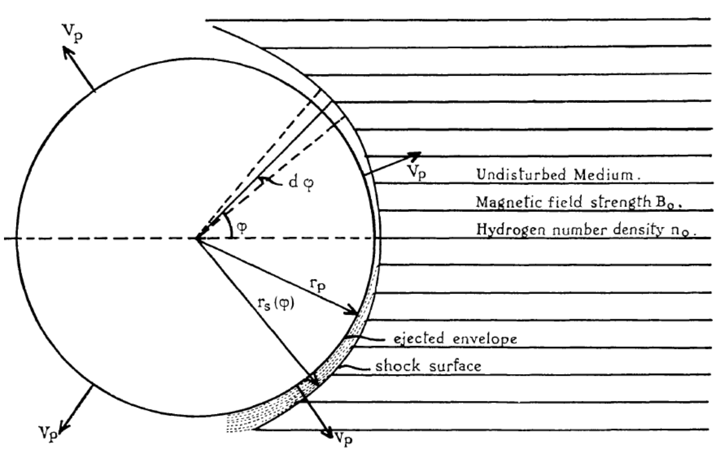
If the compression in the shell is by a factor $\boldsymbol{\eta}$ then a power-law cosmic ray spectrum $N_i(E_0)\,dE_0 = K_{0i}E_0^{-\gamma_i}\,dE_0$ is modified by the betatron effect to:

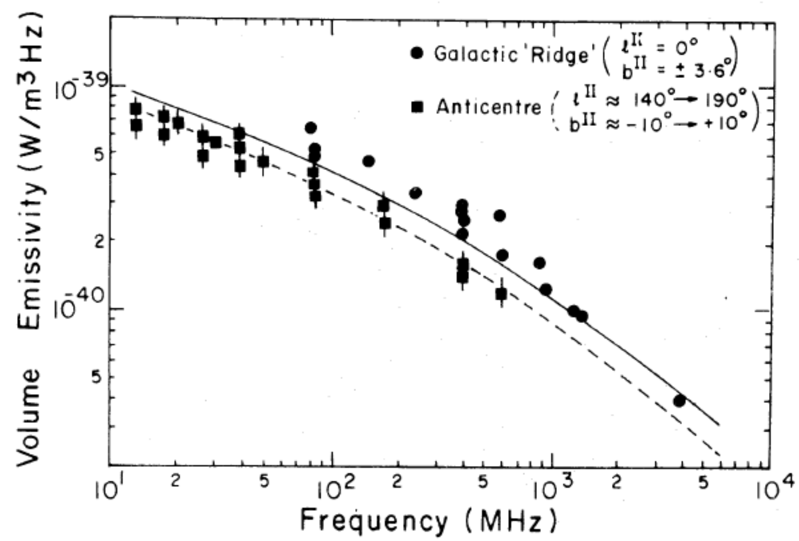
$$N_i(E')\,dE' = K_i\left[\frac{1}{2}+\frac{1}{2}\,\frac{\eta^2}{(\eta-1)(2\eta-1)^{1/2}}\,\sin^{-1}\left(\frac{\eta-1}{\eta}\right)^{1/2}\right]^{(\gamma_i-1)} E'^{-\gamma_i}\,dE'$$

after pitch-angle scattering behind the shock, where $K_i/K_{0i} = \eta^3/\{3\eta(\eta-1)+1\}$

Now calculate the distribution of $\boldsymbol{\eta}$ in the McKee-Ostriker 3-phase model of the ISM regulated by SNRs, to determine the average interstellar synchrotron emissivity …

Van der Laan, MNRAS **124**:125,1962

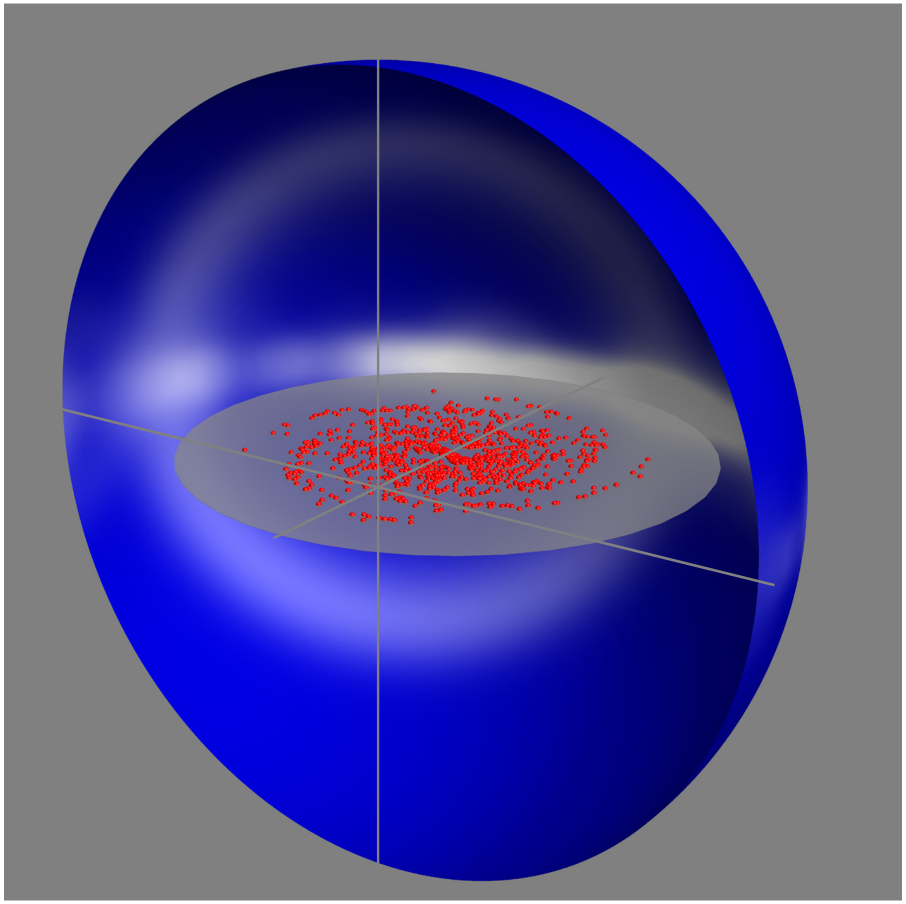


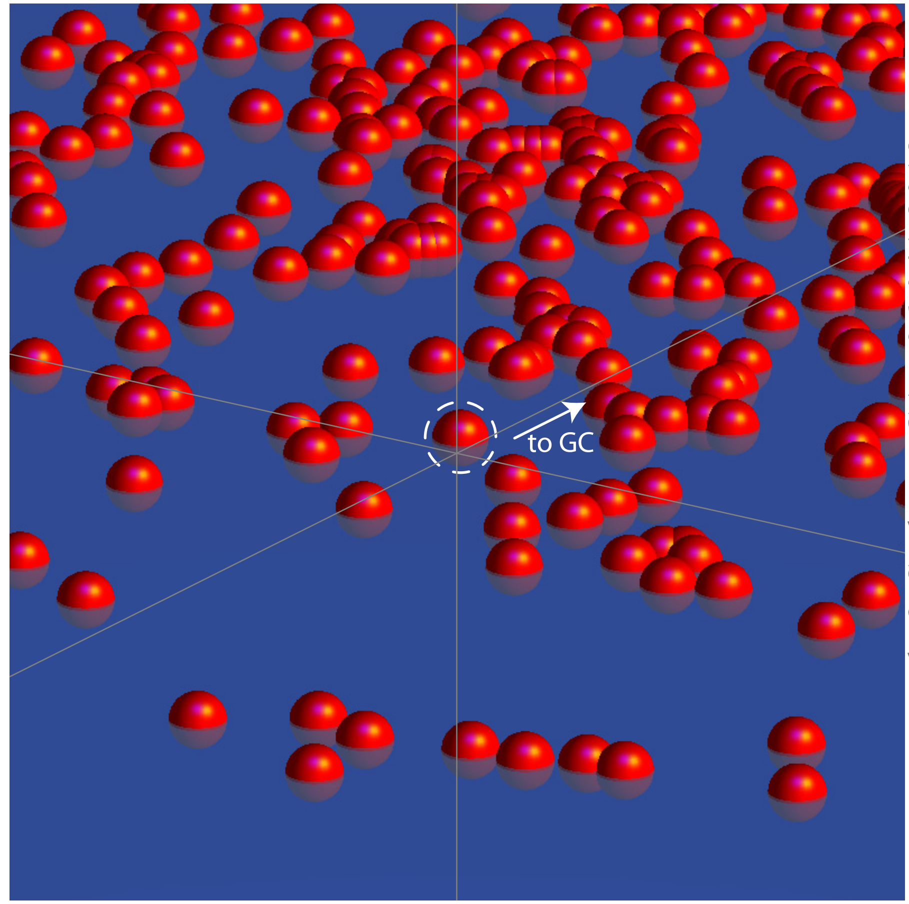


Sarkar, MNRAS **199**:97,1982

Figure 1. The average interstellar synchrotron emissivity due to old radiative supernova remnants, for a magnetic field of 1 μG in the hot interstellar medium ($n_0 = 10^{-2}$ cm^{-3}). The dashed and solid lines refer to the cases with and without pitch-angle scattering behind the shocks, respectively. Observational data are from the compilation by Daniel & Stephens (1975).

Simulating the galactic distribution of old SNRs





Mertsch & Sarkar, JCAP 06:041,2013

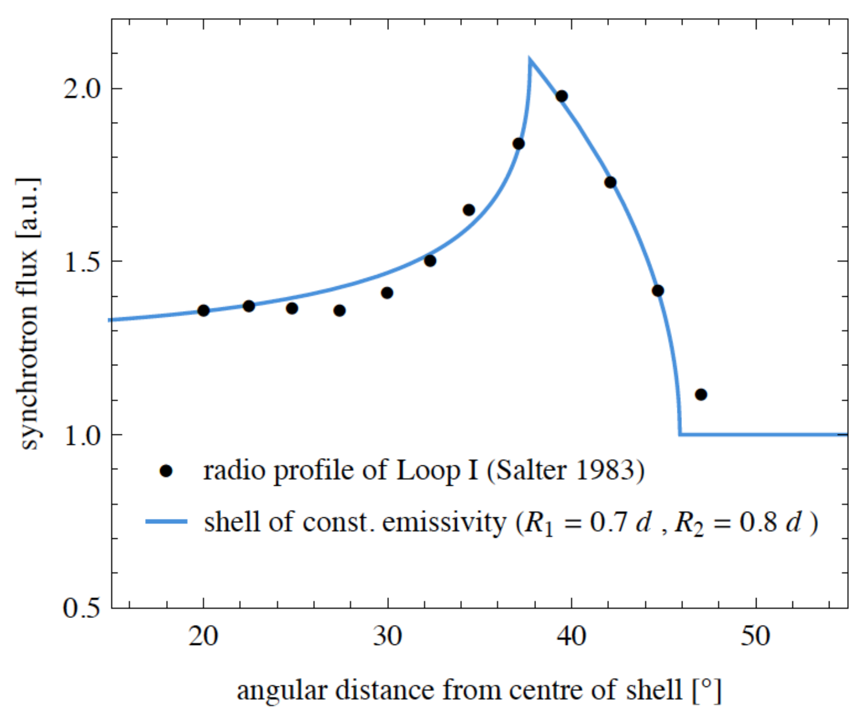
With ~3 SN/century, there must be *several thousand* old SNRs in the radiative phase of evolution … their shells will compress the interstellar magnetic field – and the *coupled* cosmic ray electrons – to high values, significantly boosting the synchrotron emissivity

Angular Power Spectrum of a SNR shell

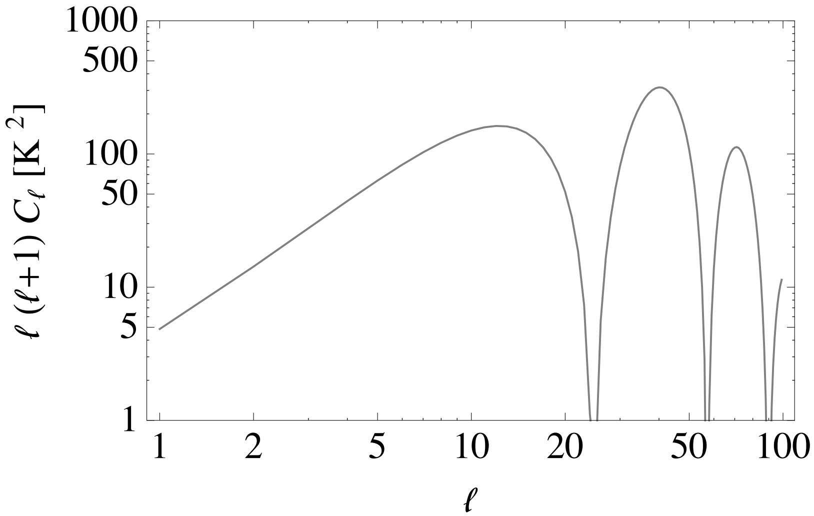
… after projection along line-of-sight, the shell of homogeneous emissivity has angular profile $g(r)$

$$C_l^j = \frac{1}{2l+1} \sum_{m=-l}^{l} \left| a_{lm}^j \right|^2$$

$$= \frac{1}{4\pi} \left(f_j(\nu) \int_{-1}^{1} \mathrm{d}z' P_l(z') g_j(z') \right)$$



Mertsch & Sarkar, JCAP 06:041,2013



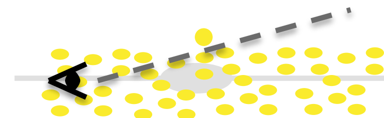
Angular power spectrum for shell i:

$$C_i(\ell) \propto \left(\mathrm{P}_l \left(\cos \frac{R_i}{d_i} \right) \right)^2$$

… thickness of shell determines cut-off

Modelling an ensemble of shells

Assumption: flux from one shell factorises into angular part and frequency part: $J_{\text{shell }i}(\nu, \ell, b) = \varepsilon_i(\nu) g_i(\ell, b)$



Mertsch & Sarkar, JCAP 06:041,2013

Frequency part: $\varepsilon_i(\nu)$

Magnetic field gets compressed in SNR shell

Electrons get betatron accelerated

Emissivity increased with respect to ISM

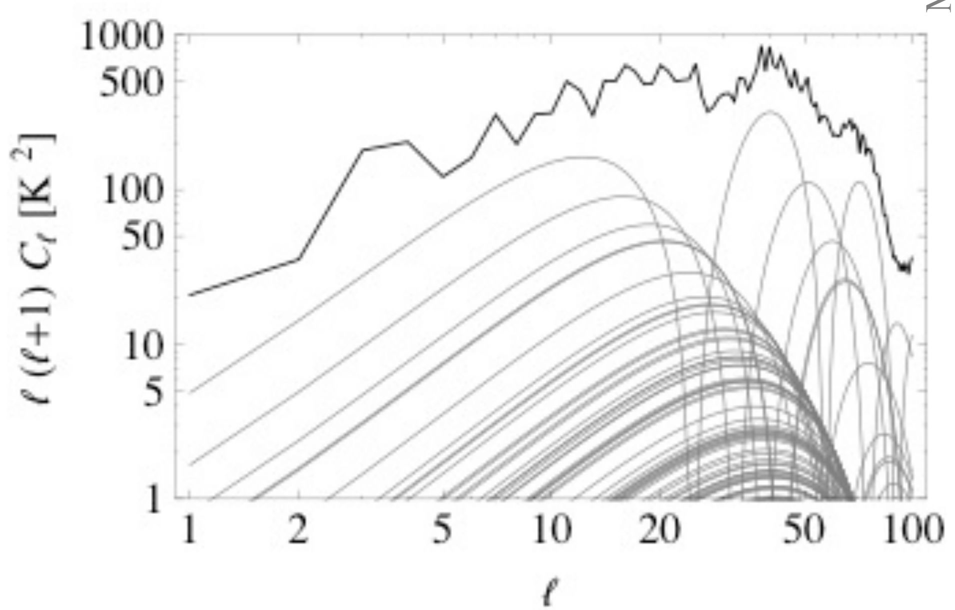
Angular part: $g_i(\cos\psi)$

Assume constant emissivity in shell:

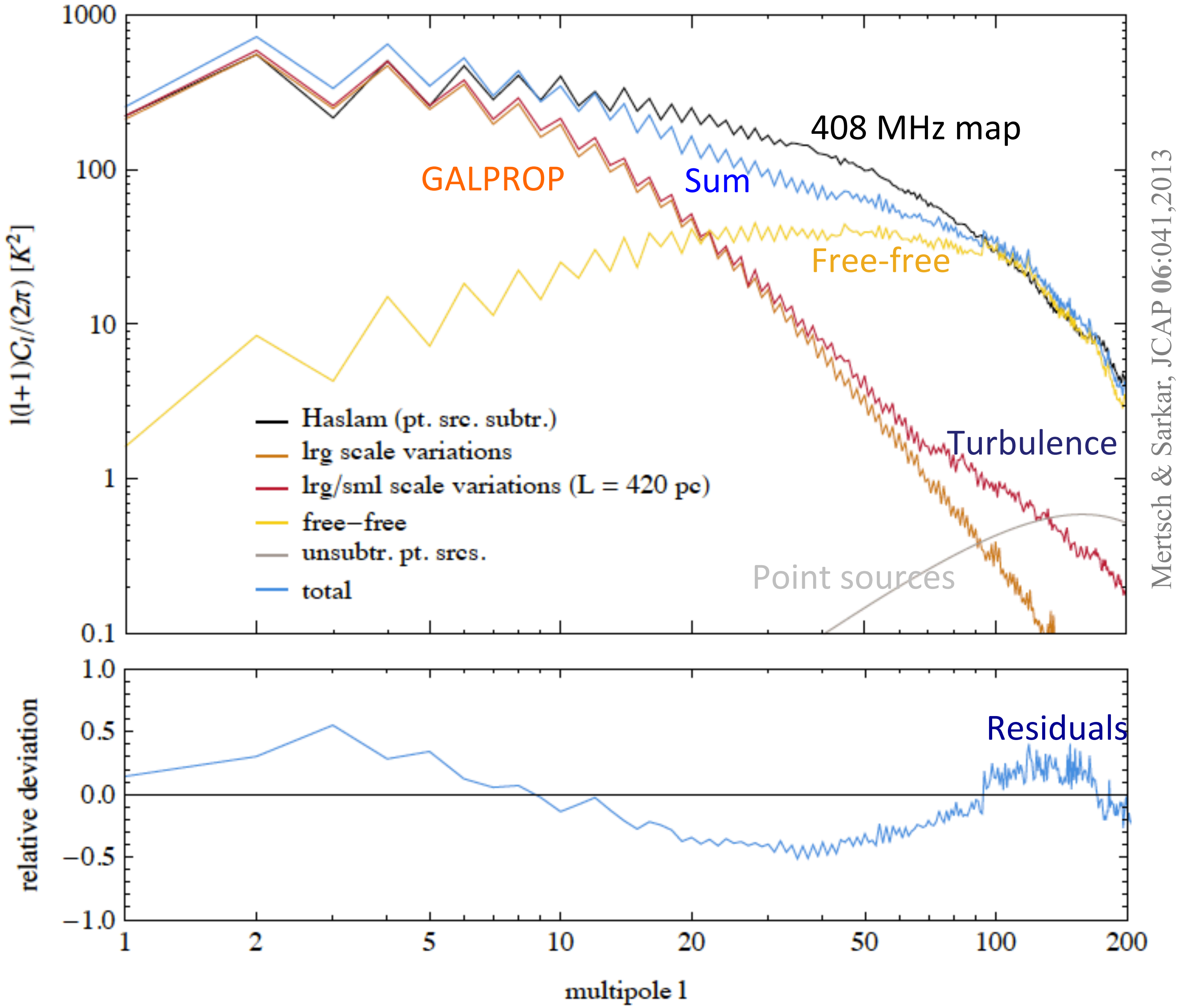
$$a_{lm}^{i}{}' \sim \varepsilon_i(\nu) \int_{-1}^{1} \mathrm{d}z' P_l(z') g_i(z')$$

Add up contribution from all shells:

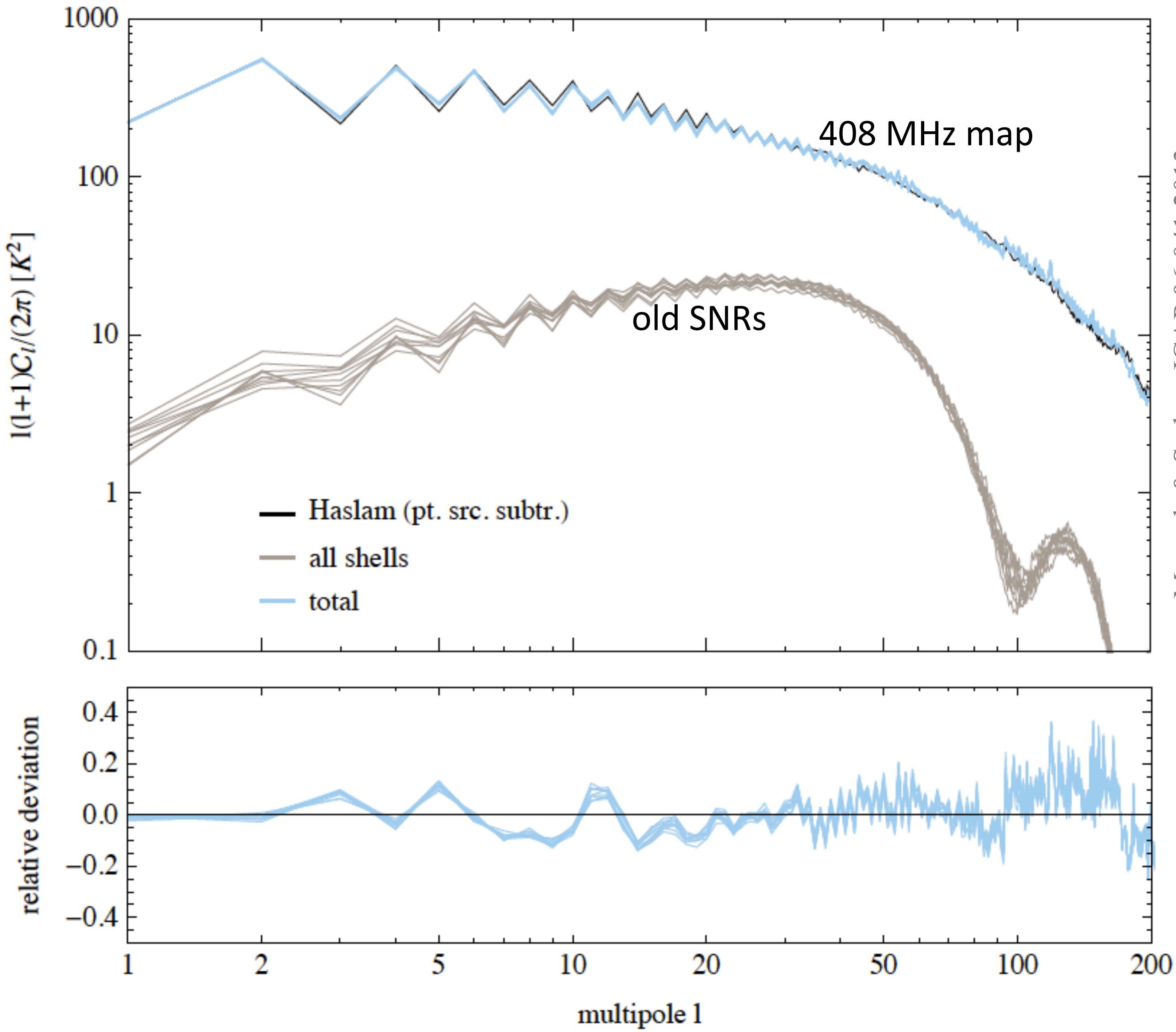
$$a_{lm}^{\text{total}} = \sum_i a_{lm}^i$$



The uniform galaxy model does *not* fit the data

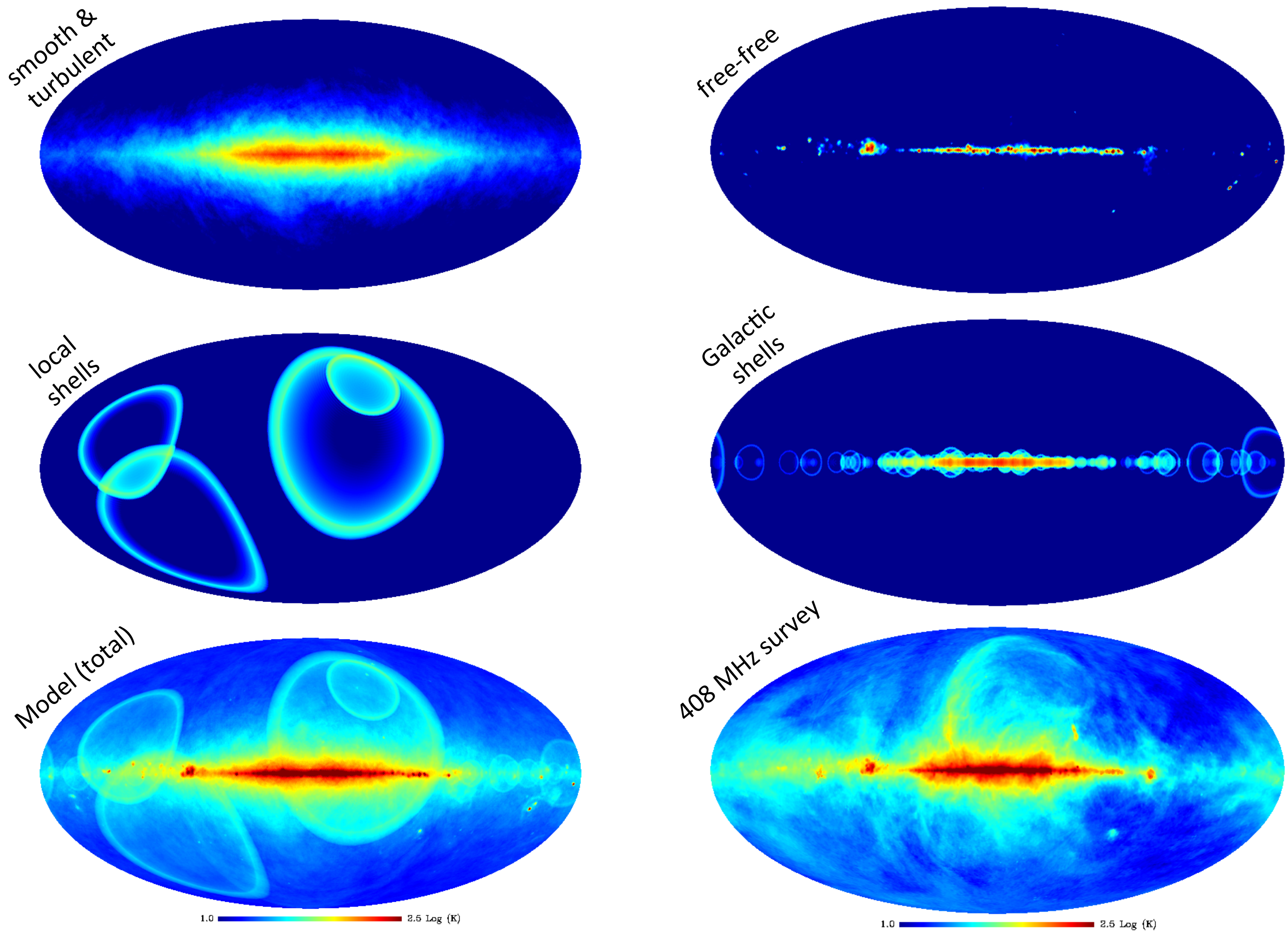


… but adding old SNR shells allows an excellent match!



Mertsch & Sarkar, JCAP **06**:041,2013

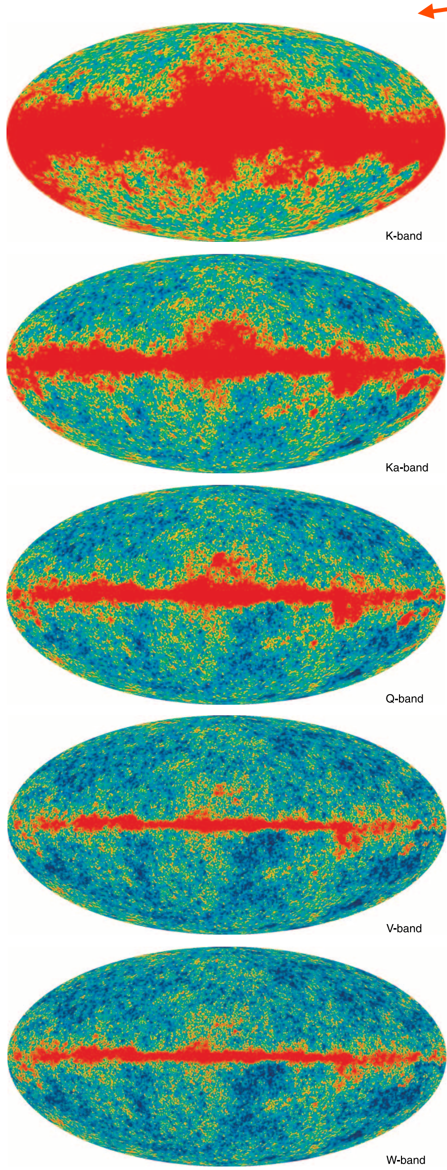
This model has structure at high latitude (like the *real* radio sky)

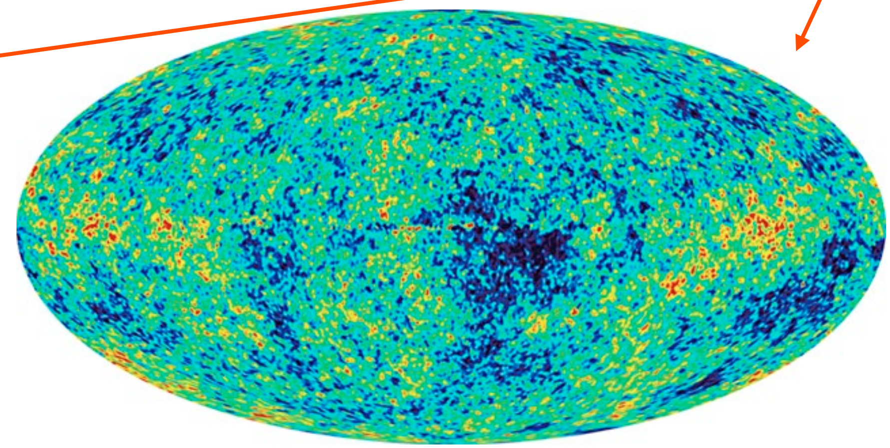


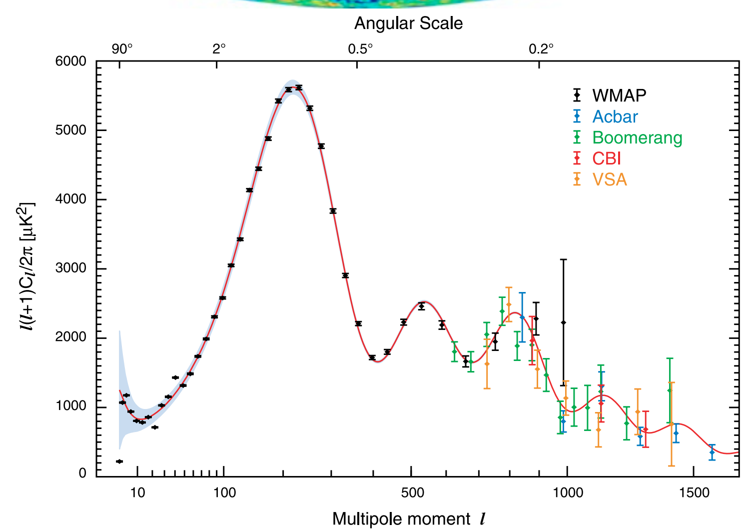
Mertsch & Sarkar, JCAP 06:041,2013

CMB foreground removal: How do we get from this to this?

Hinshaw *et al*, ApJS 170:288,2007







Answer: ILC - Internal Linear Combination (SMICA for Planck)

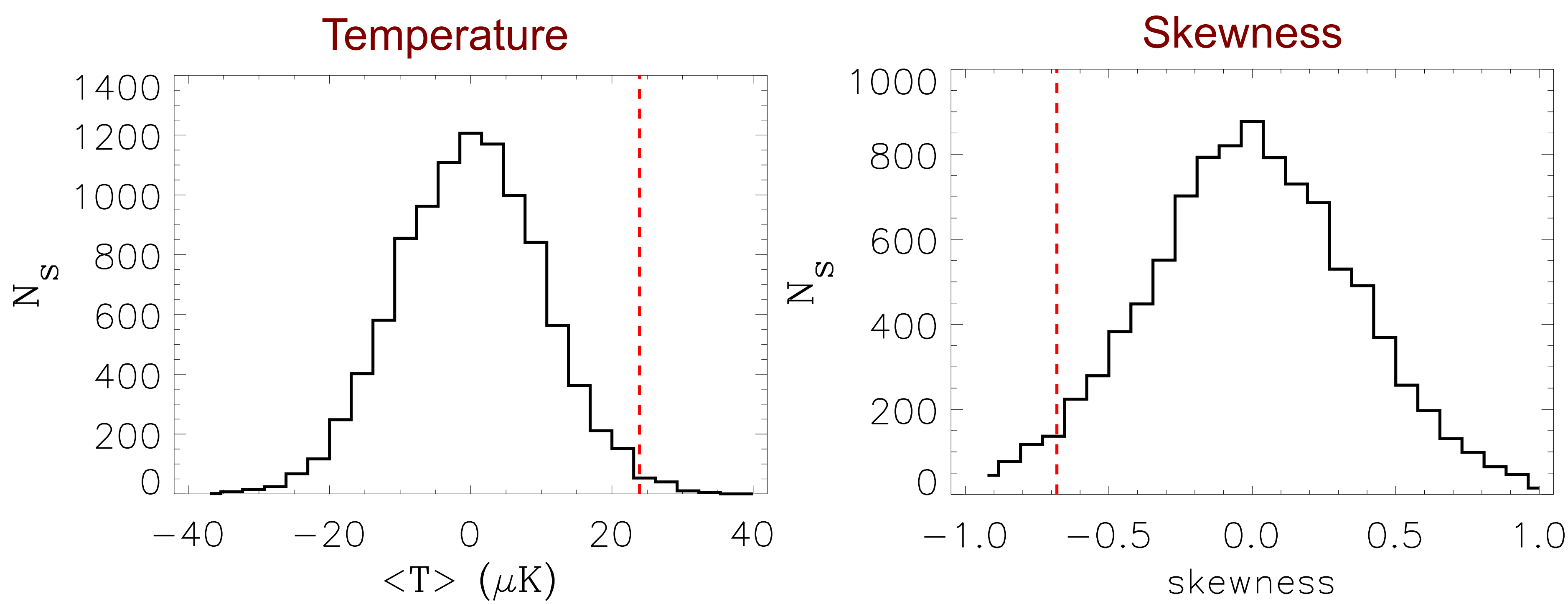
$$T_{\rm ILC} = \sum_i \zeta_i T_i = \sum_i \left(T_{\rm CMB} + S_i T_{\rm foreground}\right)$$

… and minimise the variance $\sigma^2_{\rm ILC}$

Anomalies in WMAP-9 Internal Linear Combination map ($\ell \lesssim 20$)

There is a 23 *m*K *excess* temperature in ring around Loop I

(NB: This is ~1/4 of the total *TT* signal in the 'cleaned' CMB map)

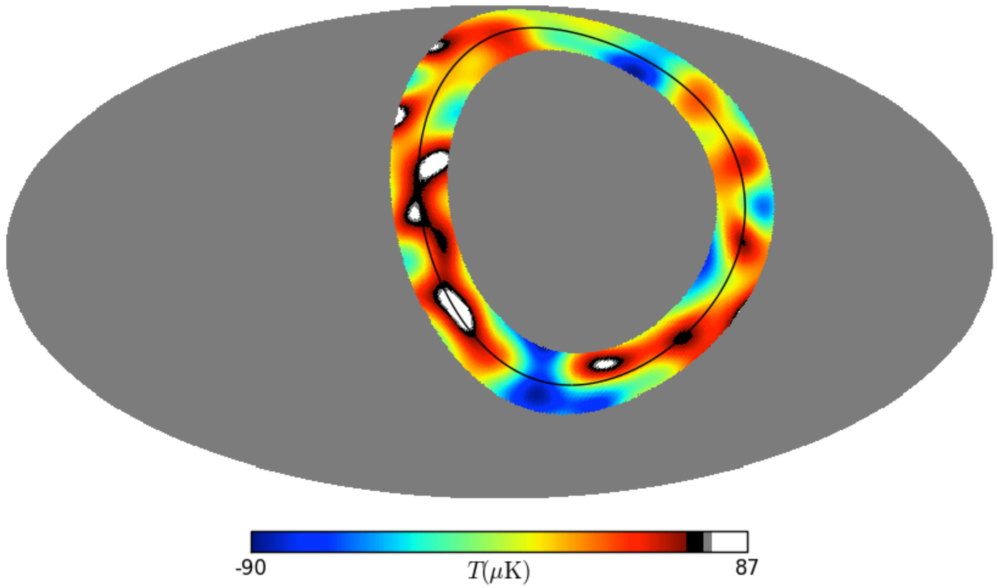


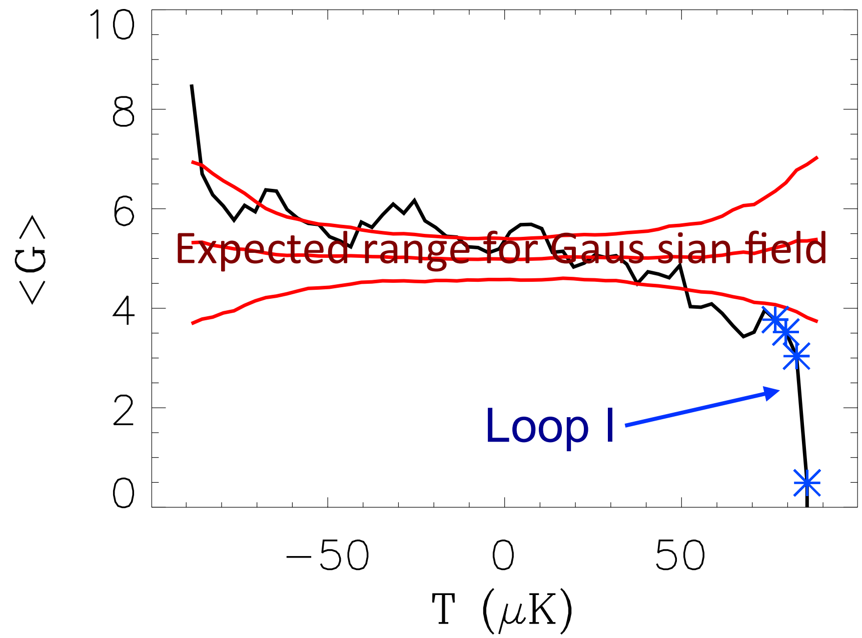
Compare with MC $\Rightarrow$ p-values of $\mathcal{O}(10^{-2})$

Liu, Mertsch & Sarkar, ApJL **789**:L29,2014

Anomalies in WMAP-9 Internal Linear Combination map ($\ell \lesssim 20$)

<u>Cluster analysis</u> (Naselsky & Novikov, ApJ **444**:1,1995): Compute for each pixel the angular distance *G* from Loop I along great circles crossing both the pixel and the loop center and compare with random realisation of best-fit ΛCDM model

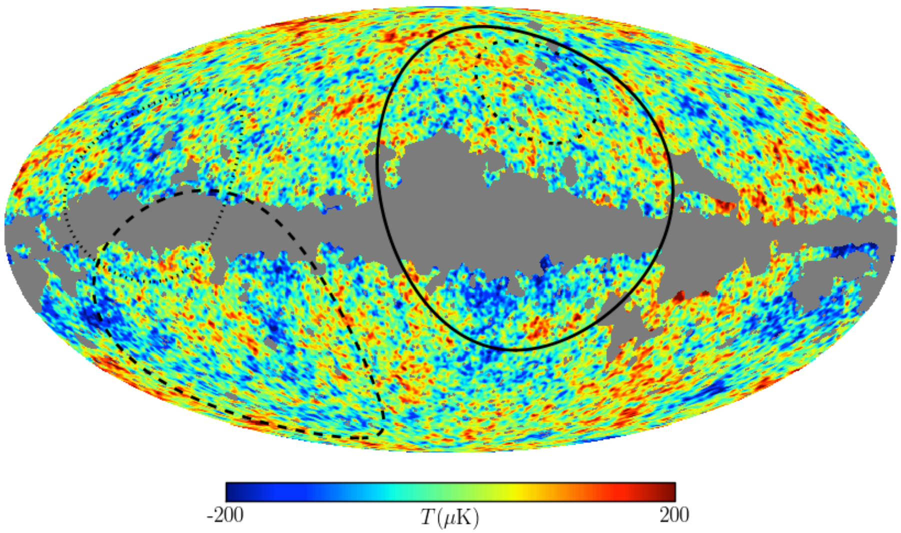




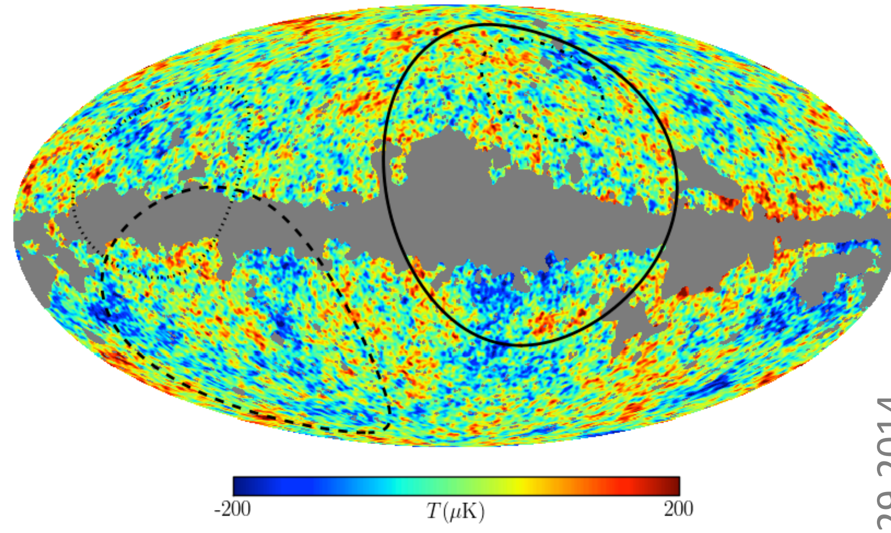
From 100,000 MC runs: probability for *smaller* $\langle G \rangle$ in last 4 bins ~ 10^{-4}

Liu, Mertsch & Sarkar, ApJL 789:L29,2014

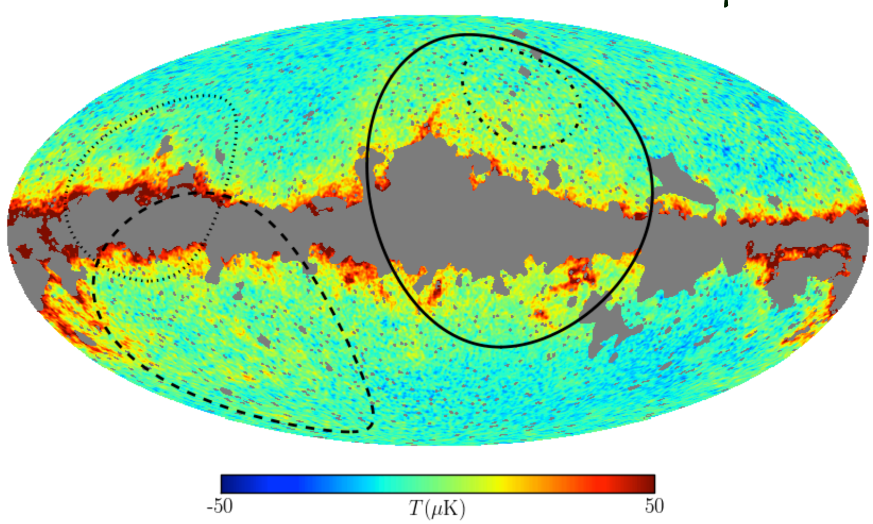
ILC coefficients from Loop I region



ILC coefficients from rest of sky



Difference $ILC_{rest} - ILC_{Loop\ I}$



Liu, Mertsch & Sarkar, ApJL **789**:L29,2014

There *is* an imprint of the radio loops in the WMAP ILC (also Planck SMICA) maps of the CMB which have *supposedly* been cleaned of all foreground emissions!

What do we know about the Loop I anomaly?

- Spatially correlates with Loop I
- *Unlikely* to be synchrotron (checked with our synchrotron model)
- Frequency dependence:

Simple toy model: $$\xi(\hat{\mathbf{n}}) = \tau(\hat{\mathbf{n}}) T_s \Theta(\nu_{\min} \leq \nu_j \leq \nu_{\max})$$

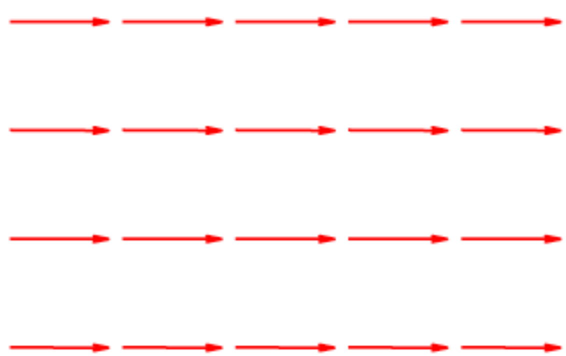
with $\tau(\hat{\mathbf{n}}) \sim 10^{-6}$ and $T_s \sim 20\,\mathrm{K}$

Liu, Mertsch & Sarkar, ApJL **789**:L29,2014

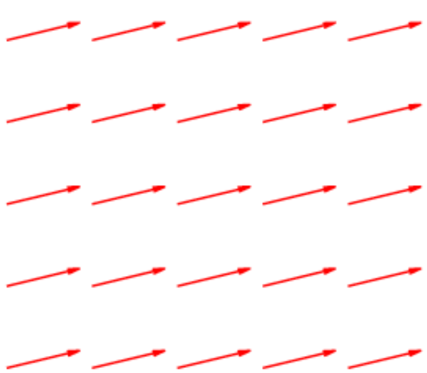
Could it be *magnetic* dipole radiation from dust (with ferrimagnetic inclusions)?

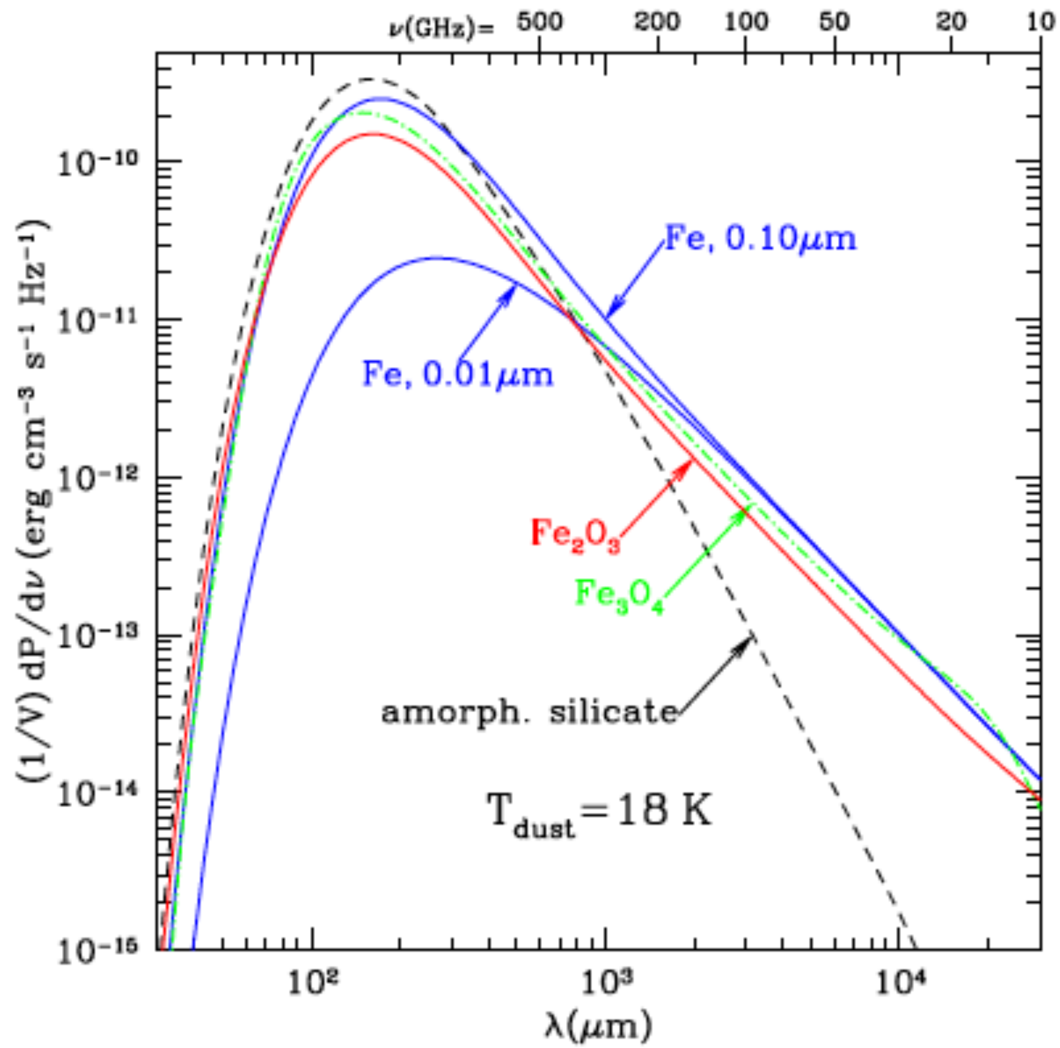
This has a **blackbody-like** spectrum so would have *evaded* foreground cleaning!

Could it be *magnetic* dipole radiation from dust in the loops (with iron or ferrimagnetic inclusions)?



Response to a fluctuation





Draine & Hensley, ApJ **765**:139,2013

Could it be *magnetic* dipole radiation from dust (with iron or ferrimagnetic inclusions)?

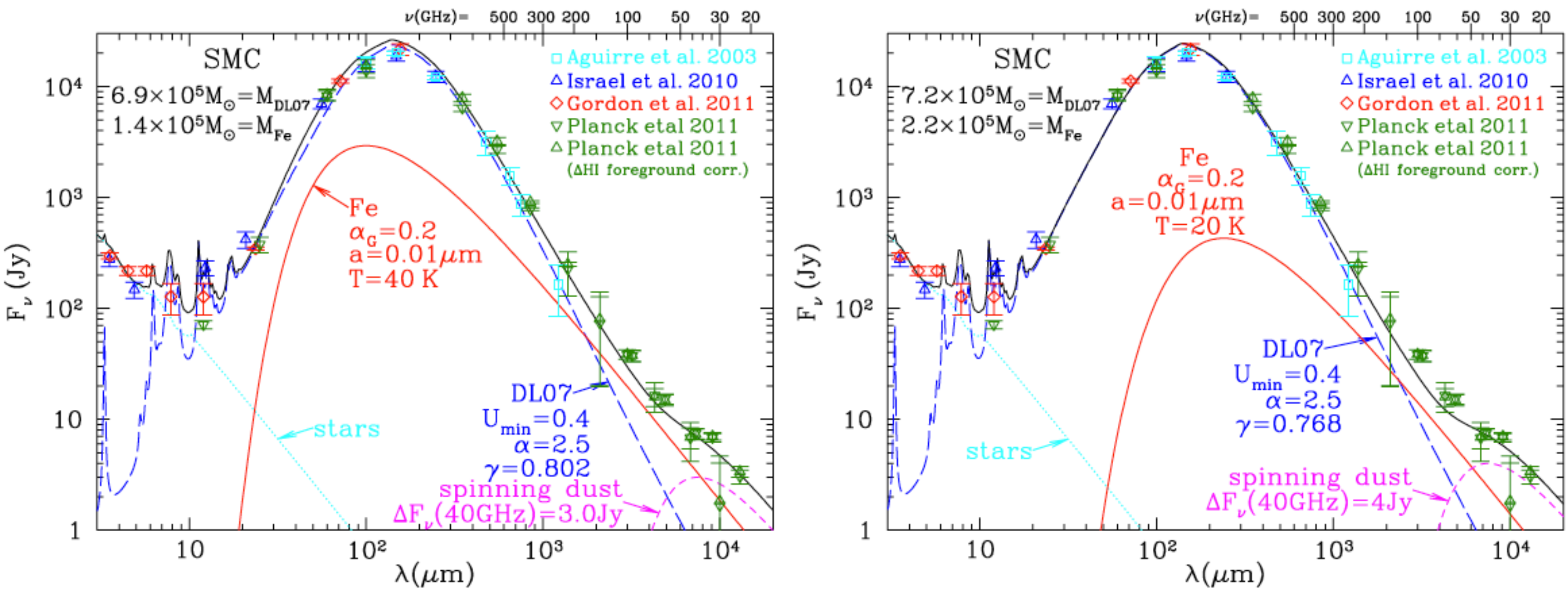
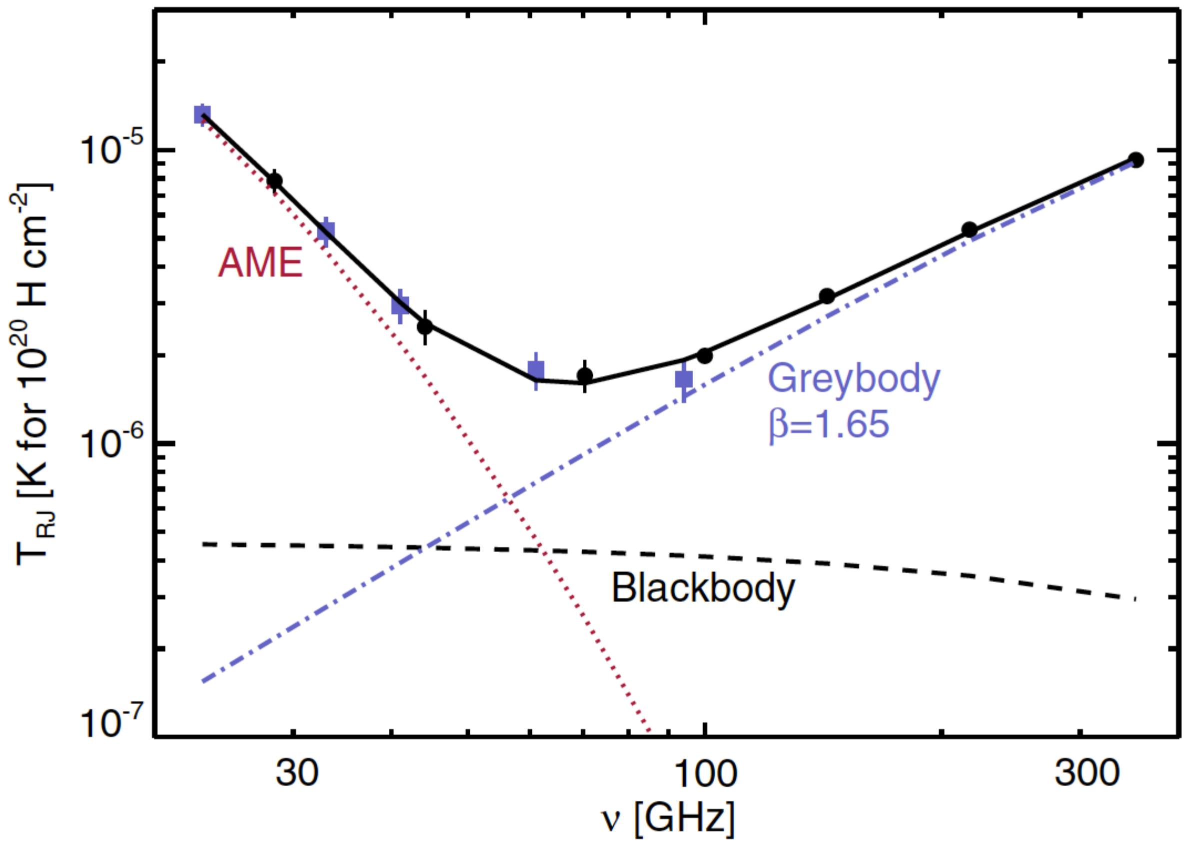


Figure 3. Similar to Figure 2, but with metallic Fe nanoparticles added to the dust model. The Fe particles are assumed to be at $T = 40$ K in Model 3 (panel a) and $T = 20$ K in Model 4 (panel b).

… the emission from the SMC may be understood if the interstellar dust mixture includes magnetic nanoparticles, emitting magnetic dipole radiation resulting from thermal fluctuations in the magnetization … If the Fe nanoparticles are, for the most part, freefliers heated by typical starlight, then $T \approx$ 40K is expected (see Figure 4 of Draine & Hensley 2012). If, on the other hand, the Fe nanoparticles are inclusions in larger composite grains, then the $T \approx$ 20K temperature is appropriate, consistent with the temperature of the "normal" dust.

Draine & Hensley, ApJ **757** (2012) 103

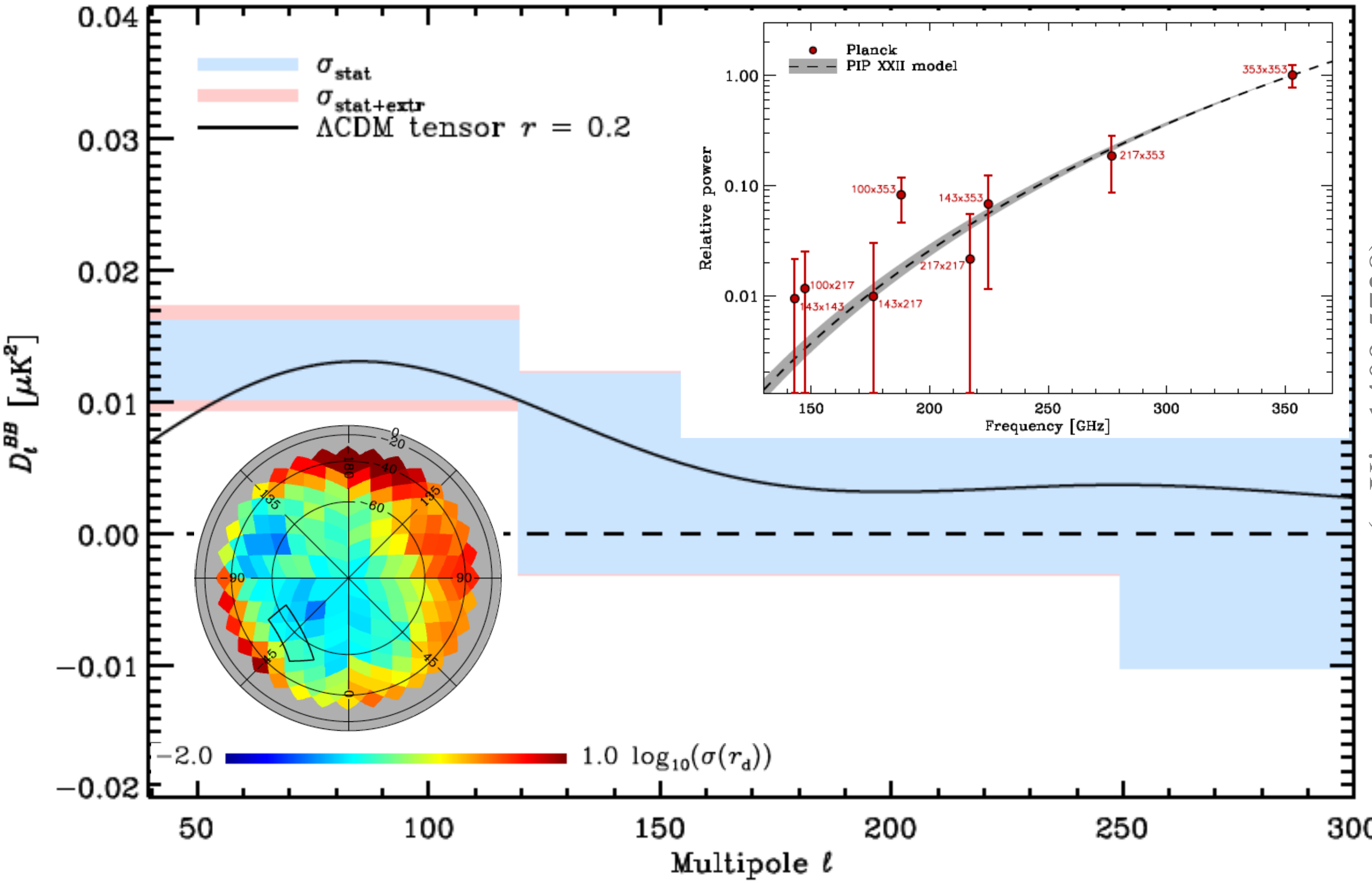
Could it be *magnetic* dipole radiation from dust (with iron or ferrimagnetic inclusions)?



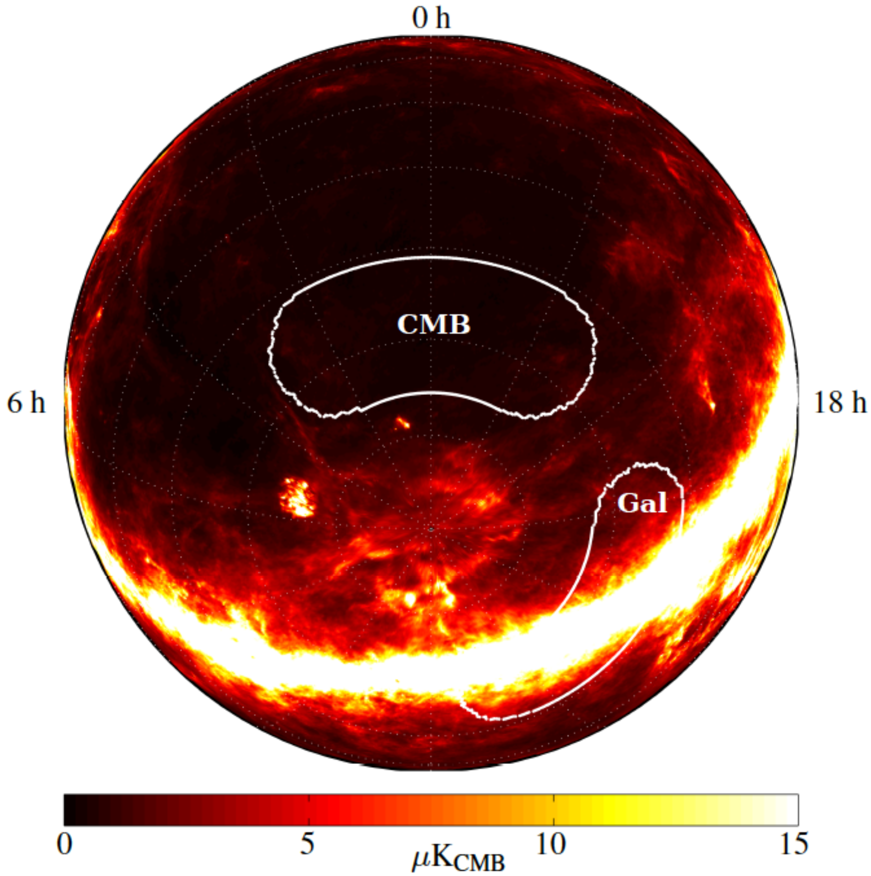
"We show that the flattening of the dust SED can be accounted for with an additional component with a blackbody spectrum. This additional component, which accounts for (26 +- 6)% of the dust emission at 100 GHz, could represent magnetic dipole emission"

Planck XVII, A&A **566**:A55, 2014 (see also arXiv:1405.0874v1)

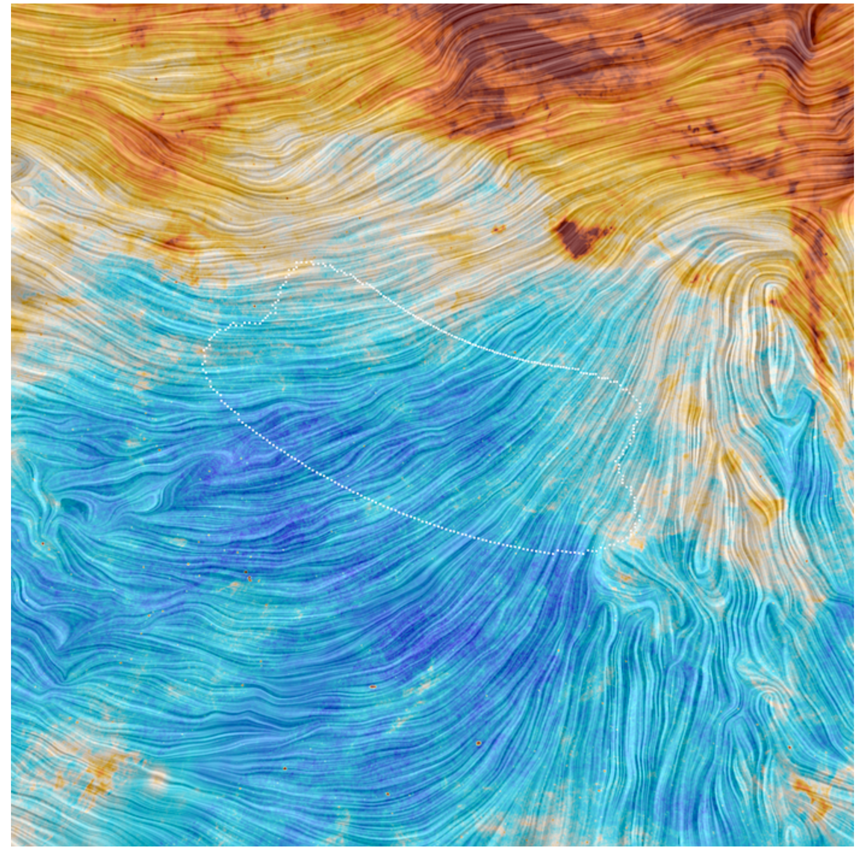
The 353 GHz polarised dust emission map from *Planck* shows high latitude emission from dust with a high polarisation fraction of ~20% - extrapolated to 150 GHz, this is comparable to the BICEP2 'signal'!



(arXiv:1409.5738)



[arXiv:1403.3985]



Planck view of BICEP2 field

"The *BICEP 2* field is centered on Galactic coordinates (l, b) = (316^0, -59^0) and was originally selected on the basis of exceptionally low contrast in the FDS dust maps (Finkbeiner et al. 1999). ~~It must be emphasized that these ultra-clean regions are very special – at least an order of magnitude cleaner than the average $b > 50^0$ level~~."

The formation of large-scale structure is akin to a scattering experiment

The **Beam:** **inflationary density perturbations**

No 'standard model' – usually *assumed* to be adiabatic and ~scale-invariant

The **Target:** **dark matter (+ baryonic matter)**

Identity unknown - usually taken to be cold (sub-dominant 'hot' component?)

The **Detector:** **the universe**

Modelled by a 'simple' FRW cosmology with parameters h, Ω_{CDM}, Ω_{b}, Ω_{Λ}, Ω_{k} …

The **Signal:** **CMB anisotropy, galaxy clustering …**

measured over scales ranging from ~ 1 – 10000 Mpc ($\Rightarrow$ ~8 e-folds of inflation)

We cannot simultaneously determine the properties of *both* the **beam** *and* the **target** with an unknown **detector**

Cosmologists adopt suitable priors wrt the 'beam' and the 'target' to break parameter *degeneracies* in determining the parameters of the assumed ΛCDM model

For example they assume that the 'beam' is a power-law spectrum … but is this what we find if we *reverse* the procedure to extract it from the data (also assuming ΛCDM)?

Deconvoluting the primordial power spectrum

Assume there are data sets with data points d_a related to the PPS $\mathcal{P}_{\mathcal{R}}(k)$ by

$$d_a = \int_{-\infty}^{\infty} K_a(\theta, k)\, \mathcal{P}_{\mathcal{R}}(k)\, \mathrm{d}\ln k + n_a = \sum_i W_{ai}(\theta)\, p_i + n_a.$$

Examples include CMB anisotropy, CMB lensing potential, galaxy clustering, Lyman α forest, cluster abundance and weak lensing data sets.

Given an estimate $\hat{\boldsymbol{\theta}}$ of the background cosmological parameters finding the PPS is an *ill-posed* inverse problem, with no unique solution.

We use the Tikhonov regularisation estimate:

$$\hat{\mathbf{p}}\left(\mathbf{d}, \hat{\boldsymbol{\theta}}, \lambda\right) = \min_{\mathbf{p}} \left[-2\ln\mathcal{L}\left(\mathbf{d}|\mathbf{p}, \hat{\boldsymbol{\theta}}\right) + \lambda R(\mathbf{p})\right].$$

Here $\mathcal{L}(\mathbf{d}|\mathbf{p}, \boldsymbol{\theta})$ is the likelihood function, $R(\mathbf{p})$ is a roughness penalty function and λ is the regularisation parameter.

Hunt & Sarkar, JCAP **01**:025, 2014

Properties of Tikhonov regularisation

Only features in $\hat{\mathbf{p}}$ required to fit data.

Estimate $\hat{\mathbf{p}}$ is biased towards *smoothness*.

A tradeoff exists between the bias and variance of $\hat{\mathbf{p}}$, governed by λ.

There is an (almost) *linear* relationship between data and $\hat{\mathbf{p}}$ – permits *analytic* error analysis.

Fast in practice, allows extensive Monte Carlo testing.

Can be modified to account for CMB lensing.

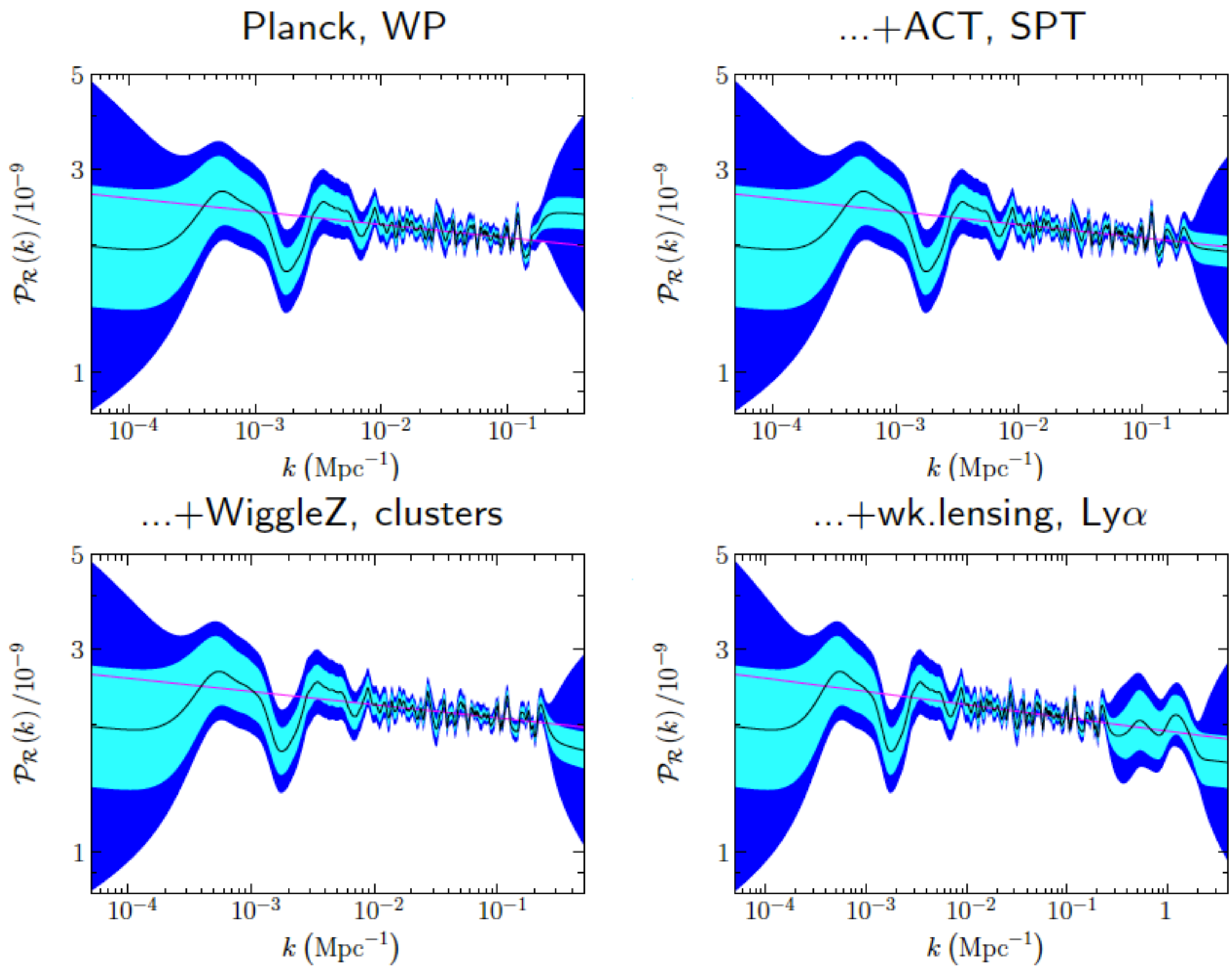
Can include positivity constraint on PPS by using $\ln \mathcal{P}_{\mathcal{R}}$.

Can recover more than one unknown function eg $\mathcal{P}_{\mathcal{R}}(k)$ & $\mathcal{P}_{\mathcal{T}}(k)$.

Can include priors on the slope of the recovered function(s) – impose inflation consistency relation $n_{\mathrm{t}} = -r/8$.

Hunt & Sarkar, JCAP **01**:025, 2014

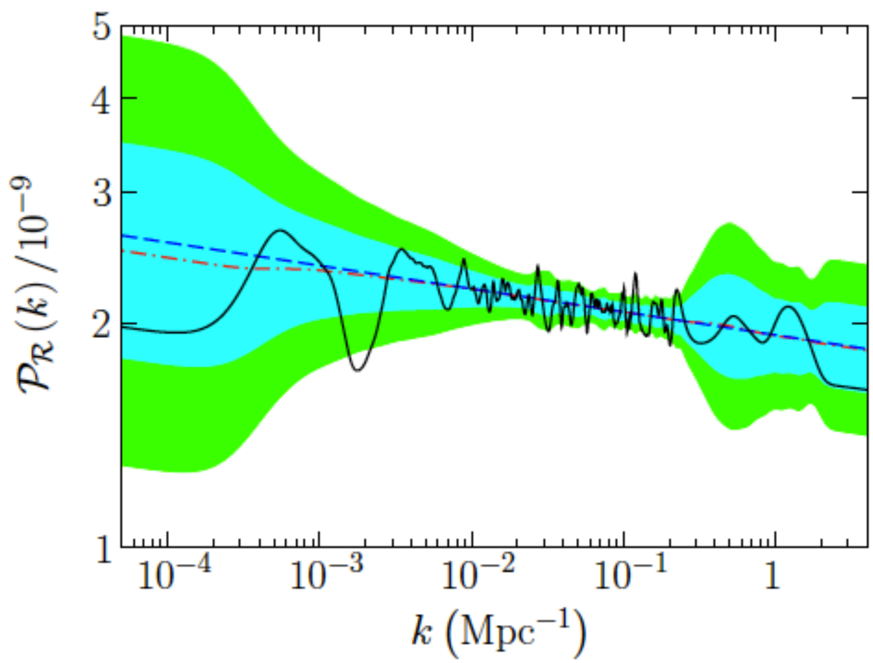
Planck results for $\lambda = 400$



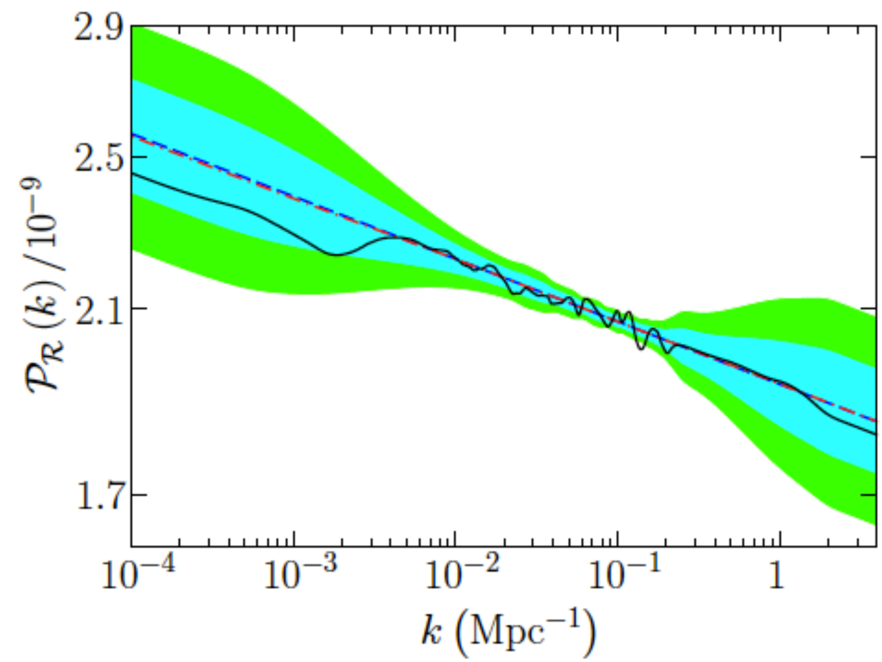
Hunt & Sarkar, arXiv:1501:03338

Is the PPS different from a power-law?

All data, $\lambda = 400$



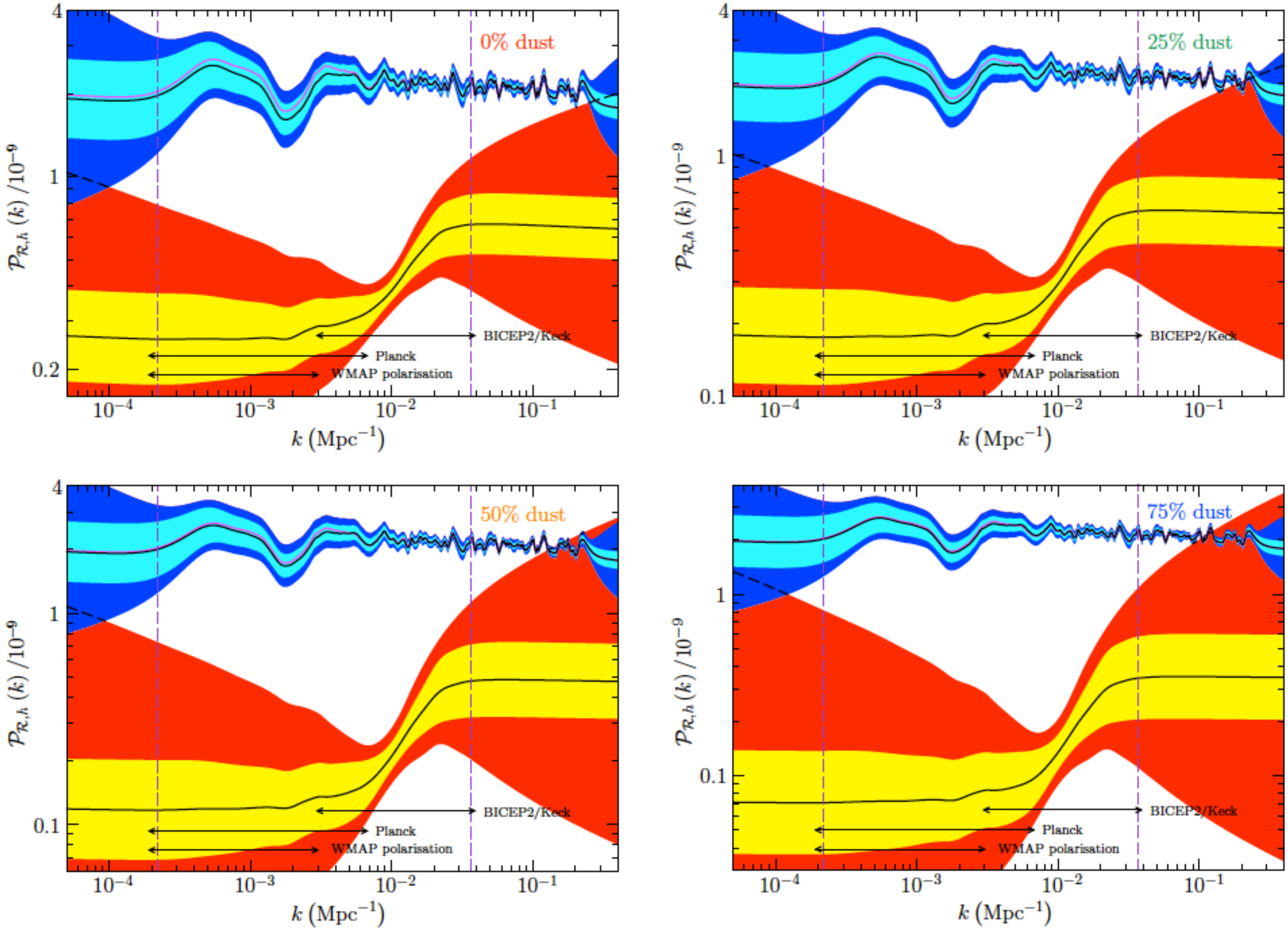
All data, $\lambda = 20000$



Hunt & Sarkar, arXiv:1501:03338

Comparison with Monte Carlo simulations assuming a power-law with $n_s = 0.969$ shows deviations significant at 2σ level. (accounting for 'look elsewhere' effect)

So no firm evidence yet … however finding even *one* feature would immediately rule out *all* slow-roll models and provide a crucial hint as to inflationary dynamics …



Hunt & Sarkar, arXiv:1501:03338

… and the recovered tensor power spectrum from BICEP data has a strong blue tilt, which is *inconsistent* with the slow-roll inflationary consistency condition: $n_t = - r/8$

Summary

Inflation driven by the slow roll of a scalar field is a convenient paradigm which enables us to engage with CMB & other data … but it is *very* challenging to realise in a physical (field-theoretical) framework without rather unnatural *fine-tuning* of parameters.

Lacking a fundamental understanding of how vacuum energy couples to gravity, inflation must in any case be considered a 'toy model'
… unless of course we detect the predicted gravitational waves!

This will however be hard unless we learn how to model the Galactic foreground emission *far more accurately* than we can at present.

Meanwhile there is an indication that the primordial spectrum of fluctuations cuts off on the scale of the *present* Hubble radius H_0^{-1} !
There are also indications of features in the spectrum which if established would immediately rule out all 'slow roll' inflation models.
This is arguably the most promising way to further probe inflation