NLO Calculation for DIS Heavy Quark Production in the General-Mass Scheme

Work in Progress

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(and, hopefully, J. Smith)

... Massive

heavy quark only in the final state, not a parton
i.e. FFNS: Fixed Flavor Number Scheme

- Inclusive heavy quark production in DIS
  E. Laenen, S. Riemersma, J. Smith, W. L. van Neerven
  Nucl. Phys. B 392 (1993) 162, 229
  " the Smith program": pthad
- Fully exclusive version:
  B. W. Harris, J. Smith
  Nucl. Phys. B452 (1995) 109 and Phys. Rev. D 57, 2806 (1998)
  HVQDIS
- Tabulated matrix elements, for F_2^c:
  S. Riemersma, J. Smith, W. L. van Neerven
  Phys. Lett. B 347 (1995) 143
  "the Riemersma program"

... Massless

$\overline{\text{MS}}$ scheme, heavy quark is a parton
i.e. ZM-VFNS: Zero-Mass Variable Flavor Number Scheme

- Inclusive hadron production in DIS
  A. Daleo, D. de Florian, R. Sassot
  Phys. Rev. D 71, 034013 (2005)
  ”the Daleo program”: TIMBA
- Also available: calculations / programs by
  P. Aurenche, R. Basu, M. Fontannaz, R. M. Godbole
  G. Kramer, B. Kniehl, M. Maniatis

FFNS VERSUS ZM-VFNS

$m \neq 0 \longrightarrow$

- correct threshold behavior
  no collinear divergences
  but terms $\propto \log(\mu/m)$ with $\mu = Q, p_T, \ldots$
- large corrections at large μ

$m = 0 \longrightarrow$

- not reliable at small μ
- mass singularities ($1/\epsilon$-poles or $\log m^2$-terms)
  absorbed in PDFs and FFs
- QCD prediction: DGLAP (RG) evolution resums
  large logarithms $\log(\mu/m)$
- more reliable at large μ

Goal: combine massive (low scale) and massless (high scale) calculations

- exploit freedom to choose an appropriate
  factorization scheme

General-Mass Scheme

- The problem:
  Conventionally, PDFs and FFs are defined in the $\overline{\mathrm{MS}}$ scheme
  $\overline{\mathrm{MS}}$ scheme is based on a massless calculation
  Massless and massive calculations contain different singularities
  Can not use $\overline{\mathrm{MS}}$ FFs in a massive calculation?
- The solution:
  Match massless and massive calculations:

$$d\sigma^{\mathrm{sub}} = \lim_{m \to 0} d\tilde{\sigma}(m) - d\hat{\sigma}_{\overline{\mathrm{MS}}}$$

The subtracted cross section (in a massive calculation)

$$d\hat{\sigma}(m) = d\tilde{\sigma}(m) - d\sigma^{\mathrm{sub}}$$

can be used with $\overline{\mathrm{MS}}$ fragmentation functions

→ The GM-VFNS (general-mass variable flavor number scheme)

→ Technical problem: need subtraction terms

- collinear $\ln(\mu^2/m_c^2)$ terms:
  subtracted from hard part (to avoid double counting) and
  resummed by DGLAP evolution equations ($\rightarrow f_c \neq 0$)
- Partons: g, u, d, s, c, (charm is a parton: $f_c \neq 0$)
- VFNS with $m_c \neq 0 \longrightarrow$ GM-VFNS
- FFs, e.g. $D_c^{D^\star}(z, {\mu'_F}^2)$ evolved

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- technically more involved:
  - calculation with $m_c \neq 0$
  - mass factorization with massive regularization
  - kinematics: factorization with massive partons

$+$ large collinear logarithms $\ln \frac{\mu^2}{m_c^2}$ resummed in evolved $f_c(x, \mu^2)$ and $D_c^{D^\star}(x, \mu^2)$

$+$ $(\frac{m_c}{p_T})^n$ included

$\Rightarrow$ good for smaller p_T: $0 < p_T^2 \lesssim m_c^2$ <u>and</u> $p_T^2 \gg m_c^2$

HARD SCATTERING COEFFICIENTS WITH HEAVY QUARK MASSES

Mass terms contained in the hard scattering coefficients:

$$\mathrm{d}\hat{\sigma}(\mu_F, \mu'_F, \alpha_s(\mu_R), \frac{m_h}{p_T})$$

Two ways to derive them:

- Compare massless limit of a massive fixed-order calculation with a massless $\overline{\mathrm{MS}}$ calculation to determine subtraction terms

OR

- Perform mass factorization using partonic PDFs and FFs

- Compare limit $m \to 0$ of the massive calculation Smith et al
  with massless $\overline{\text{MS}}$ calculation Daleo et al

$$\lim_{m \to 0} \mathrm{d}\sigma(m) = \mathrm{d}\hat{\sigma}(\overline{\text{MS}}) + \Delta \mathrm{d}\sigma$$

$\Rightarrow$ Subtraction terms

$$\mathrm{d}\sigma_{\text{SUB}} \equiv \Delta \mathrm{d}\sigma = \lim_{m \to 0} \mathrm{d}\sigma(m) - \mathrm{d}\hat{\sigma}(\overline{\text{MS}})$$

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$$d\sigma_{\text{SUB}} \equiv \Delta d\sigma = \lim_{m\to 0} d\sigma(m) - d\hat{\sigma}(\overline{\text{MS}})$$

- Subtract $d\sigma_{\text{SUB}}$ from massive partonic cross section while keeping mass terms

$$d\hat{\sigma}(m) = d\sigma(m) - d\sigma_{\text{SUB}}$$

$\rightarrow$ $d\hat{\sigma}(m)$ short distance coefficient including m

$\rightarrow$ allows to use PDFs and FFs with $\overline{\text{MS}}$ factorization $\oplus$ massive short dist. cross sections

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- Treat contributions with charm in the initial state with $m_c = 0$

Sketch of kinematics:

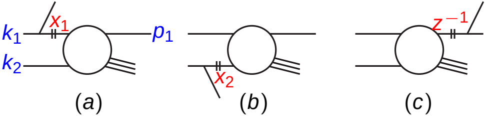


Fig. (a):

$$d\sigma^{\rm sub}(ab \to QX) = \int_0^1 dx_1\, f^{(1)}_{a\to i}(x_1, \mu_F^2)\, d\hat\sigma^{(0)}(ib \to QX)[x_1 k_1, k_2, p_1] \equiv f^{(1)}_{a\to i}(x_1) \otimes d\hat\sigma^{(0)}(ib \to QX)$$

Fig. (b):

$$d\sigma^{\rm sub}(ab \to QX) = \int_0^1 dx_2\, f^{(1)}_{b\to j}(x_2, \mu_F^2)\, d\hat\sigma^{(0)}(aj \to QX)[k_1, x_2 k_2, p_1] \equiv f^{(1)}_{b\to j}(x_2) \otimes d\hat\sigma^{(0)}(aj \to QX)$$

Fig. (c):

$$d\sigma^{\rm sub}(ab \to QX) = \int_0^1 dz\, d\hat\sigma^{(0)}(ab \to kX)[k_1, k_2, z^{-1} p_1]\, d^{(1)}_{k\to Q}(z, {\mu'_F}^2) \equiv d\hat\sigma^{(0)}(ab \to kX) \otimes d^{(1)}_{k\to Q}(z)$$

Kniehl, Kramer, Schienbein, HS, EPJC41(2005)199

Partonic PDFs and FFs

1. initial state

$$f^{(1)}_{g\to Q}(x,\mu^2) = \frac{\alpha_s(\mu)}{2\pi} P^{(0)}_{g\to q}(x) \ln\frac{\mu^2}{m^2}$$

$$f^{(1)}_{Q\to Q}(x,\mu^2) = \frac{\alpha_s(\mu)}{2\pi} C_F \left[\frac{1+z^2}{1-z}\left(\ln\frac{\mu^2}{m^2} - 2\ln(1-z) - 1\right)\right]_+$$

$$f^{(1)}_{g\to g}(x,\mu^2) = -\frac{\alpha_s(\mu)}{2\pi}\frac{1}{3}\ln\frac{\mu^2}{m^2}\delta(1-x)$$

2. final state

$$d^{(1)}_{g\to Q}(z,\mu^2) = \frac{\alpha_s(\mu)}{2\pi} P^{(0)}_{g\to q}(z) \ln\frac{\mu^2}{m^2}$$

$$d^{(1)}_{Q\to Q}(z,\mu^2) = C_F \frac{\alpha_s(\mu)}{2\pi} \left[\frac{1+z^2}{1-z}\left(\ln\frac{\mu^2}{m^2} - 2\ln(1-z) - 1\right)\right]_+$$

- Other partonic distribution functions are zero to this order in α_s
- Analogous for photon splitting: $g \to \gamma$, $\alpha_s \to \alpha$, color factors

WHERE WE ARE ...

- Massless limit of Smith's program: coefficient functions from

$$\lim_{m \to 0} d\sigma_{NLO}(m)$$

- Coefficient functions from Daleo's program:

$$d\hat{\sigma}_{NLO}(\overline{\mathrm{MS}})$$

- Subtraction terms using

$$d\sigma_{\mathrm{SUB}} \equiv \Delta d\sigma = \lim_{m \to 0} d\sigma(m) - d\hat{\sigma}(\overline{\mathrm{MS}})$$

- Subtraction terms using $\overline{\mathrm{MS}}$ mass factorization (with partonic PDFs and FFs)

$$d\sigma_{\mathrm{SUB}}^{MFac}$$

WHERE WE ARE ...

- We find differences between

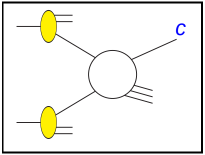
$$\Delta \mathrm{d}\sigma \quad \text{and} \quad \mathrm{d}\sigma_{\mathrm{SUB}}^{MFac}$$

  $\longrightarrow$ errors / misprints in Smith's and / or Daleo's programs?
  $\longrightarrow$different conventions?
- Implement $\mathrm{d}\sigma_{\mathrm{SUB}}^{MFac}$ for numerical evaluation
- Check numerical significance of the observed missing terms

- for the future: match to FVNS to obtain the correct $p_T \rightarrow 0$ limit
- find subtraction terms for the exclusive program HVQDIS

Backup Slides

- $m_c \neq 0$, $n_f = 3$ fixed $\longrightarrow$ FFNS
- Partons: g, u, d, s
  [NO charm parton; charm (only) in the final state]
- collinear logarithms $\ln \frac{s}{m_c^2}$ finite
  $\rightarrow$ No factorization; no conceptual necessity for FFs
  $\rightarrow$ fixed order perturbation theory; no resummation
- Usually c treated in the on-shell scheme ($\overline{\text{MS}}_m$)



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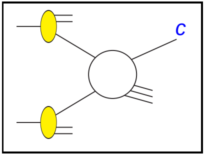

Pro and Contra:

$+$ $(\frac{m_c}{p_T})^n$ terms included; correct threshold suppression

$\Rightarrow$ **valid for $0 \leq p_T^2 \lesssim m_c^2$** $\Rightarrow$ σ_{tot} calculable

$-$ fixed order logarithms $\ln \frac{p_T^2}{m_c^2}$ large for $p_T^2 \gg m_c^2$;
resummation of these large logarithms necessary

$\Rightarrow$ **breaks down for $p_T^2 \gg m_c^2$**

$-$ non-perturbative function $D_c^H(z)$, describing the hadronisation $c \rightarrow H$ needed to match data;
$\rightarrow$ not based on factorization theorem (no AP evolution) $\rightarrow$ universal?

- $m_c = 0 \rightarrow$ 'Zero Mass'
- Number of active partons depends on scale $\mu_F \rightarrow$ ZM-VFNS
  - $\mu_F < m_c$: $n_f = 3$, Partons: g, u, d, s
  - $m_c \leq \mu_F < m_b$: $n_f = 4$, Partons: g, u, d, s, c
  - $m_b \leq \mu_F$: $n_f = 5$, Partons: g, u, d, s, c, b
- Matching conditions at transition scale $\mu_0 = m_c$ (similar at m_b): $n_f = 3 \rightarrow n_f = 4$

$$\left.\begin{array}{l} \alpha_s^{(3)} \rightarrow \alpha_s^{(4)} = \alpha_s^{(3)} + \mathcal{O}(\alpha_s^3) \\ f_i^{(3)} \rightarrow f_i^{(4)} = f_i^{(3)} + \mathcal{O}(\alpha_s^2) \end{array}\right\} @\ \mu_0 = m_c \qquad \boxed{f_c^{(4)}(x, \mu_0^2 = m_c^2) = 0}\ \text{pert.b.c.}$$

- Collinear divergences related to c lines factorized into non-perturbative PDFs and FFs

- $m_c = 0 \rightarrow$ 'Zero Mass'
- Number of active partons depends on scale $\mu_F \rightarrow$ ZM-VFNS
  - $\mu_F < m_c$: $n_f = 3$, Partons: g, u, d, s
  - $m_c \leq \mu_F < m_b$: $n_f = 4$, Partons: g, u, d, s, c
  - $m_b \leq \mu_F$: $n_f = 5$, Partons: g, u, d, s, c, b
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- Collinear divergences related to c lines factorized into non-perturbative PDFs and FFs

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Pro and Contra:

+ large collinear logarithms $\ln \frac{\mu^2}{m_c^2}$ resummed in evolved $f_c(x, \mu^2)$ and $D_c^H(x, \mu^2)$ to LL and NLL accuracy $\Rightarrow$ $\boxed{\text{good for large } \mu^2 \simeq p_T^2 \gg m_c^2}$
+ Universality of PDFs and FFs guaranteed by factorization theorem $\rightarrow$ predictive power, global data analysis

− $(\frac{m_c}{p_T})^n$ terms neglected in the hard part $\Rightarrow$ $\boxed{\text{breaks down for } p_T^2 \lesssim m_c^2}$ $\Rightarrow$ No $\sigma_{\rm tot}$

- Calculate $m \to 0$ limit of the massive 3-FFNS calculation (at partonic level) [1]
  Only keep m as regulator in $\ln \frac{m^2}{s}$
- Partonic subprocesses (3-FFNS: charm, 4-FFNS: replace c by b)
  - Leading Order:
    1. $gg \to c\bar{c}$
    2. $q\bar{q} \to c\bar{c} \quad (q = u, d, s, [c])$
  - Next-To-Leading Order:
    1. $gg \to c\bar{c}g$
    2. $q\bar{q} \to c\bar{c}g$
    3. $gq \to c\bar{c}q$
- Limiting procedure non-trivial:
  - Map from PS-slicing to Subtraction method
  - care needed to recover $\delta(1-w)$, $\left(\frac{1}{1-w}\right)_+$, $\left(\frac{\ln(1-w)}{1-w}\right)_+$
- Checks:
  - Compare Abelian parts with results in [2]
  - Numerical tests

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[1] Bojak, Stratmann, PRD67(2003)034010; FORTRAN code provided by I. Bojak
[2] Kramer, HS, EPJC22(2001)289; EPJC28(2003)495

Only light lines

1. $gg \longrightarrow qX$
2. $gg \longrightarrow gX$
3. $qg \longrightarrow gX$
4. $qg \longrightarrow qX$
5. $q\bar{q} \longrightarrow gX$
6. $q\bar{q} \longrightarrow qX$
7. $qg \longrightarrow \bar{q}X$
8. $qg \longrightarrow \bar{q}'X$
9. $qg \longrightarrow q'X$
10. $qq \longrightarrow gX$
11. $qq \longrightarrow qX$
12. $q\bar{q} \longrightarrow q'X$
13. $q\bar{q}' \longrightarrow gX$
14. $q\bar{q}' \longrightarrow qX$
15. $qq' \longrightarrow gX$
16. $qq' \longrightarrow qX$

Only light lines

1. $gg \to qX$
2. $gg \to gX$
3. $qg \to gX$
4. $qg \to qX$
5. $q\bar{q} \to gX$
6. $q\bar{q} \to qX$
7. $qg \to \bar{q}X$
8. $qg \to \bar{q}'X$
9. $qg \to q'X$
10. $qq \to gX$
11. $qq \to qX$
12. $q\bar{q} \to q'X$
13. $q\bar{q}' \to gX$
14. $q\bar{q}' \to qX$
15. $qq' \to gX$
16. $qq' \to qX$

Heavy quark initiated ($m_Q = 0$)

1. -
2. -
3. $Qg \to gX$
4. $Qg \to QX$
5. $Q\bar{Q} \to gX$
6. $Q\bar{Q} \to QX$
7. $Qg \to \bar{Q}X$
8. $Qg \to \bar{q}X$
9. $Qg \to qX$
10. $QQ \to gX$
11. $QQ \to QX$
12. $Q\bar{Q} \to qX$
13. $Q\bar{q} \to gX$, $q\bar{Q} \to gX$
14. $Q\bar{q} \to QX$, $q\bar{Q} \to qX$
15. $Qq \to gX$, $qQ \to gX$
16. $Qq \to QX$, $qQ \to qX$

List of Subprocesses: GM-VFNS

#	Only light lines	Heavy quark initiated ($m_Q = 0$)	Mass effects: $m_Q \neq 0$
1	$gg \to qX$	-	$gg \to QX$
2	$gg \to gX$	-	-
3	$qg \to gX$	$Qg \to gX$	-
4	$qg \to qX$	$Qg \to QX$	-
5	$q\bar{q} \to gX$	$Q\bar{Q} \to gX$	-
6	$q\bar{q} \to qX$	$Q\bar{Q} \to QX$	-
7	$qg \to \bar{q}X$	$Qg \to \bar{Q}X$	-
8	$qg \to \bar{q}'X$	$Qg \to \bar{q}X$	$qg \to \bar{Q}X$
9	$qg \to q'X$	$Qg \to qX$	$qg \to QX$
10	$qq \to gX$	$QQ \to gX$	-
11	$qq \to qX$	$QQ \to QX$	-
12	$q\bar{q} \to q'X$	$Q\bar{Q} \to qX$	$q\bar{q} \to QX$
13	$q\bar{q}' \to gX$	$Q\bar{q} \to gX$, $q\bar{Q} \to gX$	-
14	$q\bar{q}' \to qX$	$Q\bar{q} \to QX$, $q\bar{Q} \to qX$	-
15	$qq' \to gX$	$Qq \to gX$, $qQ \to gX$	-
16	$qq' \to qX$	$Qq \to QX$, $qQ \to qX$	-

$\oplus$ charge conjugated processes

[1] Aversa, Chiappetta, Greco, Guillet, NPB327(1989)105