Next-to-leading order QCD corrections to the $e^+e^- \to t\bar{t}$ total cross section

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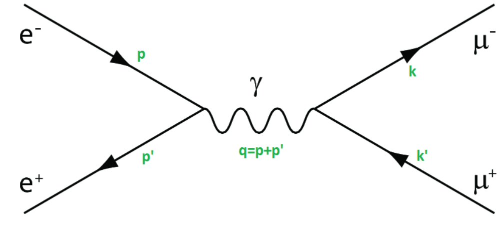
DESY Summer Student Programme 2015

FINAL PRESENTATION

OUR PROCESS: $e^+e^- \to t\bar{t}$



The contribution to **the total cross section** at tree level

Two ways of calculation:

- Direct calculation,
- Via optical theorem.

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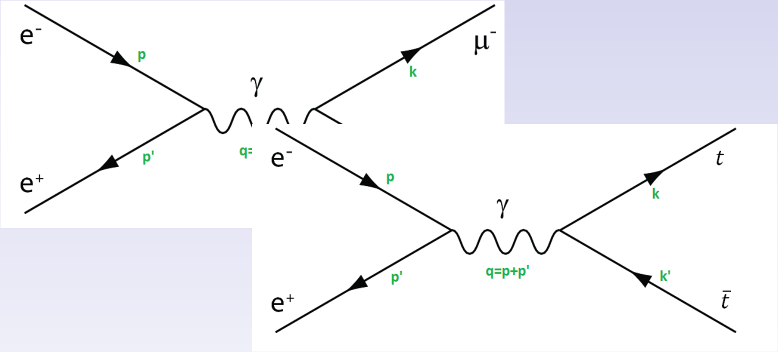

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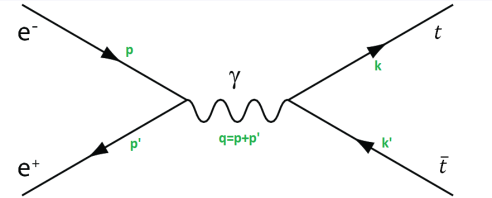

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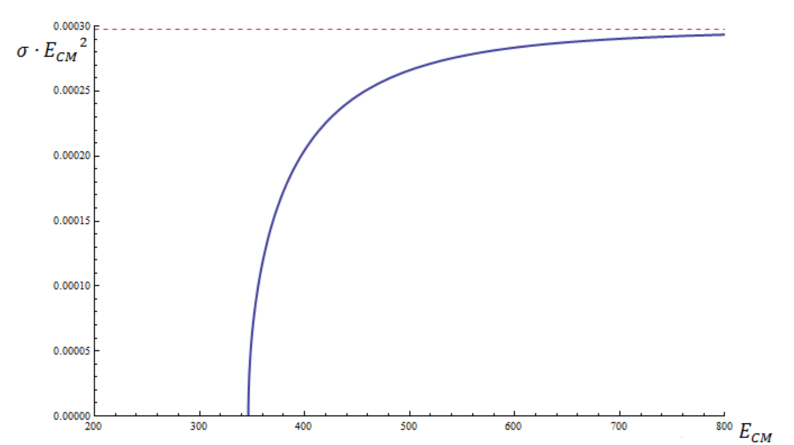

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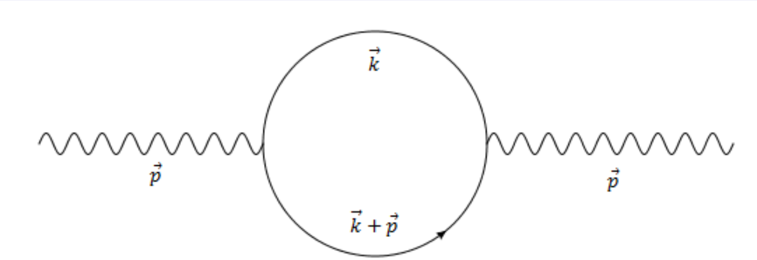

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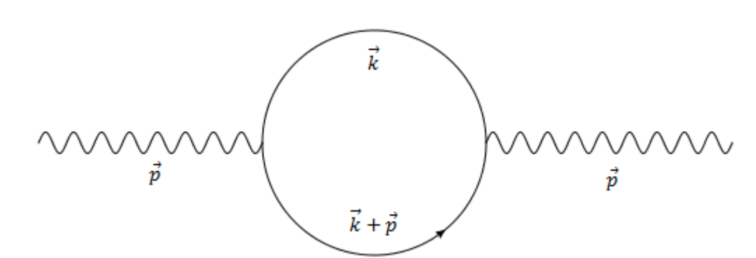

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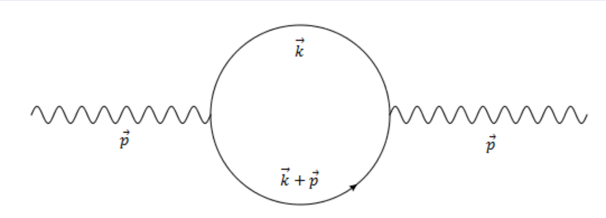

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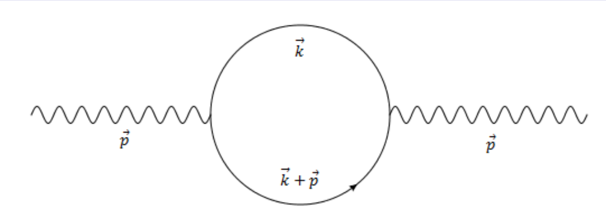

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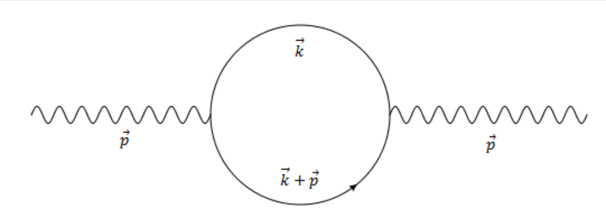

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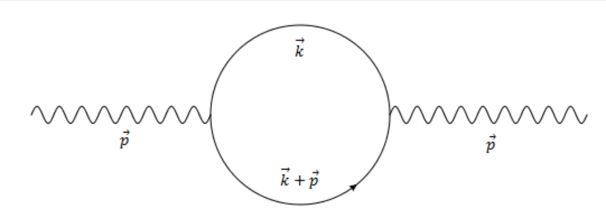

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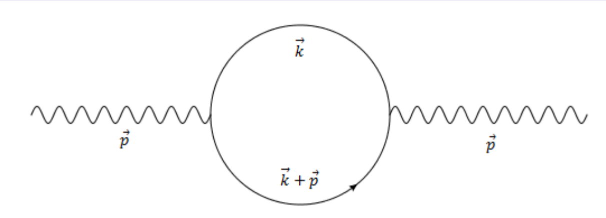

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- **Third step**: calculating integral order by order in the expansion of Laurent series

Method of differential equations and IBP relations in detail

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IBP relations

The equalities of the form are called **IBP relations**:

$$\int d^D k \frac{\partial}{\partial k^\mu} p_\mu I(n_1, n_2) = 0,$$

where p_μ–internal or external momenta

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REMARK: In order to get the solution for higher order we need to know the value of the integral for lower order.

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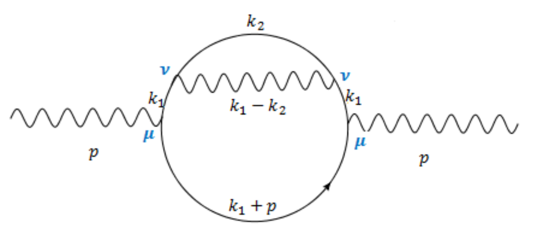

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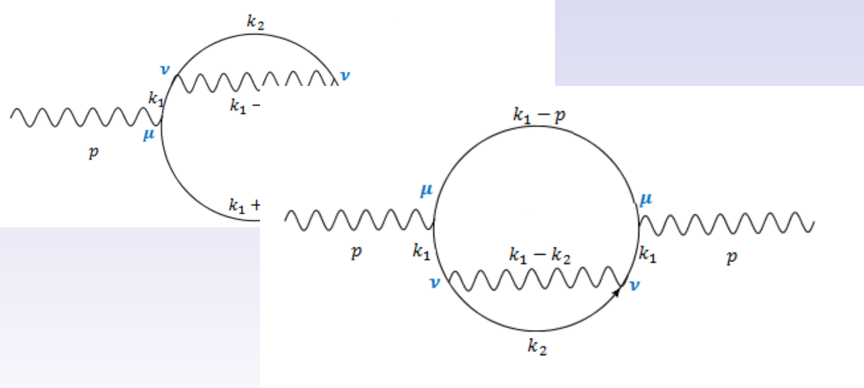

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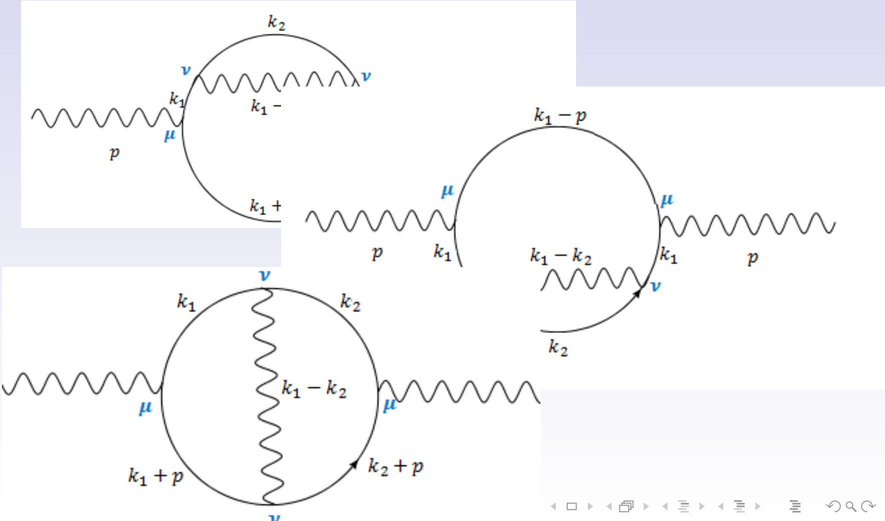

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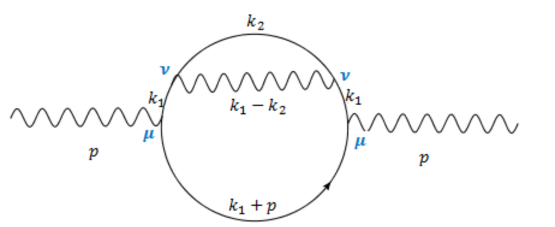

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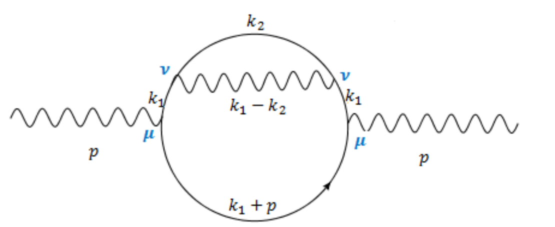

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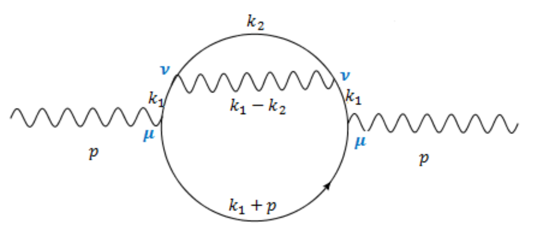

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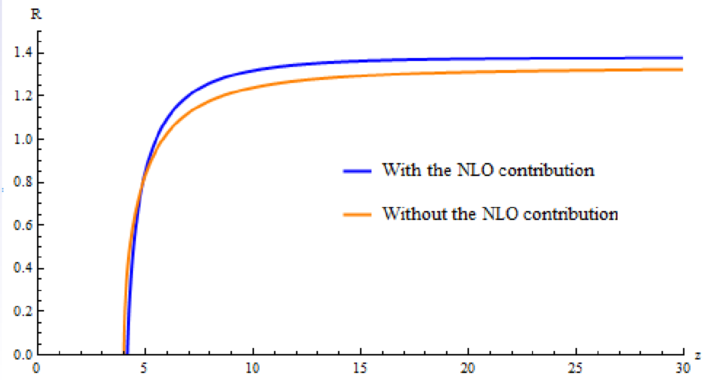

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THANK YOU FOR YOUR ATTENTION